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Motivated to some extent by the empirical significance of student loans to human capital accumulation, this paper incorporates debt-financed knowledge capital accumulation to a demand-led model of capacity utilization and growth. Average labor productivity varies positively with the average stock of knowledge capital across the labor force. Any increase in labor productivity ensuing from knowledge capital accumulation is fully passed on to the real wage, but insufficient aggregate effective demand, by generating unemployment, gives rise to underutilization of the knowledge capital capacity. Both the stability properties and financial fragility (in the Minskyan sense) of the long-run equilibrium outcome depend on how the debt servicing of working households is specified. The same dependence applies to how the rates of physical capital utilization and labor employment (where the latter also measures the rate of knowledge capital utilization) respond to changes in the ratio of working households' debt to physical capital.

Keywords: Knowledge capital; debt; capacity utilization; employment.

JEL Codes: E12; E22; E24.

Debt-financed knowledge capital accumulation, capacity utilization and economic growth*

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1. Introduction

The role of human capital accumulation as a source of economic growth has been extensively explored in mainstream growth theory. In an early contribution, Lucas (1988), building upon Uzawa (1965), assumes that individuals choose periodically how to allocate their non-leisure time between current production and skill acquisition (or schooling), where the latter raises productivity in future periods. Human capital accumulation, by involving constant returns to the existing stock of human capital, arises as a source of sustained long-run growth. Mankiw, Romer and Weil (1992) incorporate accumulable human capital, along with physical capital and labor, as an additional production factor into an otherwise standard Solow model. As a result, the level of output per worker varies positively with both the level of physical capital per worker and the level of human capital per worker. Analogously to the accumulation of physical capital, in Mankiw, Romer and Weil (1992) the rate of human capital accumulation is fully and automatically determined by the availability of savings brought about by foregone consumption by individuals.

Admittedly, such mainstream growth approach, by assuming that the economy always grows at full capacity utilization, does neglect both the role of aggregate effective demand in growth dynamics and the impact of autonomous investment in human capital formation on aggregate effective demand. Meanwhile, demand-driven approaches to growth dynamics have relegated any closer attention to human capital formation through education (and to 'knowledge' capital accumulation more broadly) as narrowly supply-sided. One notable exception is Dutt (2010), who formalizes the process of skill acquisition in a neo-Kaleckian framework so that both the number of high-skilled and low-skilled workers and their respective wages vary over time and impact upon the interaction between income distribution and economic growth. Relatedly, Lima, Carvalho and Serra (2017) embed accumulation of human capital through the provision of universal public education by a balanced-budget government in a neo-Kaleckian model of capital capacity utilization, income distribution and economic growth. The level of education, as represented by the stock of human capital, positively affects both workers' productivity in output production and (partly also as a result of it) their bargaining power in the labor market.

Given that the making of costly investments in human capital formation through education is one of the main types of productivity-enhancing knowledge accumulation, this paper explores formally some implications of debt-financed accumulation of knowledge capital, along with its resulting impact on labor productivity, in a demand-led macromodel of capital capacity utilization and growth. The model features debt-financed knowledge capital accumulation as a further source of effective demand along with expenditures in investment in physical capital and consumption. Analogously to the determination of investment plans in physical capital as independent from any existing saving, the model features a likewise independent investment function with respect to knowledge capital formation. This independence of the accumulation of knowledge capital by workers' households is accommodated by an endogenous supply of credit money, and hence the debt-financed nature of the desired investment in such another accumulable capital asset. Since the aggregate stock of knowledge capital remains uniformly distributed in the labor force, which is always in excess supply, it follows that unemployed labor also means unutilized knowledge capital. As a result, the economy operates with excess productive capacity not only in physical capital and labor, as in several demand-driven growth models, but also in knowledge capital.

Meanwhile, as for the debt servicing by workers' households who finance knowledge capital accumulation through debt, we first assume that workers' households allocate to consumption their entire wage income net of debt service. That is, working households give prior claim to debt servicing obligations on their current wage income, and therefore treat their consumption spending as a residual. Yet we further assume that consumption so determined is constrained to be positive. We later explore implications of the alternative assumption that debt servicing by working households is specified according to an income-driven repayment plan bearing some similarity to some repayment plans applicable to federal student loan payments in the U.S. For this specification, we explore dynamic implications of debt-financed accumulation of knowledge capital by drawing upon the financial instability approach developed by Minsky (1975, 1982), especially his suggestive taxonomy of hedge, speculative and Ponzi financing regimes. One interesting possibility that arises is the existence of a long run characterized by multiple equilibria, with both equilibria situated in the Ponzi financing regime. Revealingly, though the long-run equilibrium with higher debt ratio is stable, it is situated in the Minskyan regime of highest financial fragility as it is likewise the saddle-point unstable one with lower debt ratio.

Although our model is not intended to describe specifically debt-financed knowledge capital accumulation through student loans, the recent U.S. experience with student debt is arguably illustrative of the significance of working households' financing of human capital formation through debt and how this is a possible route to financial fragility in the Minskyan sense. In fact, data from the New York Fed Consumer Credit Panel reveal that student debt reached US$1.2 trillion in the U.S. in 2015, an amount which was higher than household debt with credit card and auto loans.[1] Besides, the average annual real growth between 2005 and 2015 was around 10%. The average outstanding balance also rose, to US$ 27,900 in 2015, which represented a real increase of 4% between 2005 and 2015 (see Figure 1).

More U.S. students have been borrowing and they have been borrowing more. The number of full-time undergraduates using loans to attend college increased from 45.6% in 1999-2000 to 56.7% in 2011-2012. Consequently, in 2011-2012, 84.4% of full-time college students were receiving some type of financial support to study, while this proportion was 71.9% in 1999-2000. Expectedly, younger adults are responsible for a significant part of the total outstanding balance related to student loans, and in 2015, 39% of student loan borrowers were under the age of 30 years old (see Figure 2). As a negative sign for the sustainability of student debt, the proportion of borrowers having repayment troubles shows an increasing trend. In 2015, for instance, 10.7% of them defaulted on their repayment, the same figure as in 2014, but one higher than in the previous years; in 2005, the default rate was 5.8% (see Figure 3). In fact, recent evidence has found that many borrowers have only managed their repayment due to personal savings or family support (Lochner and Monge-Naranjo, 2016). Also, given that the methodology to calculate delinquency rates on student loans does not consider those students who are out of the repayment cycle (which includes loans in deferment, in grace period or in forbearance), these results on default rates tend to underestimate effective delinquency rates (FRBNY, 2016).[2]

More recent data reveals that total household debt increased by 1.8% in the fourth quarter of 2016, rising $226 billion to reach $12.58 trillion. Balances increased across all debt varieties,

[1] FRBNY (2015, 2016) are both reports based on the New York Fed Consumer Credit Panel, which is constructed from a nationally representative random sample drawn from Equifax credit report data.

[2] FRBNY (2016) even admits that this methodology implies that among loans in the repayment cycle delinquency rates are roughly twice as high.

with a 1.6% increase in mortgage balances, a 1.9% increase in auto loan balances, a 4.3% increase in credit card balances, and a 2.4% increase in student loan balances in the same quarter. Specifically, outstanding student loan balances rose by $31 billion, and reached $1.31 trillion as of December 31, 2016. Besides, 11.2% of aggregate student loan debt was 90+ days delinquent or in default in the fourth quarter of 2016 (see FRBNY, 2017).

In this paper, we assume that any increase in labor productivity which results from knowledge capital accumulation is fully passed on to the real wage. Hence, while insufficient effective demand causes the aggregate stock of knowledge capital to be underutilized, it turns out that employed workers receive a full wage return on their knowledge capital. In fact, in our model the wage share in income, which is computed as the ratio of real wage to labor productivity, gives a simplified measure of the wage return on knowledge capital. As any increase in labor productivity brought about by knowledge capital accumulation is fully passed on to the real wage, employed workers are able to collect the full wage return on their knowledge capital. Nevertheless, we are able to explore changes in the distribution of income in the economy by using an index of income inequality given by the ratio of the income of profit-cum-interest recipients to the net income of working households. For both specifications of debt servicing explored in this paper, we find that the index of inequality varies positively with the interest rate and the working households' debt to physical capital ratio and negatively with the wage share. Meanwhile, the short- and long-run equilibrium rates of physical capital utilization and labor employment and the long-run equilibrium rate of output growth may all vary positively or negatively with the wage share.

The sequence of the paper is structured in the following way. Section 2 lays out the model structure. Section 3 solves for the short-run equilibrium values of the rates of physical and knowledge capital utilization, assuming that the existing stocks of debt (and the respective flow of debt service) and physical and knowledge capital are all given. Section 4 introduces long-run issues by following the dynamics of the ratios of physical capital to knowledge capital and working households' debt to physical capital. While in this section we assume that workers' households allocate to consumption their entire wage income net of debt service, in Section 5 we alternatively assume that debt servicing by working households is specified as following an income-driven repayment program. Finally, Section 5 concludes the paper.

2. The structure of the model

The model will deal with a closed economy that produces a single good/service for both investment (on physical and knowledge capital) and consumption. Two homogeneous factors of production are used in the production process, physical capital and labor, and the stock of knowledge capital is assumed to remain uniformly embodied in the available labor force. These production inputs are combined through a fixed-coefficient technology:

$$X = \min\left[K\nu, La(h)\right], \tag{1}$$

where X is the output level, K is the stock of physical capital, L is the employment level, ν is the knowledge capital stock to labor force ratio (or average knowledge capital) and $a(h)$ is the output to labor ratio (or labor productivity), which varies endogenously with the average knowledge capital. For simplicity, the technical coefficient ν will be normalized to a constant equal to one. In the production function in (1), we also assume that $a(0)=0$, $a'(h)>0$ and $a''(h)\leq 0$. Given that we are dealing with a single good/service economy, the 'production' of knowledge capital does not constitute another production process or even productive sector. Indeed, it is assumed herein that the single good/service that can be used for both physical capital accumulation and consumption can also be used for knowledge capital accumulation.[3] Moreover, given that unemployed workers are as skilled (or knowledge capital endowed) as employed ones, the rate of labor employment, which is determined by aggregate effective demand, measures the degree of knowledge capital utilization. Though we consider only the case in which aggregate effective demand is insufficient to yield full utilization of the existing productive capacity (in either capital or labor) at the ongoing price and wage rate, we abstract from (physical and knowledge) capital depreciation and labor deskilling. Moreover, the model is cast in real terms.

The economy is composed of two classes of households, capitalists and workers, who earn profits and wages, respectively. The functional division of aggregate income is then given by:

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[3] Indeed, a more inclusive model could drop the assumption of homogeneous labor – e.g. by bringing in low-skilled and high-skilled workers, who would be paid differently, as in Dutt (2010). However interesting, a more inclusive specification along these lines will be the subject of future research – for which we invite the reader to stay tuned.

$$X = VL + rK, \tag{2}$$

where V is wage rate and r is the rate of profit on physical capital, which is the flow of profits, R, as a proportion of the physical capital stock. From (1) and (2), the share of labor in income, σ, is given by:

$$\sigma = \frac{V}{a(h)}. \tag{3}$$

To focus on the issue of possible better employability prospects opened up by knowledge capital accumulation, we assume that any increase in labor productivity which results from knowledge capital accumulation is fully passed on to the wage rate. Hence, while insufficient aggregate effective demand causes the existing aggregate stock of knowledge capital to be underutilized, employed workers receive a full wage return on their knowledge capital. In fact, the wage share in income, as specified in (3), provides a simplified measure of the wage return on knowledge capital. Given that any increase in labor productivity brought about by knowledge capital accumulation is fully passed on to the wage rate, so that employed workers are always able to collect the full wage return on their knowledge capital, the wage share (and hence income distribution between wages and profits) remains unchanged. Nonetheless, we later draw on Dutt (2006) to discuss changes in the distribution of income in the economy by using an index of inequality given by the ratio of the income of profit-cum-interest recipients to the net income of working households.

Firms produce (and hire labor) according to aggregate effective demand. As we model only the case in which excess productive capacity (in labor and capital in general) prevails, labor employment is determined by production:

$$L = \frac{X}{a(h)}. \tag{4}$$

At a point in time, the technological parameters are given, having resulted from previous knowledge and physical capital accumulation. Over time, however, knowledge capital accumulation takes place as described later, which results in labor productivity growing at the proportionate rate $\hat{a}$. Formally:

$$\hat{a} = \rho(\hat{h}), \tag{5}$$

where $\hat{h}$ is the proportionate growth rate of the knowledge capital stock to labor force ratio, with $\rho(0) \geq 0$, $\rho'(.) > 0$ and $\rho''(.) \leq 0$.

The employment rate, $e = L / N$, is linked to the state of the market for goods/services in the following way:

$$e = \frac{L}{X} \frac{X}{K} \frac{K}{N} = uk , \tag{6}$$

where $u = X / K$ is the rate of physical capital capacity utilization and k stands for the ratio of physical capital stock to labor force in productivity units, that is, $k = K / (Na(h))$. This formal link between u and e resulting from the fixed-coefficient nature of the technology implies that a rise in output in the short run, when k is given, is necessarily accompanied by an increase in employment. Moreover, as the aggregate knowledge capital stock is uniformly distributed in the available labor force, the employment rate also measures the degree of utilization of the aggregate knowledge capital stock. For simplicity, we treat the labor force, N, as constant (but nonetheless always in excess supply) and normalize it to one, and further assume that the level of labor productivity has a one-to-one correspondence with the average stock of knowledge capital, h, so that $a(h) = h = H$ and, hence $k = K / H$. Moreover, it follows that $\hat{a} = \hat{h} = \hat{H}$.

Firms' decisions regarding accumulation of physical capital are made independently from any prior savings. The implied desired growth rate of the stock of physical capital, g_K, assuming no depreciation, is given by:

$$g_K = \frac{I_K}{K} = \alpha + \beta u , \tag{7}$$

where I_K denotes firms' desired investment in physical capital, α is a positive autonomous component and β is a positive parameter.

Analogously to the determination of investment in physical capital as independent from prior savings, working households' decisions to accumulate knowledge capital are also assumed to be so independent. This independence of knowledge capital accumulation is accommodated by an endogenous supply of credit money, which then ensures the debt-financed nature of the desired investment in such another accumulable capital asset. Working households' desired level of investment in knowledge capital is given by:

$$I_H = \gamma VL, \tag{8}$$

where γ is a positive parameter. Analogously to specifications of the desired investment in physical capital in the Cambridge U.K. tradition, which typically have total profits (or their expected value) as a positive determinant, the desired level of investment in knowledge capital in (8) varies positively with the wage bill. The implied desired growth rate of the stock of knowledge capital, g_H, assuming no depreciation, is given by:

$$g_H = \frac{I_H}{H} = \gamma \frac{VL}{X}\frac{X}{K}\frac{K}{H} = \gamma\sigma e, \tag{9}$$

where e is given by (6). Recall that the knowledge capital stock is uniformly distributed in the labor force, so that the employment rate also measures the degree of utilization of such stock. Thus, the specification in (9) can be interpreted as incorporating an accelerator effect, but applied to the investment in knowledge capital instead of physical capital.[4] Meanwhile, for future reference, working households' desired level of investment in knowledge capital as a proportion of the physical capital stock is given by:

$$\frac{I_H}{K} = \gamma \frac{VL}{X}\frac{X}{K} = \gamma\sigma u. \tag{10}$$

Following the tradition of Kalecki (1971), Kaldor (1956), Robinson (1962) and Pasinetti (1962), we assume that workers and capitalists have different consumption behavior. Workers provide labor and earn wage income, of whose net value (recall that workers have to meet debt servicing obligations) they consume a constant fraction. Although we assume that the labor force, N, is a constant normalized to one, workers are always in excess supply. Firm-owner capitalists' households receive not only profit income, which is the entire surplus over

[4] Even though we simplify matters by assuming no depreciation of knowledge capital, the expression in (9) could be interpreted as referring to the growth rate of the *net* stock of knowledge capital in the presence of de-skilling due to unemployment. In this broader interpretation, a higher employment rate yields a higher growth rate of the *net* stock of knowledge capital by implying less labor de-skilling. Yet another broader interpretation is that knowledge capital accumulation involves on-the-job learning externalities.

the wage bill, but also interest income as recipients of workers' debt servicing, and spend in consumption a constant proportion of it, $0<1-s<1$, where s denotes capitalists' saving rate. To focus more sharply on the implications of debt-financed knowledge capital accumulation, we further assume that neither capitalists nor workers borrow for consumption purposes.

3. Short-run equilibrium

The short-run is defined as the time period along which the stock of physical capital, K, the stock of knowledge capital, H, the output-labor ratio, a, and the wage rate, V (and therefore the wage share, σ), can all be taken as given. Supply-demand equilibrium in the market for goods/services is given by:

$$X = C_w + C_c + I_H + I_K, \qquad (11)$$

where C_w and C_c stand for aggregate consumption by workers' households and capitalists' households, respectively. Therefore, knowledge capital accumulation by working households is an extra source of aggregate effective demand alongside with investment in physical capital and consumption by the two classes of households. For future reference, the equilibrium condition in (11) as a proportion of the physical capital stock is given by:

$$u = \frac{C_w}{K} + \frac{C_c}{K} + \frac{I_H}{K} + g_K. \qquad (12)$$

As for debt servicing by workers' households who finance knowledge capital accumulation by means of debt, we start by assuming that workers' households allocate to consumption their entire wage income net of debt service. Later on, however, we alternatively assume that debt servicing by working households follows an income-based repayment plan bearing some similarity to certain repayment plans applicable to federal student loan payments in the U.S. Thus, working households' consumption is firstly given by:

$$C_w = VL - iD, \qquad (13)$$

where D is the stock of debt held by working households and i is the interest rate. While the former is given in the short run, but varies over time, the latter, for simplicity, is assumed to stay constant throughout. Reasonably, we assume that working households' consumption is always positive, which then requires that $VL > iD$ throughout. Given our previously made assumption that capitalists' households allocate to consumption a constant proportion, s, of their profit and interest income, it follows that:

$$C_c = (1-s)(R+iD)\,. \tag{14}$$

Meanwhile, the debt-financed nature of the desired investment in knowledge capital by working households implies that:

$$\dot{D} = I_H\,, \tag{15}$$

where $\dot{D} = dD/dt$ denotes the change in the stock of debt held by working households. Given that aggregate output is determined by aggregate effective demand, and labor (along with the knowledge capital uniformly embodied in the labor force) is always in excess supply at the ongoing wage rate, the rate of physical capital capacity utilization, u, adjusts for the equilibrium in the market for goods/services in (11) to obtain. By normalizing (13) and (14) by the physical capital stock and substituting the resulting expressions (along with (7) and (10)) into the normalized goods/services-market equilibrium condition in (12), we obtain:

$$u = \left(\frac{VL}{X}\frac{X}{K} - \frac{iD}{K}\right) + (1-s)\left(\frac{R}{K} + \frac{iD}{K}\right) + \gamma\sigma u + \alpha + \beta u\,. \tag{16}$$

Using (2) and (3) (along with the previously made assumption that $a(h) = h = H$) to re-write the rate of profit, R/K, in terms of the physical capital capacity utilization and substituting the resulting expression into (16), we can solve for the short-run equilibrium rate of physical capital capacity utilization to obtain:

$$u^* = \frac{\alpha - si\delta}{\Omega}, \tag{17}$$

where $\delta = D/K$ is the debt ratio and $\Omega = s(1-\sigma) - \gamma\sigma - \beta$. To ensure that the demand-led output-adjustment stability condition known as the Keynesian stability condition is satisfied, we further assume that saving as a proportion of the physical capital stock, which is given by $u - (C_w/K) - (C_c/K)$ is more responsive to changes in physical capital capacity utilization than investment (in both physical and knowledge capital) as a proportion of the physical capital stock, which is given by $(I_H / K) + g_K$. This condition is equivalent to a positive denominator in (17), so that we have to further assume that the numerator in (17) is positive to ensure a positive value for the short-run equilibrium rate of physical capital utilization.

Meanwhile, using (6), the short-run equilibrium employment rate, which also measures the rate of knowledge capital capacity utilization, is given by:

$$e^* = \frac{(\alpha - si\delta)k}{\Omega}. \tag{18}$$

As routinely assumed in one-good macroeconomic models featuring physical capital and labor as factors of production, we have made the conveniently simplifying assumption that the single good/service produced in the economy can be alternatively used for consumption or physical capital accumulation purposes. To add further convenience, however, we assume that such single good/service can also be used for knowledge capital accumulation. In the long-run equilibrium, therefore, the growth rate of output, g^*, can be measured by the growth rate of either kind of capital, given that both physical and knowledge capital grow at the same rate in the long-run equilibrium:

$$g^* = g_K^* = g_H^*. \tag{19}$$

By virtue of the demand-led nature of the model, the short-run equilibrium rates of physical capital capacity utilization and employment (and therefore the long-run equilibrium growth rate of output) all vary positively with the parameters α, β and γ, and negatively with the capitalists' saving rate, the interest rate and the debt ratio. Meanwhile, the effect of a rise in the wage share (which, per (3), also measures the wage return on knowledge capital) on the short-run equilibrium value of the rate of physical capital utilization is given by:

$$\frac{\partial u^*}{\partial \sigma} = u_\sigma^* = \frac{(s+\gamma)(\alpha - si\delta)}{\Omega^2} > 0. \tag{20}$$

Therefore, *ceteris paribus*, a rise in the wage share, by redistributing income from capitalists' households who save to workers' households who spend in consumption all of their net wage income, raises both consumption demand and aggregate effective demand and thereby boosts the rates of physical capital utilization and employment. In fact, since investment demand includes a double accelerator effect (one working through investment in physical capital, the second operating via investment in knowledge capital), aggregate effective demand increases even further. Also, per (19), an increase in the wage share raises the growth rate of output in the long-run equilibrium. Meanwhile, (17)-(18) show that, *ceteris paribus*, a rise in the ratio of physical capital to knowledge capital, k, despite leaving the short-run equilibrium rate of physical capital utilization unchanged, raises the short-run equilibrium rate of employment.

As we assume that employed workers are always able to collect the full wage return on their knowledge capital, the distribution of income between wages and profits remains unchanged. Yet we can still explore changes in the distribution of income by using the following index of inequality given by the ratio of the income of profit-cum-interest recipients to the net income of working households, which we borrow from Dutt (2006):

$$l = \frac{(1-\sigma)u + i\delta}{\sigma u - i\delta}. \tag{21}$$

Therefore, the wage share in income, σ, the interest rate, i, and the debt ratio, δ, affect the index of inequality in (21) both directly and, by affecting physical capital capacity utilization, indirectly. Note that, *ceteris paribus*, a rise in physical capital capacity utilization, by raising total income from production, reduces the extent of inequality. It can then be checked that in the short run the extent of inequality varies positively with the interest rate and the debt ratio, and negatively with the wage share.

4. Long-run equilibrium

In the long run we assume that the short-run equilibrium values of the variables are always attained, with the economy moving over time due to changes in the stocks of physical capital, K, knowledge capital, H, and working household's debt, D. Recall that we have assumed that the aggregate stock of knowledge capital remains uniformly distributed in the labor force (whose measure we have normalized to a constant equal to one), and that the level of labor productivity is equal to the average stock of knowledge capital, which together imply that the proportionate growth rates of the aggregate stock of knowledge capital and labor productivity remain one and the same. Thus, one way of following the behavior of the system over time is by investigating the dynamic behavior of the short-run state variables k, the ratio of physical capital stock to knowledge capital stock, and δ, the ratio of working households' stock of debt to physical capital stock.[5] It is logically possible that the debt ratio is negative in a given short run, with working households being net creditors, but we abstract from this possibility. From the definition of these two variables, we have the following state transition functions in terms of proportionate growth rates:

$$\hat{k} = \hat{K} - \hat{H}\,, \tag{22}$$

and:

$$\hat{\delta} = \hat{D} - \hat{K}\,. \tag{23}$$

Substitution of (6), (7) and (9) into (22) yields:

$$\hat{k} = \alpha + (\beta - \gamma\sigma k)u^{*}, \tag{24}$$

where u^{*} is given by (17).

Meanwhile, (23) can be re-written as follows:

$$\hat{\delta} = \frac{\dot{D}}{K}\frac{K}{D} - \hat{K} = \frac{I_H}{\delta K} - g_K\,. \tag{25}$$

Therefore, substitution from (7) and (10) into (25) yields:

$$\hat{\delta} = \frac{\gamma\sigma u^{*}}{\delta} - \alpha - \beta u^{*}, \tag{26}$$

where u^{*} is given again by (17).

Equations (24) and (26), after using (17), constitute a planar autonomous two-dimensional system of differential equations in which the proportionate growth rates of k and δ depend on the levels of k and δ and parameters of the system.

[5] Of course, in a long-run equilibrium with constant values of the ratios of physical capital stock to knowledge capital stock and working households' stock of debt to physical capital stock, the ratio of working households' stock of debt to knowledge capital stock is also constant. Also, since in a long-run equilibrium the rate of physical capital utilization (as measured by the ratio of output to physical capital) is constant, the ratio of working households' stock of debt to output is constant as well.

Solving (26) for the steady state where $\hat{\delta} = 0$ is equivalent to solving the following quadratic equation for δ:

$$(\gamma\sigma - \beta\delta)(\alpha - si\delta) - \alpha\delta\Omega = 0 . \tag{27}$$

In Appendix A-1, we demonstrate that the quadratic equation in (27) has two strictly positive solutions, δ_1^* and δ_2^*, with $\delta_1^* < \delta_2^*$. In this Appendix A-1 we demonstrate as well that the sign of the partial derivative J_{22} in (31) or (31'') below is negative (positive) when evaluated at δ_1^* (δ_2^*). However, a necessary condition for a positive J_{22} is $\delta^* > \alpha / si$, which implies a strictly negative value for the short-run equilibrium value of physical capital utilization, u^*, when the Keynesian stability condition is satisfied, as discussed right after (17). As a result, $\delta_1^* > 0$, which implies a negative sign for J_{22}, is the only economically relevant solution of the quadratic equation in (27). In fact, given that the state transition function for δ in (27) does not depend on k, δ_1^* is actually the only strictly positive long-run equilibrium value for δ which is economically relevant.

The Jacobian matrix of partial derivatives for the system of differential equations composed by (24) and (26), after using (17), and evaluated at the only economically relevant stationary point, is given by:

$$J_{11} = \frac{\partial \hat{k}}{\partial k} = -\frac{\gamma\sigma(\alpha - si\delta^*)}{\Omega} < 0 , \tag{28}$$

$$J_{12} = \frac{\partial \hat{k}}{\partial \delta} = \frac{si(\gamma\sigma k^* - \beta)}{\Omega} , \tag{29}$$

$$J_{21} = \frac{\partial \hat{\delta}}{\partial k} = 0 , \tag{30}$$

$$J_{22} = \frac{\partial \hat{\delta}}{\partial \delta} = \frac{\gamma\sigma u_\delta^*}{\delta^*} - \frac{\gamma\sigma u^*}{(\delta^*)^2} - \beta u_\delta^* . \tag{31}$$

where $u_\delta^* = (\partial u^* / \partial \delta)$, which is negative when evaluated at a positive stationary solution for δ, as discussed in the preceding section. In principle, it seems that not all of these partial derivatives can be unambiguously signed. The sign of J_{11} is unambiguously negative, given that a higher k, *ceteris paribus*, by lifting the employment rate, raises the growth rate of the stock of knowledge capital while leaving unchanged the growth rate of the stock of physical capital. The sign of J_{12} seems to be ambiguous, given that, *ceteris paribus*, a higher δ lowers both the rate of physical capital utilization (and hence the growth rate of the stock of physical capital) and the rate of employment (and hence the growth rate of the stock of knowledge capital). The reason for the sign of J_{21} is that neither workers' households desired level of investment in knowledge capital as a proportion of the stock of physical capital nor the growth rate of the stock of physical capital depend on the employment rate (and hence on k). At first sight, the sign of J_{22} is ambiguous, given that a higher debt ratio, *ceteris paribus*, by having a negative effect on the rate of physical capital utilization, lowers workers' households desired level of investment in knowledge capital as a proportion of the stock of physical capital and the growth rate of the physical capital stock. Note, however, that (31) can be re-written as follows:

$$J_{22} = \frac{\partial \hat{\delta}}{\partial \delta} = -\frac{\gamma \sigma u^*}{(\delta^*)^2} - \frac{\gamma \sigma si}{\Omega \delta^*} + \frac{\beta si}{\Omega} = -\frac{\gamma \sigma u^*}{(\delta^*)^2} - \frac{si}{\Omega}\left(\frac{\gamma \sigma}{\delta^*} - \beta\right). \qquad (31')$$

Meanwhile, setting (24) to zero to obtain a relationship between k and u^*, and then setting (26) to zero to obtain a relationship between δ and u^*, we obtain the following relationship between the long-run equilibrium values of the two state variables of the respective dynamic system:

$$k^* = \frac{1}{\delta^*}. \qquad (32)$$

Thus, we can further re-write (31') by manipulating the term in parentheses in it to obtain:

$$J_{22} = \frac{\partial \hat{\delta}}{\partial \delta} = -\frac{\gamma \sigma u^*}{(\delta^*)^2} - \frac{si}{\Omega}\frac{\alpha}{u^*} < 0, \qquad (31'')$$

which confirms that the sign of J_{22} is negative when evaluated at the economically relevant $\delta^* > 0$, as intimated earlier. Meanwhile, by similarly manipulating the term in parentheses in (29), we can re-write this equation as follows:

$$J_{12} = \frac{\partial \hat{k}}{\partial \delta} = \frac{si\alpha}{\Omega u^*}, \quad (29')$$

which confirms that the sign of J_{12} is positive.

Therefore, given that the Jacobian matrix represented by (28)-(31) has a positive determinant, $Det(J) = J_{11}J_{22} > 0$, and a negative trace, $Tr(J) = J_{11} + J_{22} < 0$, the long-run equilibrium configuration with $\hat{k} = \hat{\delta} = 0$, which is given by $(k, \delta) = (k^*, \delta^*)$, is locally stable. In fact, as portrayed in Figure 4, this long-run equilibrium is a locally stable node. Given that J_{12} is positive, the slope of the $\hat{k} = 0$ isocline, which is given by $-(J_{11} / J_{12})$, is positive. Since $\partial \hat{k} / \partial \delta$ is positive, $\hat{k}$ undergoes a steady rise as δ increases, so that the sign of $\hat{k}$ is negative (positive) to the right (left) of the $\hat{k} = 0$ locus, which explains the direction of the horizontal arrows. Meanwhile, the slope of the $\hat{\delta} = 0$ isocline, which is given by $-(J_{21} / J_{22})$, is equal to zero. Given that $\partial \hat{\delta} / \partial \delta < 0$, it follows that $\hat{\delta}$ undergoes a steady decrease as δ increases, so that the sign of $\hat{\delta}$ is positive (negative) below (above) the $\hat{\delta} = 0$ isocline, which explains the direction of the vertical arrows.

5. Alternative specification of working households' debt servicing

The specification in (13) assumes that workers' households (who accumulate knowledge capital through debt) allocate to consumption all their disposable wage income (which is the entire wage bill net of any debt service). In this section, we explore the implications of the alternative assumption that debt servicing by working households follows an income-driven repayment plan bearing some similarity to repayment plans applicable to federal student loan payments in the U.S.[6] More precisely, working households' consumption is now given by:

[6] The main safety net available to U.S. borrowers of federal student loans facing excessive monthly payments is income-driven repayment. Income-Based Repayment (IBR), available since 2009, is the most widely available such repayment plan for federal student loans (which comprise about 55% of

$$C_w = \phi VL\,, \tag{33}$$

where $0 < \phi < 1$ is a parameter, and we refer to $0 < 1-\phi < 1$ as repayment coefficient. Thus, the change in the stock of debt held by working households is now given by:

$$\dot{D} = I_H + iD - (1-\phi)VL\,. \tag{34}$$

Following the same steps leading to (17), but using (33) instead of (13), we can solve for the short-run equilibrium rate of physical capital utilization to obtain:

$$u^* = \frac{\alpha + (1-s)i\delta}{1-(1-s)(1-\sigma)-(\phi+\gamma)\sigma-\beta}\,. \tag{35}$$

To save on notation, we define the denominator in (35) as $\Psi \equiv 1-(1-s)(1-\sigma)-(\phi+\gamma)\sigma-\beta$. To ensure that the demand-led output-adjustment stability condition known as the Keynesian stability condition is satisfied, we assume that aggregate effective demand as a proportion of the physical capital stock, which is given by the sum of the three parameterized terms in Ψ, is less responsive to changes in the rate of physical capital utilization than aggregate supply as a proportion of the physical capital stock, which is equivalent to a positive value for Ψ.

Using (6), the short-run equilibrium rate of employment (or knowledge capital utilization) can be obtained:

$$e^* = \frac{[\alpha + (1-s)i\delta]k}{\Psi}\,. \tag{36}$$

Meanwhile, the growth rate of output in the long-run equilibrium is again given by (19), but with the short-run equilibrium physical capital utilization being given by (35) instead of (17).

As in the preceding section, the demand-led nature of the model implies that the short-run equilibrium rates of physical capital utilization and knowledge capital utilization (and hence

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the total stock of student loans). Monthly payments are 10% or 15% of discretionary income, and are recalculated each year based on the updated income and family size. Any outstanding balance will be forgiven if not repaid in full after 20 or 25 years. Mueller and Yannelis (2017) provide evidence that the IBR program has been successful at reducing student loan defaults. Another program is Income-Contingent Repayment Plan (ICR), in which monthly payments are the lesser of 20% of discretionary income or the amount that would be paid on a repayment plan with a fixed payment over 12 years, adjusted according to the borrower's income. Any outstanding balance will be forgiven if not repaid in full after 25 years.

the long-run equilibrium rate of growth of output) all vary positively with the parameters α, β and γ, and negatively with the capitalists' rate of saving. Yet these endogenous variables now vary positively instead of negatively with each of the separate variables which compose the debt service, which are the interest rate and the debt ratio. The intuition is that, per (33)-(34), changes in the debt service received (and partially spent on consumption) by capitalists' households does not alter the proportion of the wage income (whose net value is all spent on consumption) allocated to debt servicing. In fact, (34) shows that what represents a deduction of wage income is now the repayment amount, which depends on the repayment coefficient, $1-\phi$. Thus, if the amount of wage income allocated to debt servicing is insufficient to fully serve the outstanding debt, i.e. $(1-\phi)VL < iD$, the stock of debt held by working households will increase due not only to knowledge capital accumulation, but to insufficient repayment provision as well. Meanwhile, the impact of an increase in the wage share in income (which, per (3), also measures the wage return on knowledge capital) on the short-run equilibrium rate of physical capital utilization is given by:

$$\frac{\partial u^*}{\partial \sigma} = u_\sigma^* = \frac{\left[(\phi+\gamma)-(1-s)\right]\left[\alpha+(1-s)i\delta\right]}{\Psi^2}. \tag{37}$$

The intuition for the ambiguity in the sign of (37) is straightforward. A rise in the wage share, *ceteris paribus*, by redistributing income from capitalists' households who save to workers' households who spend in consumption all of their net wage income, may or may not raise consumption demand. The reason is that the working households' repayment provision given by $(1-\phi)VL$ represents a leakage of consumption demand analogous to any act of pure saving by workers. Meanwhile, a rise in the wage share in income, by raising working households' desired investment in knowledge capital as a proportion of the physical capital stock, expands aggregate effective demand as a proportion of the physical capital stock and thereby exerts an upward pressure on the rates of physical capital utilization and employment. Therefore, a rise in the wage share produces an increase in the short-run equilibrium rates of physical capital utilization and employment (and hence in the long-run equilibrium value of the rate of output growth) provided that $(\phi+\gamma) > (1-s)$.

Recall that the wage share, the interest rate, and the debt ratio all affect the index of inequality in (21) both directly and, by affecting physical capital utilization, indirectly. Recall also that, *ceteris paribus*, an increase in the physical capital utilization reduces the extent of inequality. In the case of the alternative specification of the debt servicing in (33), although the short-run equilibrium physical capital utilization varies positively with the interest rate and the debt ratio, it can be checked that the index of inequality in (21) ultimately varies positively with any of these two separate components of the debt servicing, as in the original specification in (13). Besides, although the alternative specification in (33) implies that the impact of a rise in the wage share on physical capital utilization is ambiguous, it can be checked that the index of inequality in (21) ultimately unambiguously vary negatively with the wage share.

5.1 Long-run dynamics again

As in the preceding section, in the long-run analysis we follow the dynamic behavior of the short-run state variables k, the ratio of physical capital to knowledge capital, and δ, the ratio of working households' stock of debt to physical capital. The state transition function for k is still given by (24), but with u^* now given by (35), whereas the new state transition function for δ, given (34), is the following:

$$\hat{\delta} = -\frac{(1-\phi-\gamma)\sigma u^*}{\delta} - \beta u^* - \alpha + i\,, \tag{38}$$

where u^* is likewise given by (35).

Equations (24) and (38), after using (35), constitute a planar autonomous two-dimensional system of differential equations in which the proportionate growth rates of k and δ depend on the levels of k and δ and parameters of the system. As in the original specification of the debt servicing in the preceding section, it follows that solving $\hat{\delta}$ for the steady state where $\hat{\delta} = 0$ implies solving a quadratic equation in δ. Yet unlike in that specification, in which one of the solutions (which are both positive) is ruled out because it implies a negative rate of physical capital utilization, the quadratic equation in (38) can have two economically relevant solutions (recall that we abstract from the logical possibility that the debt ratio is negative).

As there are reasonable parameters values that ensure the emergence of multiple long-run equilibria, in what follows we explore this interesting possibility via numerical simulation.[7]

The Jacobian matrix of partial derivatives for the system of differential equations composed by (24) and (38), after using (35), and evaluated at a positive stationary point, is given by:

$$J_{11} = \frac{\partial \hat{k}}{\partial k} = -\frac{\gamma\sigma\left[\alpha + (1-s)i\delta^*\right]}{\Psi} < 0, \tag{39}$$

$$J_{12} = \frac{\partial \hat{k}}{\partial \delta} = \frac{i(1-s)(\beta - \gamma\sigma k^*)}{\Psi}, \tag{40}$$

$$J_{21} = \frac{\partial \hat{\delta}}{\partial k} = 0, \tag{41}$$

$$J_{22} = \frac{\partial \hat{\delta}}{\partial \delta} = -\frac{1}{\Psi}\left[(1-s)\beta i - \frac{\alpha\sigma(1-\phi-\gamma)}{(\delta^*)^2}\right]. \tag{42}$$

The sign of J_{11} is negative, as a higher k, *ceteris paribus*, by boosting the employment rate, raises the growth rate of the stock of knowledge capital while leaving unchanged the growth rate of the stock of physical capital. In principle, the sign of J_{12} seems to be ambiguous, given that, *ceteris paribus*, a higher δ raises both the rate of physical capital utilization (and hence the growth rate of the stock of physical capital) and the rate of employment (and hence the growth rate of the stock of knowledge capital). Setting (24) to zero, though, it follows that $\beta - \gamma\sigma k = -\alpha / u^*$, so that the sign of J_{12} is unambiguously negative. Intuitively, the positive effect of a rise in the debt ratio is stronger on the growth rate of the knowledge capital stock than on the growth rate of the physical capital stock (compare (7) and (9), both evaluated at a positive long-run equilibrium). The justification for the sign of J_{21} is that neither working households' level of investment in knowledge capital as a proportion of the stock of physical capital nor the growth rate of the stock of physical capital depend on the employment rate (and hence on k). The sign of J_{22}, however, is ambiguous, given that a higher debt ratio,

[7] The possible such set of reasonable parameters that we use for illustrative purposes is the following: $\alpha = 0.02$; $\beta = 0.05$; $\gamma = 0.05$; $\sigma = 0.5$; $s = 0.7$; $i = 0.03$; and $\phi = 0.8$. For the plausibility of the latter value, which implies that the repayment coefficient is equal to 0.2, see footnote 6.

ceteris paribus, by positively affecting the rate of physical capital utilization, raises both working households' level of investment in knowledge capital as a proportion of the stock of physical capital and the rate of growth of the physical capital stock. Therefore, if the accelerator effect operating through investment in knowledge capital (see (9)) and the demand-injection effect operating through net wage income (see (33)-(34)), when combined, are strong enough so as to imply $\phi+\gamma>1$, it follows that J_{22} is negative. Note that the same condition is sufficient (but not necessary) for the rates of physical capital utilization, labor employment and output growth to vary positively with the wage share in income in the long-run equilibrium (see (37)). Also, J_{22} can still be negative even if $\phi+\gamma<1$ (which does not imply, also, that the the rates of physical capital utilization, employment and output growth vary negatively with the wage share in the long-run equilibrium). Though the sign of J_{22} is ambiguous, the parameters values that we assume to ensure the occurrence of multiple long-run equilibrium imply that J_{22} is positive when evaluated at δ_1^*, and negative when evaluated at δ_2^*, where $\delta_1^*<\delta_2^*$. This multiple equilibria configuration is shown in Figure 5.[8]

Let us first consider the dynamic implications of a negative sign for J_{22}. Since the Jacobian matrix represented by (39)-(42) would have a positive determinant, $Det(J)=J_{11}J_{22}>0$, and a negative trace, $Tr(J)=J_{11}+J_{22}<0$, the long-run equilibrium configuration with $\hat{k}=\hat{\delta}=0$, given by $(k,\delta)=(k_2^*,\delta_2^*)$, is a locally stable node. Since $\partial\hat{k}/\partial\delta$ is negative, $\hat{k}$ undergoes a steady fall as δ increases, so that the sign of $\hat{k}$ is negative (positive) to the right (left) of the $\hat{k}=0$ locus, which explains the direction of the horizontal arrows. Meanwhile, the slope of the two $\hat{\delta}=0$ isoclines, given by $-(J_{21}/J_{22})$, is equal to zero. As $\partial\hat{\delta}/\partial\delta$ is negative when evaluated at δ_2^*, and positive when evaluated at δ_1^*, it follows that $\hat{\delta}$ first undergoes a steady increase and later a steady decrease as δ increases. Thus, the sign of $\hat{\delta}$ is negative (positive)

[8] The set of parameter values specified in the preceding footnote yields long-run equilibrium values given by $\delta_1^*=0.898$ and $\delta_2^*=3.713$. In Appendix A-2, we show through simulation that in this situation the sign of J_{22} falls monotonically from positive to negative values as δ rises from zero to upper economically relevant values.

below (above) the $\hat{\delta}_1 = 0$ isocline, and positive (negative) below (above) the $\hat{\delta}_2 = 0$ isocline, which explains the direction of the vertical arrows. The long-run equilibrium with $\hat{k} = \hat{\delta} = 0$ given by $(k, \delta) = (k_1^*, \delta_1^*)$ is then saddle-point unstable. In fact, the Jacobian matrix in (39)-(42), when evaluated at such equilibrium, has a negative determinant, $Det(J) = J_{11}J_{22} < 0$. This unstable long-run equilibrium with lower debt ratio is also characterized by lower rates of physical capital utilization, employment and growth (and a higher index of inequality) than the stable equilibrium with higher debt ratio.

Note that the long-run configuration with multiple equilibria portrayed in Figure 5 vanishes if we alternatively assume, *ceteris paribus*, that the desired investment in physical capital in (7) has no autonomous component, so that $\alpha = 0$. In Appendix A-3, we show that there is a set of plausible parameter values ensuring that the quadratic equation in (38) has only one economically relevant solution characterized by $\delta^* > 0$ (recall that although a negative debt ratio is logically possible, in which case working households are net creditors, we abstract from this possibility). Meanwhile, this alternative assumption does not change the comparative static results for the short-run equilibrium rates of physical capital utilization and employment in (35) and (36) obtained earlier (except, of course, the results concerning the autonomous component α). The same applies to the results regarding the index of inequality associated with (35) evaluated at $\alpha > 0$, also derived earlier. Furthermore, it can be easily checked that the Jacobian matrix given by (39)-(42), when evaluated with $\alpha = 0$ and at $\delta^* > 0$, features both a positive determinant, $Det(J) = J_{11}J_{22} > 0$, and a negative trace, $Tr(J) = J_{11} + J_{22} < 0$, so that the long-run equilibrium is a locally stable node similar to the one given by $(k, \delta) = (k_2^*, \delta_2^*)$ in Figure 5.

To gain further insight on the qualitative behavior of the model, in the next sub-item we make another alternative assumption regarding the dynamics of aggregate effective demand. *Ceteris paribus*, we assume that capitalist households save all of their profit and interest income, which results in a larger leakage of aggregate effective demand. This simplifying assumption will then allow us to more tractably and transparently explore the further relevant issue of the relationship between macroeconomic *instability* and working households' *financial fragility*, as elaborated in what follows.

5.2 Minskyan taxonomy and financial fragility

The simplified version intimated in the preceding paragraph, in which capitalists' households are assumed to save all of their profit and interest income, allows us to tractably explore the significant issue of the relationship between macroeconomic *instability* (in the sense of the preceding dynamic analysis) and working households' *financial fragility*. Indeed, according to the financial instability view nicely developed by Hyman Minsky (1975, 1982), the capital development of a market economy involves exchanges of present money for future money. In the case of physical capital accumulation by firms, the present money is required to pay for resources that participate in the production of investment output, whereas the future money is the amount of profit income that will accrue to firms as their capital assets are utilized in production. Consequently, it is fitting to conceive of working households' knowledge capital accumulation via debt in a similar way. As laid out in our model, the present (credit-)money is required to finance investment spending in knowledge capital accumulation by workers, whereas the future money is the sum of wage income that workers will receive when their knowledge capital assets are *actually* employed in production.[9] Given the process by which investment in knowledge capital by workers is financed, the liabilities on their balance sheet determine a series of prior payment commitments, while their knowledge capital assets generate a series of conjectured cash inflows. Given that investment in knowledge capital accumulation is financed by credit-money generated by workers' borrowing, the flow of credit-money to workers is a response to their expectations of higher future wage income, whereas the flow of debt service from workers is financed out of wage income that is actually realized. Thus, the debt-financed nature of the investment in knowledge capital considered in this paper is consistent with Minsky's understanding of his financial instability theory as an

[9] In fact, as noted by Minsky himself: "In the modern world, analyses of financial relations and their implications for system behavior cannot be restricted to the liability structure of businesses and the cash flows they entail. Households (by the way of their ability to borrow on credit cards for big ticket consumer goods such as automobiles, house purchases, and to carry financial assets), governments… and international units…have liability structures which the current performance of the economy either validates or invalidates." (1992, pp. 4-5)

explanation of both the impact of debt on system behavior and the manner in which debt is validated.

According to Minsky's (1982) broad characterization, *hedge* financing units are those that can fulfill all of their contractual payment obligations out of their cash flows, whereas *speculative* units are those that can meet their interest payment commitments on outstanding debts, even as they are unable to repay the principle out of income cash flows. For *Ponzi* units, though, the cash flow from operations is insufficient to fulfill either the repayment of principle or the interest due on outstanding debts.[10] To formalize this taxonomy of finance regimes in terms of a working household's cash flow accounting categories, we follow the manner in which Foley (2003) and Lima and Meirelles (2007) neatly do so for a firm.

In a highly aggregated form, the relevant cash flow identity in this paper equates the working household's source of funds from net wage income, W, and new borrowing, B, to its uses of funds for investment in knowledge capital accumulation, I_H, and debt service, F:

$$W + B \equiv I_H + F\,. \tag{43}$$

Therefore, the change in debt, $\dot{D} = dD/dt$, is given by new borrowing:

$$\dot{D} = B = I_H + F - W\,. \tag{44}$$

The Minskyan taxonomy, meanwhile, can be derived as follows:

$$\textit{Hedge}\text{:}\quad \mathrm{W} \geq I_H + F \text{ or } B \leq 0\,. \tag{45}$$

$$\textit{Speculative}\text{:}\quad F \leq \mathrm{W} < I_H + F \text{ or } I_H \geq B > 0\,. \tag{46}$$

$$\textit{Ponzi}\text{:}\quad \mathrm{W} < F \text{ or } B > I_H\,. \tag{47}$$

In this section, we have $W \equiv (1-\phi)VL$ and $F \equiv iD$, so the change in the working household's debt is correspondingly given by (34), which we reproduce below for ease of reference:

$$\dot{D} = I_H + iD - (1-\phi)VL\,. \tag{34}$$

[10] Therefore, in the specification of debt servicing used in Section 3, in which working households are assumed to consume their entire wage income net of debt service (see (13)), our further assumption that working households' consumption is always strictly positive is equivalent to imposing restrictions on the parameters that ensure that workers are never in a Ponzi financing regime.

Therefore, as a proportion of the physical capital stock, the Minskyan taxonomy in (45)-(47) can be expressed as follows:

$$\textit{Hedge}:\ (1-\phi)\sigma u^* \geq \gamma\sigma u^* + i\delta. \tag{48}$$

$$\textit{Speculative}:\ i\delta \leq (1-\phi)\sigma u^* < \gamma\sigma u^* + i\delta. \tag{49}$$

$$\textit{Ponzi}:\ (1-\phi)\sigma u^* < i\delta. \tag{50}$$

As intimated in the end of the preceding section, let us assume that capitalists save all of their profit and interest income. Under this assumption, it follows from the revised version of (35) that the rate of physical capital utilization (but not the rate of employment and the equilibrium rate of output growth in the long run) becomes exogenous, though the revised version of (37) shows that a higher wage share is accompanied by higher rates of physical capital utilization, employment and output growth in the long-run equilibrium. We can solve for the respective demarcation lines in the $(k,\delta)-$space to obtain:

$$\delta_{H-S} = \frac{(1-\phi-\gamma)\alpha\sigma}{i\Theta}, \tag{51}$$

and:

$$\delta_{S-P} = \frac{(1-\phi)\alpha\sigma}{i\Theta}, \tag{52}$$

where $\Theta \equiv 1-\beta-(\phi+\gamma)\sigma$, which is assumed to be positive to ensure a positive value for the (now exogenous) rate of physical capital utilization. Note that (51) and (52) do not depend on k, so that these demarcation lines are parallel to each other and the horizontal axis.

Meanwhile, in this simplified specification the long-run equilibrium with $\hat{k} = \hat{\delta} = 0$ is given by the following pair:

$$k^* = \frac{1-(\phi+\gamma)\sigma}{\gamma\sigma}, \tag{53}$$

and:

$$\delta^* = \frac{(1-\phi-\gamma)\alpha\sigma}{(i-\alpha)\Theta-\alpha\beta}. \tag{54}$$

To ensure that (51) is positive, which implies that the speculative finance regime arguably involves positive levels of the debt ratio, we further assume that $\phi + \gamma < 1$. This condition also ensures that k^* is positive, whereas a positive value for Θ (which is required to ensure a positive rate of physical capital utilization) requires that $(\phi + \gamma)\sigma < 1 - \beta$. Meanwhile, the assumption that $\phi + \gamma < 1$ implies a positive value for the numerator in (54), so that a positive value for (54) requires $(i - \alpha)\Theta > \alpha\beta$, a necessary condition for which is $i > \alpha$. As we further assume that the condition for a positive δ^* is satisfied, it can then be checked that this further condition implies that $\delta^* > \delta_{H-S}$. In fact, there are reasonable parameters values that ensure that $\delta^* > \delta_{S-P}$.[11] As a result, the unique long-run equilibrium in this further simplified version with capitalists saving all their income is situated in the Ponzi financing regime.

This long-run configuration is shown in Figure 6. It can be checked that the Jacobian matrix in (39)-(42) features a negative determinant, so that the unique long-run equilibrium is saddle-point unstable. Given that $\partial\hat{k} / \partial k$ is negative, $\hat{k}$ undergoes a steady decrease as k increases, so that the sign of $\hat{k}$ is positive (negative) to the left (right) of the vertical $\hat{k} = 0$ locus, which explains the direction of the horizontal arrows. Meanwhile, given that $\partial\hat{\delta} / \partial\delta$ is positive, $\hat{\delta}$ undergoes a steady increase as δ increases, so that the sign of $\hat{\delta}$ is negative (positive) below (above) the horizontal $\hat{\delta} = 0$ isocline, which then explains the direction of the vertical arrows. It follows that the unstable arm of the saddle point is the vertical $\hat{k} = 0$ locus, while the stable arm (on which the system will be only by fluke) is the horizontal $\hat{\delta} = 0$ locus. Therefore, this

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[11] The possible such set of reasonable parameters that we use for illustrative purposes is the following: $\alpha = 0.02$; $\beta = 0.05$; $\gamma = 0.05$; $\sigma = 0.5$; $s = 1.0$; $i = 0.03$; and $\phi = 0.8$. For the plausibility of the latter, which implies a repayment coefficient equal to 0.2, see footnote 6. This set of parameter values yields $\delta^* = 0.35$ and $k^* = 23$, while the demarcation lines in (51) and (52) are given, respectively, by $\delta_{H-S} = 0.095$ and $\delta_{S-P} = 0.127$. Meanwhile, going back to the multiple long-run equilibria in Figure 5, in Appendix A-4 we demonstrated through simulation that both equilibria are located in the Ponzi region, as (already) shown in that figure. Moreover, these two equilibria are given by $\delta_1^* = 0.898$ and $\delta_2^* = 3.713$ (see footnote 8), while the demarcation lines are given by $\delta_{H-S} = 0.142$ and $\delta_{S-P} = 0.193$. In Figure 5, therefore, though the equilibrium with higher debt ratio is locally stable, it is nonetheless situated in the Minskyan regime of highest financial fragility.

long-run equilibrium configuration is characterized by a perverse combination of working households' financial fragility and macroeconomic instability.

6. Conclusion

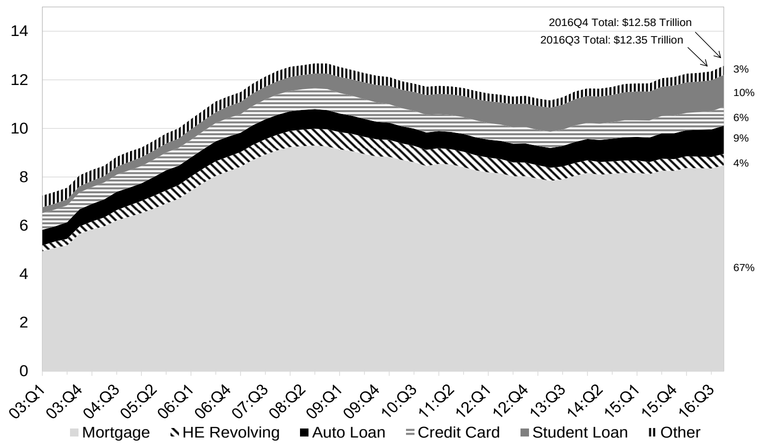
The role of human capital as a source of economic growth has been extensively explored in the literature. As it is typically assumed that economies are always growing at full capacity, though, it is neglected both the role of aggregate demand in growth dynamics and the impact of autonomous investment in human capital formation in and of itself on aggregate effective demand. Meanwhile, demand-driven approaches to growth dynamics have typically relegated any closer attention to human capital formation through education (and to 'knowledge' capital accumulation more broadly) as narrowly supply-sided.

As the making of costly investments in human capital accumulation through education is indeed one of the main forms of productivity-enhancing knowledge accumulation, this paper explores several dynamic implications of debt-financed accumulation of knowledge capital, along with its resulting positive impact on labor productivity, within a demand-led model. Although our model is not intended to describe specifically debt-financed knowledge capital formation through student loans, the recent U.S. experience with student debt is illustrative of the significance of working households' financing of human capital formation via debt and how this is a possible route to financial fragility and macroeconomic instability. The model features knowledge capital accumulation as an additional source of aggregate demand along with expenditures in consumption and investment in physical capital. Given that the aggregate stock of knowledge capital remains uniformly distributed across workers, it then follows that unemployed labor also means unutilized knowledge capital.

Although any increase in labor productivity brought about by knowledge capital accumulation is fully passed on to the real wage, the employment rate is determined by aggregate effective demand. It follows that the short- and long-run equilibrium rates of physical capital utilization and labor employment and the long-run equilibrium rate of output growth may all vary either positively or negatively with the wage return on knowledge capital. Meanwhile, we explore changes in the distribution of income by using an index of inequality given by the ratio of the income of profit-cum-interest recipients to the net income of working households. For all the specifications of working households' debt servicing explored in the paper, we obtain that the index of inequality varies positively with the interest rate and the working households' debt to physical capital ratio and negatively with the wage share and the physical capital utilization.

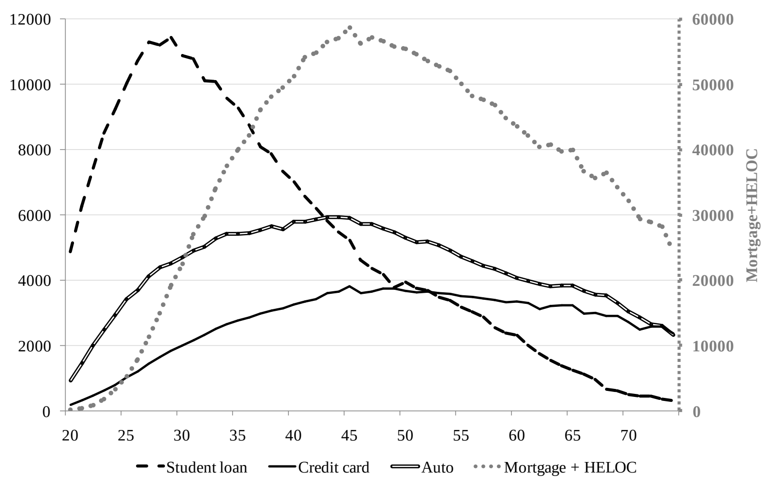
FIGURES

Figure 1: Household debt balance: total and composition (In trillions of dollars)*



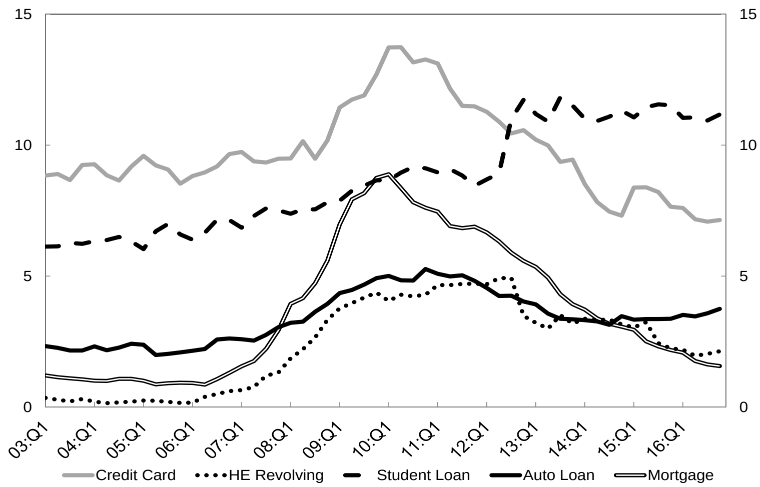
Source: FRBNY Consumer Credit Panel / Equifax.
*HE Revolving: Home Equity Revolving. Percentage values alongside the 2016Q4 bar indicate the composition of household debt in that quarter only.

Figure 2: Average Debt per Capita by Age in 2015 (in 2015 dollars)*



Source: Brown et al. (2015) with data from FRBNY Consumer Credit Panel / Equifax.
*HELOC: Home Equity Line of Credit.

Figure 3: Percent of balance 90+ days delinquent by loan type



Source: FRBNY Consumer Credit Panel / Equifax.

*HE Revolving: Home Equity Revolving.

Figure 4: Stable long-run equilibrium with standard debt repayment

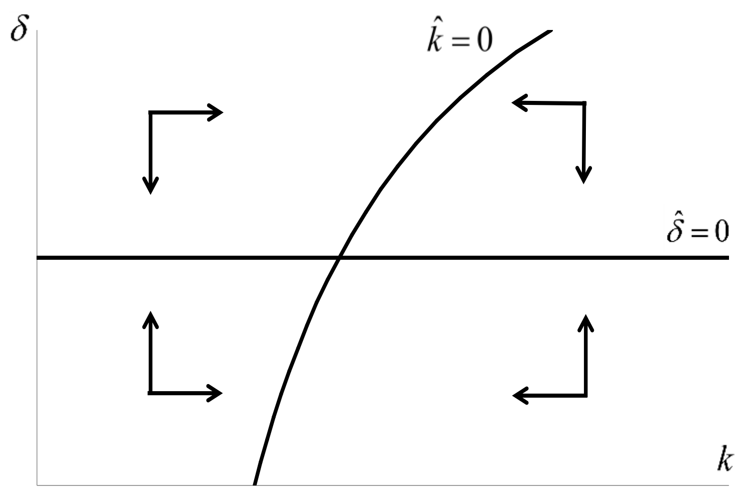


Figure 5: Multiple long-run equilibria with income-driven debt repayment

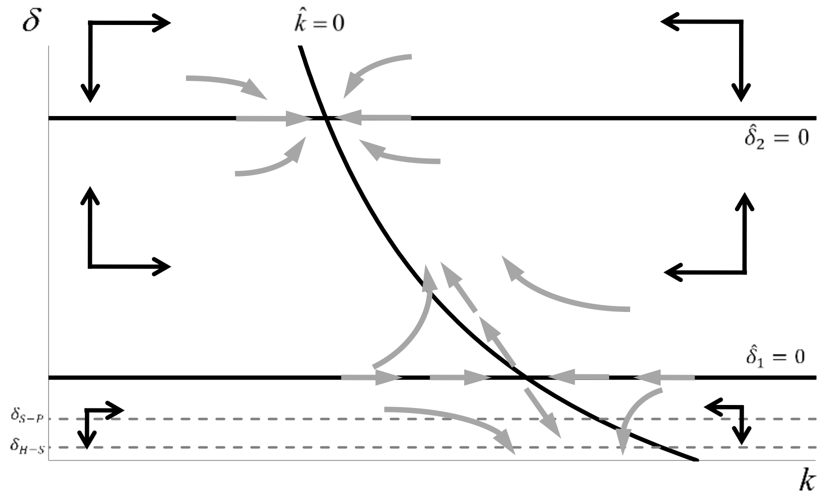
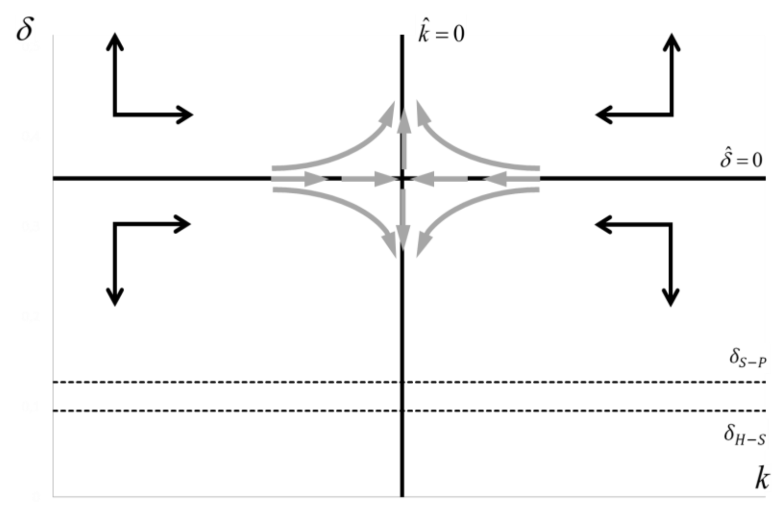


Figure 6: Saddle-point unstable long-run equilibrium with income-driven debt repayment by workers and capitalists saving all their income



References

Brown, M., Haughwout, A., Lee, D., Scally, J. and van der Klaauw, W. (2015): "Payback time? Measuring progress on student debt repayment", Federal Reserve Bank of New York, Liberty Street Economics. Acessed January 16, 2017. (http://libertystreeteconomics.newyorkfed.org/2015/02/payback_time_measuring_progress_on_student_debt_repayment.html)

Brown, M., Lee, D., Scally, J., Strair, K. and van der Klaauw, W. (2016): "The graying of American debt", Federal Reserve Bank of New York, Liberty Street Economics. Acessed January 16, 2017. (http://libertystreeteconomics.newyorkfed.org/2016/02/the-graying-of-american-debt.html)

Dutt, A. K. (2010): "Keynesian growth theory in the 21st century", in P. Arestis and M. Sawyer (Eds.): *Twenty-first Century Keynesian Economics*, Macmillan Palgrave, London, pp. 39-80.

Federal Research Bank of New York (FRBNY) (2016): "Quarterly Report on Household Debt and Credit", November.

Federal Research Bank of New York (FRBNY) (2017): "Quarterly Report on Household Debt and Credit", February.

Foley, D. (2003): "Financial fragility in developing economies", in: A. K. Dutt and J. Ros (Eds.) *Development Economics and Structuralist Macroeconomics*, Edward Elgar, Aldershot.

Kaldor, N. (1956): "Alternative theories of distribution", *Review of Economic Studies*, 23(2), pp. 83-100.

Kalecki, M. (1971): *Selected essays on the dynamics of the capitalist economy*, Cambridge University Press, Cambridge.

Lima, G. T., Carvalho, L. and Serra, G. P. (2017): "Human capital, income distribution and economic growth in a Neo-Kaleckian model", Department of Economics of the University of São Paulo, mimeo.

Lima, G. T. and Meirelles, A. J. A. (2007): "Macrodynamics of debt regimes, financial instability and growth", *Cambridge Journal of Economics*, 31(4), pp. 563-580.

Locher, L. and Monge-Naranjo, A. (2016): Student loans and repayment: theory, evidence and policy, in: E. A. Hanushek, S. Machin and L. Woessman (Eds.): *Handbook of the Economics of Education*, vol. 5, North-Holland, Amsterdam, pp. 397-478.

Lucas, R. E. (1988): "On the mechanics of economic development," *Journal of Monetary Economics*, 22(1), pp. 3-42.

Mankiw, G., Romer, D. and Weil, D. (1992): "A contribution to the empirics of economic growth", *The Quarterly Journal of Economics*, 107(2), pp. 407-437.

Minsky, H. (1975): *John Maynard Keynes*, Columbia University Press, New York.

Minsky, H. (1982): *Can "it" happen again? Essays on instability and finance*, M. E. Sharpe, New York.

Minsky, H. (1992) *The financial instability hypothesis*, The Levy Economics Institute, Working Paper No. 94, May.

Mueller, H. and Yannelis, C. (2017): 'Students in distress: Labor market shocks, student loan default, and federal insurance programs", NBER Working Paper No. 23284, March.

Pasinetti, L. (1962): "The rate of profit and income distribution in relation to the rate of economic growth", *Review of Economic Studies*, 29, pp. 267-79.

Robinson, J. (1962): *Essays in the theory of economic growth*, Macmillan, London.

Uzawa, H. (1965): "Optimal technical change in an aggregative model of economic growth", *International Economic Review*, 6(1), pp. 18-31.

Appendix

Appendix A-1

Let us demonstrate that the quadratic equation in (27) has two strictly positive solutions, δ_1^* and δ_2^*, with $\delta_1^* < \delta_2^*$, while δ_1^* is the only economically relevant one.

Note that the quadratic equation in (27) can be re-written as:

$$\beta si\delta^2 - \left[\gamma\sigma si + \alpha\left(\beta+\Omega\right)\right]\delta + \alpha\gamma\sigma = 0 . \tag{A-1.1}$$

Therefore, the solution of (A-1.1) is given by:

$$\delta^* = \frac{\left[\gamma\sigma si + \alpha\left(\beta+\Omega\right)\right] \pm \sqrt{\Delta}}{2\beta si}, \tag{A-1.2}$$

where:

$$\Delta = \alpha^2\left[s(1-\sigma)-\gamma\sigma\right]^2 + (\gamma\sigma si)^2 + 2\alpha\gamma\sigma si\left[s(1-\sigma)-\gamma\sigma-2\beta\right].$$

The condition for the solution of (A-1.1) to be unique is given by $\Delta = 0$. Imposing this condition and solving for β yields:

$$\beta = \frac{\left[s(1-\sigma)-\gamma\sigma\right]}{2} + \frac{\alpha^2\left[s(1-\sigma)-\gamma\sigma\right]^2 + (\gamma\sigma si)^2}{4\alpha\gamma\sigma si}. \tag{A-1.3}$$

Note that the assumed Keynesian stability condition related to the short-run equilibrium rate of physical capital capacity utilization in (17), which is given by $\Omega > 0$, can be re-written as follows:

$$\beta < s(1-\sigma)-\gamma\sigma . \tag{A-1.4}$$

Therefore, for both (A-1.3) and (A-1.4) to be simultaneously satisfied it is required that:

$$\beta = \frac{2\alpha\gamma\sigma si\left[s(1-\sigma)-\gamma\sigma\right] + \alpha^2\left[s(1-\sigma)-\gamma\sigma\right]^2 + (\gamma\sigma si)^2}{4\alpha\gamma\sigma si} < s(1-\sigma)-\gamma\sigma, \tag{A-1.5}$$

which can be re-written as:

$$\alpha^2\left[s(1-\sigma)-\gamma\sigma\right]^2-2\alpha\gamma\sigma si\left[s(1-\sigma)-\gamma\sigma\right]+(\gamma\sigma si)^2=\left\{\alpha\left[s(1-\sigma)-\gamma\sigma\right]-(\gamma\sigma si)\right\}^2<0.$$

Since the preceding condition cannot be satisfied, it follows that the solution of (A-1.1) is not unique. Besides, note that (A-1.3) implies that the respective condition for $\Delta<0$ cannot be satisfied either, so that we are left with $\Delta>0$. Therefore, the quadratic equation in (27) has two solutions. Given that $\left[\gamma\sigma si+\alpha(\beta+\Omega)\right]>0$, these two solutions are given by:

$$\delta_1^*=\frac{\left[\gamma\sigma si+\alpha(\beta+\Omega)\right]-\sqrt{\left[\gamma\sigma si+\alpha(\beta+\Omega)\right]^2-4\beta si\alpha\gamma\sigma}}{2\beta si}>0, \qquad \text{(A-1.6)}$$

and:

$$\delta_2^*=\frac{\left[\gamma\sigma si+\alpha(\beta+\Omega)\right]+\sqrt{\Delta}}{2\beta si}>0. \qquad \text{(A-1.7)}$$

Therefore, there are two strictly positive solutions, δ_1^* and δ_2^*, with $\delta_1^*<\delta_2^*$. Note that the sign of the partial derivative J_{22} in (31) or (31'') is negative (positive) when evaluated at the long-run equilibrium given by δ_1^* (δ_2^*). However, a necessary condition for a positive J_{22} is $\delta^*>\alpha/si$, which implies a strictly negative value for the short-run equilibrium value of physical capital utilization in (17), u^*, when the Keynesian stability condition is satisfied, as discussed right after (17). As a result, $\delta_1^*>0$, which implies a negative sign for J_{22}, is the only economically relevant solution of the quadratic equation in (A-1.1).

Appendix A-2

As regards the long-run equilibrium pictured in Figure 5, let us show through simulation that the sign of the partial derivative J_{22} falls monotonically from positive to negative values as δ rises from zero to upper economically relevant values. Using the parameter values indicated on footnote 7, note that the partial derivate in (42) has the following values as δ increases:

δ	J_{22}
0.05	1.5988
0.1	0.3988
0.2	0.0988
0.5	0.0148
0.897630456	0.003764378
1	0.0028
1.5	0.000577778
2	-0.0002
3	-0.000755556
3.713480655	-0.000909933
4	-0.00095
5	-0.00104

Parameter values: $\alpha = 0.02$; $\beta = 0.05$; $\gamma = 0.05$; $\sigma = 0.5$; $s = 0.7$; $i = 0.03$; and $\phi = 0.8$.

Appendix A-3

Let us demonstrate that there is a set of parameter values which ensures that (38) has a unique and strictly positive solution when $\alpha = 0$. Setting the expression in (38) to zero, after assuming that $\alpha = 0$, yields:

$$\hat{\delta} = -(1-\phi-\gamma)\sigma\frac{(1-s)}{\Psi} - \beta\frac{(1-s)\delta}{\Psi} + 1 = 0. \qquad \text{(A-3.1)}$$

Using the plausible parameter values indicated on footnote 7, which were already used in Appendix A-2, it follows that the expression in (A-3.1) solves for a unique long-run equilibrium debt ratio given by:

$$\delta^{*} = \frac{s[1-(\phi+\gamma)\sigma-\beta]}{(1-s)\beta} - 1 = 23.5. \qquad \text{(A-3.2)}$$

Appendix A-4

Let us show through simulation that the two long-run equilibria in Figure 5 are located in the Ponzi region, as mentioned in footnote 11. Using the same set of parameter values specified in Appendix A-2, (38) becomes:

$$-0.00045\delta^2 + 0.00208\delta - 0.00150 = 0 . \qquad \text{(A-4.1)}$$

As reported on footnote 8, the two corresponding equilibria are given by $\delta_1^* = 0.898$ and $\delta_2^* = 3.713$, whereas the demarcation lines of the Minskyan financing regimes are given by:

$$\delta_{H-S} = \frac{(1-\phi-\gamma)\alpha\sigma}{i\left[\Psi-(1-\phi-\gamma)\sigma(1-s)\right]} = \frac{(1-\phi-\gamma)\alpha\sigma}{i\left[s-\beta-(\phi+\gamma)\sigma s\right]} = 0.142 ,$$

and:

$$\delta_{S-P} = \frac{(1-\phi)\alpha\sigma}{i\left[\Psi-(1-\phi)\sigma(1-s)\right]} = 0.193 .$$

In Figure 5, therefore, though the equilibrium with higher debt ratio is locally stable, it is nonetheless situated in the Minskyan regime of highest financial fragility.