Department of Economics- FEA/USP

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Maurício Bugarin
Wilfredo Leiva Maldonado

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Maurício Bugarin (bugarin.mauricio@gmail.com)

Wilfredo Leiva Maldonado (wilfredo.maldonado@usp.br)

Research Group: NEFIN

Abstract:

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ENDOGENOUS ASYMMETRY IN SEQUENTIAL AUCTIONS[1]

Mauricio Bugarin[2]

University of Brasília

Wilfredo Maldonado[3]

University of Sao Paulo

Abstract

This paper proposes an innovative methodology for sequential auctions of homogeneous goods that creates asymmetry among participants, thereby achieving higher revenue for the auctioneer. The asymmetry arises endogenously from a competitive advantage in the second auction granted to the winner of the first auction. The analysis shows that the auctioneer's expected revenue in this scenario is higher than in auctions without a competitive advantage and approaches the expected revenue of an optimal reserve price auction. After analyzing the benefits and drawbacks of the competitive advantage mechanism, this paper concludes that it represents a more effective auction design than conventional approaches.

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[2] Mauricio Bugarin gratefully acknowledges the financial support of CNPq research grant 311586/2022-8.
E-mail: bugarin.mauricio@gmail.com; URL: https://bugarinmauricio.com.

[3] Wilfredo Maldonado acknowledges the financial support of CNPq 302856/2022-6 and 402949/2021-8, Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) 2023/07274-0, and Fundação Instituto de Pesquisas Econômicas – FIPE. Email: wilfredo.maldonado@usp.br

1. INTRODUCTION

In any country, the public sector is one of the largest consumers of goods and services. According to McHugh (2020), a typical American school consumes 320,000 sheets of paper per year; Nassar et al. (2022), calculates the number of prescriptions in England and Wales above 1,918,138 for every 100,000 inhabitants in 2019. Just as the purchase of paper for a public school and the purchase of medicines for a public hospital, a large part of public procurement of goods and services is implemented through competitive auction mechanisms, which have a repetitive character. Thus, the choice of the optimal mechanism for acquiring these materials must take into account their recurring nature.

Since the seminal paper by Vickrey (1961), the academic literature has successfully developed the theory of static auctions for a single object, which is now well understood, at least under the standard paradigm of independent private values (Myerson, 1981). The extension to dynamic auction models, on the other hand, finds less standardized results and focuses.

When considering an infinitely repeated relationship, a significant amount of studies continues to seek conditions to validate some version of the "Folk Theorem," in which cooperative behavior is induced among players that yield Pareto superior outcomes, as in the seminal works of Fudenberg and Maskin (1986) and Abreu (1988), among many contemporaneous and subsequent works. Viewed solely in the context of the players themselves, the Folk Theorem presents trigger strategies that generate highly beneficial outcomes for all, which are unattainable in a static context. However, in the case of an oligopoly, what is beneficial to the players induces an outcome equivalent to a monopoly, which can be detrimental to consumers, bringing about deadweight losses to the economy. Thus, the study of repeated games highlights the inevitability of collusion, which limits competition among oligopolistic firms aware of their long-term interactions, as extensively reviewed by Sorin (1992). Consequently, the Folk Theorem in repeated games has driven research in Public Economics, focusing on developing mechanisms to dismantle cartels of firms with market power. The attractiveness of coalitions among players and the search for mechanisms to limit them are issues that have also captured the attention of auction theory and applied researchers, particularly in the literature known as "bidding rings" (Aoyagi, 2003; Marshall & Marx, 2012; McAfee & McMillan, 1992; Phillips et al., 2003; Skrzypacz & Hopenhayn, 2004).

Another more recent strand of the literature considers finite sequential auctions, typically two auctions, and seeks to understand how this dynamic relationship alters the equilibria found in the static version of the games. A key focus of this literature is to determine whether the strategic advantage of selling multiple objects in two sequential auctions, as opposed to a single simultaneous auction, leads to an increase in the auctioneer's expected return. For example, Salant & Cabral (2019) find that in the case of unit demand by participants, sequential selling is beneficial for the auctioneer. A similar result was found by Charkraborty (2018) for the case of multiple-unit demand and by Charkraborty (2019) for the case of unit demand with risk-averse participants.

Despite being extremely rich, this literature does not apply to situations where there is no option for simultaneous sales, as in the case of recurring public procurements. Another line of research assumes that objects are sold sequentially and tries to understand how this auction format affects participant behavior when there is a relationship between the goods auctioned in the first and second auctions. De Silva et al. (2005) focus on the context of construction service bidding and argue that winning a bid results in a cost reduction if the same bidder wins the subsequent bid, due to economies of scale, for instance. In this case, the winner of the first auction bids more aggressively, benefiting the auctioneer. Similarly, Jofre-Benet & Pasendorfer (2014) consider the possibility that the goods sold in simultaneous auctions may be complements or substitutes, concluding that in the case of substitute goods, the first-price format is more desirable, and if the goods are complements, the second-price format is more desirable from the auctioneer's expected revenue perspective.

In the examples mentioned above, there is an asymmetry between the participants of the second auction depending on who won the first. This asymmetry, however, is generated exogenously and is due to a relationship between the good sold in the first auction and the one sold in the second, which affects the expected benefit of winning the second auction for the winner of the first auction, thus benefiting the auctioneer.

However, when there is no synergy between the two goods, there is no exogenous asymmetry that can benefit the auctioneer. The goal of this paper is precisely to propose a mechanism to be applied to sequential auctions without synergies that has the effect of increasing the auctioneer's expected revenue. The basic principle of the mechanism is to endogenously create asymmetry among the participants, thereby making their bids more aggressive. The mechanism, applied to two consecutive auctions, creates this asymmetry in the second auction by offering a competitive advantage in that second auction to the winner of the first auction. By introducing this advantage, the auction design increases the interest of all participants in winning the first auction, as the benefits of victory are now enhanced: in addition to receiving the item, the winner has an increase in her chance of winning the second auction. With the augmented interest in winning the first auction, all participants raise their bids, thereby increasing the auctioneer's expected revenue in the first auction and generating a higher expected revenue across both auctions than would be obtained by simply repeating the static auction without introducing the asymmetry-generating rule.

The literature on auctions has extensively researched a different type of asymmetry: the exogenous differences between participants. These differences may arise from an asymmetric ex-ante distribution of values (Kaplan & Zamir, 2012; Kirkegaard, 2022), or from participants having differential information about the value of the auctioned object (Wilson, 1966; Wilson, 1967; Milgrom & Webert, 1982; Xu & Cavallo, 2022). In the present model, the participants are ex-ante identical, and the asymmetry arises endogenously from the auction design.

In addition to this introduction, this paper is organized as follows. Section 2 first defines the basic elements of the model of independent private values with two sequential auctions, which is used to illustrate the proposed mechanism. It then presents the symmetric Bayesian Nash equilibria as well as the auctioneer’s expected revenue for the standard cases where no reserve price is used and where the reserve price that maximizes the auctioneer's revenue is included. According to auction theory, this latter mechanism is the one that maximizes the auctioneer's revenue among all possible standard auctions. Section 3 proposes the new mechanism, finds the corresponding Nash equilibrium for the sequential auctions, and calculates the auctioneer's expected revenue. Section 4 compares the revenue found with those obtained by traditional mechanisms and discusses the relative advantages of the proposed mechanism. Finally, Section 5 presents concluding remarks along with proposals for extensions of the presented model. The proofs of all propositions are detailed in the Appendix.

2. THE BASIC MODEL AND ITS CLASSIC SOLUTIONS

There are two agents (players, auction participants or bidders) who wish to acquire as many units as possible of a homogeneous good. In total, two units are sold in two consecutive auctions. The value that agent $i = 1,2$ assigns to the good is a random variable v_i uniformly distributed in the interval $[0,1]$ and is realized at each new auction. The agents' values are private and independent in each auction and across different periods. The objects sold have no value to the auctioneer, who derives all their utility from their sale. In this section, we recapitulate two classic mechanisms for selling the item.

2.1. The basic model: two consecutive and independents auctions

Suppose two independent auctions are conducted, one after the other, with no connection between them. Then, from an ex-ante perspective, the solution will be the same for each auction. According to the Revenue Equivalence Theorem (Myerson, 1981), the auctioneer's revenue will be the same regardless of the format chosen from the four traditional formats: first-price sealed-bid, second-price sealed-bid, ascending open, and descending open auctions. For simplicity, in this case we choose the Vickrey auction format (Vickrey, 1961), i.e., the second-price sealed-bid auction. In this auction, each player submits a bid in a sealed envelope to the auctioneer, who opens them and selects the highest bid as the winner. However, the winner pays only the second-highest bid, i.e., the bid of their opponent.

Thus, we can define the first model to be considered.

Definition 1. Model $\mathcal{M}_1$: two sequential auctions of second-price sealed-bid and without a reserve price

For the sake of completeness, we present the following proposition.

PROPOSITION 1. *In model $\mathcal{M}_1$ the optimal strategies of the players in each auction are given by $\beta_k(v) = v$ and the expected revenue of the auctioneer in this sequential auction is $R_1 = 2/3$.*

2.2. The basic model: using a reserve price

Now, suppose two independent auctions are conducted, one after the other, but the auctioneer sets a reserve price r, below which the item will not be sold. In this case, by choosing the reserve price optimally, the auctioneer can increase their expected revenue in each auction (Krishna, 2002; Ausubel & Cramton, 2004).

Consider, in this case, the most traditional first-price sealed-bid auction, where the player who makes the highest bid wins and pays for the item her own bid. We then define the second model considered.

Definition 2. Model $\mathcal{M}_2$: two sequential first-price sealed-bid auctions with reserve price $r > 0$.

Once again, for the completeness of the exposition, we set the next proposition.

PROPOSITION 2. *In model $\mathcal{M}_2$ the optimal strategies of the players in each auction are given by $\beta_k(v) = \frac{v}{2} + \frac{r^2}{2v}$ and the expected revenue of the auctioneer in each auction is given by $2\left[\frac{1}{6} + \frac{r^2}{2} - \frac{2}{3}r^3\right]$. Furthermore, that expected revenue attains its maximum value* *at $r = \frac{1}{2}$, thus the expected revenue of the auctioneer in this sequential auction is equal to $R_2 = \frac{2}{3} + \frac{1}{6}$.*

When comparing the result obtained with the optimal choice of reserve price to that obtained with the standard model, it becomes apparent that there has been a 1/6 increase in the auctioneer's expected revenue, corresponding to a significant gain of 25% over the previous value.

However, this benefit does not come without cost. Indeed, when $r = 1/2$, in one-fourth of the cases, the item will not be sold, meaning the probability of failure for each auction is 25%. This property of Pareto inefficiency of auctions with reserve prices is particularly delicate when considering their application in situations where the public sector is determined to ensure the auction's success, i.e., ensuring that the item (such as a public company) is necessarily sold.

3. INTRODUCING ENDOGENOUS ASYMMETRY AMONG THE PLAYERS

Consider now the two original sequential auctions without a reserve price, where the first auction is a standard first-price auction, but include in the mechanism the following Asymmetry Creation Rule, valid for the second auction:

Rule of strategic advantage: The winner of the first auction is given the chance to match the winning bid of the second auction to win the object.

Hence, we define the proposed model of this paper.

Definition 3. Model $\mathcal{M}_3$: two sequential first-price sealed-bid auctions, with the first auction without a reserve price and the rule of strategic advantage applied to the second auction.

In this section, we aim to understand the effect of this strategic advantage on the equilibrium of the auction mechanism. To do so, we proceed by backward induction to find the Perfect Bayesian Nash equilibrium of the two period game. Therefore, we first find the Bayesian Nash equilibrium in the first auction, conditional on the equilibrium of the second auction, completing the construction of the Perfect Bayesian Nash equilibrium. Finally, we calculate the auctioneer's total expected revenue as a function of the competitive advantage g.

3.1. Bayesian Nash Equilibrium of the second auction

We are searching for a Bayesian Nash Equilibrium (BNE) $\left(\beta_1(v_1), \beta_2(v_2)\right)$ where the functions $\beta_i(\cdot)$, $i = 1{,}2$ are piecewise linear and non-decreasing functions from $[0{,}1]$ to

$[0,1]$ satisfying $0 \leq \beta_i(v_i) \leq v_i$, for all $v_i \in [0,1]$. The next proposition presents such equilibrium

PROPOSITION 3. (*Bayesian Nash Equilibrium in the Second Auction with Competitive Advantage*) *In model $\mathcal{M}_3$, the piecewise linear Bayesian Nash equilibrium in the second period auction, when player 1 is the winner of the first auction and player 2 is the loser, is given by:*

$$\beta_1(v_1) = 0; \ \ \beta_2(v_2) = \frac{v_2}{2}$$

the respective expected utilities of the players are:

$$EU_1 = \frac{7}{4} \times \frac{1}{6} \ ; \ EU_2 = \frac{1}{2} \times \frac{1}{6}$$

Furthermore, the expected revenue of the auctioneer in the second auction of model $\mathcal{M}_3$ is:

$$R_{32} = \frac{1}{4}$$

It is also worth noting that $R_{32} = \frac{1}{4} < \frac{1}{3}$. Therefore, the competitive advantage mechanism reduces the auctioneer's expected revenue in the second period compared to the traditional first-price sealed-bid auction mechanism. Consequently, for this mechanism to be beneficial for the auctioneer, there must first be an additional gain in the auctioneer's expected revenue in the first auction, and moreover, this gain must be sufficiently high to offset the losses in the second auction.

3.2. Bayesian Nash Equilibrium of the first auction

In this first auction, the players are completely symmetric, which suggests the search for a symmetric Bayesian Nash equilibrium. In this equilibrium, each player will consider the effect of their bid not only in the first auction but also in the second auction, as already calculated. Proposition 4 below presents this equilibrium.

PROPOSITION 4. (*Symmetric Bayesian Nash Equilibrium in the First Auction with Competitive Advantage*) *In model $\mathcal{M}_3$, the symmetric Bayesian Nash equilibrium in the first period is given by:*

$$\beta_i(v) = \beta(v) = \frac{v}{2} + \frac{5}{4} \times \frac{1}{6}; \ \ for \ \ i = 1,2$$

In addition, the expected revenue of the auctioneer in this first auction is:

$$R_{31} = \frac{1}{3} + \frac{5}{4} \times \frac{1}{6}$$

Therefore, the total revenue of the auctioneer with the mechanism (model) $\mathcal{M}_3$ is:

$$R_3 = R_{31} + R_{32} = \frac{2}{3} + \frac{1}{8}$$

It is worth comparing the solution found with the one where there is no advantage, in which $\beta(v_1) = \frac{v_1}{2}$; $\beta(v_2) = \frac{v_2}{2}$. In this case, the prospect of a future advantage has the effect of increasing the bids of both players in the first auction. Therefore, the competitive advantage mechanism increases the competitiveness of both players in the first period, causing them to choose higher bids, which will increase the auctioneer's revenue in the first period.

The following corollary summarizes the Perfect Bayesian Equilibrium of the two-auction game.

COROLLARY 4. (*Perfect Bayesian Nash Equilibrium in the two-auction game with Competitive Advantage*) *In model $\mathcal{M}_3$, the symmetric Perfect Bayesian Nash equilibrium is given by the following strategies.*

In the first auction, both players follow the big strategy:

$$\beta(v) = \frac{v}{2} + \frac{5}{4} \times \frac{1}{6}$$

In the second auction, the winner of the first auction follows the bid strategy:

$$\beta_w(v) = 0,$$

and match the winning bid if it is not above the value she gives to the object.

And the loser of the first auction follows the bid strategy:

$$\beta_l(v) = \frac{v}{2}$$

4. DISCUSSION: The advantages of the mechanism that generates asymmetry vis-à-vis traditional mechanisms

As we saw earlier, without the use of a reserve price, standard auctions yield a return $R_1 = \frac{2}{3} \cong 0{,}6667$.

On the other hand, the highest return that the auctioneer can expect using the optimal reserve price mechanism in the two sequential auctions is: $R_2\left(\frac{1}{2}\right) = \frac{2}{3} + \frac{1}{6} \cong 0{,}8333$.

Finally, the proposed mechanism of endogenous asymmetry generation yields an expected return of approximately $R_3 = \frac{2}{3} + \frac{1}{8} \cong 0{,}7917$.

The result is not better than using the optimal reserve price mechanism. However, it is superior to using no reserve price at all and it closely approximates the expected return with optimal reserve prices. Indeed, $\frac{R_2 - R_3}{R_2} = 0{,}05$, i.e., a difference of 5%.

It is worth noting that, according to Engelbrecht-Wiggans (1987), when there is an endogenous decision to participate in the auction and a sunk participation cost, including a reserve price can reduce the number of participants, thereby reducing competition which, in turn, reduces the auctioneer's return. From this perspective, the comparison should be made precisely with the model without a strategic reserve price, model $\mathcal{M}_1$, which generates a clearly inferior return compared to the proposed model, $\mathcal{M}_3$.

Furthermore, the item is always sold in the proposed auction format, even though possibly at lower prices in the second auction. That makes this mechanism effective, unlike the one that includes a strategically chosen reserve price. However, note that just like in the mechanism with optimally chosen reserve price, the proposed mechanism is not efficient, albeit for different reasons. While in the traditional mechanism, inefficiency results from the possibility of the item not being sold when it should be (Myerson 1981; Riley & Samuelson, 1981), in the proposed mechanism, the item is always sold, but in the second auction, it might be sold to a participant (winner of the first auction) who values it less than her competitor (loser of the first auction).

In spite of this, the proposed mechanism has the potential to approximate the auctioneer's expected revenue to that of the optimal reserve price in each auction. Additionally, it effectively ensures the sale of the item. This latter property of the proposed mechanism is especially relevant in applications to the public sector, where a failed auction, one in which the item is not sold, brings with it a wide range of additional costs, from the cost of organizing another bidding process to reputational costs, for example (Casady et al., 2023). Faced with these costs, the literature even finds situations where the public manager may distort behavior to favor cartel formation (Tanaka & Hayashi, 2016). This type of concern disappears when using the competitive advantage mechanism.

Another advantage, especially relevant for the public sector, is that the public manager does not have discretion in the strategic choice of the reserve price to be used in the auction. In fact, this price is typically determined based on accounting studies that seek to assess the value of the company. Now, Auction Theory convincingly shows that, at least in the context of independent private values[4], the optimal reserve price is strictly above the item's value for the auctioneer (Myerson, 1981; Riley & Samuelson, 1981), and this difference can be substantial. For example, in the parameterization studied in this article, the item's value for the auctioneer was zero while the reserve price was ½, which is the expected value that any participant assigns to the item. By replacing the discussion regarding the reserve price with a discussion about the competitive advantage, the public manager gains greater freedom in fine-tuning the mechanism.

Furthermore, the literature is not consensual with respect to the use and properties of the optimal reserve price when the number of participants varies. Menicucci (2021) shows that when the virtual valuation function (Myerson 1981) is not monotonic, the optimal reserve price weakly increases with the number of participants. Conversely, when participation is endogenous and costly, Engelbrecht-Wiggans (1987) shows that if potential participants only learn the value of the object when they incur the participation cost, then a null reservation price may yield higher expected returns to the auctioneer, because positive reserve prices reduce participation. These results reinforce support for an alternative model to the strategically chosen reserve price model.

Additionally, by creating a competitive advantage for the winner of an auction, the mechanism establishes a dynamic link between this player and the auctioneer, causing the player to have an interest in behaving appropriately to maintain the privilege guaranteed by the victory until the next auction. Suppose, for example, that what is being auctioned is a public-private partnership (PPP). The literature on the subject finds, both theoretically and empirically (Laffont 2005, p.245; Bugarin & Ribeiro 2021; Estache & Quesada, 2001; Gagnepain et al. 2013; Guasch et al. 2006, 2007, 2008), ample evidence of non-compliance with contractual commitments throughout the concession, leading to costly renegotiation processes. The competitive advantage mechanism can become another incentive for compliance with commitments by including the possibility of losing the competitive advantage in the next PPP auction if the winner fails to fulfill the contractual commitments in the current partnership.

Finally, the competitive advantage also has the potential to reduce the chances of adventurers who do not intend to fulfill contractual commitments from winning the bidding processes, both in the first auction, as they know they will lose the advantage in

[4] When there is correlated information, as in the case of the common value model or the affiliated private value model, Levin & Smith (1996) show that the seller's optimal reservation price converges to his true value, as the number of bidders increases.

the second if they win and fail to meet the conditions, and in the second auction, as the serious winner will have an advantage over them.

5. CONCLUSIONS

This article starts from the fact that many economic relationships implemented through auctions are repeated, such as the purchases of office supplies in the public service, the hiring of maintenance services for public parks, the organization of yearly fairs, and so on.

The present research aims to leverage the repeated interaction between the auctioneer and the players to create a mechanism that induces greater competition among the participants, resulting in higher expected revenue for the auctioneer. The new proposal for repeated auctions has the following format: the first auction is conducted as a standard first-price auction. However, there is a modification in the second auction that deviates from the standard format. The first auction is implemented as a standard first-price auction. However, there is a modification in the second auction that deviates from the standard format. This modification, called the "asymmetry creation rule" or the "competitive advantage mechanism," allow for the winner of the first auction to match the winning bid of the second auction, thereby winning the object.

The article demonstrates that in the context of symmetric independent private values, identically distributed according to the uniform distribution, the competitive advantage mechanism generates an expected return well above that of simply applying two standard auctions consecutively without a reserve price. If we compare to the auction using the optimal reserve price that maximizes the auctioneer's expected return, the mechanism generates an expected revenue close to this optimal mechanism.

Although it does not reach the auctioneer's expected revenue when the optimal reserve price is used, the competitive advantage mechanism has several properties that make it particularly desirable in practical applications, especially in the public sector.

One significant benefit is that it ensures that there will always be sales of the items, which cannot be ensured in the case of using a reserve price. This property is especially relevant when considering the different organizational and reputational costs associated with a "failed" auction, where the item is not sold.

In real-world implementations in the public sector, reserve prices are typically determined by an accounting analysis of the item's value to be auctioned. This estimate of the item's value for the auctioneer is unlikely to coincide with the optimal value that maximizes the auctioneer's revenue. Indeed, Myerson (1981) and Riley & Samuelson

(1981) show that the optimal reserve price is higher than the auctioneer's reservation value in the risk-neutral independent-private-values (IPV) auction model that we study here. Thus, the competitive advantage mechanism may, in practice, generate expected revenue for the auctioneer higher than the mechanism with a reserve price equal to the item's value for the auctioneer.

Finally, the proposed mechanism, by creating a dynamic bond between the auctioneer and the winner, incentivizes the winner to fulfill all commitments made in the first auction to avoid losing the strategic advantage in the subsequent auction. This provides greater control by the public sector over the auction winner, a recurring concern in mechanism design applied to the public sector.

Naturally, there are questions that can be raised regarding the implementation of the proposed mechanism.

One concern is the mechanism violates the principle of equality, as one participant has an advantage over the others in the second auction. This characteristic suggests that some participants (losers in the first auction) may challenge the asymmetry creation rule in the second auction in court. For this reason, great care must be taken to convince society, and especially the Judiciary, of the advantages of the mechanism before implementation.

Furthermore, and associated with the above questioning, although the proposed mechanism ensures that the item will always be sold, it does not guarantee that the item will be sold to the participant who values it the most, i.e., the mechanism is not efficient, just like the mechanism with the optimal reserve price, albeit for different reasons.

This article makes an original contribution to the literature on repeated auctions by shifting the focus from issues related to discouraging collusion among participants to the search for practical alternative mechanisms that can reinforce the auctioneer's expected revenue. This research can be extended in multiple directions, some of which are presented below as suggestions for future research.

The model presented assumes that the auctioneer assigns zero value to the item. Therefore, the most natural comparison is indeed with the traditional model with a reserve price of zero, a comparison that is clearly favorable to the proposed model. Considering that in the public sector, the reserve value is the value of the item for the auctioneer, the proposed model should also be compared with an implementation including a reserve value identical to the value of the item for the auctioneer, and the two possible implementations should be compared.

Given that sequential public tenders usually take significant periods of time to complete, intertemporal discounting between the two auctions is another area for extension. It is also necessary to understand how the results found here are affected by the number of participants, particularly when this number is endogenous. Additionally, a model with more than two consecutive auctions or where the objects to be sold have some degree of complementarity or substitutability should be considered.

Finally, the model should be extended to more general probability distributions of participants' values in the symmetric independent private values model, and its applicability to situations where values are possibly affiliated should be verified. These extensions are our suggestions for future research.

REFERENCES

Abreu, D. (1988). On the theory of infinitely repeated games with discounting. *Econometrica: Journal of the Econometric Society*, 383-396.

Aoyagi, M. (2003). Bid rotation and collusion in repeated auctions. *Journal of Economic Theory*, *112*(1), 79-105.

Ausubel, L. M., & Cramton, P. (2004). Vickrey auctions with reserve pricing. *Economic Theory*, *23*(3), 493-505.

Bugarin, M., & Ribeiro, F. (2001). The paradox of concessions in developing countries. *Brazilian Review of Econometrics* 41(1): 69-100. https://doi.org/10.12660/bre.v41n12021.83397.

Casady, C. B., Petersen, O. H., & Brogaard, L. (2023). Public procurement failure: The role of transaction costs and government capacity in procurement cancellations. *Public Management Review*, 1–28. https://doi.org/10.1080/14719037.2023.2231945.

De Silva, D. G., Jeitschko, T. D., & Kosmopoulou, G. (2005). Stochastic synergies in sequential auctions. *International Journal of Industrial Organization*, *23*(3-4), 183-201.

Engelbrecht-Wiggans, R. (1987). On optimal reservation prices in auctions. *Management Science*, *33*(6), 763-770.

Estache, A., & Quesada, L. (2001). Concession contract renegotiations: Some efficiency versus equity dilemmas (Policy Research Paper No. 2705). Washington, DC: The World Bank. https://openknowledge.worldbank .org/handle/10986/19495

Fudenberg, D., & Maskin, E. (1986) The folk theorem in repeated games with discounting and with incomplete information', *Econometrica*, *54*: 533-554.

Gagnepain, P., Ivaldi, M., & Martimort, D. (2013). The cost of contract renegotiation: Evidence from the local public sector. American Economic Review, *103*(6), 2352–1383. http://dx.doi.org/10.1257/aer.103.6.2352.

Guasch, J. L., Laffont, J.-J., & Straub, S. (2006). Renegotiation of concession contracts: A theoretical approach. Review of Industrial Organization, *29*, 55–73. http://dx.doi.org/10.1007/s11151-006-9109-5.

Guasch, J. L., Laffont, J.-J., & Straub, S. (2007). Concessions of infrastructure in Latin America: Government-led renegotiations. Journal of Applied Econometrics, *22*(7), 1267–1294. http://dx.doi.org/10.1002/jae.987.

Guasch, J. L., Laffont, J.-J., & Straub, S. (2008). Renegotiation of concession contracts in Latin America: Evidence from the water and transport sectors. International Journal of Industrial Organization, *26*(2), 421–442. http://dx.doi.org/10.1016/j.ijindorg.2007.05.003.

Jofre-Bonet, M., & Pesendorfer, M. (2014). Optimal sequential auctions. International Journal of Industrial Organization, 33, 61-71.

Kaplan, T. R., & Zamir, S. (2012). Asymmetric first-price auctions with uniform distributions: analytic solutions to the general case. *Economic Theory*, *50*(2), 269-302.

Kirkegaard, R. (2022). Efficiency in asymmetric auctions with endogenous reserve prices. *Games and Economic Behavior*, *132*, 234-239.

Krishna, V. (2002). Auction theory. *San Diego: Academic Press*.

Laffont, J.-J. (2005). Regulation and development. Cambridge: Cambridge University Press.

Levin, D., & Smith, J. L. (1996). Optimal reservation prices in auctions. *The Economic Journal*, *106*(438), 1271-1283.

Marshall, R. C., & Marx, L. M. (2012). The economics of collusion: Cartels and bidding rings. MIT Press.

McAfee, R. P., & McMillan, J. (1992). Bidding rings. *The American Economic Review*, 579-599.

McHugh, R. (2020) How Much Paper Schools Use & How Much It Costs Them. *Record Nations,* Jan. 14, 2000. https://www.recordnations.com/blog/how-much-paper-schools-use-how-much-costs/

Menicucci, D. (2021). In the basic auction model, the optimal reserve price may depend on the number of bidders. *Journal of Economic Theory*, *198*, 105371.

Milgrom, P., & Weber, R. J. (1982). The value of information in a sealed-bid auction. *Journal of Mathematical Economics*, *10*(1), 105-114.

Myerson, R. B. (1981). Optimal auction design. *Mathematics of operations research*, *6*(1), 58-73.

Naser, A.Y., Alwafi, H., Al-Daghastani, T. *et al.* Drugs utilization profile in England and Wales in the past 15 years: a secular trend analysis. *BMC Prim. Care*, *23*, 239 (2022). https://doi.org/10.1186/s12875-022-01853-1

Nash, J. F. (1950). The bargaining problem. *Econometrica*, *18*(2): 155-162.

Phillips, O. R., Menkhaus, D. J., & Coatney, K. T. (2003). Collusive practices in repeated English auctions: Experimental evidence on bidding rings. *American Economic Review*, *93*(3), 965-979.

Riley, J. G., & Samuelson, W. F. (1981). Optimal auctions. *American Economic Review*, *71*(3), 381-392.

Salant, D. J., & Cabral, L. (2019). Sequential auctions and auction revenue. *Economics letters*, *176*, 1-4.

Skrzypacz, A., & Hopenhayn, H. (2004). Tacit collusion in repeated auctions. *Journal of Economic Theory*, *114*(1), 153-169.

Sorin, S. (1992). Repeated games with complete information. *Handbook of game theory with economic applications*, *1*: 71-107.

Tanaka, S., & Hayashi, S. (2016), Collusion between public procurers and suppliers in the context of Japan's public procurement: The role of the risks of "unsuccessful procurement", *Journal of Public Procurement*, Vol. 16 No. 3, pp. 291-311. https://doi.org/10.1108/JOPP-16-03-2016-B003.

Vickrey, W. (1961). Counterspeculation, auctions, and competitive sealed tenders. *The Journal of Finance*, *16*(1), 8-37.

Wilson, R. (1966). *Competitive bidding with disparate information.* Working paper No. 114. Graduate School of Business, Stanford University.

Wilson, R. (1967). Competitive bidding with asymmetric information. *Management Science*, *13*(11), 816-820.

Xu, H., & Cavallo, R. (2022). The Strange Role of Information Asymmetry in Auctions—Does More Accurate Value Estimation Benefit a Bidder? *Proceedings of the AAAI Conference on Artificial Intelligence 36*(5), pp. 5236-5243.

APPENDIX

PROOF OF PROPOSITION 1.

Each auction admits a (weakly) dominant strategy for each player, which is to bid their own value. Therefore, the expected revenue of the auctioneer is given by:

$$2\int_0^1\int_0^{v_1} v_2 dv_2 dv_1 = 2\int_0^1 \frac{(v_1)^2}{2} dv_1 = \frac{1}{3}$$

Since two auctions are conducted, also assuming, for simplicity, the absence of intertemporal discounting, then the expected return in the two auctions is $R_1 = 2/3$. □

PROOF OF PROPOSITION 2.

Suppose player 1 assigns the value v_1 to the item and makes the bid b. Further suppose that player 2's strategy is $\beta_2(v_2)$.

Let $r \in (0,1)$ be the auctioneer's reserve price. If $v_i < r$, then any $\beta_i(v_i) \in (0,r)$ is an optimal response.

Suppose $v_i > r$, let's find the optimal response of player i given player's j strategy $\beta_j(\cdot)$. If player i bids b, then her utility is:

$$u_i(b, \beta_j(v_j); v_i) = \begin{cases} v_i - b; & b \geq \beta_j(v_j) \text{ and } b \geq r \\ 0; & \text{otherwise} \end{cases}$$

Her expected utility results:

$$\int_0^{\beta_j^{-1}(b)} (v_i - b) dv_j = (v_i - b)\beta_j^{-1}(b); \text{ for } b \geq r$$

and the first order condition (FOC) for interior solution is:

$$-\beta_j^{-1}(\beta_i(v_i)) + (v_i - \beta_i(v_i)) \times \frac{1}{\beta_j'\left(\beta_j^{-1}(\beta_i(v_i))\right)} = 0$$

As we search for a symmetric solution, we consider $\beta_i = \beta_j = \beta$:

$$-\beta^{-1}(\beta(v)) + (v - \beta(v)) \times \frac{1}{\beta'\left(\beta^{-1}(\beta(v))\right)} = 0 \Rightarrow \frac{v - \beta(v)}{\beta'(v)} = v$$

$$\Rightarrow v = \beta(v) + \beta'(v) \times v = \left(v\,\beta(v)\right)' \Rightarrow \frac{v^2}{2} + k = v\,\beta(v) \Rightarrow \beta(v) = \frac{v}{2} + \frac{k}{v}$$

if $v = r$, the player may bid $\beta(r) = r$, since her utility will be zero, anyway.

Using this boundary condition we are able to calculate k:

$$r = \beta(r) = \frac{r}{2} + \frac{k}{r} \Rightarrow k = r^2$$

thus, the Bayesian Nash Equilibrium results:

$$\beta(v) = \begin{cases} \frac{v}{2} + \frac{r^2}{v}; & v \geq r \\ \in (0, r) & v < r \end{cases}$$

but then, the expected revenue of the auctioneer in each auction is given by:

$$2\int_{r}^{1}\int_{0}^{v_1} \beta(v_1) dv_2 dv_1 = 2\left[\frac{1}{6} + \frac{r^2}{2} - \frac{2}{3}r^3\right]$$

The expression above is maximized at $r = 1/2$, with the auctioneer's revenue in this case being $\frac{1}{3} + \frac{1}{12}$. Since two consecutive auctions are conducted, the auctioneer's expected revenue is $R_2\left(\frac{1}{2}\right) = \frac{2}{3} + \frac{1}{6}$. □

PROOF OF PROPOSITION 3.

First, let us calculate the Bayesian Nash Equilibrium. To this end, suppose, without loss of generality, that player 1 has an advantage in this auction, i.e., player 1 won the first auction. Further, suppose that player 1 assigns the value v_1 to the item and makes the bid b_1. Finally, suppose that player 2's strategy is $\beta_2(v_2)$.

Then player 1 will win this second auction by making a bid $b_1 \geq 0$ if $b_1 \geq \beta_2(v_2)$ or if $b_1 < \beta_2(v_2)$ but $v_1 \geq \beta_2(v_2)$.

Note that if $b_1 > \beta_2(v_2)$, then 1 will win the auction and pay b_1, whereas it could have won the object for $\beta_2(v_2)$ had he chosen $b_1 = 0$. Therefore, 1 has a weakly dominant strategy, which is to always bid zero.

Therefore, $\beta_1(v_1) = 0, \forall v_1 \in [0,1]$ and 1 will use its ability to match player 2's bid if $v_1 \geq \beta_2(v_2)$.

Suppose player 2, who lost the first auction, assigns the value v_2 to the item and makes the bid $b_2 \leq v_2$, while player 1 is using the above strategy. Then, player 2 wins if and only if $v_1 \leq b_2$.

Therefore, 2's expected utility is:

$$u_2(b_2) = \int_0^{b_2} (v_2 - b_2)\, dv_1 = (v_2 - b_2)b_2$$

Player 2's expected utility is maximized at: $b_2 = \frac{v_2}{2} = \beta_2(v_2)$.

Next, let us calculate the expected revenue of the auctioneer in the second auction. Given the equilibrium strategy, the object is always sold, and the payment is always player 2's bid, regardless of which player actually receives the object. Therefore, the seller's expected return is:

$$R_{32} = \int_0^1 \frac{v_2}{2} dv_2 = \frac{1}{4} < \frac{1}{3}.$$

To conclude, let us calculate the expected utilities of each player in the second auction.

The winner in the first auction will win the second auction whenever $v_1 \geq \beta_2(v_2) = \frac{v_2}{2}$, in which case she will pay $\frac{v_2}{2}$. Note that $v_1 \geq \frac{v_2}{2} \Leftrightarrow v_2 \leq 2v_1$. Therefore, player 1's expected payoff is:

$$EU_1 = \int_0^{\frac{1}{2}} \int_0^{2v_1} \left(v_1 - \frac{v_2}{2}\right) dv_2 dv_1 + \int_{\frac{1}{2}}^{1} \int_0^{1} \left(v_1 - \frac{v_2}{2}\right) dv_2 dv_1 = \frac{7}{4} \times \frac{1}{6}$$

The loser in the first auction will win the second auction whenever $v_1 < \beta_2(v_2) = \frac{v_2}{2}$, in which case she will pay $\frac{v_2}{2}$. Note that $v_1 < \frac{v_2}{2} \Leftrightarrow v_2 > 2v_2$. Note that this can only be the case when $v_1 < \frac{1}{2}$Therefore, player 2's expected payoff is:

$$EU_2 = \int_0^{\frac{1}{2}} \int_{2v_1}^{1} \left(v_2 - \frac{v_2}{2}\right) dv_2 dv_1 + \int_{\frac{1}{2}}^{1} \int_0^{1} \left(v_1 - \frac{v_2}{2}\right) dv_2 dv_1 = \frac{1}{2} \times \frac{1}{6} \ \square$$

PROOF OF PROPOSITION 4.

First, let us calculate the symmetric Bayesian Nash Equilibrium of the first auction in the model with competitive advantage. Consider player 1 and suppose she assigns the value v_1 to the item and bid b_2. Further, suppose that player 2's strategy is $\beta_2(v_2)$. Then, the expected utility for player 1 is:

$$\int_0^{\beta_2^{-1}(b_1)} \left[(v_1 - b_1) + \frac{7}{4} \times \frac{1}{6}\right] dv_2 + \int_{\beta_2^{-1}(b_1)}^{1} \left[\frac{1}{12}\right] dv_2$$

$$= (v_1 - b_1)\beta_2^{-1}(b_1) + \frac{7}{4} \times \frac{1}{6}\beta_2^{-1}(b_1) + \frac{1}{12}\left(1 - \beta_2^{-1}(b_1)\right)$$

$$= \left(v_1 - b_1 + \frac{5}{2} \times \frac{1}{12}\right)\beta_2^{-1}(b_1) + \frac{1}{12}$$

The first-order condition corresponds to:

$$-\beta_2^{-1}(b) + \left(v_1 - b + \frac{5}{2} \times \frac{1}{12}\right)(\beta_2^{-1})'(b) = 0$$

As we are seeking a symmetric equilibrium, $\beta_1(v) = \beta_2(v) = \beta(v)$, and in player 1's optimal choice, $b = \beta(v_1)$. Therefore, the equation above can be rewritten as:

$$v_1\, \beta'(v_1) = v_1 - \beta(v_1) + \frac{5}{2} \times \frac{1}{12}$$

Then we can rewrite the first order condition as:

$$v_1\, \beta'(v_1) + \beta(v_1) = v_1 + \frac{5}{2} \times \frac{1}{12}$$

Therefore, the equilibrium bidding function must satisfy the following condition, where k is a constant of integration to be determined:

$$v_1\, \beta(v_1) = \frac{v_1^2}{2} + \frac{5}{2} \times \frac{1}{12} v_1 + k$$

Substituting $v_1 = 0$ into the equation above, we find the value of $k = 0$, so that the Bayesian Nash equilibrium of the game is given by the bidding function: For $i = 1{,}2$

$$\beta(v_i) = \frac{v_i}{2} + \frac{5}{2} \times \frac{1}{12}$$

To complete the proof of the proposition, let us calculate the auctioneer's total expected revenue (for both auctions).

Given the strategy profile $\beta_i(v_i) = \frac{v_i}{2} + \frac{5}{2} \times \frac{1}{12}, i = 1{,}2$, the expected revenue of the auctioneer in the first auction is given by:

$$R_{31}(g) = 2\int_0^1 \int_0^{v_1} \beta(v_1) dv_2 dv_1 = \frac{1}{3} + \frac{5}{4} \times \frac{1}{6}$$

Namely, there is a gain of $\frac{5}{4} \times \frac{1}{6}$ in the expected revenue of the auctioneer in the first auction when compared to the situation where there is no potential linkage between the two auctions. However, it has been previously observed that this mechanism reduces the expected revenue in the second period.

Therefore, the total revenue in both auctions when the competitive advantage mechanism is used is:

$$R_3 = R_{31} + R_{32} = \frac{1}{3} + \frac{5}{4} \times \frac{1}{6} + \frac{1}{4} = \frac{2}{3} + \frac{1}{8}. \square$$