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Labor Supply in Pandemics Environments: An Aggregative Games Approach

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We analyze the effects that pandemic processes have on labor supply decisions using an aggregative game framework. The individual payoff depends on her labor supply and on the probability of being infected, which in turn, depends on the aggregate labor supply. We show the effects of social and sanitary public policies on the Nash equilibrium and analyze its expectational stability. The results indicate that compensating policies and sanitary policies can attenuate the damaging effect of pandemic and stabilize expectations regarding the aggregate decision of labor supply. We also find a set of parameters where two-period cycles for the expectations revision map may arise, implying the oscillating behavior of the probability of contagion in this class of models.

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JEL Codes: C72, J28, J38

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1 Introduction

The event of Covid-19 constituted an economic shock that affected countries in different ways. Besides the challenges related to quick responses in order to avoid a massive mortality rate, the fact that the disease is mainly spread by social contact called for the design of new economic policies. In particular, decision makers tried to balance the goal of avoiding high mortality with the problem related to decrease of economic activity, since policies such as lockdown, tracking and quarantine directly affected the labor market.

The new environment after Covid-19 largely affected policy makers, firms and workers. The latter had to balance their decisions based on the trade-off between receiving higher wages and being susceptible of infection. Moreover, the interaction of workers' decisions was evident, as the higher the number of active workers in the market, the higher the probability of contagion. This situation of collective and individual strategic decisions fits well on an aggregative game framework, as we propose in this paper.

Many studies have been done in order to assess how some government responses to the Covid-19 pandemic were effective to contain its spread. Aum et al. (2021) propose a SIR (Susceptible, Infectious, or Recovered) epidemiological model to evaluate the effectiveness of policies that determine lockdowns, testing individuals for their state of infection and the combination of tracking and quarantine. They use data from South Korea and the UK to calibrate and test their model. Birinci et al. (2021) analyze the effects of different labor market policies, namely, unemployment insurance expansions and payroll subsidies. They show that the introduction of payroll subsidies alone is preferred over unemployment insurance expansions and estimate an optimal mix of policies, with 20% of the budget directed to payroll subsidies and the remainder to unemployment insurance expansions. Bradley et al. (2021) propose a matching model in which the decision of whether the worker should work from home is a function of the joint surplus of a match. They show that policies that impose lockdowns are costly to young workers and more beneficial to the old, whose rate of mortality decreases. More recently, Dergiades et al. (2022) analyze the impact of public policies to contain the spread of Covid-19 on the U.S. labor market. They classify policies as non-pharmaceutical interventions (NPIs) or economic support measures (ESM) and show that NPI's quickly cause an increase in the number of unemployment claims, while the reduction of unemployment caused by ESMs does not occur immediately.

Famiglietti and Leibovici (2022) analyze the relation between the spread of Covid-19, non-pharmaceutical interventions, and economic activity, showing that the adoption of both policies together contribute to the mitigation of economic negative impacts caused by NPIs. Eichenbaum et al. (2022) have similar results, with a positive interaction between social distancing/mask use and testing/quarantining.

There is no much work yet that evaluates the effect of policies on the labor market. Miller (2020) estimates the optimal level of wage reduction that minimizes the weighted sum of the average of the cost imposed onto workers who experience the wage reduction and the variance of this cost across workers. Data from a university in the United States is used to verify how the weights a planner puts on minimizing the average pain, compared to equalizing pain across employees, affect the optimal policy. If the worker is infected, her wage suffers a reduction corresponding to the income compensation she will receive; otherwise, she will receive her total wage. The probability of infection depends on the total labor supply and sanitary policies. Thus, when both kinds of policies are implemented (wage compensation policies and sanitary policies), each worker is confronted with the decision of how much labor she will supply.

This paper sheds fill some gap in the literature to understand the rationale of workers' decisions in pandemic environments. We model their decisions as strategies in an aggregative game. For quite general functional forms and a large set of parameters there is a Nash equilibrium in which best response functions' second-order conditions are automatically satisfied. Stability properties of the equilibrium are analyzed: we show the conditions under which workers increase their individual labor supply whenever they perceive a decrease in the probability of contagion. In this case the probability of infection increases, with recurrent oscillations.

2 The Model

Consider a labor market in which $N \geq 2$ workers are at risk of infection by a contagious disease. Let $\mathcal{N} = \{1, 2, \cdots, n\}$, be the set of workers. A worker $n \in \mathcal{N}$ decides to supply $x_n \in [0, 1]$ units of labor, paid at rate w_n per unit. If infected, the worker is forbidden to work, but eligible to receive a compensating wage $\lambda_n w_n$, where $\lambda_n \in (0, 1)$ is a public policy parameter defined by the government to mitigate the economic impact of the pandemic.

Since the contact with other people increases contagion, the total labor supply $X = \sum_{n=1}^{N} x_n$ determines the probability that a worker is infected, defined as $\pi = \pi(X)$. We assume that $\pi : [0, N] \to [0, 1]$ is a continuous and strictly increasing function, and define the labor supply of all workers but n as $X_{-n} = X - x_n$.

Each worker $n \in \mathcal{N}$ has preferences represented by the Von Neumann-Morgenstern utility function $u_n : \mathbb{R}_+ \to \mathbb{R}$. Given a profile $(x_1, x_2, \cdots, x_n)$ of labor supply, the payoff of worker n is:

$$U_n(x_n, X_{-n}) = \pi(X_{-n} + x_n)\, u_n(\lambda_n w_n x_n) + (1 - \pi(X_{-n} + x_n))\, u_n(w_n x) \tag{1}$$

Individuals decide how much to work before knowing whether they are contaminated. The sets of individual strategies, S_n, and payoffs, $U_n(\cdot,\cdot)$, define the aggregative game $\Gamma = \{S_n, U_n\}_{n=1}^{N}$, where for all $n, S_n = [0, 1]$ and $U_n : [0, 1] \times [0, (N - 1)] \to \mathbb{R}$ follows definition (1).

The maximization of $U_n(\cdot, X_{-n})$ for a given X_{-n} generates the best-reply function $x_n^*(X_{-n}) = \arg\max U_n(\cdot, X_{-n})$ of worker n. Since $\pi(\cdot)$ and $u_n(\cdot)$ are continuous functions, so is $U_n(\cdot,\cdot)$. Therefore, the best reply functions are non-empty and upper hemi-continuous. However, because $U_n(\cdot, X_{-n})$ is not necessarily a quasiconcave function, the best-reply values may not define a convex set. To avoid this problem, we consider isoelastic utility functions and log-linear distributions of probability of contagion.

3 Existence and stability of the equilibrium for some specific functional forms

In this section we analyze the existence and stability of the Nash equilibrium of the game $\Gamma = \{S_n, U_n\}_{n=1}^{N}$ for some widely used classes of utility functions and probabilities of contagion. We state the conditions for obtaining best-response functions of the individuals, existence and equilibrium stability.

Before stating the results of this section, let us recall the following concepts of replacement functions and Perceived-to-Actual maps, proposed by Cornes et al. (2021), to define and analyze the expectational stability of the Nash equilibrium in aggregative games. The replacement function of individual n is defined as the function $r_n : [0, N] \to [0, 1]$ that satisfies:

$$U_n(r_n(X), X - r_n(X)) \geq U_n(z, X - z);\ \forall z \in S_n.$$

Thus, $r_n(X)$ is the individual optimal participation of n in the total contribution X of the game. According to this approach, also used by Okuguchi (1993) and Cornes and Hartley (2007), the expectational stability of the Nash equilibrium occurs when $|\rho'(X^*)| < 1$, where $\rho(X) = \sum_{n=1}^{N} r_n(X)$ is the perceived-to-actual map. The equilibrium is expectationally unstable if $(|\rho'(X^*)| > 1)$.

3.1 Isoelastic utility functions and log-linear probabilities of contagion

There is some evidence that the probability of contagion is increasing on social contact. Using data for Atlanta, Boston, Chicago, New York (NYC), and Philadelphia, Glaeser et al. (2022) estimate that total Covid-19 cases per capita decreased on average by approximately 20% percent for every ten percentage point fall in mobility between February and May 2020. With support from their findings, we assume that $\pi(X)$ follows a distribution that can be estimated according to the specification below:

$$\ln \pi = \beta_0 + \beta_1 \ln X,$$

where $\beta_0 = -\sigma \ln N$ and $\beta_1 = \sigma$. Note that the latter is affected by policies aimed to decrease the contagion rate, with large values of σ associated with more effective policies. Therefore, expected utility functions and probability of contagion follow the specification below:

$$u(z) = \frac{z^{1-\gamma} - 1}{1-\gamma};\quad \pi(X) = \left(\frac{X}{N}\right)^{\sigma}, \tag{2}$$

where $0 < \gamma < 1$ and $\sigma > 1$. Without loss of generality, we assume that all individuals share the same value of γ. In this case:

$$U_n(x_n, X_{-n}) = \left(\frac{X_{-n} + x_n}{N}\right)^{\sigma} \left(\frac{(\lambda_n w_n x_n)^{1-\gamma} - 1}{1-\gamma}\right) + \left(1 - \left(\frac{X_{-n} + x_n}{N}\right)^{\sigma}\right) \left(\frac{(w_n x_n)^{1-\gamma} - 1}{1-\gamma}\right)$$

$$= w_n \left[\left(\frac{X_{-n} + x_n}{N} \right)^{\sigma} \left(\frac{(\lambda_n x_n)^{1-\gamma}}{1-\gamma} \right) + \left(1 - \left(\frac{X_{-n} + x_n}{N} \right)^{\sigma} \right) \left(\frac{{x_n}^{1-\gamma}}{1-\gamma} \right) \right] - \frac{1}{1-\gamma}$$

Maximization of the payoff function above does not depend on w_n, and the best-response function is the solution of:

$$\max_{x} \left[\left(\frac{X_{-n} + x}{N} \right)^{\sigma} \left(\frac{(\lambda_n x)^{1-\gamma}}{1-\gamma} \right) + \left(1 - \left(\frac{X_{-n} + x}{N} \right)^{\sigma} \right) \left(\frac{{x_n}^{1-\gamma}}{1-\gamma} \right) \right] \tag{3}$$

The following proposition states that the objective function of problem (3) is strictly concave for an open set of parameters:

Proposition 1. *Suppose that the payoff functions U_n given in (1) of the game Γ have functional specifications as in (2), and its parameters belong to the set*

$$\mathcal{C} = \left\{ \left(\gamma, \sigma, (\lambda_n)_{n=1}^{N} \right) \in (0,1) \times (1, +\infty) \times (0,1)^N ; (\sigma - \gamma) \left(1 - \lambda_n^{1-\gamma} \right) < \gamma (\sigma - 1) \right\}. \tag{4}$$

Then, the objective function of the maximization problem (3) is strictly concave for all $n \in \mathcal{N}$.

Proposition 1 states sufficient conditions for the existence of a unique solution of the workers' maximization problem. Note that if $\left(\bar{\gamma}, \bar{\sigma}, (\bar{\lambda}_n)_{n=1}^{N} \right) \in \mathcal{C}$, then $\left(\bar{\gamma}, \sigma, (\bar{\lambda}_n)_{n=1}^{N} \right) \in \mathcal{C}$ for all $\sigma > \bar{\sigma}$. Therefore, if a police against dissemination of the disease is associated with strictly concave payoff functions, so are all other more effective policies. Similar relations are valid for the parameters $\bar{\lambda}_n$ and $\bar{\gamma}$ in the set $\mathcal{C}$, since for any $\lambda_n > \bar{\lambda}_n$ and for any $\gamma > \bar{\gamma}$, $\left(\bar{\gamma}, \bar{\sigma}, (\lambda_n)_{n=1}^{N} \right) \in \mathcal{C}$ and $\left(\gamma, \bar{\sigma}, (\bar{\lambda}_n)_{n=1}^{N} \right) \in \mathcal{C}$. Therefore, increases in the compensating wage rates or risk aversion preserve the uniqueness property of problem (3). To present the results of existence of equilibrium, we define:

$$K(\gamma, \lambda) = \sum_{n=1}^{N} \left[\frac{1-\gamma}{1 - \lambda_n^{1-\gamma}} \right] \text{ for } \lambda = (\lambda_1, \cdots, \lambda_n) \text{ and } \gamma \in (0,1) \tag{5}$$

In the case of $\gamma = 1$, we have:

$$K(1, \lambda) = - \sum_{n=1}^{N} \left[\ln (\lambda_n) \right]^{-1}$$

Note that $K(\gamma, \lambda) > 0$ for all $\gamma \in (0, 1]$, since for all $n \in \mathcal{N}$, $\lambda_n \in (0, 1)$. We can also observe that $K(\gamma, \lambda)$ is strictly increasing in any λ_n.

Proposition 2. *Consider the game $\Gamma = \{S_n, U_n\}_{n=1}^{N}$, with payoffs functions given in (1) and the specification (2). If the parameters of the game belong to the set $\mathcal{C}$, then the replacement functions are:*

$$r_n(X) = \left(\frac{1-\gamma}{\sigma}\right)\left(\left(1-\lambda_n^{1-\gamma}\right)^{-1}\left(\frac{X}{N}\right)^{-\sigma} - 1\right)X \tag{6}$$

and the Perceived-To-Actual (P-T-A) map is:

$$\rho(X) = \left(\frac{K(\gamma, \lambda) N^{\sigma}}{\sigma}\right)X^{1-\sigma} - \frac{(1-\gamma)}{\sigma}NX. \tag{7}$$

Furthermore, the aggregate labor supply Nash equilibrium is:

$$X^* = \left(\frac{K(\gamma, \lambda)}{\sigma + (1-\gamma)N}\right)^{1/\sigma} N. \tag{8}$$

From Proposition 2 and equations (6) and (7), the replacement and P-T-A functions are increasing on the parameters λ. Therefore, individual and collective responses to the perception of the total labor supply are larger if the workers receive a larger compensation rate in case of infection. Moreover, as the P-T-A function is decreasing in the perception of the aggregate labor supply. From Proposition 2, definition (5) and equation (8), we can verify that the aggregate labor supply Nash equilibrium X^* is strictly increasing in the compensation policy.

An interesting impact of the wage compensation policy is its cross effect among individuals. Consider a worker n whose compensation λ_n increases. Because X^* strictly increases, there must be a reduction in the labor supply of all other workers whose compensation does not change. Therefore, individual n is the only worker who bears the increase in the total labor supply, an effect not to be missed in the design of wage compensation policies.

From equation (8), the aggregate labor supply Nash equilibrium X^* is also strictly increasing in σ. Therefore, labor supply is positively affected by higher efforts of fighting the spread of the disease.

Finally, we can observe the effect of the population size N and the sanitary policy parameter σ on the probability of contagion in equilibrium $\pi(X^*) = K(\gamma, \lambda)/(\sigma+(1-\gamma)N)$. The greater these parameters, the lower the probability of contagion.

To end this subsection, we establish some results of expectational stability for the specification (2), which include the existence of expectational cycles around the Nash equilibrium for some parameter values.

Proposition 3. *Consider the game $\Gamma = \{S_n, U_n\}_{n=1}^{N}$, with payoffs functions given in (1) and the specification (2). Assume that its parameters belong to the set $\mathcal{C}$. Then the Nash equilibrium X^* presented in Proposition 2 is:*

(i) Expectationally stable if $\sigma + (1-\gamma)\,N < 2$;

(ii) Expectationally unnstable if $\sigma + (1-\gamma)\,N > 2$;

(iii) If $\sigma_0 + (1-\gamma_0)\,N_0 = 2$, there exists a neighborhood of those parameters where there are cycles of period 2 around the expectationally unstable X^*.

According to Part (i) of Proposition 3, in the case of a large population, if the relative risk aversion is sufficiently close to one and the curvature of the contagion probability is sufficiently low, then the equilibrium is expectationally stable. In the proof provided in the Appendix, we show that $\rho'(X^*) < 0$, thus the stability is oscillating. Therefore, the convergence to X^* occurs through greater and lower values of the aggregate labor supply, oscillating in the contagion probability. Although this oscillation is empirically sound, the more realistic case that combines a strong policy against the pandemic with not so low relative risk aversion is unstable, as shown in part (ii) of the proposition. In other words, when workers are confident on the sanitary policies, the aggregate labor supply exceeds the corresponding equilibrium.

Part (iii) of the proposition is the most interesting result, as it shows the existence of two-period cycles, which is a consequence of the period-doubling bifurcation theorem in Devaney (1989). This result is illustrated by recurrent oscillations that can be observed in pandemic processes. Workers increase their individual labor supply whenever they perceive a decreasing in the probability of contagion. As a result, the probability of infection increases, restarting the cycle.

3.2 Logarithmic utility functions and general contagion probability functions

Since the assumption of $\gamma \neq 1$ is crucial in most of the results given in the previous subsection, we will extend the analysis for the case of logarithmic utilities of the individuals. As we will see, this case allows us the analysis of more general forms of the probability of contagion and clearer conclusions on individual and aggregate labor supply.

The next proposition applies to the following specification:

$$u(z) = \ln z; \quad \pi'(X) > 0, \; \pi''(X) > 0 \tag{9}$$

Proposition 4. *Suppose that payoff functions U_n assume the functional specifications (9). Then the objective function of the maximization problem (3) is strictly concave for all $n \in \mathcal{N}$.*

With logarithmic utilities, a unique solution for the individual problem can be obtained for any distribution of probability of contagion that satisfies strict monotonicity and strict convexity. We characterize the replacement functions, perceived-to-actual maps and Nash equilibrium as:

Proposition 5. *Consider the game $\Gamma = \{S_n, U_n\}_{n=1}^{N}$, with payoffs functions (1) and specification (9). Its replacement function and P-T-A map are given by:*

$$r_n(X) = \frac{1}{\pi'(X)\ln\left(\lambda_n^{-1}\right)}; \quad \rho(X) = \frac{K(1.\lambda)}{\pi'(X)} \tag{10}$$

If, in addition, $\pi'(N) \geq \frac{K(1,\lambda)}{N}$, then there exists a unique equilibrium that satisfies:

$$X^* = \frac{K(1.\lambda)}{\pi'(X^*)} \tag{11}$$

Similarly to Proposition 2, increases in the compensation parameter λ_n are associated with a larger total labor supply X^*. As a consequence, the individual labor supply of workers whose compensation does not change decreases.

Figure 1 shows the typical shape of the Perceived-To-Actual map for the logarithmic utility function. The decreasing behavior of this map indicates the countercyclical response of the actual labor supply to the perceived labor supply. The intersection with the diagonal corresponds to the Nash equilibrium aggregate labor supply. As stated in Proposition 3 for the case of isoelastic utility functions and log-linear probability of contagion with respect to the total labor supply, there can be cycles around the Nash equilibrium.

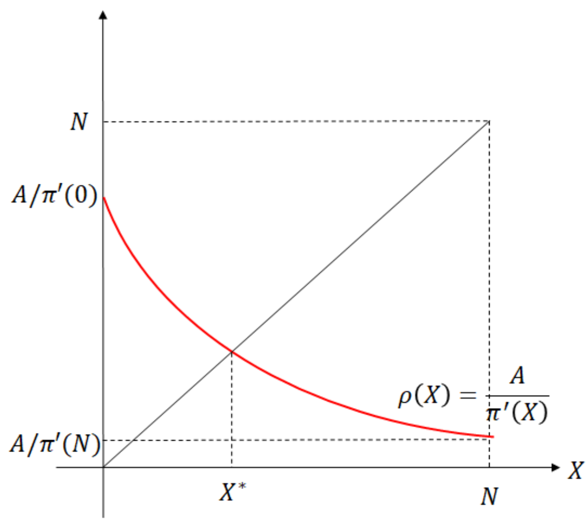

Figure 1: The Perceived-To-Actual map for logarithmic utility and general form of the probability of contagion.

Finally, the following proposition establishes the conditions to obtain expectational stability of the Nash equilibrium:

Proposition 6. *If a game* $\Gamma = \{S_n, U_n\}_{n=1}^{N}$*, with payoffs functions given in (1) is specified as (9), its Nash equilibrium is expectationally stable if and only if:*

$$\frac{X^{*}\pi''\left(X^{*}\right)}{\pi'\left(X^{*}\right)} < 1$$

Analysis of the existence of bifurcations from an expectationally stable to an expectationally unstable Nash equilibrium and the corresponding existence of attracting cycles around it would require more information about the function π.

4 Final Remarks

The pandemic triggered by Covid-19 brought challenges to the scientific community, not restricted to fields directly related to Health Sciences. The massive impacts of the pandemic on economic growth and workers' income demanded a quick response from policy makers. As it seems that pandemic will be present for a long time, policymakers must consider sanitary and economic policies to face this new economic environment.

From the individual point of view, the pandemic affects workers' decisions of labor supply. At the aggregate level, individuals' choices impact the probability of contagion, which is taken as a focal point for a worker's decision that depends on her risk aversion. Since the probability of contagion is clearly affected by the aggregate decision of labor supply, aggregative games constitute a simple, although embracing framework to model the dynamics of these decisions.

The model presented in this work assumes that the capacity to work of infected individuals is reduced, implying a wage decrease or even its elimination. The government must define a social policy to compensate the wage loss of infected workers. At the same time, the aggregate decision of labor supply affects the probability of contagion, which can also be influenced by sanitary policy decisions. All of these aspects are considered in the model to analyze individual and aggregate labor supply decisions.

The first result shows the existence of a large set of parameters of the proposed functional forms for which the second-order conditions of best-response functions are automatically satisfied. This feature of the model is not always overcome in aggregative games. Following the result of existence, we explicitly calculate the individual and collective responses to the perception of the aggregate labor supply, the so-called replacement functions, and the perceived-to-actual map of the game. This exercise is important for the Nash equilibrium solution of the game, since it is defined as the fixed point of its perceived-to-actual map. The explicit forms of these functions allow us to perform some static comparative analysis, where the Nash equilibrium presents very intuitive responses to parameter variations. Lastly, we study the expectational stability of the equilibrium. We show that the equilibrium is unstable for a large set of parameter values related to sanitary policy, population size, and risk aversion. As expected, contagion follows a rapid increasing path. We also prove the existence of two-period cycles around the Nash equilibrium, which is compatible with the persistent oscillations around some levels of stationary aggregate labor supply.

These results shed light on the best social and sanitary policies design. From the theoretical point of view, the model provides a clear analysis of how the Nash equilibrium is affected by workers's decisions and public policies. Empirically, it suggests particular functional forms that may be consistent with the observed data available since Covid-19 became a pandemic.

Appendix

Proof of Proposition 1

To prove this proposition it suffices to show that the objective function of problem (3) is strictly concave. Its first and second derivatives are, respectively:

$$f'(x) = (1-\gamma)\, x^{-\gamma} \left\{ \frac{\sigma u(\lambda_n)}{N^\sigma} x X^{\sigma-1} + (1-\gamma)\, u(\lambda_n) \frac{X^\sigma}{N^\sigma} + 1 \right\}$$

and:

$$\frac{f''(x)}{1-\gamma} = \frac{\sigma(\sigma-1)\, u(\lambda_n)}{N^\sigma} X^{\sigma-2} x^2 - \frac{2(1-\gamma)\, \sigma u(\lambda_n)}{N^\sigma} X^{\sigma-1} x - \gamma \left\{ (1-\gamma)\, u(\lambda_n) \frac{X^\sigma}{N^\sigma} + 1 \right\};$$

recall that $u(\lambda_n) = (1-\gamma)^{-1}(\lambda_n^{1-\gamma} - 1)$. Since the coefficient of x^2 is strictly negative (because $\lambda_n < 1$ and $\sigma > 1$), $f''(x) < 0$ if and only if the discriminant of the right-hand side is negative, namely:

$$\left[\frac{(1-\gamma)\, \sigma u(\lambda_n)}{N^\sigma} X^{\sigma-1} \right]^2 + \left(\frac{\sigma(\sigma-1)\, u(\lambda_n)}{N^\sigma} X^{\sigma-2} \right) \gamma \left((1-\gamma)\, u(\lambda_n) \frac{X^\sigma}{N^\sigma} + 1 \right) < 0$$

Simplifying $\sigma, N^\sigma, u(\lambda_n)$ and $X^{\sigma-2}$, we have:

$$\begin{aligned} & (1-\gamma)^2 \sigma u(\lambda_n) \tfrac{X^\sigma}{N^\sigma} + \gamma(\sigma-1) \left((1-\gamma)\, u(\lambda_n) \tfrac{X^\sigma}{N^\sigma} + 1 \right) > 0 \\ \Leftrightarrow \quad & X < \left[\tfrac{\gamma(\sigma-1)}{(\sigma-\gamma)\left(1-\lambda_n^{1-\gamma}\right)} \right]^{1/\sigma} N, \end{aligned}$$

Note that because $u(\lambda_n)$ is negative, the inequality sign is reversed. It follows that $f''(x) < 0$, as long as the parameters $\left(\gamma, \sigma, (\lambda_n)_{n=1}^{N}\right) \in \mathcal{C}$. We conclude that the payoff function is strictly concave.

□

Proof of Proposition 2

Proposition 1 guarantees that the first order condition characterizes the solution of problem (3). Therefore, the replacement function $x = r_n(X)$ satisfies:

$$\frac{\sigma u(\lambda_n)}{N^{\sigma}} x X^{\sigma-1} + (1-\gamma) u(\lambda_n) \frac{X^{\sigma}}{N^{\sigma}} + 1 = 0$$

$$\Rightarrow x^* = r_n(X) = \left(\frac{1-\gamma}{\sigma}\right)\left(\left(1-\lambda_n^{1-\gamma}\right)^{-1}\left(\frac{X}{N}\right)^{-\sigma} - 1\right) X$$

Summing up the last equation above, we obtain the P-T-A function:

$$\rho(X) = \left(\frac{K(\gamma, \lambda) N^{\sigma}}{\sigma}\right) X^{1-\sigma} - \frac{(1-\gamma)}{\sigma} N X$$

Finally, after solving the equation $\rho(X^*) = X^*$ we find the aggregate supply Nash equilibrium:

$$X^* = \left(\frac{K(\gamma, \lambda)}{\sigma + (1-\gamma) N}\right)^{1/\sigma} N.$$

□

Proof of Proposition 3

Part (i): From Cornes et al. (2021), the expectational stability of the equilibrium occurs if an only if $|\rho\prime(X^*)| < 1$. Then,

$$\rho'(X^*) = \left(\frac{(1-\sigma) K(\gamma, \lambda) N^{\sigma}}{\sigma}\right)(X^*)^{-\sigma} - \frac{(1-\gamma)}{\sigma} N$$

$$= \left(\frac{(1-\sigma) K(\gamma, \lambda) N^{\sigma}}{\sigma}\right)\left(\frac{K(\gamma, \lambda)}{\sigma + (1-\gamma) N}\right)^{-1} N^{-\sigma} - \frac{(1-\gamma)}{\sigma} N$$

$$\Rightarrow \rho'(X^*) = 1 - \sigma - (1-\gamma) N$$

Since $\sigma > 1$ and $\gamma > 0$, the above expression is negative. Therefore, a necessary and sufficient condition for the expectational stability of the equilibrium is:

$$\sigma + (1-\gamma)\,N - 1 < 1 \Leftrightarrow \sigma + (1-\gamma)\,N < 2$$

Part (ii): The proof of this part follows the same steps as the proof of Part (i).

Part (iii): For this part, we must show that: $\left.\frac{\partial\rho^{(2)}}{\partial\gamma}\right|_{X=X^*} \neq 0$ and the same for the other parameters (σ and N), where $\rho^{(2)}(X) = \rho(\rho(X))$, which can be done after some tedious calculations. The existence of attracting two-cycles is a consequence of the fact that the Schwarzian derivative of ρ is negative, since $\rho''' < 0$ (see Devaney (1989) for details). □

Proof of Proposition 4

The payoff function in specification (9) is:

$$\begin{aligned} & f(x) = \pi(X_{-n} + x)\ln(\lambda_n w_n x) + (1 - \pi(X_{-n} + x))\ln(w_n x) \\ = \; & \pi(X_{-n} \;+ x)\ln(\lambda_n) + \ln(x_n) + \ln(w_n) \end{aligned}$$

Its second derivative, $f''(x) = \pi''(X)\ln(\lambda_n) - \frac{1}{x^2}0$, is negative, which implies that f is a concave function.

□

Proof of Proposition 5

The payoff function of individual n (up to a constant) is:

$$f(x) = \pi(X)\ln(\lambda_n) + \ln(x)$$

Since $\lambda_n \in (0,1)$, the second-order derivative satisfies:

$$f''(x) = \pi''(X)\ln(\lambda_n) - 1/x^2 < 0$$

Therefore, the first order condition defines the replacement function:

$$f'(x) = \pi'(X)\ln(\lambda_n) + \frac{1}{x} = 0 \Rightarrow x_n \equiv r_n(X) = \frac{1}{\pi'(X)\ln(\lambda_n^{-1})}$$

and the P-T-A map is:

$$\rho(X) = \sum_{n=1}^{N} r_n(X) = \frac{K(1.\lambda)}{\pi'(X)}$$

To prove the existence and uniqueness of X^* in (11), consider the function $\phi : [0, N] \to \mathbb{R}$ defined by:

$$\phi(y) = y\pi'(y) - K(1, \lambda).$$

The function $\phi(\cdot)$ is strictly increasing. Since $\phi(0) = -A < 0$ and $\phi(N) = N\pi'(N) - A = N\left(\pi'(N) - \frac{A}{N}\right)$, $\pi'(N) \geq \frac{A}{N}$ implies $\phi(N) \geq 0$. Therefore, there exists $X^* \in (0, N]$ such that $\phi(X^*) = 0$, and

$$X^* = \frac{A}{\pi'(X^*)}$$

is an equilibrium of Γ. Strict monotonicity of ρ guarantees uniqueness.

□

Proof of Proposition 6 The expectational stability is characterized by $|\rho'(X^*)| < 1$. The derivative of ρ evaluated in the Nash equilibrium is:

$$\rho'(X^*) = -\frac{K(1, \lambda)\pi''(X^*)}{(\pi'(X^*))^2} = -\frac{K(1, \lambda)}{\pi'(X^*)}\left(\frac{\pi''(X^*)}{\pi'(X^*)}\right) = -\frac{X^*\pi''(X^*)}{\pi'(X^*)}$$

where the last equality results from equation (11). Since $\pi' > 0$, $\pi'' > 0$, and $\frac{X^*\pi''(X^*)}{\pi'(X^*)} < 1$, the equilibrium is expectationally stable.

□

References

Aum, S., S. Y. T. Lee, and Y. Shin (2021, FEB). Inequality of fear and self-quarantine: Is there a trade-off between GDP and public health? *Journal of Public Economics 194*.

Birinci, S., F. Karahan, Y. Mercan, and K. See (2021, FEB). Labor market policies during an epidemic. *Journal of Public Economics 194*.

Bradley, J., A. Ruggieri, and A. H. Spencer (2021, SEP). Twin peaks: Covid-19 and the labor market. *Euopean Economic Review 138*.

Cornes, R., L. Fiorini, and W. Maldonado (2021). Expectational stability in aggregative games. *Journal of Evolutionary Economics 31*, 235–249.

Cornes, R. and R. Hartley (2007). Aggregative public good games. *Public Economic Theory 9*(2), 201–219.

Dergiades, T., C. Milas, and T. Panagiotidis (2022, AUG). Unemployment claims during COVID-19 and economic support measures in the U.S. *Economic Modelling 113*.

Devaney, R. (1989). *An Introduction to Chaotic Dynamical Systems*. Addison-Wesley Publishing Company.

Eichenbaum, M. S., S. Rebelo, and M. Trabandt (2022, MAY). The macroeconomics of testing and quarantining. *Journal of Economic Dynamics & Control 138*.

Famiglietti, M. and F. Leibovici (2022, MAY). The impact of health and economic policies on the spread of COVID-19 and economic activity. *European Economic Review 144*.

Glaeser, E. L., C. Gorback, and S. J. Redding (2022, JAN). JUE insight: How much does COVID-19 increase with mobility? evidence from new york and four other U.S. cities. *Journal of Urban Economics 127*(SI).

Miller, K. (2020, NOV). Sharing the sacrifice, minimizing the pain: Optimal wage reductions. *Economics Letters 196*.

Okuguchi, K. (1993). Unified approach to cournot models: oligopoly, taxation and aggregate provision of a pure public good. *European Journal of Political Economy 9*, 233–245.