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Abstract:

The present-day standard textbook narrative on the history of growth theory usually takes Robert Solow's 1956 contribution as a key starting point, with extensions on the savings decision (done by David Cass and Tjalling Koopmans in 1965) being the next important development. However, such account is historically misleading because it organizes past developments based on theoretical concerns. Our goal is to tell a richer story about the developments of growth theory from the 1950s to the mid 1960s, in the activity analysis literature that started before Solow's model and never had him as a central reference. We stress the role played by Edmond Malinvaud, and take his travel from the French milieu of mathematical economics to the Cowles Commission in 1950-1951 and back to France as a guiding line. The rise of turnpike theory in the end of the 1950s generated a debate on the choice criteria of growth programs, opposing the productive efficiency typical of these models to the utilitarian approach supported by Malinvaud and Koopmans. The Vatican Conference of 1963, where Koopmans presented a first version of his 1965 model, was embedded in this debate. We argue that Malinvaud's (and Koopmans's) contributions were crucial to steer the activity analysis literature towards a utilitarian analysis of growth paths.

Keywords: Edmond Malinvaud; Optimal Growth; Tjalling Koopmans; History of Growth Theory

JEL Codes: B21; B22; B23

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Matheus Assaf & Pedro Garcia Duarte[1]

Introduction

A flood tide at the macroeconomics shores. That was a shared perception of the inhabitants of those economic lands who were concerned with economic growth in the 1960s. For instance, Bernard Okun (then at Princeton University) and Richard Richardson (at the International Monetary Fund) wrote that "interest in economic growth has been sufficiently contagious that since World War II the literature on this subject has reached flood proportions" (Okun and Richardson 1961, v). A similar sense of the vastitude of this literature was exposed in James Meade's (1961) book and in the acclaimed survey by Frank Hahn and Robin Matthews (1964, 779), echoed in the follow up survey by Ronald Britto (1973, 1343). By the end of the decade, Edwin Burmeister and Rodney Dobell (both of whom graduated in 1965 from MIT), in one of the first graduate textbooks on economic growth, recognized that the field was "very much in flux" (Burmeister and Dobell 1970, xi). The 1960s was also the time of growthmanship, a political goal of achieving high economic growth, and MIT was its key site in the US (Boianovsky and Hoover 2014).[2]

Growth was then mangled with development as two important concerns of the postwar period, both very much connected with planning and without stable meanings.[3] As an anecdotal evidence, on the one hand one could find a book on economic

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[2] Likewise, Mauro Boianovsky and Kevin Hoover (2009, 2-3) argued that growth economics "exploded" in the 1960s. Writing in retrospect, William Nordhaus and James Tobin (1972, 1) stated that "[a] long decade ago economic growth was the reigning fashion of political economy. It was simultaneously the hottest subject of economic theory and research, a slogan eagerly claimed by politicians of all stripes, and a serious objective of the policies of governments." See also Schmelzer (2016) for a study of the transnational harmonization of growth policies and the role of the Organisation for Economic Co-operation and Development (OECD) in forging it.

[3] To substantiate this claim quantitatively, we can refer to the work of François Claveau and Yves Gingras (2016), who took the large corpus of economics articles available in the Web of Science (Thomson Reuters) from 1956 to 2014 and presented economics as the evolution of networks of specialties, based on the sharing of references by any two documents. In their analysis, economic growth is a stable specialty of economics from their starting date to the mid 1970s (see their fig. 4, p. 566), closely connected to development in the 1960s (check the network graph of their web platform available at: http://www.digitalhistoryofscience.org/economics/; accessed on February 5 2018).

development relating it not only to problems of "underdeveloped countries", but also "to the growth problems of the 'advanced countries'" (Okun and Richardson 1961, v). On the other hand, we have a book titled *Economic Growth* that deals instead with development (Nelson 1960).[4]

The literature on economic growth was not only large but also multifarious: part of it was interested on actual growth of economies and its historical specificities (see Abramovitz's 1952 survey), another part placed growth in the context of planning and economic policy (cf. Hickman 1965, Fox, Sengupta and Thorbecke 1966, ch. 10), while other authors were interested in the theory and models of economic growth (the object of Hahn and Matthews's 1964 survey), or even in mathematical models and theories (Tinbergen and Bos 1962, Burmeister and Dobell 1970). And these interests could overlap, as in the case of Roy Radner's (1963) research for the Office of Naval Research bringing mathematical growth models to planning.

It is thus not surprising to see these economists organizing the developments in the field of economic growth in different ways, given their diverse interests in this literature. Hahn and Matthews (1964) would survey a vast theoretical literature and place the Harrod-Domar model centrally as the point of departure. Britto (1973) would instead focus on the neoclassical growth model, taking Knut Wicksell as a precursor, but placing Robert Solow and Trevor Swan as the starting points, contrasting this model with the models of Nicholas Kaldor and Luigi Pasinetti that featured two social classes with different propensities to save. The same opposition between neoclassical and Keynesian growth models was earlier explored by John Green (1963), who was completely silent about the linear growth models discussed by Hahn and Matthews (1964).

As for the books, Haavelmo (1954) framed his analysis around a central distinction between deterministic and stochastic approaches to growth and evolution. Tinbergen and Bos (1962) structured their mathematical analysis in terms of the number of sectors considered (from one to several sectors, in this order) and, within each case, the number of scarce factors and the technology of the production function. This book is frugal in references and in organizing the ideas around great minds. The economic growth part of R. G. D. Allen (1968) started with capital accumulation and the basic Harrod-Domar model to then consider two-sector models, technological change, the neo-classical model, vintage models and Kaldor models, without discussing anything of the von Neumann literature. In their turn, Burmeister and Dobell (1970) organized the

[4] Jan Tinbergen and Hendricus Bos (1962) discuss in their book "the optimum rate of development" (pp. 24-31), while the literature analyzed here would refer to this as optimum growth rate, which testifies to the unsettled nature of the vocabulary and concepts. Almost a decade earlier, Trygve Haavelmo (1954) would present "models of economic growth" in his book on the "theory of economic evolution," concerned with economic dissimilarities across different regions of the globe.

book from the one-sector, neoclassical growth model to extensions going to, among others, two-sector models, multi-sector models, and the optimal growth literature.

Such varied ways or organizing the then recent developments in the economic growth literature contrasts strikingly with the present-day historical account of those outcomes in textbooks such as Robert Barro and Xavier Sala-i-Martin's (2004, 16-21). The modern narrative usually takes Solow's 1956 contribution (with exogenous saving rate) as a key starting point, that David Cass (1965) and Tjalling Koopmans (1965) later extended by allowing agents to choose the saving rate from an intertemporal utility maximization problem (this is the so-called "Ramsey-Cass-Koopmans model"). Such account is simply silent about multi-sector models.

As it is often the case, the potted histories that typically appear in textbooks are historically misleading. The road connecting Solow to the Ramsey-Cass-Koopmans model is not so straightforward. In particular, the contributions of Koopmans came out of the activity analysis and multi-sector models literature of the late 1940s and the 1950s, before Solow's own contribution.[5] And here Edmond Malinvaud played an important role in developing a utilitarian intertemporal framework to produce a welfare analysis of resource allocation. Being a student of Maurice Allais, very familiarized with Allais's utilitarian intertemporal approach, and knowing the 1945 multi-sector model of John von Neumann of a linear production economy, Malinvaud insisted that activity analysis had to move beyond the production efficiency discussion typical of this literature, to one that included the consumption of individuals.

Our goal here is to tell a richer story about the developments of growth theory from the 1950s to the mid 1960s, in a branch of the literature that started before Solow (1956) and never had him as a central reference, stressing the role played by Malinvaud and taking his travel from the French milieu of mathematical economics to the Cowles Commission in 1950-1951 and back to France as a guiding line (sections 1 and 2). It was during his North-American sojourn that Malinvaud interacted with Koopmans right after the famous 1949 activity analysis conference and wrote his paper published in *Econometrica* in 1953. This paper had the strong support of Koopmans in his 1957 book, where he placed Malinvaud's model as a central tool for studying economic growth in the context of the activity analysis literature. However, Malinvaud was eventually eclipsed by the influential 1958 book by Robert Dorfman, Solow and Paul Samuelson, which pushed the literature along the von Neumann path of the so-called "turnpike theorem," with a focus on productive efficiency (section 3). From Europe, when turnpikes thrived, Malinvaud engaged with the activity analysis literature through

[5] The fact that Solow (1956) was not a central reference in the activity analysis literature doesn't mean that the latter developed disconnected from economic growth. If we take again Claveau and Gingras's (2016) online platform (its polygon graph), the activity analysis literature (row 7) and growth (row 8) are two separate specialties from 1956 to the early 1970s, but not far from each other and with thicker links (because they share references).

publications and conferences. He organized a conference in Cambridge in 1963, bringing together Koopmans, Allais and others working on this subject, an opportunity that he and Koopmans took to express once again their concerns with the lack of a utilitarian analysis to the study of economic growth (section 4). Just a few months after this conference, both economists went to another meeting, at the Vatican City, where they presented optimal growth models with utility maximization as the basic optimality criteria (section 5). It was exactly this paper that secured Koopmans the role of a founding father of the Ramsey-Cass-Koopmans model, nowadays understood as an extension of Solow (1956).

1. At a critical crossroad: Malinvaud, Allais, and von Neumann

One of Malinvaud's first publications, his 1953 *Econometrica* paper, is important not only for its posterior impact on economics but also for being at the crossroad of French and American developments in mathematical economics. For highlighting this, some biographical information is helpful.[6] Malinvaud entered the *École Polytechnique* in 1942 and in parallel he studied law, where he encountered economics courses. With studies interrupted by World War II, he was an autodidact in economics. He obtained his law degree and diploma from *Polytechnique* in 1946 and spent two years at the *École d'application de l'INSEE,* the *Institut National de la Statistique et des Études Économiques*, when he became an economist.[7] It was here that Malinvaud encountered two important figures: the mathematician and statistician Georges Darmois and Allais, who taught him economics in his second year and introduced him in the modern Anglo-Saxon literature of John Hicks, Samuelson, and von Neumann, among others.[8] In 1948 Malinvaud joined for two years the informal group of young economics around Allais, the *Groupe de Recherches Economiques et Sociales* (GRECS), whose participants included Marcel Boiteux and Gerard Debreu (Krueger 2003, 183).[9] In 1950, Malinvaud received, with Allais's recommendation letter, a Rockefeller fellowship and went to the Cowles Commission at Chicago where

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[6] This information comes from Renault (2016, 2-9).

[7] The *École d'application de la statistique* was created in 1942 with the mission of training public servants in statistics and economics (Renault 2016, 2, fn. 2). The INSEE, established in 1946, is a French institution in charge of not only coordinating the statistical system and producing public statistics, but also studying the trends of the French economy. See Malinvaud's views on the INSEE in Krueger (2013, pp. 184-185).

[8] According to Malinvaud, Allais made him organize his prior economics knowledge and learn general equilibrium (Krueger 2003, 182-183). It is worth pointing out that, Allais was in no way a celebrity in French academia, with Debreu describing him as a "mathematically mad person" (Debreu in Weintraub 2002, 137).

[9] At that time, from 40 to 80 people (eventually including foreign economists) attended the meetings which were held in a café up to 1953. See Laudereau and Diemer (2010).

Debreu was a research associate.[10] Malinvaud spent fifteen months at Cowles, then directed by Koopmans, having arrived right after the important 1949 conference on activity analysis (analyzed by Düppe and Weintraub 2014). After this visit to Cowles, he returned in 1951 to Paris and the INSEE's director made him lead the team charged to produce the French national accounts.[11]

On the French side of Malinvaud's background, the INSEE and Allais are really important. Soon after Malinvaud's arrival at INSEE, Allais published his influential book *Économie et Intérêt* (Allais 1947), which had important utilitarian and general equilibrium elements that instigated Malinvaud. Allais was himself a graduate from the *École Polytechnique*, where he studied from 1931 to 1933, moving afterward to the *École Nationale Supérieure des Mines de Paris*, with its "strong tradition in economics and a special curriculum for graduates of the Ecole Polytechnique" (Munier 1991, 180). In 1940, after the armistice, Allais resumed research. In 1944 he became professor at the *École Nationale Supérieure des Mines*, and two years later a researcher at CNRS (*Centre National de la Recherche Scientifique*). Allais was, together with Pierre Massé, a key actor in renewing the French tradition of mathematical economics. However, they were part of a very small group placed outside traditional professional circles such as economics departments and research institutes (Drèze 1964, Arena 2000). They were, instead, in engineering schools or statistics departments, in the *Grandes Écoles*, in research or executive divisions of the nationalized industries, or in the government. Several were engineers (a few were mathematicians) with little training in economics and they published in technical journals not read by most French economists (Drèze 1964, 4-8; Arena 2000, 972-973).

It was at the INSEE that Malinvaud interacted with yet another important figure in the French mathematical economics and operations research scene, Georges-Théodule Guilbaud, who was a professor at the INSEE from 1948 to 1955 (Barbut 2008, 12). From him Malinvaud (1953, 236 n. 5) borrowed a technical term for his analysis, *chronic* (see below). Guilbaud was Allais's contemporary: in 1931 he was admitted to both the *École Normale Supérieure* and the *École Polytechnique*, choosing the former, graduating in 1932 and getting his *agrégation de mathématiques* in 1935. From 1947 to 1955 Guilbaud was a researcher at the *Institut de Science Économique Apliquée* (ISEA), then directed by François Perroux, period in which he was also teaching not only at INSEE but, starting in 1949, also at *Sciences Po* and at the *Institut Statistique de l'Université de Paris* (ISUP) (Barbut 2008, 12). It is in the late 1940s and

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[10] Circa 1948 Boiteux and Debreu applied to the same Rockefeller Fellowship and Allais allegedly tossed a coin and chose Debreu (Düppe 2012, 425, n. 5). See Düppe (2012, 422-425) for more details on the friendship of Debreu with Malinvaud and Boiteux, and their contact with Allais.

[11] Renault (2016, 4-5) explains that Malinvaud and Allais had a very strong relationship prior to the former's trip to the US. Upon his return to France the intensity of their relationship weakened and Malinvaud felt that GRECS's (that existed until 1969) dynamism was never the same of the period 1948-1950.

early 1950s that Guilbaud published several articles and books on mathematical economics, statistics, game theory, and cybernetics.[12] Guilbaud was a central actor institutionalizing operations research in France: in 1953 he created an operations research seminar at ISUP and helped organize a CNRS international colloquium on econometrics in Paris, while three years later he created and directed at ISUP the *Bureau Universitaire de Recherche Óperationelle* (BURO) and helped create and presided the French Society for Operations Research, where practitioners and researchers met in the mid-1950s (Roy 2006, 28). Guilbaud's renown was not confined by national borders: he became a Fellow of the *Econometric Society* in 1951, after Allais's election in 1949 and one year before Darmois's.

According to Roy (2006), in the 1950s operations research was actively pursued in France, in companies and in the military, particularly by engineers. But the French university system at large was not so receptive, and even hostile, to this field, with the important exception of the *Université de Paris* with ISUP and, moreover, the *Institut Henri Poincaré* (IHP) where linear programming, game theory, stochastic processes, mathematical economics and econometrics were taught (both Guilbaud and Darmois taught at IHP).[13] The Econometric Society meetings and the different seminars organized by Allais (GRECS), Guilbaud and René Roy could have been the main points of contact of those developing mathematical economics and the economists, "but few general economists ever attend such meetings, for lack of interest and/or mathematical background" (Drèze 1964, 6).

Malinvaud was thus part of a small academic world where there was an enthusiasm with mathematical economics. Certainly Guilbaud and Allais shared this sentiment.[14] Allais in his 1947 book paid homage to Irving Fisher and pursued a Paretian analysis of intertemporal economics based several times in what later came to be known as the overlapping-generations model. And he praised very much the mathematical approach to economics (although keeping the more sophisticated mathematical presentation to appendices for the sake of making his analysis reach a broader audience):

> Mathematical thinking is the wonderful tool that, by freeing the spirit of darkness, confusion and helplessness of the verb, helps overcome gradually, and without exceptional effort, any difficulty in an unparalleled flow of light and clarity. Only those unaware may persist

[12] The list of Guilbaud's works was published in 2008 in the issue 183 of the journal *Mathématiques et Sciences Humaines*, pp. 17-23.

[13] Roy (2006, 27) argued that it was only after 1968 that elective operations research courses became part of university curricula outside ISUP.

[14] In this respect it is noteworthy that in the period 1959-1963 Guilbaud was professor at the law and economics faculty of the *Université de Paris* and there he specified and organized the mathematics teaching, turning it compulsory to economics students.

> without it. They do not know what they're missing! As for those who undertake the journey of initiation, they will never think again to return to the land of verbal metaphysics and they will pursue an ever more fruitful route to an ever greater light. (Allais 1947, 534-535, our translation)[15]

And what Malinvaud could find of an intertemporal analysis in Allais's book? One important discussion is the equilibrium welfare. He first considered a Pareto optimal characterization of a static economy with a given demographic structure and capital stock. Here, a Pareto optimal equilibrium is one that maximizes what he called *rendement social*, a problem that he characterized as being of economic technique, independent of the social-political conception that one may have about the distribution of goods. He then moved to an intertemporal context (reaching his *rendement social généralisé*), considering an economy in a finite time interval, perfect foresight, and given demographic structure at each instant of time, identical property structure each time, and given capital stock at the initial and terminal time periods. Allais argued that the Pareto optimum is not absolutely satisfactory in this context, as it ignores future satisfactions. For him, there is as much difference between the satisfactions of a given individual in different time periods as between the satisfactions of different individuals in a given instant: intertemporal satisfactions are not comparable. He then generalized the Pareto concept to include not only individuals in a given period of time but also the same individuals in different time periods.

Another important point in Allais (1947) was his discussion of the search of an optimal economic structure, in contrast to the optimal satisfaction in a given structure (*rendement social généralisé*): his theory of "social productivity" (*productivité sociale*). He considered an economy with several sectors, a finite time interval, and what he characterized as a "permanent economic regime," one in which the initial and final stocks of capital vary in the same amount. The question is then what is the (Pareto) optimal distribution of economic factors and of consumption across sectors, given that the *rendement social* is already maximal (Allais 1947, 208). Although the two concepts, of *rendement social* and of social productivity, apply to distinct situations (though it is possible to maximize them simultaneously), the two are analogous in a particular sense. Both concepts are related to the search of an optimum for the economic conditions of production, leaving it entirely aside distributional issues (among different individuals or for given individuals in different time periods).

If in Allais (1947) Malinvaud could find an articulated intertemporal welfare discussion, the situation was rather different in the growth model that John von

[15] And he was sure that this would be the only way to doing economics in the near future: "Every day, our method is conquering new minds and this is a definitive conquest. This movement is slow but steady, in a single and irreversible direction: it will end in the near future by winning all minds" (Allais 1947, 534, our translation).

Neumann published originally in German in 1938, but which was translated into English only in 1945 (von Neumann [1938] 1945-46). Von Neumann developed a general equilibrium model in which goods are produced by other goods with a number of technically possible production processes in excess of the number of goods. He wanted to determine which of the technically possible processes will be deemed profitable and thus used, and what prices result from such decision. Clearly, the two questions depend on one another, and the solution had to use a fix-point theorem (von Neumann used Brouwer's theorem) instead of the typical counting of equations and unknowns. He assumed, for simplicity, constant returns to scale and supposed an economy that can expand but with an unchanged structure: each of the possible production processes will be used in certain intensities, and the ratios of the different intensities remain unchanged. Thus, the economy can expand by scaling up or down all intensities by the same factor (the so-called "coefficient of expansion of the whole economy," von Neumann [1938] 1945-46, 2).

More importantly, von Neumann introduced the consumption of goods only through the production process (which lasts one time period): workers consume (for subsistence) while producing and the resulting income is net of consumption. As a consequence, he ended up with a technical decision of characterizing the profitable production processes, the prices, and the coefficient of expansion of the economy. As the output is net of consumption, no clear normative analysis is possible in his model, something Malinvaud found problematic. Von Neumann's model became really important for the activity analysis literature that was vigorously debated at the Cowles Commission after the 1949 activity analysis conference, exactly when Malinvaud visited Chicago and interacted with Koopmans and others.[16] Koopmans was instrumental in bringing new mathematical tools, linear programming, to extend the wartime research on resource allocation into a "more general economic theory of production" (Düppe and Weintraub 2014, 454).

So here we have Malinvaud, trained at *École Polytechnique*, who was a student of Allais and Guilbaud, key figures pushing mathematical economics (and operations research) in France, moving to the US for the hotbed of mathematical economics, the Cowles Commission. It was in the US that he initiated his first contributions to activity analysis with capital accumulation informed by utilitarian concerns that helped shape the later utilitarian model of economic growth of Cass (1965) and Koopmans (1965).

2. Across the Atlantic: Malinvaud and Efficient Capital Accumulation

During his brief period working at the Cowles Comission, Malinvaud produced three discussion papers in 1951 with the basic setup used in his 1953 *Econometrica*

[16] Model-wise, Allais and the activity analysis were very distinct, the former with an overlapping-generations model while the latter employed a linear production model.

paper. At that time, growth theory was still a nascent field in economics and Malinvaud was engaged with the theory of resource allocation and welfare economics. The latter was clearly a hot topic then, as evidenced in the Commission's annual report for the period June 1950-July 1951 that in the "Staff Meetings and Seminars" section recorded: "[i]t was possible to arrange most of the seminar sessions of the 1950-1951 period into two sequences: 'Utility and Probability' (papers by Carnap, Savage, Marschak, Arrow, Chernoff, van Dantzig, and Thurstone) and 'Economics of Welfare' (papers by Hurwicz, Koopmans, Slater, Debreu, and Hildreth)" (Cowles Commission 1951, 24-25). The report also states that the invitations to the seminars were extended to research personnel as well as to the public and to interested graduate students.

At Cowles, Malinvaud's (1951a) first discussion paper started by noting that static and dynamic problems are completely different because including more time periods in a welfare analysis is equivalent to a multiplication of the number of goods taken into consideration. If the study of intertemporal equilibrium problems amounted to simply adding more time periods on a static problem, two important questions would be left unanswered: first, does it make sense to sacrifice present satisfaction for better future satisfactions? Second, what is the optimal amount of capital and how to reach it? The latter could be answered without a utilitarian consideration, by simply determining the amount of capital in each time period that promotes productive efficiency, like in von Neumann ([1938] 1945-46). However, the optimal amount of capital is also the one that produces the highest amount of consumption goods. Therefore, Malinvaud analyzed capital accumulation in an economy with three types of goods: consumption goods, production goods and investment goods, and argued that the economic resources at disposal of society are maximized if the investment goods are only enough to compensate depreciation, making the level of production goods stable. Malinvaud called this a "z-optimum state," a productive efficient state that doesn't consider the amount of consumption goods produced. However, Malinvaud also used another efficiency metric called "s-optimal state." In an s-optimal state, the satisfaction derived from the supply of consumption units is maximized. But in his first discussion paper, the theorems he proved are somewhat confusing, and the framework he used is very different from his 1953 paper.[17]

Two months later, Malinvaud (1951b) published his second discussion paper investigating the issue of capital accumulation as a contribution to the literature of welfare economics rather than to growth theory: he was concerned with the validity of the fundamental theorem of welfare economics in an intertemporal framework, showing

[17] On a supplement to the discussion paper published two weeks later, Malinvaud addressed the limitations of his sketchy model. The theorems, he argued, only account for stationary states (i.e., no growth). But since most modern capitalistic economies are essentially progressive, meaning that capital accumulation and technical progress occur, it should be essential to determine what characteristics of a stationary economy could be used for a progressive one, an issue he did not study in his first working paper.

that here the efficient outcome is one in which "firms should maximize their net profit defined as the discounted value of all their present and future incomes" using competitive prices and interest rate vectors (Malinvaud 1951b, 1).[18] Malinvaud (1951b) stressed that the only proof of this proposition until then was presented by his advisor Allais (1947), however in a very simplified model. Malinvaud considered his model to be a generalization of von Neumann's, because the structures of both models are very similar and he relaxed von Neumann's restrictive assumptions: the absence of consumption, the linearity of production, and the absence of technical change.

Without directly discussing what would be the optimal rate of capital accumulation, Malinvaud (1951b) proved a version of the first welfare theorem applied to an intertemporal framework where there is a capital structure that defines the production. The challenge here is how to compare present and future consumptions, with some discounting being necessary. Malinvaud further developed this topic in his third Cowles discussion paper, which has the same title of the 1953 paper, "Capital Accumulation and Efficient Allocation of Resources." He investigated the validity of the fundamental welfare theorem and placed the meaning of the interest rate at the center of the inquiry that addressed two questions: would the competitive interest rate work as a competitive price that allows an efficient allocation of resources? And, if yes, would the interest rate be a price that reflects time only, being the same for all commodities?

It was only in his third discussion paper that Malivaud explored (in the last sections of the paper) what would be the optimal amount of capital in a stationary economic system, a concept much emphasized by Allais (Malinvaud 1951c, 26, n. 8). A stationary state associated to an optimum vector of capital is defined as the one that delivers a consumption vector that is greater or equal than any other possible stationary state, with strict inequality in at least one good. Based on some hypotheses about the production possibility set, Malinvaud (1951c) showed that the optimum capital vector exists and is associated with a zero interest rate.

In the 1953 *Econometrica* paper Malinvaud brings together the different problems and results that he explored in the 1951 Cowles discussion papers. He opens the paper stressing that, among the many problems regarding the accumulation of capital, the most important one is to understand when it is efficient to save today in exchange for more future consumption. This question is relevant both to capital theory and welfare economics, two branches of economics that were closely linked according to Malinvaud. He lists Ramsey (1928) and Allais (1947), among others, as works that had previously tackled the problem stated above.

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[18] It is worth stressing that up until the 1940s, mathematicians and economists (including those at Cowles) working with optimizing dynamic models had not reached an agreement on what is the proper objective function that a firms maximize, if discounted or undiscounted profits, or the average rate of return on capital over the life of the enterprise (see Duarte 2016, 270-285).

When modeling production, Malinvaud (1953) followed the structure that he had already used in the previous discussion papers. He defines a *Chronic* as "a quantitative description of the economics activity occurring during all future periods."[19] The chronic doesn't require the definition of any standard of value, such as a vector of prices, to be completely defined. Four vectors define a chronic: b_t is a vector of outputs, a_t is a vector of inputs, x_t a vector that represents the difference between units consumed and labor used in production and z_t is vector of natural resources used in production. Those four vectors are related as shown in Figure 1. Production happens during one time period. Vector a_t and vector b_{t+1} are connected by a production possibility set T ($(a_t, b_{t+1}) \in T_t$, for all t) with the same properties of von Neumann's model.

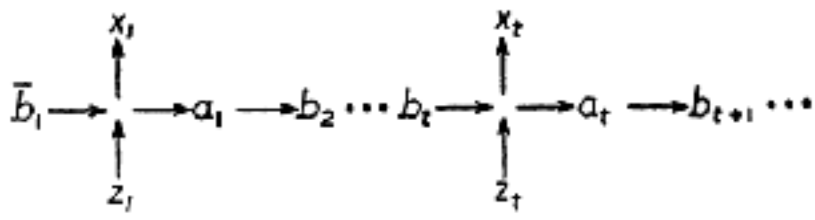

Figure 1: Diagram of a chronic (Malinvaud 1953, 237)

Efficiency, however, is defined differently than in von Neumann's productive efficiency: it is the chronic that generates the highest vector of consumption. In addition to an efficient chronic, Malinvaud also defined an optimal chronic as the one that generates a consumption plan that is Pareto optimal. So he studied efficiency (maximum consumption) and (Pareto) optimality relating growth theory and welfare economics by deriving the properties of such chronics and showing that, under certain hypotheses, every optimal vector of capital is associated with an efficient chronic. Malinvaud (1953) noted that nothing was assumed regarding the rhythm of expansion in the economy. This represents a very different focus from what growth theory came to be at the end of the 1950s and the beginning of the 1960s, as it will be explored in the next sections.

At the end of the paper, Malinvaud acknowledged that he was not the first to relate capital theory to welfare economics. He discussed the works of such economists as Fisher, Wicksell, William Stanley Jevons, Friedrich Hayek, Léon Walras, and Allais (1943), placing his work as part of something like a "French Walrasian lineage," that he was sure to transplant to American soil at Cowles.

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[19] The production side of the model of his second discussion paper (Malinvaud 1951b) is identical to the "Chronic" defined here, although without this label. Malinvaud (1953) credited Guilbaud (1951) for introducing this term ("Chronique" in the original).

3. (Not) Spreading the Word: Malinvaud, Koopmans, and Dorfman, Samuelson and Solow

Koopmans, as the research director of Cowles, was the one who took the lead and organized the 1949 activity analysis conference, under a RAND Corporation research contract (Düppe and Weintraub 2014). The following year he welcomed Debreu as a research associate (after a short visit that Debreu made to Cowles; cf. Düppe 2012, 426), and Malinvaud as a Rockefeller fellow. In pushing forward activity analysis and general equilibrium, Koopmans wrote an influential book, *Three essays on the state of economic science*, published in 1957 trying to popularize the mathematical tools of convex sets used in activity analysis, helped with visual proofs and examples. Koopmans was presenting this work in seminars in 1955 at Carnegie Mellon (then Carnegie Tech) and Stanford and at the Econometric Society Meeting (Ann Arbor), where he met with David Gale (Brown University).[20] After they discussed in person, Koopmans referred Malinvaud's (1953) paper to Gale as a useful "discussion of the use of an interest rate as a means of allocating capital to production process. This might clarify that the rate of interest has a meaning deeper than that of a payment by one individual to another for a loan, in that it also guides the best use of such a loan."[21] Koopmans was also lecturing on those tools to mathematicians, trying to "increase the frequency and effectiveness of communication between mathematicians and economists engaged in research" (Koopmans and Bausch 1959, 79-80).

In the first chapter of that book, Koopmans (1957) presented price theory using an activity analysis approach and reserved a long section to Malinvaud's 1953 paper, showing how important as a reference on growth theory it was for the activity analysis community. However, Koopmans's enthusiasm was not to rest alone for long. Just one year later, in 1958, Dorfman, Samuelson and Solow (DOSSO) published another influential book, *Linear Programming and Economic Analysis*, following the wave on activity analysis, which eventually eclipsed Malinvaud's contribution. In this book, also financed by the RAND Corporation, the authors went back to von Neumann ([1938] 1945-46) and introduced a different framework for studying growth theory, centered on the turnpike theorems.

On chapter 12, Dorfman, Samuelson and Solow present their version of a turnpike theorem that would be important for the work on growth theory among the

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[20] Koopmans was member of the program committee and Gale was a discussant at a session (Econometric Society 1956).

[21] Letter from Koopmans to Gale, Sept. 27 1955, Box 24, folder 488 ("von Neumann Model"), Tjalling Koopmans Papers, Yale University. The same folder contains other letters from which the information on the seminars he presented come from. Interestingly, Gale chose not to mention Malinvaud (1953) in his 1960 book, *The Theory of Linear Economic Models*, where there is just a little discussion on von Neumann. Finally, Koopmans was really discussing things with Malinvaud. In a 1956 research report to Tobin (director of Cowles) Koopmans informed that the draft of the last section of the first essay was with "Fellner, Debreu, Malinvaud and Tinbergen" (Box 22, folder 434, "Research Reports").

activity analysis community in the following years. The authors opened the chapter with a discussion of welfare results in intertemporal problems, arguing that they are not extensions of welfare results from atemporal problems, and not properly developed at the time —a concern of Malinvaud as well and many others. DOSSO discussed that a necessary condition for optimality in an intertemporal problem that doesn't appear on a single period problem is that the marginal rate of substitution between two goods produced in a given period must be equal to the marginal rate of substitution between the same two goods considered as inputs in the next period. This means that a growth path could be efficient in every period considered by itself, but sub-optimal intertemporally.

The turnpike theorem is introduced in the section of the book that discusses capital accumulation on models without consumption, or models where consumption appears only as an input for labor, when they compared von Neumann to Thomas Malthus, Karl Marx and Wassily Leontief (DOSSO 1958, 325). The literature that emerged after the publication of this book used a model similar to von Neumann's original model, mostly based on an exposition made by the mathematician Samuel Karlin (1959).[22] To characterize the efficient capital accumulation, DOSSO reduced the set of possible optimum paths to the balanced growth ones. A balanced growth path is one in which the stock of capital maintains the same proportions of the factors of production through time. They named this type of growth path as "Malthus-Cassel-Harrod balanced growth." Among these, those satisfying the intertemporal efficient production conditions, the optimum balanced growth paths, were labelled the "von Neumann path."

The authors then proceed to analyze what are the implications of the "von Neumann path" to the growth of an economy with finite (but long enough) time horizon, given an initial and final conditions for the structure of capital.[23] They then presented the turnpike theorem and sketched its prooft:[24]

> "Society has decided in what proportions it would like to possess capital stocks at the end of the planning period [given any initial capital structure]. (...) Then if the programming period is very long, the

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[22] However, the model in DOSSO has some differences with von Neumann's and those developed afterwards. Continuous time and substitutability of the factors of production are the major differences from the original model.

[23] It is worth pointing out how common it was to consider dynamic problems in a given finite interval of time. As we saw, Allais (1947) had done this with his permanent economic regime in the context of an overlapping-generations model. Duarte (2016) discusses how welfare considerations related to discounting future utilities were developed in the postwar period, an issue that appeared both when the planning horizon was finite and when it was infinite. Recursive methods developed by Richard Bellman and others helped popularize infinite horizon problems in growth theory.

[24] Samuelson (1965) argued that he had already postulated this conjecture in a research memorandum of the RAND Corporation in 1949, but it only gained attention after the publication of the 1958 book.

> corresponding optimal capital program will be describable as follows: The system first invests so as to alter its capital structure toward the special von Neumann proportions. When it has come close to these proportions, it spends most of the programming period performing steady growth at the maximal rate (…). The system expands along or close to the von Neumann ray until the end of the programming period approaches. Then it bends away from [the von Neumann ray] and invests in such a way as to alter the capital structure to the desired terminal proportions, arriving at the [final condition] as the period ends." (DOSSO 1958, 330-331)

It is analogous to a "turnpike paralleled by a network of minor roads" and with origin and destination far apart, when it is best to go to the turnpike, "cover distance at the best rate of travel, even if this means adding a little mileage at either end." Therefore, the theorem has a "real normative significance" to "maximal von Neumann growth:" "The best [temporarily optimal] intermediate capital configuration is one which will grow most rapidly, even if it is not the desired one ." (DOSSO 1958, 331)

With Malinvaud, on the one hand, and Dorfman, Samuelson and Solow, on the other, the relationship between capital accumulation problems and intertemporal efficiency gained importance in activity analysis and welfare economics. This stimulated other economists working mainly with general equilibrium theory, rather than macroeconomics, to further study capital accumulation. Radner and Lionel McKenzie were very influential on the turnpike theorem literature in the beginning of the 1960s.

Radner published an important paper in the *Review of Economic Studies* in February of 1961, in the very same issue of Michio Morishima's proof of a turnpike theorem. Radner's paper was used as a starting point to most works on turnpike theorems, as Koopmans (1964) pointed out. Radner's model, based on the exposition of Karlin (1959), is closer to von Neumann's original model than to DOSSO's. However, he modeled production based on Gale's (1956) closed model of production, considering it a generalization of von Neumann's production. Radner maintained some characteristics of von Neumann's model, as discrete time and non-aggregated capital, and most of the turnpike literature continued to use those elements.

Therefore, in the peak of the activity analysis literature, in the 1950s to the early 1960s, welfare considerations focused mostly on productive efficiency that came out of von Neumann's model. Malinvaud (1951a, b, c; 1953) called for taking into account consumption (and indirectly, utility) in a growth context, and had Koopmans (1957) as an important supporter. However, this concern was put aside by the return to von Neumann that DOSSO (1958) promoted. General equilibrium theorists trying to extend their static results to a dynamic context, generalized the von Neumann model in the burgeoning literature of turnpike theorems. It was only in the late 1960s that economists such as Gale and Mackenzie brought explicit intertemporal utility maximization to that literature (Gale 1967, 1971; Mackenzie 1968; cf Duarte 2016, 287-291). But before this, Malinvaud once again pushed for his concern with consumption at a conference he organized to bring together part of the activity analysis literature.

4. Subsequent active engagements

Since his return to France in 1951, Malinvaud headed a research team at INSEE responsible for producing national accounts (1951-1956) and, charged by the director if INSEE who wanted to transform its *École d'application* (where Malinvaud studied), he founded the teaching center *Centre des Programmes Economiques* (CEPE) (Renault 2016, 6-7). As a way out of the rigid structure of the French higher education system, CEPE was designed to train civil servants and executives of public firms in applied economics, trying to bridge the macroeconomic and microeconomic analyses (taught by Malinvaud and by Boiteux). In November 1960 the *École d'application de l'INSEE* becomes the *École nationale de la statistique et de l'administration économique* (ENSAE), and opens its programs to graduate students from law schools and economics (in addition to statisticians). Malinvaud became its director from 1962 to 1966, with the major concern in strengthening the economics education (Renault 2016, 8). Despite his multiple institutional engagements and concerns with economics teaching, Malinvaud did not abandon the international academic scene: in addition to his new publications, he was elected Fellow of the *Econometric Society* in 1955, one year after Debreu, and was a visiting professor at the University of California, Berkeley twice, in 1961 and 1967 (the second of which he had Debreu as a colleague, who moved to Berkeley in January of 1962).

Malinvaud's first reaction to approaching economic growth through von Neumann's model was published in April 1959 in *Econometrica*, in French. Based on Kemeny, Morgenstern and Thompson (1956), he analyzed some implications of including a "final demand good" in the von Neumann model, as an output that is consumed rather than used as an input in the following period. Malinvaud then argued that maximal rate of growth program doesn't always correspond to an efficient program and thus becomes a misleading criteria. He also argued that Kemeny, Morgenstern and Thompson's (1956) interpretation that the introduction of a final demand does not alter the results of the standard von Neumann model does not hold: with a final demand in the model, the price vector no longer allows an effective decentralization of decisions. Additionally, he proved two propositions that show that the rate of expansion of final demand should always be proportional to the maximum rate of expansion of the economy, to conclude that, in models with consumption, the rate of expansion is too restrictive a criteria since it would suppose the proportionate expansion of final demand a priori. It is important to note that Malinvaud's (1959) model was somewhat different from later models on turnpike theory based on Karlin (1959). However, since the elements of Kemeny, Morgenstern and Thompson's (1956) model that he criticized were also present on the turnpike theory literature of the 1960s, the criticisms were extendable to it as well.

Malinvaud did not give up and continued to criticize the lack of consumption and of a utilitarian criteria in the study of economic growth with the von Neumann model. In June 1961, he published a paper with the explicit aim of reviewing the theory of resource allocation as applied to growth programs. He discussed the existing alternative choice criteria used for growth programs, beginning by the commonly used Pareto optimality, which requires listing all consumers and knowing all of their utility functions. The utility functions should represent the present preferences for each consumer regarding all future consumption, which raises two problems. First, if the programming period is very long, the list of consumers at the beginning of the program should include the unborn and their utility function should be well represented even before they exist. Second, present preferences aren't usually good enough to evaluate the relative importance of present and future consumption, as preferences may change over time. Because Pareto optimality relies too much on present preferences and neglects distributional problems, its usefulness would be limited. As an alternative, Malinvaud pointed to Allais's (1947) criterion of optimality as a possibility that helps addressing those two problems.[25] He argued that Allais's optimality implies Pareto optimality, but the converse is not necessarily true.

Malinvaud (1961) then discussed a third criterion of optimality that, he wrote, was usually adopted in the growth theory literature: the rate of expansion of an economy, firstly used by von Neumann ([1938] 1945). This suggests that he was referring to the nascent turnpike literature as the growth theory that used that criterion. Although maximizing the rate of expansion seems quite sensible for judging problems of production, he continued, it is not sufficient to assess optimality. First of all, growing production would be meaningless if the goods aren't economically useful. For example, an economy specialized in breeding rabbits would have a high expansion rate, but couldn't be considered optimal. Second, a theory of resource allocation needs a criterion of optimality that doesn't depend on any particular vector of prices. If various products grow at different rates, it is necessary to weight them by some measure of the overall growth rate, and there is no clear way of doing this a priori. Malinvaud (1961) argued

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[25] Allais's optimality criterion has the consistency of choices as its first requirement. In every period, the consumer can optimize his present utility function considering all future consumption stream. The preferences of each consumer is given by a set of T utility functions, one for each time period. Denoting ${}_tx$ as the sequence of consumption after period t. ${}_tx=\{ x_{t+1}, x_{t+2}, \ldots, x_T\}$, the consistency condition states that if a consumer in a period t chooses a future consumption plan ${}_tx^1$ over ${}_tx^2$, then at period $t-1$ he would choose the future consumption plan $(x_t, {}_tx^1)$ over the sequence $(x_{t-1}, {}_tx^2)$. In an economy with k consumers, a program x is Allais optimal if there isn't another possible program x^1 such that $U_{kt}({}_tx^1) \geq U_{kt}({}_tx)$ for all (k, t), with at least one strict inequality for a pair (k, t).

that one could reduce the possible paths of growth to only the balanced ones, where all goods expand at the same rate. But then it is necessary to stick to an a priori pattern of consumption, which is also a problem. He concluded that "after these considerations, one does not see how in practice one could rely only on the growth rate criterion for choosing among production programs" (Malinvaud 1961, 149).

Malinvaud (1961) also expressed his dissatisfaction with the use of end conditions in growth analysis, which imply that everything that happens after the final period is ignored by the model. Not only a terminal capital stock would forbid one to assess if its composition is efficient for production after the programming horizon:

> "An intertemporal model of competitive equilibrium with a finite horizon will look somewhat artificial since it will necessarily contain consumers for the terminal capital stock. Thus, one may also look for a model with an infinite horizon. To my knowledge, this model has not even been formulated so far. I suspect that one will encounter there some mathematical difficulties. But, the question really requires more thought than I have been able to put into it." (Malinvaud 1961, 152)

It is interesting to notice the timing of the publication of this paper. It was published just one issue after Radner's and Morishima's first contributions to turnpike theory. Malinvaud harshly criticized the optimality criterion used by this literature. He also decried the use of end conditions in growth models, which is a fundamental element of turnpike theory models as well (and also present in Allais, his mentor). Apparently his campaign wasn't very successful, given that a series of other papers insisted on working with that approach, for example McKenzie (1963a, b, c; 1967), Furuya and Inada (1962), Hicks (1961), Nikaido (1964), Sau (1965), Drandakis (1966).

Nevertheless, Malinvaud's unflagging attempts to engage with the turnpike literature were not restricted to academic publications (see also Spear and Young 2014). In July of 1963, in Cambridge, UK, he co-organized with Michael Bacharach an International Economic Association conference to discuss the activity analysis approach applied to growth theory.[26] The contributions to the conference and the discussions that followed each presentation were published in 1967 in the volume *Activity Analysis in the Theory of Growth and Planning* (Malinvaud and Bacharach 1967). Participants included Koopmans, Malinvaud, Allais, McKenzie, Radner, Dorfman, and Leonid Hurwicz, whose previous works were more closely related to activity analysis than to

[26] Bacharach was a graduate student at the University of Cambridge at the time of the conference. He was responsible for registering the discussions that followed each presentation. He got his PhD in 1965, and his doctoral dissertation was published in 1970 under the title *Biproportional Matrices and Input-Output Change*. Bacharach was a member of the editorial board of the *Review of Economic Studies* from 1968 to 1973.

growth theory.[27] Malinvaud defined activity analysis in the volume as an approach to deal with theoretical and applied economic problems, and not as a specific branch of economics. He wrote that, while in the marginalist approach every product or factor of production could be substituted by another product of factor of production, the activity analysis modeling was defined by fixed proportions between inputs and outputs, described by a technical coefficient vector. An activity could operate in a larger or smaller scale, but with fixed coefficients. As different activities are operated at the same time, the choice studied is what would be the appropriate level of the operation of each activity.

Koopmans's contribution to the conference was also published in the *Quarterly Journal of Economics* in 1964, and contained a simple growth model following von Neumann, with only two goods. He used it to demonstrate the recent results of the turnpike literature, identifying two types of contributions. The first, represented mainly by Radner (1961) and Nikaido (1964), builds a model that considers that the production set is strictly convex. The second, represented by Morishima (1961) and McKenzie (1963c), didn't rely on that hypothesis and concluded that there is no unique maximal balanced growth path, which can be less linear than those of Radner's model. In his expository effort, Koopmans consolidated the activity analysis literature and connected the different models into one single modeling approach, defined by the use of disaggregated capital, linear programming and the absence of utility maximization. However, Koopmans wasn't satisfied with this last element. In the introduction he wrote that von Neumann's model was "poor economics" because of the negligence of consumption as an important variable in growth models. This echoed Malinvaud's (1953, 241) point that "economic organization aims at satisfying consumers' needs; hence, the technical process by which this is done is irrelevant to social choice."

During the discussions that took place after Koopmans's presentation, there was some thoughts on how to include utility in the von Neumann's framework. Radner was the first to suggest a way to include it without losing the turnpike results. He conjectured that it would be possible to consider utility as a commodity in the model, produced by a sequence of goods $y(t)$. Using this strategy, one could maximize the function $\sum_0^T \alpha^t.\mathrm{u}[y(t)]$, where α is a time discount rate. Equivalently, the objective function could be expressed as $\sum_0^T \alpha^{-t}.\mathrm{u}[y(T\text{-}t)]$ and be reinterpreted as the terminal stock of utility. Once produced, the utility good would grow at a rate α^{-t}, higher than 1 if $0 < \alpha < 1$. His idea was to change the final objective of the model from reaching a given stock

27 On April 1, 1963, Malinvaud wrote to Koopmans saying that he had read his "clear and wonderful exposition of turnpike theorem" in a 1963 Cowles Foundation Discussion Paper (no. 152) and invited him to the conference ("without consulting Dorfman on this idea"). Koopmans answered on April 10, accepting the invitation but warning Malinvaud that he intended to publish his paper in the *Quarterly Journal of Economics*, as a token of appreciation for the year of research he had spent in Harvard (1960-61). The Tjalling Charles Koopmans Papers, Yale University, box 5, folder 80 ("Activity Analysis").

of capital to attaining a maximum utility stock. Radner argued that if the utility function u(.) is a homogeneous function of degree one, then all results obtained in the original model would hold. However, in a context of general equilibrium it is desirable to assume that the marginal utility is decreasing. In this case, as utility is a produced good, that would imply a decreasing return of scale (homogeneity of degree less than one). Malinvaud responded that it was possible to maintain the results previously obtained only if the decreasing rate of marginal utility was small enough.

In the introduction of the volume, while pondering the discussions at the conference, Malinvaud argued that "the formulation of social objectives for long-term development" was a problem that should be faced by economists interested on growth theory (Malinvaud and Bacharach 1967, xiii). Although the activity analysis approach provided good models for understanding productive operations, he argued that more time should be spent on thinking about how to model a social objective function, something still not appropriately modeled because the difficulties raised by the aggregation of individual preferences. For some economists, he continued, this was so problematic that they preferred to abandon altogether any attempt to model a social objective function, without even suggesting "any meaningful alternative" (p. xiii). Additionally, as the results obtained about the qualitative properties of optimal intertemporal programmes mostly concern the case of no consumption, he stressed that the literature ought to start dealing with less particular growth paths than those studied at the conference (productive efficient balanced growth paths).

5. Malinvaud's alternative

Malinvaud would present his alternative to the turnpike approach to growth shortly after the 1963 Cambridge conference. In October of the same year, a study week on the econometric approach to development planning was held at the Vatican City, sponsored by the Pontifical Academy of Sciences.[28] Theoretical models of economic growth were discussed in a particular day of the conference, with contributions made by Koopmans, Malinvaud, Morishima and Pasinetti. Koopmans's paper was the first presentation of his optimal growth model that eventually gained canonical status, along with Cass (1965), in growth economics.

The optimal growth models of Koopmans and Malinvaud had their similarities, as the latter notes in his oral presentation (Malinvaud, 1965, 301-302). Both models had an infinite time horizon and used an intertemporal utility function (having consumption as argument, and labor as well in the case of Malinvaud) as the optimality criterion, in

[28] Among participants of the Vatican Conference were Malinvaud, Koopmans, Allais, Dorfman, who also attended the Cambridge Conference in July, and Leontief, Pasinetti, Morishima, Haavelmo, Ragnar Frisch (who was in the program of the Cambridge conference but did not attend), Henry Theil, Franklin Fisher, and Herman Wold.

contrast to the typical turnpike model.[29] Despite the similarities, Malinvaud claimed during the conference that there were some differences in their motivations. Koopmans, who was in this period also interested in axiomatizing time preferences (cf. Duarte 2016, 292-293), wrote that the aim of his paper was to illustrate the usefulness of mathematical programming to optimal growth and to argue against the separation of the ethical or political choice of an objective function from the investigation of the set of feasible paths, since this could lead to a search for a nonexistent optimum (Koopmans 1965, 228-229). In his oral presentation, Malinvaud (1965, 301) asserted that, likewise Koopmans, he wanted to understand the logical problems raised by choices between intertemporal programs. Besides, he wanted to explore the relations between the models of Ramsey (1928), Tinbergen (1960), Radner (1966) and Srinivasan (1964).[30] However, during the discussion of Morishima's paper, Malinvaud added that although he shared Koopmans's motivations to "explore the consequences of assuming a particular kind of utility for choices over time," he "was still more strongly motivated by the need to see clearly what we should do when we use models with several periods (…)" in which arbitrary terminal conditions are imposed, making this analysis inadequate (Morishima 1965, 566). In order to study intertemporal problems with an infinite horizon, Malinvaud chose to use a simplified one-sector model, instead of using the typical multisectoral framework of the turnpike theory.

While Malinvaud's call for a utilitarian criterion to ordering growth paths was in line with his French training, he departed from it when he criticized the usage of finite terminal conditions, a hypothesis that Allais often adopted in his works. The infinite horizon he so much wanted to consider brings with it the need for some discounting of future utilities, an issue discussed at the Vatican conference. After Allais claimed that time discounting had no economic justification, Malinvaud (1965, 382) responded that some authors have suggested dropping the assumption of an infinite horizon in order to avoid the inexistence of an optimal program, but he "[could not] accept this point of view." Without an infinite horizon, it would not be possible to reject paths that appear as optimal with finite horizon. Koopmans agreed with his position by noting that problems that arose in infinite time would also show up in finite time with very large horizon (as typically assumed in turnpike models) (Malinvaud 1965, 383).

At the Vatican Conference, Malinvaud had another opportunity to criticize the turnpike literature. Morishima presented a turnpike model, but now one that included a

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[29] On the technical differences between Koopmans and Malinvaud, Spear and Young (2014, 229-231) argued that the latter pointed out in his paper that the postulate of a transversality condition was necessary in the model. Comparing the original Cowles Discussion Paper (Koopmans, 1963) to its published version, they argued that Koopmans included a transversality condition in his model only after Malinvaud provided a counterexample to his questioning at the conference whether paths that didn't meet the condition would violate in a finite time the sign restrictions on capital or consumption (Malinvaud 1965, 379-380).

[30] In the case of Radner, Malinvaud cited a working paper, published in 1962 by the Center for Research in Management Science at the University of California, Berkeley.

utilitarian approach, as a response to the earlier criticisms by Malinvaud and Koopmans. Morishima designed a model where workers spent their income in consumption and capitalists were responsible for the savings in the economy, resembling Uzawa's (1961) two-sector model.[31] Morishima's effort to include utility maximization in his turnpike model did not appease Malinvaud, who asserted that he was "not very happy about the hypothesis that savings come only from capitalists," because it was not "very well suited for the practical questions which is our ultimate air to answer" (Morishima, 1965, 560). For Malinvaud, Morishima's inclusion of a utility function is not a sufficient break from the earlier turnpike literature because Morishima "have tied up consumption to the general growth of the economy by other rules" (Morishima, 1965, 557). Malinvaud does not elaborate on what are these rules, but he was probably referring to the chosen functional form of the utility function, that guaranteed a proportional rise on demand of commodities when per capita income increased, since he had previously criticized the unrealism of this demand dynamics (Malinvaud 1959, 223; 1961, 149).

During the discussion of Morishima's paper, Malinvaud went further in his criticisms and, in line with his previous objections to the turnpike models (Malinvaud, 1961), he questioned the usefulness of Morishima's model:

> Malinvaud: May I question Professor Morishima on the implications of his analysis? (…) I should like to know exactly your intentions. Do you explore indications for programming? Or do you describe what happens in a capitalist economy?
>
> Morishima: The aim of this study is to extend the recent results of growth economics (especially the turnpike theorem) to a model with endogenous population growth and flexible consumption demands.
>
> Malinvaud: Yes, I understand that it is your immediate intention (…) You want to find new extensions of the turnpike theorem; but the final purpose of the exercise is not clear to me. (…) If it is purely descriptive, then we should be careful that the hypotheses provide, at least as a first approximation, a proper description of what happened during the process of growth. If it is oriented toward planning, then we must look at whether too many constraints have not be (*sic*) imposed; because, if such were the case, the results might have little significance for planning. (Morishima 1965, 557-558)

Although here Malinvaud is discussing a particular model and not the entire literature, his criticism is coherent with what he had been claiming in previous analyses of turnpike models. The problem of using Morishima's model for descriptive or normative ends could as well be extended to other turnpike models. Thus, the Vatican

[31] Morishima wrote that he followed instead Joan Robinson and von Neumann, although he does not refer to any particular work by them.

conference exposed once more the unsettled relationship between growth and welfare economics, and the usefulness of the turnpike results. While Pasinetti sided with Malinvaud when he dedicated an appendix of his paper to criticize the unrealism of the proportional economic growth in the turnpike theory, they split when the former considered von Neumann's model "as a very important first analytical step" (Morishima 1965, 561). Likewise, Allais did not side with Malinvaud or Koopmans when he argued that from a theoretical standpoint, "it is very interesting to separate the problem of optimal economic growth into two problems (…), the study of what happens if we limit ourselves to the production function [and] (…) the introduction of preference functions" (Malinvaud 1965, 381).

6. Conclusions

There surely were many tidal forces generating the flood at the economic growth shores in the 1960s. Solow (1956) was influential and helped boost interest on macroeconomic models of economic growth (Boianovsky and Hoover 2009). Yet, another very important tidal force came from activity analysis and the attempts to build a theory of allocation over time and of equilibrium growth, with which Malinvaud and Koopmans were deeply involved. This period was also the heyday of general equilibrium theory and the extensions of the basic model to include elements such as time and study whether the welfare properties of equilibrium would hold (Weintraub 1974, chs. 5-6).

Malinvaud was trained in the rather exclusive mathematical milieu in France, being a student of Allais and moving to the Cowles Commission, then directed by Koopmans, the year following the famous 1949 activity analysis conference. Malinvaud knew both the model of von Neumann ([1938] 1945-46) and its concern with productive efficiency and also Allais's utilitarian approach to intertemporal models. Starting from the working papers written during his North-American sojourn, Malinvaud campaigned for a utilitarian analysis in activity analysis models, being strongly supported by Koopmans and interacting closely with him. Notwithstanding Koopmans's (1957) endorsement, the utilitarian approach to activity analysis didn't stand the wave of turnpike theorems and productive efficiency that followed Dorfman, Samuelson and Solow (1958). Back in Europe, Malinvaud didn't flag and campaigned through several published papers and by organizing one important conference in Cambridge in 1963 and by attending the Vatican conference a few months later, both of which Koopmans attended.

We see from the preceding analysis that Malinvaud and Koopmans helped steer the activity analysis literature towards a utilitarian analysis of growth paths in a period when several issues weren't stabilized. First, in this literature the very domains of micro and macroeconomics were somewhat in flux. While Malinvaud (1953) connected his microeconomic results to Ramsey's (1928) macroeconomic analysis, Koopmans and Bausch (1959, 80) portrayed economics in a four-by-four table opposing micro and macroeconomics, and statics and dynamics. They placed activity analysis in the realm of microeconomics, separated from business cycle and economic growth (without really saying a word about what they understood to be economic growth models). Turnpike theory, which could be classified as activity analysis and economic growth, was introduced in the micro section. Koopmans's (1965) growth paper with utility maximization was presented at the Vatican conference and bridged micro and macroeconomics by taking the one-sector growth models as the basis for his welfare discussion.[32] Second, the issue of the general applicability of linear programming in economics and its connection to welfare economics (see Dorfman 1953). Third, the appropriate time horizon in growth problems and the need for discounting future utilities (Duarte 2016), and more generally the intertemporal welfare implications of general equilibrium analysis. All these concerns had nothing to do with what Solow (1956) wanted to discuss, and informed very much Malinvaud's and Koopmans's contributions to this literature.

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[32] Koopmans (1965) followed Phelps and others and briefly mentioned Harrod (and Ramsey) for his stance against discounting future utilities, but Solow is nowhere cited.

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