Department of Economics- FEA/USP

Challenging Lucas:

from overlapping generations to infinite-lived agent models

MICHAËL ASSOUS

PEDRO GARCIA DUARTE

WORKING PAPER SERIES Nº 2017-03

DEPARTMENT OF ECONOMICS, FEA-USP
WORKING PAPER Nº 2017-03

Challenging Lucas: from overlapping generations to infinite-lived agent models

Michaël Assous (Michael.Assous@univ-paris1.fr)

Pedro Garcia Duarte (pgduarte@usp.br)

Abstract:

The canonical history of macroeconomics, one of the rival schools of thought and the great economists, gives Robert Lucas a prominent role in shaping the recent developments in the area. According to it, his followers were initially split into two camps, the "real business cycle" theorists with models of efficient fluctuations, and the "new-Keynesians" with models in which fluctuations are costly, and the government has a role to play, due to departures from the competitive equilibrium (such as nominal rigidities and imperfect competition). Later on, a consensus view emerged (the so-called new neoclassical synthesis), based on the dynamic stochastic general equilibrium (DSGE) model, which combines elements of the models developed by economists of those two groups. However, this account misses critical developments, as already pointed out by Cherrier and Saïdi (2015). As a reaction to Lucas's 1972 policy ineffectiveness results, based on an overlapping generations (OLG) model, a group of macroeconomists realized that a competitive OLG model may have a continuum of equilibria and that this indeterminacy justified government intervention for competitive cycles that emerged even in deterministic models. We can identify here two distinct, but related, groups: one of the deterministic cycles of David Gale, David Cass, and Jean-Michel Grandmont, and another of the stochastic models and sunspots of Karl Shell, Roger Guesnerie, Roger Farmer and Costas Azariadis (Lucas's PhD student). Here, the OLG was the workhorse model. Following from these works, a number of authors, including Michael Woodford, argued that similar results could occur in models with infinitely lived agents when there are various kinds of market imperfections. With such generalization, some of these macroeconomists saw that once these imperfections are introduced, nothing important for business cycle modeling was lost and they could therefore leave the OLG model aside as a model of business fluctuations, to the dismay of authors such as Grandmont, Robert Solow and Frank Hahn.

In this paper, we scrutinize the differences between the deterministic cycles and sunspot groups and explore the many efforts of building a dynamic competitive business cycle model that implies a role for the government to play. We then assess the transformation process that took place in the late 1980s when several macroeconomists switched from OLG to infinite-lived agents models with imperfections that eventually became central to the DSGE literature. With this we hope to shed more light on the origins of new neoclassical synthesis.

Keywords: overlapping generations model; Robert Lucas; Michael Woodford; DSGE model; new neoclassical synthesis

JEL Codes: B22; B23; E32

Challenging Lucas:
from overlapping generations to infinite-lived agent models

Michaël Assous (*Université Paris 1: Panthéon-Sorbonne*; Michael.Assous@univ-paris1.fr)
Pedro Garcia Duarte (*University of São Paulo*; pgduarte@usp.br)

January 2017

Abstract:

The canonical history of macroeconomics, one of the rival schools of thought and the great economists, gives Robert Lucas a prominent role in shaping the recent developments in the area. According to it, his followers were initially split into two camps, the "real business cycle" theorists with models of efficient fluctuations, and the "new-Keynesians" with models in which fluctuations are costly, and the government has a role to play, due to departures from the competitive equilibrium (such as nominal rigidities and imperfect competition). Later on, a consensus view emerged (the so-called new neoclassical synthesis), based on the dynamic stochastic general equilibrium (DSGE) model, which combines elements of the models developed by economists of those two groups. However, this account misses critical developments, as already pointed out by Cherrier and Saïdi (2015). As a reaction to Lucas's 1972 policy ineffectiveness results, based on an overlapping generations (OLG) model, a group of macroeconomists realized that a competitive OLG model may have a continuum of equilibria and that this indeterminacy justified government intervention for competitive cycles that emerged even in deterministic models. We can identify here two distinct, but related, groups: one of the deterministic cycles of David Gale, David Cass, and Jean-Michel Grandmont, and another of the stochastic models and sunspots of Karl Shell, Roger Guesnerie, Roger Farmer and Costas Azariadis (Lucas's PhD student). Here, the OLG was the workhorse model. Following from these works, a number of authors, including Michael Woodford, argued that similar results could occur in models with infinitely lived agents when there are various kinds of market imperfections. With such generalization, some of these macroeconomists saw that once these imperfections are introduced, nothing important for business cycle modeling was lost and they could therefore leave the OLG model aside as a model of business fluctuations, to the dismay of authors such as Grandmont, Robert Solow and Frank Hahn.

In this paper, we scrutinize the differences between the deterministic cycles and sunspot groups and explore the many efforts of building a dynamic competitive business cycle model that implies a role for the government to play. We then assess the transformation process that took place in the late 1980s when several macroeconomists switched from OLG to infinite-lived agents models with imperfections that eventually became central to the DSGE literature. With this we hope to shed more light on the origins of new neoclassical synthesis.

Introduction

In the 1970s, Robert Lucas, Thomas Sargent, Neil Wallace and other new classical followers brought a new approach to macroeconomics: a rational expectations, competitive equilibrium view of business fluctuations. Their main result was that economic policies were ineffective to systematically alter the equilibrium values of real variables such as output and employment (see Hoover 1988, ch. 4). This sweeping implication was initially attributed to the rational expectations hypothesis, but Stanley Fischer (1977) and Edmund Phelps and John Taylor (1977) showed that it did not survive in rational expectations models with sticky prices or wages. In the 1980s, a dispute between new Keynesians and real business cycle theorists on the role of sticky prices and the real effects of monetary policy was intense, as much stressed by macroeconomists and historians of macroeconomics.[1]

However, Lucas and the policy ineffectiveness result were challenged not only by macroeconomists who introduced price stickiness into rational expectations models. Economists such as David Cass, Karl Shell, Jean-Michel Grandmont, and Michael Woodford, among others, challenged Lucas by exploring the multiplicity of equilibria of the overlapping generations model (OLG) with competitive markets and flexible prices, a model first discussed by Maurice Allais (1947) and Paul Samuelson (1958).[2] In fact, in the late 1970s and the 1980s different communities of economists, with different but related concerns, came together thanks to the properties of the OLG model.

Initially, general equilibrium theorists have been deriving macroeconomic implications from their unworldly Walrasian models (Hansen 1970, chs. 8-18; Weintraub 1974, chs. 5-6). One such a way was to make their models speak about the general price level (and the inflation rate) by adding an equation of exchange (which postulates that the nominal value of goods equates the flow of money in the economy). However, challenges appeared to doing this in a consistent way, as epitomized by Don Patinkin's (1956) book, leading to the recognition that money is not easily incorporated into a general equilibrium model —what

[1] There are too many references, but see for instance: Backhouse and Boianovsky 2013, pp. 84-86, 165-177; Blanchard 2000, 2009; Blinder 1989; De Vroey 2015; De Vroey and Duarte 2013; Duarte 2012; Goodfriend and King 1997; Gordon 1990; Mankiw 1990, 2006; Snowdon and Vane 2005; Woodford 1999, 2009.

[2] Cherrier and Saïdi (2015) provide a detailed historical analysis of these macroeconomists who challenged Lucas (1972) as a research community, with great details about their impact and the dissemination of their ideas. In particular, they stress the importance of a few institutions for the development of this community: "the Center for Analytical Research in Economics and the Social Sciences (CARESS), at the University of Pennsylvania, and the Centre pour la Recherche Economique et ses Applications (CEPREMAP) in France" (p. 3). We have a narrower focus in the present paper, but see our contribution as complementary to theirs.

came to be known as the "Hahn's problem" after Frank Hahn's works (see Hahn 1973 for a survey).[3]

To general equilibrium economists the OLG model brought new insights and questions about a monetary economy with fiat money: in such model there are multiple equilibria, with the equilibrium in which money has positive value (monetary equilibrium) coexisting with the one in which money has no value (barter equilibrium). Additionally, the barter equilibrium can be Pareto inferior in relation to the monetary equilibrium and there are no market forces that help the economy coordinate around the preferred equilibrium. In other words, the OLG model seemed to make two pivotal results of the general equilibrium literature vanish: the so-called fundamental welfare theorems, that assert that under certain circumstances the competitive equilibrium is Pareto optimal (first theorem) and that Pareto optimal equilibria can be reached by market forces after lump-sum wealth redistributions (second theorem). To many general equilibrium theorists, because the OLG model assumes that the economy continues indefinitely, its results, as Cass and Yaari (1966b, 1) wrote, "must be related to the presence of 'infinity.'"[4] This and other issues of the OLG model became a hot area of research for this first group, with concerns about how general the results thus obtained are, and several of our protagonists came from this general equilibrium community.

In the early 1980s, when the real business cycle macroeconomists brought one single model (a perfectly competitive growth model with infinite-lived agents, flexible prices, and perfect information) to bear on any macroeconomic issue, several macroeconomists were working with OLG models and addressing business fluctuations matters. Besides the efficiency issue, the model seemed to have much more to offer. New classical economists such as Wallace and Lucas saw in the OLG model the possibility to develop new microfounded models of fiat money without postulating that money balances enter the utility function of agents. At about the same time, other macroeconomists discovered that OLG models give room for either deterministic or stochastic oscillatory trajectories. Endogenous cycles and chaos as well as sunspot equilibria were then shown to occur in the presence of perfectly competitive product market devoid of any nominal price rigidities. Gradually over time, important contributors to that literature — most notably, for our interests here, is Woodford — strove to transfer OLG conclusions to infinite-lived agents models. In this

[3] See Bridel (2002) and Mehrling (2002) for a detailed analysis of Patinkin's monetary contributions.
[4] Cass and Yaari (1966a, b) argue that this is not the case: the OLG results derive from the lack of enough intermediation to intergenerational trade in these models. Shell (1971) further analyzed the consequences of infinity to general equilibrium models.

context, new dynamic models with market imperfections were developed, initially with flexible prices, that eventually became the hallmark of the sticky price, DSGE macroeconomics (earlier referred to as the new neoclassical synthesis).

Our analysis of the different communities of macroeconomists who used OLG models, their concerns and policy views, makes two contributions to the history of recent macroeconomics, complementing Cherrier and Saïdi (2015). First, it sheds light on the development of macroeconomic modeling strategies often ignored in present-day textbooks and even by historians of macroeconomics.[5] Second, it questions the view of practicing macroeconomists about the origins of the DSGE literature, according to which the dynamic models mainly came from the real business cycle (RBC) economists and the nominal rigidities came from the new Keynesian camp (see Goodfriend and King 1997, 232; Blanchard 2000, 1381-82, 1388; Goodfriend 2004, 21-22, 24; Mankiw 2006, 39; Goodfriend 2007, 60; Woodford 2009, 269).[6] While this view is partly true, it misses important elements by not paying attention that, in the late 1970s and 1980s, several economists challenged Lucas with flexible prices models and eventually became key contributors to the DSGE literature. By illuminating the context of particular dynamic models of the early 1980s and why central players switched from OLG to infinite-lived agents models, our paper enriches our understanding of the origins of the DSGE literature.

1. Starting Point: Lucas, Samuelson and indeterminacy

In the 1970s, in the attack against Keynesian macroeconomics launched by new classical economists, a critical issue became the statistical properties of the so-called Phillips curve (inflation-output tradeoff), as emphasized early on by Lucas (1972, 1973). His 1972 paper used a monetary overlapping generations model in which there is imperfect information across the different markets that are physically separated (his "islands model"). He discussed whether the market behavior implied by his model resembles "certain aspects of the observed business cycle" (p. 117): in particular, he discussed money neutrality by looking into the

[5] Even the richer classification of monetary economics proposed by Arestis and Mihailov (2011), OLG models appear as essentially a money demand model, with no mentions to endogenous fluctuations.

[6] Mankiw (2006, 39) is perhaps the clearest about this: "Like the neoclassical–Keynesian synthesis of an earlier generation, the new synthesis attempts to merge the strengths of the competing approaches that preceded it. From the new classical models, it takes the tools of dynamic stochastic general equilibrium theory. Preferences, constraints, and optimization are the starting point, and the analysis builds up from these microeconomic foundations. From the new Keynesian models, it takes nominal rigidities and uses them to explain why monetary policy has real effects in the short run."

inflation-output tradeoffs.[7] It is noteworthy that his 1972 paper is one of the very few published works in which Lucas used an OLG model. Here he did not explore the possibility of multiple equilibria, a result well discussed in Samuelson's (1958) analysis of interest rate determination (under perfect foresight).[8] In the OLG model, one possible equilibrium is that agents consume their endowment and money has no value, but another is one in which money is valued and can implement a Pareto optimum equilibrium. For Samuelson (1958, 475 n. 8) "[t]here is nothing surprising about multiple solutions in economics," and the OLG model pointed to an important market failure:

> It points up a fundamental and intrinsic deficiency in a free pricing system, namely, that free pricing gets you on the Pareto-efficiency frontier but by itself has no tendency to get you to positions on the frontier that are ethically optimal in terms of a social welfare function; only by social collusions — of tax, expenditure, fiat, or other type — can an ethical observer hope to end up where he wants to be. (Samuelson 1958, 479)

In addition to not discussing the multiplicity of equilibria, Lucas was not deeply committed to the OLG model at a time when his research was mostly related to labor market and to investment problems solved by firms.[9] The OLG model was just an interesting way of modeling a monetary economy. Contrary to many economists of our story, such as Grandmont and Cass who were much committed to OLG models, at this time Lucas used freely different types of models. In a paper presented at a conference in 1978 at the Federal Reserve Bank of Minneapolis, Lucas (1980) examined a monetary model where the infinite-lived representative agent maximizes lifetime utility and is constrained to hold money before

[7] Lucas (1973) also focused on the statistical implications of his theory for the Phillips curve, but did so in an aggregate supply-aggregate demand model of the separated markets, without explicit micrcofoundations. Similarly, in his 1975 paper he presented an equilibrium business cycle model (based on a neoclassical growth model) in which he postulated the asset demands instead of deriving them from utility maximization, which would have been "[t]he most satisfactory way to do this, from some points of view" (Lucas 1975, 1116).

[8] In fact, as Cherrier and Saïdi (2015) argue, there was a non-trivial mistake in Lucas's (1972) proof of his first theorem that prevented him to find the multiplicity of equilibria in his model. Grandmont, with the help of the French statistician Christian Gourieroux, was the one finding this mistake. See also Lucas's (1983) corrigendum.

[9] Although not committed to OLG models in his research, Lucas was at this time teaching it. When he spent the academic year of 1974-75 as Ford Foundation Visiting Professor of Economics at the University of Chicago, Lucas taught two monetary courses which featured "the pure consumption loan model" and "other intergenerational models" (letter from Lucas to Miss Kathi Bates, May 30 1974, box 2, folder "1974 - 2 of 2," Robert Lucas Papers, David M. Rubenstein Rare Book and Manuscript Library, Duke University. The OLG model and its multiple equilibria remained an important topic in Lucas's monetary economics teaching at least up until the early 1980s (see the syllabuses and problem sets in the two folders "Monetary Theory," Box 36, Robert Lucas Papers).

he can buy goods (a cash-in-advance constraint) — a model that he explored further with Nancy Stokey (Lucas and Stokey 1983, 1987).[10]

Part of this group of new classical macroeconomists joined forces with general equilibrium theorists in the search for a microfounded model of money that did not resort to the usual tricks of monetary economists: of adding real money balances as an argument of the utility function, or of resorting to the cash-in-advance constraint, or yet of assuming that money is a factor of production (as surveyed by Stanley Fischer (1975)). This was precisely the motivation of the 1978 Minneapolis conference organized by John Kareken and Wallace, as explicitly recognized by Mark Willes, the President of the Minneapolis Fed (Kareken and Wallace 1980, vii) and emphasized by the organizers in the introduction they wrote to the conference volume, in which they praised the OLG model (Kareken and Wallace, 1-2).[11] Wallace in his own contribution argued that there was consensus that "one must generalize the Walrasian model by including in it some sort of *friction*" but that no agreement existed on the sort of friction needed. His personal view was that Samuelson's overlapping generation model "gives rise to the best available model of fiat money" (Wallace 1980, 50). Moreover, he hoped to show "how models built on this friction can be made to confront virtually every long-standing problem in monetary economics" (51) and that it can be used to talk about business cycle by introducing Lucas's imperfect information assumption and deriving the statistical properties of the inflation-output tradeoffs. José Alexandre Scheinkman, also coming from the general equilibrium literature with a PhD from the University of Rochester in 1974 under Lionel McKenzie, addressed the issue of multiplicity and tenuousness of equilibria in this type of rational expectation model, a result that, according to him, cut across alternative monetary models (money in the utility function and cash-in-advance economies).[12]

Lucas also saw the OLG model as a good way of having a microfounded monetary model. In a set of notes on OLG models that he sent to Robert Townsend in 1977, arguing

[10] Although Lucas was not much connected to the general equilibrium community dealing with the challenges of building an intertemporal Walrasian model, in the late 1960s, when both were at Carnegie Mellon University, Prescott and him worked on valuation equilibrium with infinite dimensional commodity space (published as Prescott and Lucas 1972).

[11] Kareken and Wallace (1980, 2) even asserted that "of all the models so far fashioned by monetary economists, [the OLG models] are the most satisfactory. Developing them further may be the best research strategy for the period immediately ahead." This conference is also analyzed by Cherrier and Saïdi (2015).

[12] Scheinkman (1980, 91) wrote: "I show that tenuousness of equilibria in which fiat money has positive value seems to be related to the possibility of the economy operating in the absence of fiat money. The mathematical conditions that insure the absence of tenuousness are very similar in all three classes of models." Brock returns to that point in his 1988 survey on OLG noting that he had shown in 1974 "how multiple steady-state equilibria can arise when the utility function is not separable in consumption and real balances" (Brock 1988, 278) while Calvo (1979, 87-91) had indicated that nonuniqueness can arise in models where real balances enter the production function."

that efficiency can be restored in these models by adding a competitive banking system, Lucas wrote:

> This effort is more than an exercise in criticism because of the central importance of the Samuelson-Diamond-Cass-Yaari framework to the study of monetary economies. In contrast to other recent contributions to monetary growth theory, these simple intergenerational schemes provide examples of technologies in which the services of money are shown rather than merely postulated. While perhaps not a crucial for the study of the demand for money, this feature is central to the study of privately supplied, or inside, money: without a precise specification of the way monetary services are produced, one cannot hope to study the conditions under which they will be privately supplied. Since money in advanced economies is, on most definitions, largely private, this handicap is a severe one.[13]

When taking part of the 1978 conference Lucas was enthusiastic about the monetary models developed by Wallace:[14]

> I think it is no accident that of the hundreds of people who are fully capable of manipulating intergenerational models it turns out to be you (and not, say, Cass or Grandmont) who is using them to reopen basic questions in monetary theory.[15]

However, the enthusiasm with OLG monetary models was not shared so widely: James Tobin discussed Wallace's paper and was very critical of the capacity of this model to provide a reasonable theory of why money exists in human society.[16] As a result of this and other criticisms, Cass and Shell wrote a paper after the conference reacting against them. The authors argued that money has zero price in finite-horizon models with certain terminal time period, and that, echoing Wallace and Lucas, the tricks to introduce money in Walrasian

[13] Undated notes, pp. 1-2, box 22, folder "Diamond Inefficiency 1977," Robert Lucas Papers, David M. Rubenstein Rare Book and Manuscript Library, Duke University.

[14] Lucas even suggested to Kareken the kind of scholars they should invite to the conference: "I do have some opinions as to who would be good participants. My general theme, I guess, would be that we should try to avoid overdosing on theoretical purity and try to involve more people who have some idea as to what they would do with a good monetary theory if they should be so lucky as to find one." (Letter from Lucas to Kareken, Nov. 3 1977, box 3, folder "1977 - 1 of 2," Robert Lucas Papers, David M. Rubenstein Rare Book and Manuscript Library, Duke University.

[15] Letter from Lucas to Wallace, Oct. 18 1978, box 3, folder "1978 - 3 of 4," Robert Lucas Papers, David M. Rubenstein Rare Book and Manuscript Library, Duke University.

[16] Tobin's (1980, 83) views on the matter were: "I do not believe that the overlapping generations model is the key to the theory of money. The 'consumption loan' parable is valuable and instructive, but it should not be taken seriously as an explanation of the existence of money in human society."

models are unacceptable. For them, the OLG model was the way to go, Nonetheless, it is a very simplified, "skeletal" model that "only suggest (or illustrate) possible theorems" (Cass and Shell 1980, 253). Moreover, the multiple equilibria typical of these models are of "crucial importance to the theory and practice of macroeconomic policy" because they establish that "monetary policy matters very much!" (p. 254). The government can "pursue a policy of counteracting the resultant market disruption" generated by sunspot activity (p. 255), a typical view shared by other economists working in the 1970s and 1980s.

Before moving to that literature it is important to highlight that the main innovation in it was not to discuss multiple equilibria and indeterminacy per se, but to connect this feature of dynamic models with cyclical fluctuations. Since the early 1970s rational expectations macroeconomists knew well that, as Olivier Blanchard (1979, 114) summarized, "[i]n models where anticipations of future endogenous variables influence current behavior, there exists an infinity of solutions under the assumption of rational expectations" (see also Shiller 1978, pp. 22-41).[17] In the case of the OLG model, in the early 1970s the mathematician David Gale was working with Peter Diamond's (1965) model and discovered that it exhibited multiple equilibria. At the time, he had difficulties to discuss his model with economists, with the exception of Menahem Yaari (PhD in economics and statistics from Stanford, 1962) and Robert Solow.[18] In letters exchanged with Solow, who brought Diamond to the discussion, Gale struggled with them over the economic meaning of the multiple equilibria and he was aware of the challenges of doing Walrasian dynamic models:

> My paper just takes general equilibrium theory a la Walras or Arrow-Debreu and applies it literally to dynamic models like Diamond's national debt model. If one does that carefully, one gets the two steady states. This struck me as an interesting fact. Likewise, the stuff about stability. If these results turn out not to make "economic sense", then one is saying that Walrasian equilibrium is not a suitable tool for analyzing these models.[19]

[17] In the case of monetary models, Black (1972, 1974), Brock (1974), Sargent and Wallace (1973) and Sidrauski (1967) made earlier discussions of price level determination with either perfect foresight or rational expectations.

[18] "Dear Bob: First I want to express my real appreciation for your taking the trouble to look at my growth paper. I've really had sort of a problem over the past year in getting people to react to the stuff I've been doing. Manny Yaari is the only one here who has taken an interest." Letter from Gale to Solow, Apr. 6 1971, Box 3, folder "1971, April-August," Robert Solow Papers, David M. Rubenstein Rare Book and Manuscript Library, Duke University.

[19] Letter from Gale to Solow, May 28 1971, box 3, folder "1971, April-August," Robert Solow Papers, David M. Rubenstein Rare Book and Manuscript Library, Duke University. In his exchange with Diamond, Gale explained that, according to him, Diamond did not get multiple steady states in his model due to some particular assumptions: "I didn't mean to say that my analysis of your model is Walrasian and yours is not. … What I

The challenges and implications of the OLG model made it a central model for those diverse groups of economists, also giving rise to new understandings of business fluctuations related to deterministic cycles and to sunspot equilibria that we now explore.

2. "Inherent instability" in OLG Models (1973-1988)

While new classical economists were not keen to further investigate the multiplicity of equilibria typical of overlapping generations models as the one used by Lucas (1972), Cass and Shell (1983), Grandmont (1985) and many others saw this as a way of having business cycle models with perfectly competitive product markets and no nominal rigidities that had new types of dynamics and policy implications. We have here two groups of models. The first, we shall label "deterministic cycles" and is associated with Jesse Benhabib, Richard Day and Grandmont. In these models there are expectations driven cycles in absence of any shocks. [20] The second, that we shall label the "sunspot" program and is associated with Costas Azariadis (Lucas's PhD student), Cass, Roger Farmer, Shell and Woodford.[21] These models exhibit equilibrium indeterminacy and among the large set of possible equilibria are ones in which the state of the economy oscillates forever due to arbitrary events or "sunspots."[22]

2.1 OLG models and deterministic cycles

Gale (1973) provides the first general equilibrium example of the possibility of deterministic endogenous cycle by studying an exchange economy populated by a sequence of generations of agents living two-period, young and old, with one unity of a perishable consumption good per period. The young agents can either save or borrow and therefore they may carry claims or debts into the second period. Assume that this is done by means of a

---

claim is, however, that you do not get the full set of Walrasian equilibria in your analysis because of the additional behavioral assumptions you impose, i.e., that people save by investing." (June 10 1971 letter from Gale to Diamond). The discussion was not settled by mail, and Diamond offered to continue it in person during that Summer, which Diamond would spend in Berkeley.

[20] "[…] this class of examples has been crucial for the development of modern interest in the endogenous cycle hypothesis, since it provided the first general equilibrium examples of the possibility of chaotic economic dynamics, through the work of Benhabib and Day (1982) and Grandmont (1985)." (Boldrin and Woodford 1990, 210)

[21] See Boldrin and Woodford (1990), Grandmont (1992) and contributors of the new Palgrave such as Benhabib in his entry on Chaos, Shell in his entry on Sunspot and John Geanakoplos in his entry on OLG models.

[22] Woodford (1990) called endogenous fluctuations models from both programs arguing that "a simple definition might characterize endogenous models of economic fluctuations as ones in which persistent fluctuations occur despite an absence of variation in exogenous economic 'fundamental' such as tastes and technology and government policies." (Woodford 1990, 1). Boldrin and Woodford (1990, 191) complement that "we regard indeterminacy in cases of this sort to indicate a type of instability of the competitive process in that arbitrary events can determine which equilibrium occurs." As Cherrier and Saïdi (2015) present, Shell was the one to create the label "sunspots" borrowing from Stanley Jevons' business cycle theory.

universally accepted paper asset called money (or checking account). Two possible steady states can then occur: "a golden rule" path (which is Pareto-optimal) and a "balanced" path in which there is zero net indebtedness and no intergenerational trade.[23] Gale named "classical" the case in which the young are impatient and borrow and "Samuelson" the opposite. For the classical case, the golden rule path is stable while the non-optimal, non-monetary balanced path is unstable. For the Samuelson case, it is the other way around. Note that the type of utility function determines which of the two cases are obtained. Gale (1973, 16, 23) understood that limit cycles around the golden rule steady state were possible when discussing his results, something that he exemplified by assuming a quadratic utility function when young agents are impatient (classical case). However, he did not dwell on the macroeconomic implications of that case, noting only in passing:

> This is an amusing example of a "business cycle" which has nothing to do with expectation. There are no ex posts or ex antes. Every one has perfect foresight but cycling nevertheless occurs as a consequence of the equilibrium price mechanism. (Gale 1973, 27).

Later, Cass, Okuno and Zilcha (1979) showed that deterministic cycles could be of arbitrary period while Benhabib and Day (1982) established that depending on how consumption of old agents is related to consumption of young, Gale's model can generate erratic dynamics. It was quite a surprise that "that such trajectories [could] arise from a very wide class of utility functions that are robust with respect to perturbations in the parameters of the system" (Benhabib and Day 1982, 38).[24]

It is handy to illustrate that argument with the simplification used by Brock (1990).[25] Let us assume that young receives endowment w_y in the first period of life and endowment w_o in his old age. At the beginning of time the old hold fiat money in the amount of M. Following Gale, equilibria can be depicted with the help of an offer curve. First, draw, at each date t, the offer curve O of the young born at date t. That is, consider the problem:

$$\max U\left[c_y(t), c_O(t+1)\right] \tag{1}$$

subject to

[23] Samuelson identified "the social contrivance of money" as a method of initiating and facilitating non-zero indebtedness and hence (hopefully optimal) intergeneration trade.
[24] According to Benhabib (2008): "When a new literature in the 1980s showed that endogenous cycles and chaos can arise in equilibrium models in economics, it came as a surprise."
[25] See also Rosser (1990).

$$p(t) \cdot c_y(t) + p(t+1) \cdot c_O(t+1) = p(t) \cdot w_y + p(t+1) \cdot w_o$$

where $p(t)$, $p(t+1)$, $c_y(t)$ and $c_O(t+1)$ are respectively price of goods at date t and at date $t+1$, consumption by the young in time t and consumption of the old in time $t+1$. The old in time 1 faces the constraint:

$$p(1) \cdot c_0(1) = p(1) \cdot w_o + M \tag{2}$$

A set of perfect foresight equilibria is given by a sequence of prices $\big(p(t), p(t+1)\big)$ satisfying (1) and a set of consumption levels $\Big(c_y(t) + c_O(t+1)\Big)$ satisfying (2).

Now draw the Ricardian production possibility frontier, $R = \big\{\big(c_y, c_O\big): c_y + c_O = w_y + w_o\big\}$. Let L be the golden rule path determined at the intersection of R and O whose shape depends on the marginal rate of substitution between individuals' present consumption and future consumption:

$$\frac{U_y\Big(c_y(t), c_O(t+1)\Big)}{U_O\Big(c_y(t), c_O(t+1)\Big)} = \frac{w_o - c_O(t+1)}{c_y(t) - w_y}$$

where U_y and U_O are the partial derivatives of U with respect to c_O and c_y. Benhabib and Day's point is that greater variability of this ratio increases the "humpiness" of the intergenerational offer curve which beyond a certain limit generates chaotic non-stationary behavior. In the classical Gale model, the path starting from the endowment equilibrium can either cycle around the steady state (figure 1) or present erratic dynamics (figure 2):

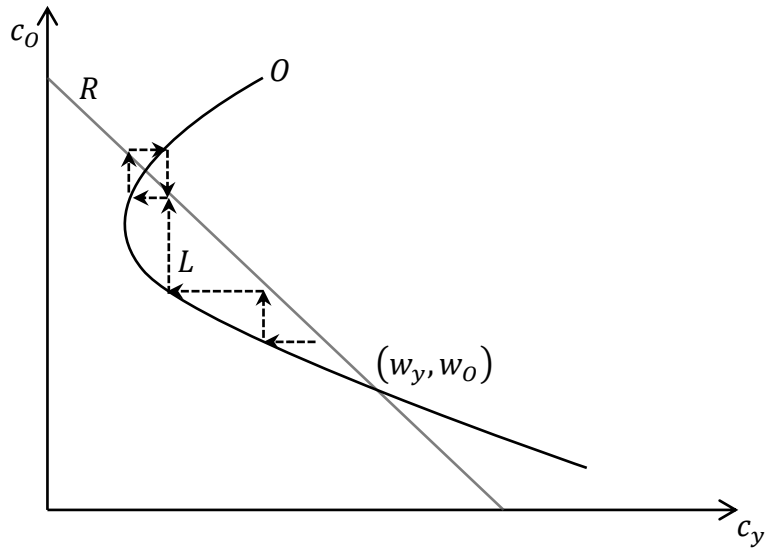

Figure 1. Gale limit cycles

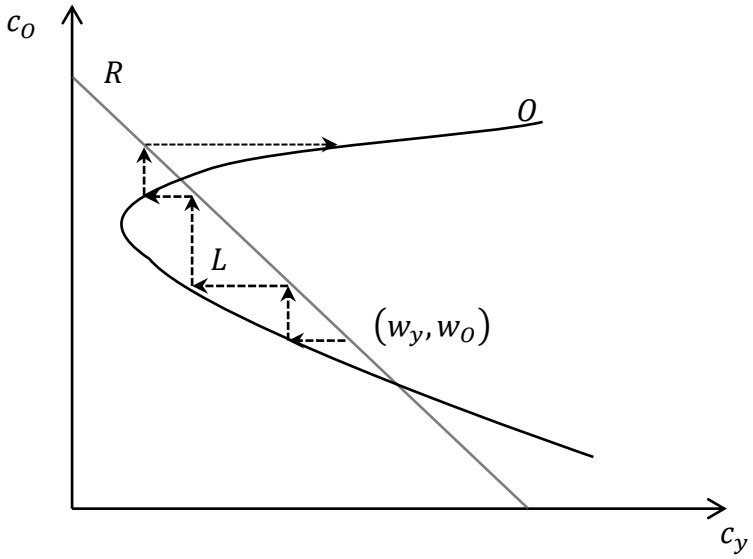

Figure 2. Erratic dynamics

These arguments led directly to the influential paper of Grandmont (1985), which left a stronger impact on the macroeconomics profession than the preceding works based on OLG models. Grandmont's motivation was also to challenge Lucas's policy ineffectiveness conclusion. It is thus no coincidence that he resorted to the very same model that Lucas developed in 1972.

Grandmont (1985) strove to identify the conditions under which regular deterministic cycles may be dampened (or created) by appropriate monetary and fiscal policies. This provides the key for systematic stabilization policy in Grandmont's model along with his conclusion of a negative relationship between long-run equilibrium output and the long-run equilibrium interest rate and a positive relationship between the latter and the long-run equilibrium level of real money balances. Grandmont argues that the government can use proportional intergenerational money transfers to pin down expectations about interest rates and real money balances and, therefore, can guide the economy to the golden rule steady state:

> "the central point here is that there are typically many long run periodic equilibria that coexist under *laissez-faire*, and that policies may be designed which force the economy to settle at only one of these – here the stationary state" (Grandmont 1985, 998)

Outside of the steady state, especially when dynamics are chaotic, agents will fail to make perfect forecasts. There will be an "expectations coordination problem." Known and credible deterministic policy rules resolve this, fix the real interest rate and eliminate cycles. Of course, all of this presumes the government can know the shape of the intergenerational offer curve, which may be a rather optimistic presumption.

Authors have generalized and improved upon these results. Farmer (1986) has considered a variation of the basic OLG model where capital is introduced both as a means of production and as an asset: he proved that, when government debt is present to finance a deficit of fixed value, periodic orbits may be obtained for the two-dimensional discrete time system that describes the economy's evolution. The technique used here was that of Hopf bifurcation for maps on the plane. The role of production was more fully analyzed in Reichlin (1986). In particular, the author was able to show that, when a nontrivial technology is present, one does not need the empirically unappealing assumption made by Grandmont which requires saving to be a decreasing function of the interest rate when the latter is high enough (strong wealth effect) in order to obtain complicated dynamic behaviors. In fact, by means of simple production functions (either fixed coefficient or CES) that use labor and the invested amount of the homogenous good to produce new output, Reichlin obtains a dynamic system for the capital stock which is represented by a map of the plane into itself. He also uses the Hopf theorem to prove the existence of a limit cycle. The result is obtained even if saving is a monotonically increasing function of the rate of interest, as long as the elasticity of substitution between factors is low enough. In Reichlin (1987), the same OLG economy is considered with a two-sector technology. Finally, Aiyagari (1987) proves the existence of periodic orbits for an exchange economy with overlapping generations that do not live only for two periods but for finitely many ones.

2.2 Indeterminacy and sunspots equilibria

Michael Woodford is a key player of the sunspot research program. He started his career in economics concerned with dynamic general equilibrium theory. After receiving a doctor of law degree (J.D.) from Yale in 1980, Woodford entered the graduate program in economics at MIT, defending his thesis in 1983 under the supervision of Timothy J. Kehoe and the co-supervision of Solow. Kehoe came from the general equilibrium community as a student of Herbert Scarf and completed his thesis at Yale in 1979 (having Andreu Mas-Collel as co-supervisor). After spending two years at Wesleyan University, Kehoe became a professor at MIT's economics department where he stayed from 1980 to 1984. According to

his own recollections, in his thesis he "used mathematical tools from differential topology to develop necessary and sufficient conditions for a general equilibrium model to have a unique equilibrium" and he became interested in macroeconomics and general equilibrium theory during his period at MIT, "mostly as a result of [his] interaction with graduate students such as David Levine and Michael Woodford."[26] Also during his years at MIT, Kehoe became a friend of Frank Hahn, who was again visiting MIT in 1982 for a semester (his first visit was in 1956).[27]

With Woodford we see once again how the overlapping generations model connected general equilibrium theorists with macro and monetary economists. In his thesis, Woodford studied intertemporal general equilibrium models in order to have "more rigorous foundations for monetary theory" (Woodford 1983, 1). In the first two chapters, co-written with his classmate Walter J. Muller, Woodford extended the literature on OLG models by considering models with production (in contrast to the pure exchange economy of Balasko and Shell 1980, 1981a,b, Cass and Shell 1983, and of Kehoe and Levine 1983) in which finite-lived and infinite-lived agents coexist and in which there is a nondepreciating asset called "land".[28] They then established the conditions for the existence and local stability of perfect foresight equilibrium and of steady state equilibrium. Importantly, the introduction of infinite-lived agents or land imply that monetary equilibria are impossible, but does not rule out the possibility of indeterminate equilibrium (local indeterminacy). Interestingly, a sufficiently large population of infinite-lived agents, among other possibilities, guarantees "that any equilibria that exist are locally determinate" (Woodford 1983, 2), a desirable property of models if one wants to use the second welfare theorem and implement the optimal equilibrium by wealth redistribution, letting the markets operate. Lastly, in the third chapter, Woodford used an OLG model to derive a precautionary demand for money balances (instead of the other well-known motives for money demand, transactions and speculative ones) and discussed the optimal monetary policy in this context, showing that there are "welfare gains from a moderately inflationary policy" (Woodford 1983, 149).

[26] Kehoe's "Career Narrative" (Sept. 2014, p. 1), available at http://users.econ.umn.edu/~tkehoe/papers/careernarrative.pdf (accessed on April 2, 2015). David Levine has a BA in mathematics and an MA in economic from UCLA and obtained his PhD at MIT in 1981.

[27] *MIT Bulletin 1982-83*, p. 223 (course catalogue available at http://dome.mit.edu/handle/1721.3/98435; accessed on March 28, 2016). Kehoe, in his "Career Narrative", wrote that "Hahn convinced me to spend the 1983-1984 academic year in Cambridge [at University of Cambridge, U.K.], which I did" (p. 1).

[28] From their theses, Muller and Woodford published one joint article (Muller III and Woodford 1988).

It is very telling of the place of OLG models in the interconnection between general equilibrium and monetary economics communities the list of people who Woodford thanked in his thesis. Besides his advisor, Tim Kehoe, "who introduced [him] to many of the concepts and methods employed in the first two essays," and his co-advisor, Bob Solow, who "helped [him] to distinguish important problems from exercises of merely technical interest," particularly important for the third essay, Woodford thanked Frank Hahn for the inspiration derived from his lectures, for discriminating "between the aspiration to rigorous formalization and the casual invocation of abstract theory in policy debates," and for convincing him "that a deeper understanding of intertemporal equilibrium models could contribute to the reconstruction of monetary theory." In addition, Woodford thanked "Stanley Fischer for first interesting [him] in overlapping generations models, and Thomas Sargent for convincing [him] of their usefulness for macroeconomic analysis" (Woodford 1983, 3).

In 1984 Woodford circulated a lengthy working paper in which he surveyed the issue of indeterminate equilibrium in OLG models, arguing that local uniqueness of competitive equilibrium is necessary for using the second welfare theorem. However, in OLG models with finite-lived agents "there may be an uncountably infinite set of competitive equilibria arbitrarily near … a given equilibrium" (Woodford 1984, 2). In such situations of indeterminacy, which are not a result of any missing markets, "preferences, endowments, and technology alone may not suffice to determine the allocation of resources in a dynamic economy," existing a role "for the belief of agents in the determination of economic outcomes" even in deterministic models (Woodford 1984, 2). Even if "the desired allocation can be supported as a competitive equilibrium" and the welfare theorems hold, the government can have a role to play: an active stabilization policy can "render the desired allocation a locally unique competitive equilibrium" (Woodford 1984, 2-3). This was explicitly an opposition to Lucas's (1972) result of policy ineffectiveness.

In this survey, Woodford makes very explicitly the point that indeterminate equilibria do not occur only in monetary OLG models and that there is "no general connection between indeterminacy of equilibrium and Pareto inefficiency" (Woodford 1984, 3).[29] Such connection is present in one-good monetary OLG models such as Gale (1973) even when agents live

[29] Woodford (1984, 24-29) discusses the example of an OLG model with production, in which he shows that: (1) indeterminacy is a possibility; (2) indeterminacy has no necessary connection with Pareto inefficiency (meaning that in this model we can have either an indeterminate Pareto optimal steady state or a determinate inefficient steady state, or a combination of these attributes); and (3) the indeterminate steady states "are not the only kind of large multiplicities of perfect foresight equilibria that may occur" (p. 29): even with determinate steady states, we can have an uncountable infinity of perfect foresight equilibria associated with sunspots.

several periods. A few years later, Kehoe and Levine (1984) analyzed whether or not Gale's result hold in an exchange economy with *n* goods per period concluding that the one-good case is a rather special because in it there never is local non-uniqueness of non-monetary equilibrium, and a non-monetary steady state is indeterminate if and only if it is inefficient. In contrast, Kehoe and Levine show that with $n>1$ goods there can be a continuum of non-monetary equilibria converging to a non-monetary steady state, which can or cannot be efficient. In particular, it is possible to have a Pareto optimal and indeterminate non-monetary steady-state.

Woodford (1984) discusses the conditions for indeterminacy of perfect foresight equilibrium, which require, among other possibilities, that the number of infinite-lived agents in the economy is small. Moreover, there were conditions under which indeterminacy implied the existence of equilibrium cycles and of "stationary rational expectations equilibrium in which 'sunspots matter'" (Woodford 1984, 4). In models with sunspot equilibria "prices and allocations are affected by random variables which are observed by agents but do not affect preferences, endowments or technology" and these equilibria are inefficient "since they involve unnecessary randomization of the equilibrium allocation" (Woodford 1984, 73). In another unpublished working paper Woodford (1986) establishes the conditions for the existence of stationary sunspot equilibria near a deterministic steady state of a stationary economy. He did so in a much wider range of dynamic models than what existed at the time in the literature, including not only different specifications of OLG models but also the infinite-lived agent monetary model of Lucas and Stokey (1987).[30]

Simply put, stationary sunspots matter when we have indeterminacy of a particular kind: of "a continuum of [stationary] equilibria all converging asymptotically to the same steady state" (Woodford 1984, 10).[31] Woodford is very explicit about this indeterminacy being more important than the indeterminacy of the "Samuelson case" (Gale 1973), in which non-stationary equilibria converge to a steady-state. Why so? Because for him, perfect foresight or rational expectations "may only be plausibly assumed in a stationary equilibrium" (Woodford 1984, 10). Thus, assuming rational expectations meant ruling out the indeterminacy of non-stationary equilibrium (but not ruling out the sunspot equilibria):

[30] Woodford (1986) cites a 1984 working paper version of Lucas and Stokey (1987).

[31] If the indeterminate steady state implies the existence of sunspot equilibria, then there is no guarantee that the economy will eventually reach "the steady state allocation in every rational expectations equilibrium" (Woodford 1984, 73). It was clear at the time that stationary sunspot equilibria was a possibility either in monetary OLG models or in models without fiat money, as demonstrated by Azariadis (1981).

> Some readers may not find the indeterminacy of perfect foresight equilibrium … a serious problem. For, they might argue, perfect foresight equilibrium as a solution concept only makes sense in the case of stationary equilibria, in which case one may expect a rational agent to have learned what to expect. (Woodford 1984, 73)

For Woodford indeterminate stationary equilibria are not isolated cases, but rather possible under many configurations of intertemporal general equilibrium models:

> In summary, the present section has shown that the economies for which equilibrium is indeterminate are not isolated cases. First, the analysis of Kehoe and Levine shows that the property of having an indeterminate steady state depends only upon certain inequalities being satisfied by the derivatives of excess demand functions. Hence, examples of economies with indeterminate steady states are robust in the sense that small changes in preferences or endowments will not affect the result. Second, the analysis of Muller and Woodford shows that many such examples can even be perturbed by adding land or infinite lived agents to the economy without changing the dimension of the indeterminacy. This makes it clear that the problem does not arise only in the case of models that are extremely special in some respect.
>
> On the other hand, there are [two] assumptions about the world that, if one is willing to grant them, would suffice to rule out the indeterminacy problem[:] … a sufficient number of agents [behaving] like infinite lived agents[;] … preferences of finite lived agents [with]… substitution effects … much stronger than income effects [(when the focus is solely on Pareto optimal equilibria)]… Ideally, of course, one would want to be able to empirically test the validity of such assumptions. (Woodford 1984, 64-65)

The problem with indeterminate rational expectations due to sunspots is that one cannot use the method of comparative statics to investigate the effects in the economy of shocks and policy intervention. However, in situations of indeterminacy, stabilization policies can render the equilibrium determinate and, therefore, avoid the economic inefficiency of sunspot equilibria, a "point of view of greatest appeal to [Woodford]" (Woodford 1984, 89). In this sense, the OLG model was interesting because of (local) indeterminacy and, therefore, it was the antithesis of another dynamic model, the infinite-lived agent model that became typical in the real business cycle literature and that Lucas himself used to discuss either

money demand or business cycle fluctuations in some of his papers (Lucas 1975, 1980; Lucas and Stokey 1983, 1987). Very interestingly, following Cass and Shell (1980), Woodford (1986, 68 n. 4) pointed out that sunspot equilibria "may be taken as formal representations of the Keynesian idea that entrepreneurial 'animal spirits' can be an independent causal factor, in addition to economic 'fundamentals' such as technology, consumer preferences, and the like."

It was exactly with this Keynesian interpretation of sunspots that Woodford (1988) opened his book chapter, stating that though Keynes placed volatility of long run expectations at the center of his investment analysis and, thus, of the effective demand, earlier business cycle economists such as Frederick Lavington and Ralph G. Hawtrey were also concerned with expectations volatility. Woodford dismissed the two grounds for neglecting expectations volatility in modern business cycle models: that sunspots imply that we are not able to explain or predict market phenomena and in practical terms they mean adding stochastic terms to the economic relations; and that microfounded rational expectations models do not leave room for spontaneous movements in expectations (i.e., movements unrelated to "fundamentals") — i.e., that sunspots implied agents to be irrational:

> I would like to argue, to the contrary, that such an account of the nature of aggregate fluctuations is entirely consistent with the aims and methods of modern equilibrium business cycle theory as represented, in particular, by the work of Lucas and Kydland and Prescott. … I propose that these [sunspot] equilibria be interpreted as representations of repetitive fluctuations in which spontaneous revisions of agents' expectations are self-fulfilling; i.e. they produce a changed outcome such that the changed expectations are validated. (Woodford 1988, 231)

Going back to the discussion about the generality of indeterminacy and sunspot results in OLG models of a decade earlier, several authors decided to move away from stylized examples of the OLG models and initiated a shift to infinite-lived agent models.[32]

3. Breaking new grounds by extending OLG conclusions to infinite lived agents models

The 1986 issue of the *Journal of Economic Theory* reflects the shift from the OLG to infinite-lived agents models.[33] As noted by Grandmont and Malgrange (1986, 5) in the

[32] The concern about the generality of the equilibrium results coming from OLG models was also stressed earlier by Cass and Shell (1980), among many others discussed in this section: "there are virtually no well-established general theorems for the overlapping generations model available in the literature" (p. 255); "[t]he question of what constitutes a general result rather than a special case is at best nebulous" (p. 256).

introduction of the issue, an important point of the symposium was to show that complex deterministic dynamics and sunspot equilibira "may arise in models where agents optimize over an infinite horizon, and are not confined to, say, overlapping generation models with 'short-lived' traders as in Benhabib and Day (1982) and Grandmont (1985)."

Woodford was central in that evolution.[34] First, he developed a model with market imperfections exhibiting sunspot equilibria or deterministic cycles: one of "stationary, infinite-horizon, competitive economies, with a small number of infinite-lived agents and financial constraints of various types" (Woodford 1988, 232). For him, this representative, infinite-lived agent model would be an alternative to the model of the real business cycle literature, but with a remarkably similar structure with the latter and also having a representation in terms of linear stochastic difference equations used in time series econometrics (something that was missing in the earlier general equilibrium literature):

> I believe that this class of examples is of particular interest in demonstrating that models of aggregate fluctuations resulting from self-fulfilling expectations represent a promising approach to equilibrium business cycle theory, an alternative to the more familiar approaches that assume an intrinsically stable economy which is, however, subject to repeated exogenous shocks to tastes, technology, or the like. (Woodford 1988, 232)

Woodford (1988) very explicitly wanted to distance sunspots from the OLG model which brought them to macroeconomics. There were four main reasons for this. First, Woodford wanted to take into account Robert Barro's (1974) argument that introducing bequest motives into OLG models makes the consumption and savings programs chosen by agents be the same that of an infinite-lived agent in a neoclassical growth model.[35] If demand bequest is positive, economic agents with finite lives may choose an infinite horizon consumption and saving program like that of a single infinite-lived agents. But if desired bequests are negative, the analogy remains valid only if one allows the transfer of debt from one generation to the other. Now, if one assumes that such transfer are not possible, the

[33] As Cherrier and Saïdi (2015) noted, this issue came out of a conference in Paris that Grandmont and Pierre Malgrange organized in Paris in 1985.

[34] See, among others, the papers by Deneckere and Pelikan on competitive growth models and by Boldrin and Montruchio on neoclassical growth models published in the same issue.

[35] He was well aware that Gale (1985) argued that there were technical difficulties with Barro's argument, as bequest economies may have an infinite number of equilibria (with only one among them being that analyzed by Barro).

similarity between OLG and infinite-lived agent models makes sense only if one introduces financial constraints to infinite-lived agents. So, "once the borrowing constraints are introduced [in an infinite-lived agent model], it is not clear that one's qualitative conclusion are much changed by assuming infinite-lived representative agents rather than OLG"[36] (Woodford 1988, 234). Second, Woodford wanted to have a better theory of money demand than the store of wealth that is present in OLG models (in which money is no longer demanded when one introduces an interest bearing asset). He then goes to the cash-in-advance, infinite-lived agents economy of Lucas and Stokey (1987). Third, the challenge was to show that endogenous fluctuations may arise at frequencies that have nothing to do with human life. OLG models were empirically unappealing because sunspot equilibria appear in models with two-period-lived agents, in which it is difficult to know how to interpret what a "period" is here and, thus, how to interpret the structural parameters.

Finally, for Woodford (1988), discussing stationary sunspot equilibria — which represent small fluctuations around a deterministic steady state — in his infinite-lived agent model was very appealing for rendering it amenable to a (locally) linear representation that is very useful for deriving quantitative predictions and testing it with time series techniques. Even if one disregards this econometric testing, one easily derives from models with linear representations predictions for variances and covariances between macroeconomic variables, which are important stylized facts about business fluctuations. Real business cycle macroeconomists such as Finn Kydland and Edward Prescott did resort to linear approximations of their models. And here Woodford (1988, 255 n. 3) distanced himself from the deterministic equilibrium cycles that Grandmont was concerned with, as their existence depended critically upon nonlinear aspects of the model's equilibrium conditions. Therefore, such cycles could not resort to the linear techniques of summarizing the observed properties of business cycles and estimating and testing dynamic models.

The argument that his model had many interesting empirical testable predictions and is useful for comparing alternative policy prescriptions, despite of having multiple equilibria, was very important for Woodford. For a particular parameter specification, Woodford (1988, 251-255) derived impulse response functions for output, consumption, price level, and investment. He then argued that they had many properties of the impulse response functions

[36] The infinite sequence of budget constraints breaks up the optimization problem of the infinite lived representative agent into an infinite sequence of independent finite horizon optimization problems, resulting in the possibility of dynamics formally analogous to those of an OLG model. But here the "periods" have nothing to do with human biology.

estimated from the US data: output and consumption had "the familiar 'hump shaped'" format; investment and consumption responses are similar "to those characteristic of a multiplier-accelerator model" (p. 252); investment is several times more volatile than consumption (but still less volatile than actually observed in the data); unexpected price increases coincide with investment increase and lead by one period an increase in output, as predicted by a Lucas supply curve (though with a completely different causality); anticipated price movements correlates negatively with output changes (similar to Prescott's claim that detrended price level is countercyclical); the aggregate fluctuations persistence predicted by the model is very similar to that found in the US data. Woodford (1988, 254-255) then concluded by bringing back his concern with designing stabilization policies that eliminate sunspot fluctuations: "In this way, business cycle theory can yield useful policy prescriptions despite an inability to predict (because of the existence of multiple equilibria) exactly what outcome must result from some possible policy interventions."[37]

Later, key contributors to the development of OLG models saw in product market imperfections a new opportunity to show the possibility of endogenous fluctuation in infinite-lived agent model. Kehoe, Levine and Romer (1989) explain how the presence of increasing returns, if one assumes monopolistic competition, may imply indeterminacy of perfect foresight equilibrium and the existence of stationary sunspot equilibria. Woodford (1990) — by developing a model in which firms support costs of acquiring information about prices of other firms and face a kinked demand curve — made the case that random events creating expectation of higher future aggregate demand may result in higher current aggregate demand.[38] In addition, he established that firms do not need to believe that other firms will base their decision on exactly the same random event. It is sufficient that each event be correlated with those that will impact their future price and investment decisions. Reflecting on "exactly which features of the model play a crucial role in generating endogenous fluctuations" (Woodford 1990, 18), he finally concluded that aggregate instability may have nothing to do with the absence of contingent claims markets, the existence of fiat money and human biology, all key elements in OLG models. Market imperfections are enough. "For, by an argument first made by Cass and Shell (1983), under some sort of market imperfection, a sunspot equilibrium is necessarily not Pareto optimal" (Woodford 1990, 21) and it may thus

[37] Woodford (1994a) once again stressed the practical significance of sunspot equilibrium and the design of "policy regimes that were [not] subject to endogenous instability" (p. 324).
[38] That paper was published in Mankiw and Romer 1990 landmark book, *New Keynesian economics*.

arise as soon as the first welfare theorem does not apply.[39] Hence, as soon as market imperfections are introduced into an otherwise standard infinite-lived agent model, indeterminacy has to be considered: "One might conclude that sunspot equilibria are likely to be the rule rather than the exception in rational expectation general equilibrium models. …the *burden of proof has to be reversed*; in all models outside the scope of the ineffectivity theorems, sunspot equilibria have to be considered, unless an explicit contrary proof is given" (Guesnerie and Woodford, 1992, 318).

The 1994 *Journal of Economic Theory* special issue was precisely about demonstrating how a wide variety of market imperfections could provide new ways to "have indeterminacy in equilibrium models with infinitely lived agents" (Benhabib and Rustchini 1994, 1), and about the empirical relevance of this literature.[40] Most of the contributions were based either on monopolistic competition or increasing returns or on both (such as Guo and Farmer 1994). Galí (1994) developed a model in which firms producing either consumption or investment goods face downward sloping demand curves and charge a markup over marginal costs. Because average markups vary over the business cycle in response to changes in the composition of aggregate demand between investment and consumption, Galí concluded that equilibrium may be indeterminate. Furthermore, he discusses numerical values for the various demand elasticity of consumption and investment goods and presents some empirical evidence on the relation between markups and the investment share in the US economy.

The implications for the understanding of stabilization policies of these imperfect-goods-market models were significant. Guesnerie and Woodford (1992) felt however the urgency to argue that "micro intervention" aimed at making markets more competitive would certainly not easily rule out endogenous fluctuations:

> It has not been established that there is any monotonic relationship between the severity of fluctuation and the degree of the market imperfection (e.g., the severity of the constraints upon financial intermediation) so that partial elimination of market imperfection (e.g., opening of a new single financial market) might have an ambiguous effect on the stability of the economy. (Guesnerie and Woodford 1992, 311)

[39] Rotemberg and Woodford (1992) show that when firms collude – in absence of price rigidity - markups may vary in response to change in aggregate demand an generate endogenous fluctuations. Although the mechanism was somewhat different, the message remained the same as in the other imperfect market models discussed here.
[40] Benhabib and Galí (1995) further explored the empirical relevance of endogenous fluctuations models.

Behind the technical details for the search of the conditions under which endogenous fluctuations occur in ever more general models, there is an important group of macroeconomists with a new understanding of business fluctuations, endogenously caused by "extrinsic uncertainty" (shocks to expectations, not to fundamentals or to economic policies).[41] In this context, as Grandmont (1986) also discussed, the government has a role to play with policies that stabilize the endogenous fluctuations:

> But policy regimes may exist that succeed in making a non-fluctuating equilibrium the unique equilibrium; these can be referred to as *stabilization policies*. It is a striking feature of the kind of models considered here that stabilization may require no actual intervention, but only the threat of intervention that, if credible, will not have to be implemented. (Guesnerie and Woodford 1992, 380)

The endogenous fluctuations group counteracted the view initiated by Lucas and his followers that economic policies are ineffective to systematically alter the real allocation of resources. This ineffective view was pushed forward, in the early 1980s, by quantitative dynamic models in which fluctuations originated from shocks to fundamentals ("intrinsic uncertainty"), particularly technology shocks as stressed by RBC macroeconomists who characterized fluctuations as efficient responses of agents to such shocks.

Sometimes the debate becomes between two groups with antagonistic views of fluctuations, granted that you proved that endogenous fluctuations are a theoretical possibility: you either explain the data with models of extrinsic uncertainty or with models with intrinsic uncertainty. However, Guesnerie and Woodford (1992, 303) (and Woodford 1992 too) are rather pragmatic, arguing that "there is in any event no reason to restrict the hypothesis of 'endogenous fluctuations' to mean that there are no important shocks to economic fundamentals."[42]

It is noteworthy that the endogenous fluctuations group started working with the "surprising" results of the OLG model, always inquiring how general they were, subsequently showing that they survive in several models in which agents have infinite lives and there are market imperfections. And all this shift from OLG to infinite-lived agent models was done in

[41] The terms extrinsic and intrinsic uncertainty comes from Cass and Shell (1983) and are widely used in this literature.

[42] Guesnerie and Woodford (1992, 294) also saw the work of studying endogenous fluctuations models in isolation as an important step for getting "strong foundations for the future task of merging the 'exogenous' and 'endogenous' viewpoints."

the context of flexible prices, but it eventually informed the sticky price literature that evolved into the DSGE (or new neoclassical synthesis) macroeconomics, as we now discuss.[43]

4. New perspective on the origins of Woodford's 2003 book

Several macroeconomists and historians of macroeconomics place Woodford's (2003) contribution to the sticky price literature of the DSGE models as a development coming out of the earlier opposition between RBC dynamic, flexible price models and New Keynesian static, sticky price models.[44] While this strand illuminates part of the developments of the DSGE literature, a question remains as to Woodford's involvement, coming out of the endogenous fluctuations literature and moving to a framework in which sunspots are not central and shocks to fundamentals and policies are important (a point briefly noted by Cherrier and Saïdi 2015, 25).

Woodford explicitly made clear in the preface of his book that it emerged out of two struggles that, according to him, has "engaged [him] since graduate school" (Woodford 2003, xiii): providing microfoundations to macroeconomics; and "reconciling central bankers' understanding of what they do with the way that monetary policy is conceived in theoretical monetary economics" (p. xiii). And nowadays central bankers use interest rates to pursue their goals. Therefore, Woodford (2003) wanted to grant interest-rate rules a central place in monetary economics and, thus, providing "foundations of a theory of monetary policy," as the subtitle of his book states.

Woodford thus places very explicitly his book within the literature on monetary rules in models with rational expectations (see Woodford 2003, ch. 1, esp. 44-55). In particular, he wanted to prove that interest-rate rules do not necessarily imply that rational expectations equilibria are indeterminate, a result that goes back to Sargent and Wallace (1975) and the subsequent literature that analyzed the conditions under which the price level is determinate with rational expectations and, therefore, not subject to "speculative bubbles" (a common term in this literature, as used by Sargent and Wallace 1975, 248).[45]

Speculative bubbles here are exactly the same as sunspots: the equilibrium values of endogenous variables are pinned down by random variables unrelated to economic

[43] See Duarte (2012), De Vroey and Duarte (2013), and references therein for a historical discussion of this literature.
[44] See references cited in footnote 1.
[45] For just a few references in this literature that are connected to the present narrative, see Blanchard (1979) and Phelps and Taylor (1977).

fundamentals. Therefore, when models exhibit speculative bubbles, there are infinite equilibria (one for each of such random variables, so to speak), i.e. equilibrium is indeterminate. Nonetheless, the earlier rational expectations literature is not explicitly related to the sunspot literature that was very important to Woodford in the 1980s: those specific rational expectations models have no microfoundations; they are dynamic IS-LM models. Therefore, they do not connect to the concerns of doing intertemporal general equilibrium analysis with overlapping generations models, although these two literatures share a common result: models in which expectations are not fully pinned down by economic fundamentals.[46]

Given this earlier bubble literature, Woodford (2003) studied macroeconomic models with explicit microfoundations in the context in which prices are rigid and fluctuations are costly. He also wanted to show that his results do not depend on the way one introduces money into these models, usually with one of the tricks that monetary general equilibrium theorists such as Wallace also wanted to avoid in the late 1970s (in particular money in the utility function), as we discussed earlier. For this, he considered a "cashless economy" in which money plays no role besides being a "unit of account in term of which prices (of both goods and financial assets) are quoted" (Woodford 2003, 63). Although variations in money supply in this context have no effect on interest rates and prices, the central bank controls the equilibrium prices of goods in terms of money by an interest-rate rule.

Moreover, Woodford's (2003) analysis of equilibrium determinacy is almost exclusively local to the steady state, using log-linear approximations and solution methods originally discussed by Blanchard and Kahn (1980). Although sunspots are not prominent in the book, Woodford does devote a fifteen page section in Chapter 2 (section 4) to discussing global determinacy and the possibility of self-fulfilling inflations and deflations. He chose to do so in a very simplified context of a completely frictionless and competitive markets (goods and financial), endowment economy populated by infinitely lived agents. The focus is, as in the rest of the book and in his earlier contributions discussed here, on designing economic policies that would prevent the undesirable self-fulfilling (or sunspot) equilibria. He argues that global determinacy is a refinement to local determinacy in the sense that it can help choose among alternative policy regimes that are "equally consistent with the same desired equilibrium, and equally serve to make it locally determinate" (Woodford 2003, 123). But this

[46] This bubble literature also used the OLG model to argue that asset bubbles are not possible for empirically plausible conditions (Tirole 1985). Santos and Woodford (1997) extended this result to a wider class of models. However, this negative result, as Guesnerie and Woodford (1992, 319-320) argued, "has little to say about the possible importance of equilibrium fluctuations due to extrinsic uncertainty" (p. 320).

requires a nonlinear approach to equilibrium conditions that is rather complex and not the focus of his book:

> The question of global uniqueness requires that I return to a consideration of the exact, nonlinear equilibrium conditions… This makes a complete treatment of the issue rather complex and beyond the scope of the present study. However, a simple example serves to illustrate how global multiplicity of equilibrium is possible, despite local determinacy. I also give examples of policy regimes that would resolve this problem. (Woodford 2003, 123)

Woodford does cite, among others, works by Grandmont (1985), Grandmont and Laroque (1986), and, the more recent work by Benhabib *et al.* (2001), and insists that local determinacy may be enough to allow expectations to coordinate upon the steady state equilibrium in which inflation is stabilized at the target level. Therefore, the requirements of the Taylor rule to have nominal interest rate responding more than one to one to inflation changes (the so-called "Taylor principle") may be enough for a determinate equilibrium. But self-fulfilling inflations and deflations can occur in monetary models and do not appear only in the context of interest-rate rules, a result that he established a decade earlier (Woodford 1994b):[47]

> … the problem of self-fulfilling inflations and deflations should probably not be dismissed out of hand. But it is also important to note that this problem is in no way special to the formulation of monetary policy in terms of an interest-rate feedback rule. In particular, exactly the same sort of problems may arise in the case of monetary targeting. (Woodford 2003, 129)

For the case of an interest-rate rule, he presents the following figure illustrating the possibility of self-fulfilling deflation, but arguing that the correct reading of the nonlinear equation drawn in the graph is of pinning down expected future inflation in terms of current inflation rate, and not the reverse as the path starting from inflation Π_0 indicated in the figure suggests:

[47] It is noteworthy that this article by Woodford was published in a symposium "Determinacy of Equilibrium under Alternative Policy Regimes," co-organized by him, David Levine and Bruce Smith, containing three other contributions than Woodford's: by Smith, Chris Sims, and Aditya Goenka. In the introduction to this symposium Woodford clearly connects his current interests with the literature on sunspots and deterministic cycles.

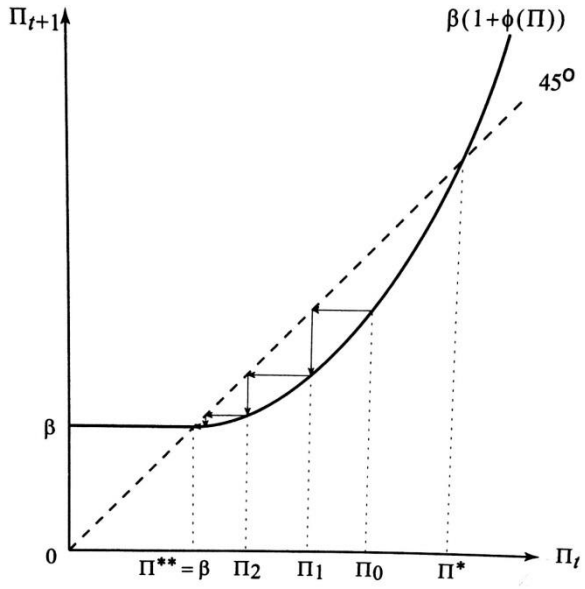

Figure 2.4 *A self-fulfilling deflation under a Taylor rule.*

Source: Woodford (2003, 126)

In discussing the characteristics of policies that would prevent self-fulfilling equilibria, Woodford (2003, pp. 131-138) brings to the fore the interaction of monetary and fiscal policies, an aspect not much explored in the earlier literature on speculative bubbles, arguing that earlier results (including by Lucas 1986 in an OLG model) that self-fulfilling equilibria are not possible were due to implicitly specifying a particular fiscal policy — called "non-Ricardian" by Benhabib *et al.* (2001), in which the transversality condition does not hold for any path of the endogenous variables —, in combination with the particular monetary policy considered in each paper. Changing fiscal policy design makes self-fulfilling equilibria possible, but this can be avoided by appropriate monetary policy regimes. In the context of this model, it is interesting to note that Woodford made no connection between the possibility of sunspots and the existence of endogenous cycles that was so important in the context of the OLG model and infinite-lived agent models with imperfections.

Before closing, it is interesting to note that Woodford (2003) himself, in the preface, placed his book in a series of efforts of producing a microfounded theory of interest-rate rules, going back to his graduate years. As we saw earlier, Woodford's thesis was based on microfounded general equilibrium monetary OLG model, with no room to interest-rate rules. However, price stickiness was not a concern of his research in his thesis and in the papers he published in the subsequent decade and a half, roughly speaking. In this sense, Woodford's (2003, xiii) suggestion that his book comes out of a long standing concern with price stickiness should be interpreted with caution:

> My advisor, Bob Solow, always insisted on the unity of microeconomics and macroeconomics, and wore both hats with equal flair. He challenged me, while I was still writing my dissertation, to try to integrate sticky prices into the kind of intertemporal general-equilibrium models that were then becoming the dominant paradigm for macroeconomic analysis. (Woodford 2003, p. xiii)

As we saw, Woodford was co-advised by Tim Kehoe and Solow, with a work following closely the general-equilibrium literature mastered by Kehoe and going to the intertemporal intricacies of the OLG model in a context of flexible prices. In his 2003 book price stickiness in an infinite-lived agent model is the staple, with fiscal and monetary policies designed to guarantee equilibrium determination and to prevent sunspot fluctuations. In this world, only exogenous shocks (intrinsic uncertainty) matter. If policies fail to be designed in the proper way, self-fulfilling equilibria are possible, but not centrally discussed in the book in relation to Woodford's contributions of the 1980s and 1990s.

Conclusions

A canonical reading of the developments in macroeconomics since the 1970s is that new Keynesians challenged Lucas's (1972) policy ineffectiveness result by moving from a flexible price to a sticky price environment. This was what in fact helped spread the rational expectations hypothesis, according to Alan Blinder (1989, 104). In the early 1980s, RBC macroeconomists advanced Lucas's flexible price approach with a dynamic general equilibrium model where technological shocks drove the business cycle. The opposition in this period was between this group and some new Keynesians with their static models of price stickiness. When this latter group went to dynamic models, they contributed to the development of the DSGE macroeconomics.

This reading illuminates some developments in mainstream macroeconomics in the 1980s, but it has major limitations. Here in this paper, complementing Cherrier and Saïdi (2015), we argued that a very important reaction to Lucas was done with multiple equilibria models with flexible prices. This started, after Gale's (1973) seminal contribution, with the very model used by Lucas (1972), an OLG model, and it developed into two strands: first the deterministic cycles, and second sunspots. The OLG model was really a workhorse model in the 1980s, acting as a kind of trading zone object (Galison 1999) that brought together diverse communities of economists with different backgrounds and interests: those general

equilibrium theorists wanting to extend their analysis to intertemporal models, monetary economists searching for microfoundations of money demand, and other more eclectic economists not really committed to a single macroeconomic model such as Lucas.

However, there was a clear effort by such endogenous fluctuations economists to analyze how general their results were. Deterministic cycles or sunspots can occur in more general models, and macroeconomists such as Benhabib, Woodford, Farmer, Kehoe and Levine moved from OLG to infinite-lived agent models with market imperfections to establish this. Price stickiness was not a major concern of this literature well into the 1990s. An important concern of the Keynesian view of fluctuations of this literature was with the design of policy regimes that could ameliorate inefficiencies, that was also key to Woodford's 2003 book of an economy modeled with infinite-lived agent, price stickiness and an interest rate rule. And Woodford is not the only important player in the DSGE literature who was very involved with the endogenous fluctuations literature: Galí is another.[48] This eventual communication between endogenous fluctuation literature to the DSGE macroeconomics has not received much attention so far.

Our historical analysis, besides shedding light on important modeling strategies and on important historical developments in macro in the 1980s, brings to the fore two issues. One is about the way macroeconomists organize past developments in order to emphasize a current developments. This is very clear in the many surveys of the endogenous fluctuations literature that practitioners wrote over time (cf. Woodford 1984, 1992; Boldrin and Woodford 1990; Guesnerie and Woodford 1992; Benhabib and Galí 1995; Benhabib 2008; Farmer 2016), in which the centrality of the OLG models as the birthplace of such an understanding of fluctuations faded away over time, with this model becoming just one case among the many presented in the surveys. Moreover, the typical organization of theories in schools of thought fails to be as appealing now, because several of such schools (Keyenesian, monetarist, new classical, RBC) become part of the same group, the exogenous fluctuations one (or intrinsic uncertainty), to be contrasted to the endogenous fluctuations (or extrinsic uncertainty) group —which encompasses both deterministic cycles and sunspots, as well as other developments in chaos not discussed here (see Cherrier and Saïdi 2015).

Another is the issue of the fragmentation and eventual marginalization of the sunspot literature during the 1980s, as forcefully argued by Cherrier and Saïdi (2015). When

[48] Woodford advised other economists who contributed to the DSGE literature, such as Stephanie Schmitt-Grohé and Martin Uribe, both PhDs from Chicago in 1994.

contrasted to the RBC literature, as they do, this is indeed the case: sunspot macroeconomists failed to be as influential as RBC economists in terms of citations and journal visibility, presence in textbooks, centrality of academic institutions occupied, etc. However, this is a rather complex issue. Many other groups may be considered marginal when subjected to this comparison, but no indisputable standard to defining "success" exists and the very "importance" of the RBC network has many dimensions of their practices. Nonetheless, even groups that were marginalized may have left undeniable marks in the development of mainstream macroeconomics. If the readers will grant us the case for a richer understanding of the DSGE literature, it is hard not to see the endogenous fluctuations literature as having been very influential on the issue of policy stabilization, and on contributing in particular ways to the spread of the infinite-lived agent models in mainstream macroeconomics —which does not conflict with the view that other issues that were important to the sunspot literature were marginalized.

References:

Allais, Maurice (1947). *Économie et Intérêt*. Paris: Imprimerie Nationale.

Arestis, Philip and Alexander Mihailov (2011). Classifying Monetary Economics: Fields and Methods from Past to Future. *Journal of Economic Surveys*, 25 (4): 769-800.

Azariadis, Costas (1981). Self-fulfilling Prophecies. *Journal of Economic Theory*, 25 (3): 380-396.

Aiyagari, S Rao (1987). Stationary Deterministic Cycles in a Class of Overlapping Generations Models with Long Lived Agents. *Working Paper #319,* Federal Reserve Bank of Minneapolis.

Backhouse, Roger E. and Mauro Boianovsky (2013). *Transforming Modern Macroeconomics: Exploring Disequilibrium Microfoundations, 1956-2003*. Cambridge, UK: Cambridge University Press.

Barro, Robert J. (1974). Are Government Bonds Net Wealth? *Journal of Political Economy*, 82 (6): 1095-1117.

Benhabib, Jess (2008). Chaotic dynamics in economics. In: Steven N. Durlauf and Lawrence E. Blume (eds.), *The New Palgrave Dictionary of Economics*. Second Edition. Palgrave

Macmillan, 2008. The New Palgrave Dictionary of Economics Online, accessed on 06 September 2016.

Benhabib, Jess and Richard Day (1982). A characterization of erratic dynamics in the overlapping generation model. *Journal of Economic Dynamics and Control*, 4 (1982): 37-55

Benhabib, Jess and Jordi Galí (1995). On Growth and Indeterminacy: Some Theory and Evidence. *Carnegie-Rochester Conference Series on Public Policy*, 43 (): 163-211.

Benhabib, Jess and Aldo Rustichini (1994). Introduction to the Symposium on Growth, Fluctuations, and Sunspots: Confronting the Data. *Journal of Economic Theory*, 63 (1): 1-18.

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe (2001). The Perils of Taylor Rules. *Journal of Economic Theory*, 96 (1-2): 40-69.

Black, Fischer (1972). Active and Passive Monetary Policy in a Neoclassical Model. *Journal of Finance*, 27 (4): 801-814.

Black, Fischer (1974). Uniqueness of the Price Level in Monetary Growth Models with Rational Expectations. *Journal of Economic Theory*, 7 (1): 53-65.

Blanchard, Olivier (1979). Backward and Forward Solutions for Economies with Rational Expectations. *American Economic Review*, 69 (2): 114-118.

Blanchard, Olivier (2000). What Do We Know about Macroeconomics that Fisher and Wicksell Did Not? *Quarterly Journal of Economics*, 115 (4) : 1375-1409.

Blanchard, Olivier (2009). The State of Macro. *Annual Review of Economics*, 1: 209-228.

Blanchard, Olivier and Charles M. Kahn (1980). The Solution of Linear Difference Models under Rational Expectations. *Econometrica*, 48 (5): 1305-1312.

Blinder, Alan S. (1989). *Macroeconomics under Debate*. New York: Harvester Wheatsheaf.

Boldrin, Michele and Michael Woodford (1990) Equilibrium Models Displaying Endogenous Fluctuations and Chaos: A survey. *Journal of Monetary Economics*, 25 (2):189-222

Bridel, Pascal (2002). Patinkin, Walras and the 'money-in-the-utility- function' tradition. *European Journal of the History of Economic Thought*, 9 (2): 268-292.

Brock, William A. (1974). Money and Growth: The Case of Long Run Perfect Foresight. *International Economic Review*, 15 (3): 750-777.

Brock, William A. (1990). Overlapping generations models with money and transactions costs. In Benjamin Friedman and Frank Hahn (eds.), *Handbook of Monetary Economics*, vol. 1. Amsterdam: North-Holland, pp. 263-295.

Cass, David and Karl Shell (1980). In Defense of a Basic Approach. In: Kareken and Wallace (eds.), pp. 251-260.

Cass, David and Karl Shell (1983). Do Sunspots Matter? *Journal of Political Economy*, 91 (2): 193-227.

Cass, David and Menahem E. Yaari (1966a). A Re-examination of the Pure Consumption Loans Model. *Jounal of Political Economy*, 74 (4): 353-367.

Cass, David and Menahem E. Yaari (1966b). A Note on the Role of Money in Providing Sufficient Intermediation. *Cowles Foundation Discussion Paper*, no. 215, Sept. 23.

Cherrier, Béatrice and Aurélien Saïdi (2015). The Indeterminate Fate of Sunspots in Economics. *Working Paper*, draft 4.8, Nov., available at SSRN (https://a1papers.ssrn.com/sol3/papers.cfm?abstract_id=2684756, accessed on Jan. 15, 2016).

De Vroey, Michel (2015). *A History of Macroeconomics – from Keynes to Lucas and Beyond*. Cambridge: Cambridge University Press.

De Vroey, Michel and Pedro G. Duarte (2013). In Search of Lost Time: The Neoclassical Synthesis. *B.E. Journal of Macroeconomics*, 13 (1): 1-31.

Diamond, Peter (1965). National Debt in a Neoclassical Growth Model. *American Economic Review*, 55 (5, Part 1): 1126-1150.

Duarte, Pedro G. (2012). Not going away? Microfoundations in the Making of a New Consensus in Macroeconomics. In Pedro Garcia Duarte and Gilberto Tadeu Lima (eds.), *Microfoundations Reconsidered – The Relationship of Micro and Macroeconomics in Historical Perspective*. Cheltenham, UK: Edward Elgar, pp. 190-237.

Farmer, Roger E. A. (1986). Deficits and Cycles. *Journal of Economic Theory*, 40 (1): 77-88.

Farmer, Roger E. A. (2016). The Evolution of Endogenous Business Cycles. *Macroeconomic Dynamics*, 20 (2): 544–557.

Fischer, Stanley (1975). Recent Developments in Monetary Theory. *American Economic Review*, 65 (2): 157-166.

Fischer, Stanley (1977). Long Term Contracts, Rational Expectations, and the Optimal Money Supply Rule. *Journal of Political Economy*, 85 (1): 191-205.

Gale, David (1973). Pure Exchange Equilibrium of Dynamic Economic Models. *Journal of Economic Theory*, 6 (1): 12-36.

Gale, David (1985). The Strategic Analysis of Bequest Behavior: A Critique of the Ricardian Equivalence Theorem. *CARESS Working Paper*, no. 85-31, University of Pennsylvania.

Galí, Jordi (1994). Monopolistic Competition, Business Cycles, and the Composition of Aggregate Demand. *Journal of Economic Theory*, 63 (1): 73-96.

Galison, Peter (1999). Trading Zone: Coordinating Action and Belief. In: Mario Biagioli (ed.), *The Science Studies Reader*. New York: Routledge, pp. 137-160.

Goodfriend, Marvin and Robert G. King (1997). The New Neoclassical Synthesis and the Role of Monetary Policy. *NBER Macroeconomics Annual*, 12 (1997): 231-283.

Gordon, Robert J. (1990). What Is New-Keynesian Economics? *Journal of Economic Literature*, 28 (3): 1115-1171.

Grandmont, Jean-Michel (1985). On Endogenous Competitive Business Cycles. *Econometrica*, 53 (5): 995-1046.

Grandmont, Jean-Michel (1986). Stabilizing Competitive Business Cycles. *Journal of Economic Theory*, 40 (1): 57-76.

Grandmont, Jean-Michel and Guy Laroque (1986). Stability of Cycles and Expectations. *Journal of Economic Theory*, 40 (1): 138-151.

Guesnerie, Roger and Michael Woodford (1992). Endogenous Fluctuations. In: Jean-Jacques Laffont (ed.), *Advances in Economic Theory Sixth World Congress*, vol. II. Cambridge: Cambridge University Press, pp. 289-412.

Guo, Jang Ting and Roger E. A. Farmer (1994). Real Business Cycles and the Animal Spirits Hypothesis. *Journal of Economic Theory*, 63 (1): 42-72

Hahn, Frank H. (1973). On the Foundations of Monetary Theory. In Michael Parkin and A. R. Nobay (eds.), *Essays in Modern Economics*. New York: Harper & Row, pp. 230-242.

Hansen, Bent (1970). *A Survey of General Equilibrium Systems*. New York: McGraw-Hill.

Hoover, Kevin (1988). *The New Classical Macroeconomics*. Oxford: Basil Blackwell.

Kareken, John H., and Neil Wallace (eds.) (1980). *Models of Monetary Economies*. Minneapolis: The Federal Reserve Bank of Minneapolis.

Kehoe, Timothy and David Levine (1984). Regularity in Overlapping Generations Exchange Economies. *Journal of Mathematical Economics*, 13 (1): 69-93.

Lucas, Robert E., Jr. (1972). Expectations and the Neutrality of Money. *Journal of Economic Theory*, 4 (2): 103-124.

Lucas, Robert E., Jr. (1973). Some International Evidence on Output-Inflation Tradeoffs. *American Economic Review*, 63 (3): 326-334.

Lucas, Robert E., Jr. (1975). An Equilibrium Model of the Business Cycle. *Journal of Political Economy*, 83 (6): 1113-1144.

Lucas, Robert E., Jr. (1983). Corrigendum. *Journal of Economic Theory*, 31 (1): 197-199.

Lucas, Robert R., Jr. (1986). Adaptive Behavior and Economic Theory. *Journal of Business*, 59 (suppl.): 401-426.

Lucas, Robert E., Jr., and Nancy Stokey (1983). Optimal Fiscal and Monetary Policy in an Economy without Capital. *Journal of Monetary Economics*, 12 (1): 55-93.

Lucas, Robert E., Jr., and Nancy Stokey (1987). Money and Interest in a Cash-in-Advance Economy. *Econometrica*, 55 (3): 491-513.

Mankiw, N. Gregory (1990). A Quick Refresher Course in Macroeconomics. *Journal of Economic Literature*, 28 (4): 1645-1660.

Mankiw, N. Gregory (2006). The Macroeconomist as Scientist and Engineer. *Journal of Economic Perspectives*, 20 (4): 29–46.

Mehrling, Perry (2002). Don Patinkin and the origins of postwar monetary orthodoxy. *European Journal of the History of Economic Thought*, 9 (2): 161-185.

Muller III, Walter J. and Michael Woodford (1988). Determinacy of Equilibrium in Stationary Economies with Both Finite and Infinite Lived Consumers. *Journal of Economic Theory*, 46 (2): 255-290.

Phelps, Edmund S. and John B. Taylor (1977). Stabilizing Powers of Monetary Policy under Rational Expectations. *Journal of Political Economy*, 85 (1): 163-190.

Prescott, Edward C. and Robert E. Lucas, Jr. (1972). A Note on Price Systems in Infinite Dimensional Space. *International Economic Review*, 13 (2): 416-422.

Reichlin, Pietro (1986). Equilibrium Cycles in an Overlapping Generations Economy with Production. *Journal of Economic Theory*, 40 (1): 89-102.

Reichlin, Pietro (1987). Endogenous Fluctuations in a Two-Sector Overlapping Generations Economy. Working Paper 87/264, European University Institute, Florence.

Rosser, Barkley J. (1990). Chaos Theory and the New Keynesian Economics. *The Manchester School*, LVIII (3): 265-291.

Samuelson, Paul A. (1958). An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money. *Journal of Political Economy*, 66 (6): 467-482.

Santos, Manuel S. and Michael Woodford (1997). Rational Asset Pricing Bubbles. *Econometrica*, 65 (1): 19-57.

Sargent, Thomas J. and Neil Wallace (1973). The Stability of Models of Money and Growth with Perfect Foresight. *Econometrica*, 41 (6): 1043-148.

Sargent, Thomas J. and Neil Wallace (1975). "Rational" Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule. *Journal of Political Economy*, 83 (2): 241-254.

Scheinkman, José Alexandre (1980). Discussion. In: Kareken and Wallace (eds.), pp. 91-96.

Shell, Karl (1971). Notes on the Economics of Infinity. *Journal of Political Economy*, 79 (5): 1002-1011.

Shiller, Robert J. (1978). Rational Expectations and the Dynamic Structure of Macroeconomic Models: A Critical Review. *Journal of Monetary Economics*, 4 (1): 1-44.

Sidrauski, Miguel (1967). Inflation and Economic Growth. *Journal of Political Economy*, 75 (6): 796-810.

Snowdon, Brian and Howard R. Vane (2005). *Modern Macroeconomics: Its Origins, Development And Current State*. Cheltenham, UK: Edward Elgar.

Tirole, Jean (1985). Asset Bubbles and Overlapping Generations. *Econometrica*, 53 (6): 1499-1528.

Tobin, James (1980). Discussion. In: Kareken and Wallace (eds.), pp. 83-90.

Wallace, Neil (1980). The Overlapping Generations Model of Fiat Money. In: Kareken and Wallace (eds.), pp. 49-82.

Weintraub, E. Roy (1974). *General Equilibrium Theory*. London: Macmillan.

Woodford, Michael (1983). *Essays in Intertemporal Economics*. Ph.D. thesis, Department of Economics, MIT.

Woodford, Michael (1984). Indeterminacy of Equilibrium in the Overlapping Generations Model: A Survey. Unpublished working paper, available at http://www.columbia.edu/~mw2230/Woodford84.pdf (accessed on March 17, 2015).

Woodford, Michael (1986). Stationary Sunspot Equilibria: The Case of Small Fluctuations Around a Deterministic Steady State. Unpublished working paper, available at http://www.columbia.edu/~mw2230/Sunspot.pdf (accessed on March 17, 2015).

Woodford, Michael (1988). Expectations, Finance and Aggregate Instability. In: M. Kohn and S.-C. Tsiang (eds.), *Finance Constraints, Expectations and Macroeconomics*. Oxford: Oxford University Press, pp. 230-261.

Woodford, Michael (1990). Self-fulfilling Expectations and Fluctuations in Aggregate Demand. *NBER Working paper*, no. 3361.

Woodford, Michael (1992). Equilibrium Models of Endogenous Fluctuations: An Introduction. In: Alessandro Vercelli and Nicola Dimitri (eds.), *Macroeconomics — a survey of research strategies*. Oxford: Oxford University Press, pp. 183-235.

Woodford, Michael (1994a). Determinacy of Equilibrium Under Alternative Policy Regimes: Introduction to the Symposium. *Economic Theory*, 4 (3): 323-326.

Woodford, Michael (1994b). Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy. *Economic Theory*, 4 (3): 345-380.Woodford, Michael (1999). Revolution and Evolution in Twentieth-Century Macroeconomics. (Available at: http://www.columbia.edu/~mw2230/macro20C.pdf, accessed on Aug. 25, 2016.)

Woodford, Michael (2003). *Interest and Prices — foundations of a theory of monetary policy*. Princeton: Princeton University Press.

Woodford, Michael (2009). Convergence in Macroeconomics: Elements of the New Synthesis. *American Economic Journal: Macroeconomics*, 1 (1): 267-279.