Department of Economics- FEA/USP

Legal Enforcement, Default and Heterogeneity of Project Financing Contracts

GABRIEL DE ABREU MADEIRA

WORKING PAPER SERIES Nº 2012-31

DEPARTMENT OF ECONOMICS, FEA-USP
WORKING PAPER Nº 2012-31

Legal Enforcement, Default and Heterogeneity of Project Financing Contracts

Gabriel de Abreu Madeira (gmadeira@usp.br)

Abstract: This paper employs mechanism design to study how imperfect legal enforcement impacts simultaneously on the availability (or scale) of credit for investment and interest rates. The analysis combines two standard ingredients of the development and contract literatures: limited commitment, which encapsulates the idea that contract enforcement is imperfect, and asymmetric information about cash flows, which justifies debt contracts and default under some circumstances. Costly use of courts may be optimal, which differs from most limited commitment models, where punishments are just threats, never applied in optimal arrangements. Paradoxically, liquidation by courts only happens in equilibrium when courts are imperfect. Numerical solutions for several parametric specifications, allowing for heterogeneity on initial wealth are provided. In all such solutions, wealthier individuals borrow with lower interest rates and run higher scale enterprises, which is consistent with stylized facts. The reliability of courts has a consistently positive effect on the scale of projects. However its effect on interest rates is subtler and depends on the degree of curvature of the production function.

Keywords: Limited Commitment, Credit Constraints, Legal Enforcement, Mechanism Design.

JEL Codes: D02, D82, L26, O12, O16

Legal Enforcement, Default and Heterogeneity of Project Financing Contracts

Gabriel de Abreu Madeira*

University of São Paulo

November 2012

Abstract

This paper employs mechanism design to study how imperfect legal enforcement impacts simultaneously on the availability (or scale) of credit for investment and interest rates. The analysis combines two standard ingredients of the development and contract literatures: limited commitment, which encapsulates the idea that contract enforcement is imperfect, and asymmetric information about cash flows, which justifies debt contracts and default under some circumstances. Costly use of courts may be optimal, which differs from most limited commitment models, where punishments are just threats, never applied in optimal arrangements. Paradoxically, liquidation by courts only happens in equilibrium when courts are imperfect. Numerical solutions for several parametric specifications, allowing for heterogeneity on initial wealth are provided. In all such solutions, wealthier individuals borrow with lower interest rates and run higher scale enterprises, which is consistent with stylized facts. The reliability of courts has a consistently positive

*This paper is a revised version of one of the chapters of my thesis at the University of Chicago. I am very grateful to the members of my committee, Lars Hansen, Roger Myerson and especially the chairman Robert Townsend for useful comments and discussions. I also thank Flavio Cunha, Weerachart Kilenthong, Mario Macis, Ricardo Madeira, Esteban Puentes, Daniel Santos, Sergio Urzua and participants of seminars at the University of Chicago, the University of São Paulo, IBMEC - São Paulo and the University of the Thai Chamber of Commerce for helpful comments. I am very thankful to CNPq for sponsoring my graduate studies at the University of Chicago. All errors are mine.

effect on the scale of projects. However its effect on interest rates is subtler and depends on the degree of curvature of the production function.

1 Introduction

Both credit markets and the quality of institutions are believed to play a key role in the development of countries. These roles are often interconnected in the economic literature [1]. As pointed out by North (1981), solid institutions provide support for private contracts. In particular, a good institutional environment enhances the capacity of agents to commit, which enlarges the set of contractual arrangements available, thus raising opportunities for risk protection and project financing. This idea has been widely explored by development economists, which, typically, investigate the impacts of limited commitment on the capacity of agents to invest and protect themselves from adverse shocks [e.g. Banerjee and Newman (1993), Evans Jovanovic (1989), Lloyd-Ellis and Bernhardt (2000), Ligon, Thomas and Worral (2002) and Genicot and Ray (2003)]. On the other hand, the risks associated with ineffective legal enforcement are many times presented as suspects of causing high bank spreads in developing countries [Laeven and Madjnoni (2003), Costa and Mello (2006) and Visaria (2005)]. Here, the focus moves from income and consumption behavior to the shape of contracts. A natural theoretical benchmark for this idea stands in another branch of the theoretical literature, which departs from stylized descriptions of the economic environment and derives optimal transfer schedules among contracting parts [eg. Townsend (1979), Krasa and Villamil (2000), De Marzo Fishman (2007)]. From this perspective, it seems plausible that a combination of imperfect information (which generates debt contracts) and high judicial risks would produce an environment where feasibility requires high interest rates. However, bank spreads and the volume of credit interact in non-trivial manner in response to changes in the quality of the legal system. Is it necessarily the case that an inefficient legal systems increase interest rates? Isn't it possible that, under judicial risk, lenders anticipate high probabilities of default and credit is rationed so that risky contracts are avoided? The empirical evidence linking institutions and bank spreads is far from being robust, which suggests that the effects of institutions on interest rates may be subtle.

[1] Some examples are Acemoglu and Johnson (2005), LaPorta *et. al.*(1998), Knack and Keefer(1995) and Mauro (1999).

Bangladesh and Colombia, for instance, rank amongst the 5 countries with lower institutional rating according to a measure constructed by Leaven and Madjoni (2003). However, their average bank spreads [2], of 6.39% and 5.92% respectively, are relatively low. Further, the countries with higher bank spreads (Brazil, Uruguay, Russia, Ukraine and Bolivia) are not those with lower measures of quality of the legal system.

Most of the literature does not explain how legal enforcement affects credit contracts in a broader sense, looking at scale of borrowing and repayment schedules simultaneously. The purpose of this paper is to help filling this gap. The model presented here borrows ingredients both from the development literature on limited liability and the literature on dynamic financial contracts with asymmetric information. As in Evans and Jovanovic(1989) and Banerjee and Newman(1993), individuals - that are heterogeneous on wealth - can invest in projects, but their ability to finance them is subject to their capacity to commit to repay. Ex-post, the only incentive for repayment is the threat of asset seizure by courts. But courts are imperfect, in the sense that they are costly and may fail to liquidate even when contracts request liquidation. This limited commitment ingredient, encompassed by imperfect courts, is combined with uncertainty and ex-post asymmetric information. Although firms retain no private information initially, by the time repayment is requested the productivity of projects is observed exclusively by borrowers. Asset seizure is modelled as liquidation of projects, but the cash flows that these projects generate can be hidden by borrowers.

The arrangements that emerge from this combination of limited commitment and asymmetric information possess several realistic features. Even when randomization is allowed as a contractual policy, debt contracts are optimal. Those are contracts where the amount borrowers are requested to repay in order to avoid some probability of asset seizure (or liquidation of projects) is deterministic[3]. This deterministic amount determines the interest rates (and thus bank spreads) of project financing contracts implied by the model. But repayment does not

[2] Bank spreads are here defined as the difference between lending interest rates and deposit interest rates, as reported by the IMF. See Leaven and Majnoni (2002).

[3] This feature is typical of costly state verification models (e.g. Townsend (1979)), but when randomization is allowed debt contracts may not be optimal. Krasa and Villamil (2000) show that a combination of costly state verification and renegotiation generate debt contracts even when renegotiation is allowed. Here, debt contracts follow from the fact that cash flows are never observed, and incentive for repayment comes from a discrete threat, liquidation.

necessarily happen. The model generate three possible scenarios that resemble events from real world credit markets. First there is the possibility of repayment. Second, there it is possible that defaulters are sued by lenders and taken to courts (this scenario is henceforth called default). Finally, and consistently with the limited commitment literature, borrowers are allowed to transfer their liquidated assets voluntarily to lenders, which prevents costs with lawsuits. All of these scenarios emerge as possible outcomes in optimal contracts but, paradoxically, liquidation by courts in optimal contracts will only happen when courts are imperfect, in the sense that they may fail to liquidate.

Since these results are derived from optimal contracts conditional on the environment, they embody the policy message that no improvement can be made without changing the environment, that is, without improving the legal system. Indeed, a theorem characterizes both the increase in court reliability - a higher probability of liquidation when contracts call for liquidation - and lower costs of courts as welfare increasing institutional improvements. This policy implication reinforces the importance of understanding the patterns that should be expected from credit data if a bad legal system is to account for high bank spreads and limited access to credit.

The predictions about how institutions impact on heterogeneous agents are studied by numerical simulations, developed for several parameter specifications. A large set of simulation results relating the quality of institutions with interest rates, amounts of borrowing, scale of projects and probability of default or voluntary liquidation is presented. These results are potentially useful for empirical analysis of credit markets. Many of them are consistent with the existing literature and stylized facts. Interest rates are higher and scale of enterprise are lower for poorer investors[4]. Legal enforcement affects mostly poor individuals. Improvements in the reliability of courts always result in more investment and higher output. However, the combination of limited commitment and asymmetric information introduces some original and sometimes surprising results. For instance, differently from most of the limited liability literature, individuals do not borrow as much as they can. They take into account the fact that,

[4]As reported by Araújo and Rodriguez (2003), microdata from credit markets in Brazil (which is the country with higher bank spread in the data set used by Leaven and Madjnoni (2003)) reveal that interest rates are strongly dependent on the characteristics of borrowers. In their dataset, bank spreads are considerably higher for small firms and small loans.

if they borrow too much, their projects will have higher risk, which increases interest rates they have to pay if they are successful. More strikingly, the intuitive idea that interest rates are decreasing with the quality of legal enforcement is not always confirmed by the model results. This happens when the scale of projects is fixed or the production function has a high degree of curvature. However, with low curvature of the production function, there are several examples where institutional improvement produce simultaneously increases in the amounts borrowed and in the interest rates.

The paper is organized as follows. Section 2 presents the model, the characterization of the optimal contracts and some propositions describing their shape and welfare properties. Section 3 shows some results from linear programming solutions for the general case where risk aversion is allowed. Section 4 specializes the model to risk neutrality and present some numerical solutions for this case. Section 5 summarizes and discusses the results and confronts them with some findings from the empirical related literature. It also points out directions for future research.

2 The Environment

The economy consists of investors, that live for three periods and receive an endowment (or initial wealth) w in the first period of life, a risk neutral lender and a court.

The utility of investors (or borrowers) is given by :

$$U(c_1) + \beta U(c_2) + \beta^2 U(c_3),$$

where $0 \leq \beta \leq 1$, and c_t is consumption in period t, $U' > 0$ and $U'' \leq 0$. At each period investors can save in an outside market with an exogenous interest of r.

In their first period of life, investors can invest in a risky project. When they invest an amount k in the first period, the project produces a cash flow of $\theta f(k)$ in the following two periods, where $\theta \in \Theta$ is a random project quality parameter, with p.d.f. $h(\theta)$. The distribution $h(\theta)$ is assumed to be common knowledge. The parameter θ is unobserved when investment takes place: it is revealed to the investor only after the first cash flow. The parameter distribution $h(\theta)$ is the source of ex ante heterogeneity in the expected quality of projects, while the realization θ is an ex-post (after the contract is defined) shock. The project can be liquidated after the first cash flow, producing an outcome ik in the third period. I assume that $i < (1+r)^2$, implying that it is not worth to invest in a project just to liquidate it.

Investors can borrow from the lender, that has the capacity to commit to actions determined in a contact, and are willing to accept any contract that generates nonnegative profits (defined as the present value of transfers from the borrower to the lender). However, the borrowing contracts are constrained by a combination of asymmetric information and limited commitment from the investor. When borrowing takes place, a contract can specify default conditions. These are conditions under which the lender has the right to require the liquidation of the firm by a court. When a court liquidates a project, the outcome of liquidation (or the collateral), ik, is transferred to the lender after one period. But courts are imperfect: they require the payment of a fixed cost $c \geq 0$ to be activated, and when they are activated they liquidate the project with a probability $\lambda \leq 1$. Savings and cash flows of the borrower cannot be observed by the lender and the courts. After observing the cash flow, a borrower can produce a message about his type, that may be used in contracts. Borrowing contracts must be subject to a zero profits condition: the present value of transfers from the lender to the borrower is not bigger than zero. The interest rate used to define this present value is r, so the borrower and the lender face the same lending interest rate.

A key feature of the model is the possibility of strategic default. After θ is observed the contract determine transfers from the borrower to the lender in three scenarios, that correspond to possible strategies for the borrower in the second period. First repayment, that guarantees that the firm will not be liquidated. Second default, that implies liquidation by the court with probability λ. Third, voluntary liquidation. Investors can liquidate their projects without being enforced by the court. This last possibility makes the model richer, since the possibility of non-repayment without the use of courts is allowed. It also makes it clear that, in the model, institutions affect credit markets only through their effect on enforcement. Courts are not the only entity with access to a liquidation technology. Their only purpose is to enforce liquidation, something that investors can do by themselves.

Contracts can specify either positive or negative transfers from the borrower to the lender in each of these scenarios. The only constraint on these transfers is the limited commitment constraint, determining that these transfers cannot be positive in the case of default. Under default, only the collateral (the liquidation value of the firm) can be transferred to the lender. And this happens with probability λ,when the court enforce liquidation.

All transfers between the lender and the borrower take place in the first and second periods. This is not restrictive, since in the third period the threat of liquidation cannot be used to extract payments from the borrowers *and*, as the borrower can save in the second period facing the same interest rate as the lender, there is no need for the lender to make transfers to the borrower in the third period.

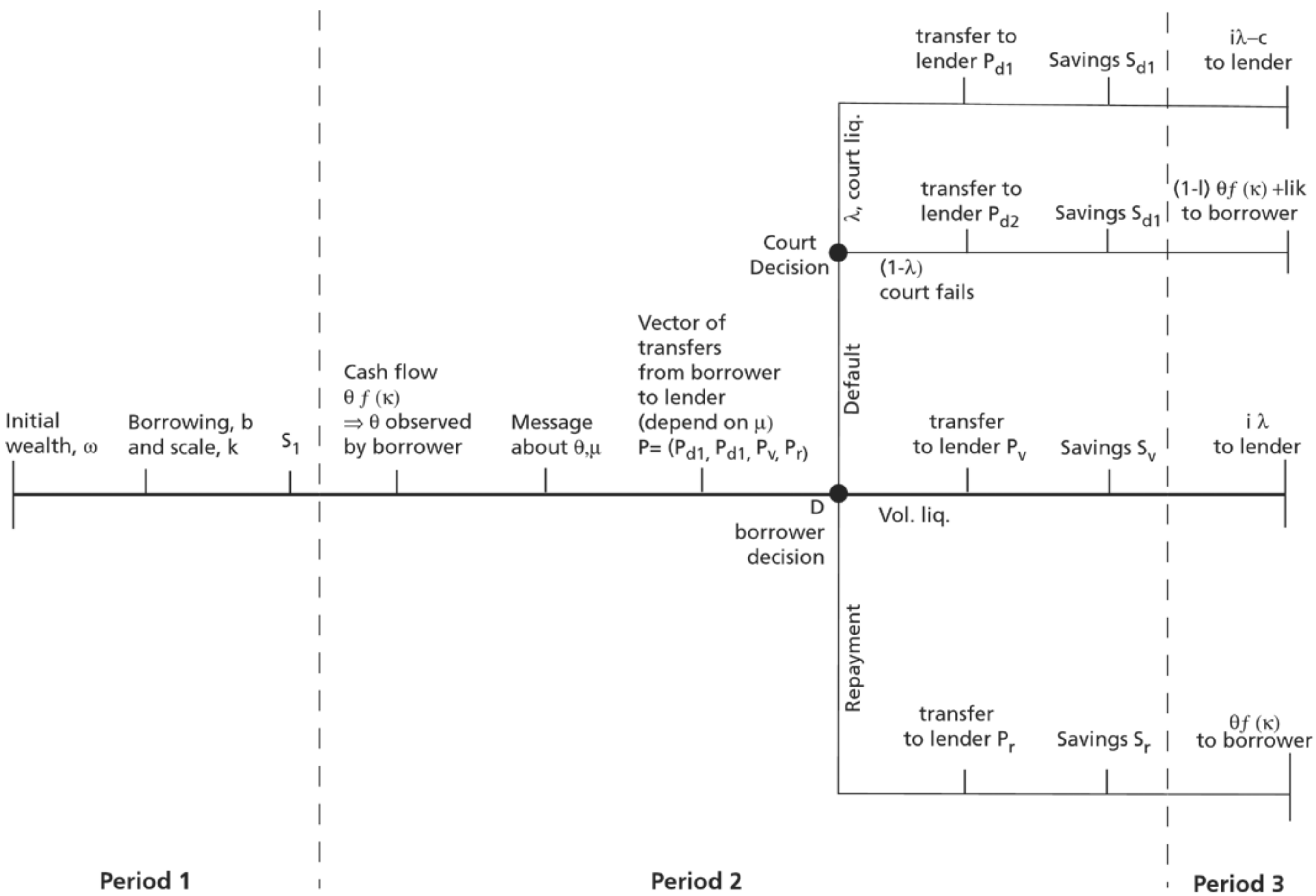
A contract defines a borrowing amount $b \in B \subseteq \Re$, a scale of project $k \in K \subseteq \Re^2_+$ and a vector $p \equiv (p_r, p_v, p_{d1}, p_{d2}) \in P$ describing the transfers form the borrower to the lender in the second period under repayment (p_r), voluntary liquidation (p_v), default and liquidation by a court (p_{d1}) and default and no liquidation by the court (p_{d2}). As it was argued above, p_{d1} and p_{d2} cannot assume positive values[5] Therefore, $P \subseteq \Re^2 \times \Re^2_-$. For generality, the vector p is allowed to be a random function of the message about the cash flow. Notice that k must be specified in the contract since it plays the role of collateral.

The timing of events is described in the timeline presented in figure 1. In the first period, investors receive a bequest w. Then a borrowing contract defines an amount of borrowing b and a scale of project, k. Finally borrowers save an amount s_1. In the second period, the borrower receive a cash flow and thus observes θ, issues a message μ about the cash flow, and, based on μ , a vector p is defined. After p is defined, the borrower takes the decision between voluntarily liquidating, repaying or defaulting (decision node D). If the decision is default, the court have a probability λ of liquidating and a probability $(1 - \lambda)$ of not liquidating. Notice that in the second period, optimal savings are different for each branch of the three. I denote savings under repayment, voluntary liquidation, default with liquidation and default without liquidation respectively by s_r, s_v, s_{d1} and s_{d2}.

When investors are taken to court but the court fails to liquidate, they have the possibility to liquidate the project and obtain ik or wait for a cash flow. This is expressed by the binary

[5] Alternative equivalent formulation could characterize as decision variables to be defined in the contracts the discrete choice between default, voluntary liquidation and repayment, d, and the amount of second period transfers p, and impose an alternative limmited comitment constraint that any choice for the this decision variable must produce at least the utility of default with zero transfers.

variable l, that takes value 1 if the choice is liquidation and 0 if it is no liquidation. A proposition to be stated below shows that under reasonable conditions, borrowers will never liquidate when courts fail to do so. This means that l will always be equal to zero in optimal contracts. Therefore, throughout the paper, the term voluntary liquidation refers to liquidation before investors are taken to courts. The inclusion of the possibility of voluntary liquidation after courts - that could also be thought as another decision node after court decision in the tree below - makes the model more realistic, and also helps in the derivation of properties of optimal contracts.



2.1 The Optimal Contract

Definition *A borrowing contract is a triple* $\{b, k, \pi(p \mid \theta)\}$*, where* $b \in B \subseteq \Re$ *is the level of borrowing,* $k \in K \subseteq \Re^+$ *is the scale of the project financed, and* $\pi : P \times \Theta \to (0, 1)$ *is the probability of* p *conditional on* θ*.*

Notice that savings in the first period, s_1, are not part of the contract. In principle, contracts could include s_1 as a choice variable. If this were the case, the contract would need to produce incentives for individuals to adopt the prescribed level of savings. But assuming that s_1 is equal to zero, and including in the contract incentives for people not to save positive amounts does not constrain the problem. As the lender and the borrower face the same saving interest rate, savings may be implicitly provided by the lender. Indeed, suppose a contract specifies an amount of borrowing b, a saving amount of $s_1 > 0$ and a distribution of p conditional on θ of $\pi(p \mid \theta)$. An alternative contract with borrowings $\widetilde{b} = b - s_1$, and distribution of transfers $\widetilde{\pi}(p + s_1(i + r) \mid \theta) = \pi(p \mid \theta)$, and $s_1 = 0$, would have investors and lenders facing the same resources in each state as in the first contract. Therefore, if they did not have incentives for hidden savings in the first contract, they would not also have incentives for so in the new one. Notice that this transformation may require a negative value of "borrowing ", $\widetilde{b}$. This is not ruled out of the set of possible contracts. From now on, I assume that $s_1 = 0$. But an incentive constraint determining that no hidden savings ($s_1 > 0$) are desired by agents must be added to the contract.

The solution to the problem is constructed backward. After transfers are given in the second period, the borrower defines second period savings. Unlike in the first period, savings in the second period may be bigger than zero, as there are no transfers between the borrower and the lender in the third period. In the case of default and no liquidation by courts, individuals also choose an optimal value of l, that is equal 1 if there is liquidation after the court fails and zero otherwise. The second period savings decision and the choice of l determine indirect utilities from the second period on conditional on transfers. The indirect utilities in the second period under repayment, voluntary liquidation, default with liquidation by courts and default without liquidation by courts are respectively denoted by: $V_2^r(\theta f(k), p_r)$, $V_2^v(\theta f(k), p_v)$, $V_2^{d1}(\theta f(k), p_{d1})$, and $V_2^{d2}(\theta f(k), p_{d2})$[6]. Notice that by the concavity of U, all of these functions are concave on both arguments. I denote the indirect utility under default as:

[6]The derivation of these indirect value funcitons is trivial:

$V_2^r(\theta f(k), p_r) = \max_{s_r} U(\theta f(k) - p_r - s_r) + \beta U(\theta f(k) + (1 + r)s_r)$;

$V_2^v(\theta f(k), p_v) = \max_{s_v} U(\theta f(k) - p_v - s_v) + \beta U((1 + r)s_v)$;

$V_2^{d1}(\theta f(k), p_{d1}) = \max_{s_{d1}} U(\theta f(k) - p_{d1} - s_{d1}) + \beta U((1 + r)s_{d1})$ and

$V_2^{d2}(\theta f(k), p_{d2}) = \max_{s_{d2}} U(\theta f(k) - p_{d2} - s_{d2}) + \beta U(\max\{\theta f(k), ik\} + (1 + r)s_{d2})$

$V_2^d(\theta f(k), p_{d1}, p_{d2}) \equiv V_2^{d1}(\theta f(k), p_{d1})\lambda + V_2^{d2}(\theta f(k), p_{d2})(1-\lambda)$.

At the decision node D, investors take the utility maximizing decision. Therefore, the indirect utility in the second period given the vector p is:

$V_2(\theta f(k), p) = \max\{V_2^d(\theta f(k), p_{d1}, p_{d2}), V_2^v(\theta f(k), p_v), V_2^r(\theta f(k), p_r)\}$.

The discrete decision at the node D is described by the indicator functions $I_r(\theta f(k), p)$, that has value 1 if there is repayment and 0 otherwise, $I_d(\theta f(k), p)$, that has value 1 if there is default and 0 otherwise, and $I_v(\theta f(k), p)$, that has value 1 if there is voluntary liquidation and 0 otherwise.

A first step in the characterization of the optimal contract is the definition of the optimal distribution of transfers in the second period conditional on θ, when b , w and k are given. The choice variable in this program is $\pi(\theta \mid p)$, the probability distribution of p conditional on θ. The choice of $\pi(\theta \mid p)$ is subject to the following conditions.

First, the transfers policy must be such that investors have no incentives to hide cash flows: $\forall \overline{\theta}, \widehat{\theta} < \overline{\theta}$.

$$\sum_p \pi(p \mid \overline{\theta}) V_2(\overline{\theta} f(k), p) \geq \sum_p \pi(p \mid \widehat{\theta}) V_2(\overline{\theta} f(k), p). \tag{1}$$

Notice that this constraint determine that individuals with high θ have no incentive to report a lower value of θ. It is assumed, for simplicity, that individuals cannot pretend to have a value of θ that is higher than the one they actually had. Individuals can hide cash flow, but they cannot pretend they had a cash flow that is higher than the one they actually had.

Another condition on π is that individuals should have no incentives to make hidden savings. In the case of risk neutrality this condition is innocuous. But if investors are risk averse, the effect of savings on second period utilities depend on θ, p and the decision between repayment, default and voluntary liquidation. With positive savings and risk aversion, (1) may not be a correct characterization of incentives for individuals to report the truth cash flows. The condition for no hidden savings is necessary for (1) to be an accurate incentive constraint. This condition is:

$\forall \mu : \Theta \to \Theta$ with $\mu(\theta) \leq \theta$ and $\forall\ s_1 \in (0, b + w - k)$,

$$\sum_{p,\theta} \pi(p \mid \theta)h(\theta)(U(B+w-k)+\beta V_2(\theta f(k),p)) \geq \tag{2}$$
$$\sum_{p,\theta} \pi(p \mid \mu(\theta))h(\theta)(U(B+w-k-s_1)+\beta V_2(\theta f(k),p+(1+r)s_1)).$$

This condition states that for any reporting strategy, μ, there is no incentives for hidden savings.

The participation condition for the lender, or zero profit condition, is:

$$B(1+r) \leq \sum_{p,\theta} \pi(p \mid \theta)h(\theta)[I_r(\theta f(k),p)p_r + I_d(\theta f(k),p)(\lambda(p_{d2}+\frac{ik}{1+r}) \tag{3}$$
$$+(1-\lambda)p_{d1}-c)+I_v(\theta f(k),p)(p_v+\frac{ik}{1+r})],$$

The conditions for π to be a probability distribution are:

$$\pi \geq 0, \qquad \sum_{p,\theta} \pi(p \mid \theta) = 1. \tag{4}$$

Notice that given w, conditions (1) to (4) cannot be fulfilled for some values of k and b. They can only be defined for a set of feasible borrowing-scale combinations $\Gamma(w)$. This is the set of values of k and b such that:

(a). (1) to (4)are valid for some π given w, b and k.

(b). $b+w-k. \geq 0$.

Clearly, $\Gamma(w)$ is not empty. Indeed, setting $k=w$, $b=0$, $p_{d1}=p_r=p_v=0$, and $p_{d2}=-ik$, all constraints of the problem are satisfied. This implies that at least one contract is available to any individual.

Given w, for any $(b,k) \in \Gamma(w)$, the optimal transfers in the second period are defined by the following program:

Program 1

$$\widetilde{V}_1(b,k,w) = \max_{\pi} U(b+w-k) + \beta \sum_{p,\theta} \pi(p \mid \theta)h(\theta)V_2(\theta f(k),p), \tag{5}$$

s.t. (1) to (4).

Given the function $\widetilde{V}_1(b,k,w)$, it is possible to define the program determining the choices of b and k given w and $h(\theta)$. This program is:

Program 2 $\widetilde{V}(w) = \max\limits_{(b,k)\in\Gamma(w)} \widetilde{V}_1(b,k,w).$

2.2 Some Properties of the Solution

This subsection presents some general properties of optimal contracts. The propositions depend on assumptions on the second period indirect utility functions. Although these assumptions are stated in terms of the indirect utilities, they ultimately depend on the function U. They are all valid for standard specifications of the utility function U such as $CARA$ and $CRRA$. Most of them (assumptions (a), (b), (c), (d) and (e)) are also valid with linear utility (risk neutrality). The assumptions used in the derivation of the results are:

Assumptions

(a) $V_2^r(\theta f(k), p)$ is concave on p and $\theta f(k)$.

(a') $V_2^r(\theta f(k), p)$ is strictly concave on p and $\theta f(k)$.

(b) $(\partial^2 V_2^r(\theta f(k), p)/\partial^2 p)/\ (\partial V_2^r(\theta f(k), p)/\partial p)$ is nonincreasing with $\theta f(k)$.

(c) $-(\partial^2 V_2^r(\theta f(k), p)/\partial^2 p)/\ (\partial V_2^r(\theta f(k), p)/\partial p)$ is nonincreasing with p.

(d) $(\partial^2 V_2^r(\theta f(k), p)/\partial p \partial \theta f(k)) \leq 0$.

(d') $(\partial^2 V_2^r(\theta f(k), p)/\partial p \partial \theta f(k)) < 0$.

(e) $(\partial^2 V_2^v(\theta f(k), p)/\partial^2 p)/(\partial V_2^v(\theta f(k), p)/\partial p)$ is non increasing with $\theta f(k)$.

Assumption (a) is a consequence of U being concave. Condition (b) states that the absolute risk aversion of repayers with respect to transfers received in the second period is nonincreasing with θ. Condition (c) that it is nonincreasing with transfers received in the second period. Condition (d) states that the marginal utility of receiving transfers in the second period is non-increasing with θ. Condition (e) states that the indirect utility of liquidators has nonicreasing absolute risk aversion with respect to transfers in the second period. An absolute risk aversion that is not increasing with θ guarantees that if repayment or liquidation is certain, it is possible to substitute lotteries for nonrandom utility equivalents without producing additional incentives for high θ individuals to misreport their cash flows. Nonincreasing absolute risk aversion with transfers received in the second period guarantees that this can be done without additional incentives for hidden savings.

The following proposition states that there is a pooling value of repayment for all types that pay with certainty. This is what makes the optimal contracts resemble debt contracts.

Proposition 1 *Let b, k and w be given. Suppose that assumptions (a) to (d) are valid and that Θ is finite. Then, if an optimal contract implies that types θ_1 and θ_2 choose repayment with probability one, there exists an optimal contract in which both types repay the same amount $\widehat{p_r}$ with probability 1. If (a') and (d') are valid (which follows from risk aversion), this is a necessary result: an optimal contract where types θ_1 and θ_2 repay with certainty must have both types repaying the same amount with certainty.*

Proof. See appendix 1 ■

This follows from asymmetric information about cash flows. Ideally, with risk aversion, it would be desirable to extract higher payments from individuals with high cash flows. But this is not possible when cash flows can be hidden. Individuals with high cash flow realizations would have incentives to misreport their cash flows. Therefore, the amount of repayment that guarantees zero probability of project liquidation does not depend on θ. This bunching value of repayments can be used to define borrowing interest rates as:

$$r_b = \frac{\widehat{p_r}}{b} - 1, \tag{6}$$

where $\widehat{p_r}$ is the pooling amount of repayment defined in proposition 1. Notice that this borrowing interest rate differs from r, the outside market saving interest rate.

The presence of a unique value of repayment for types that repay with certainty is directly related to the fact that sometimes randomization is optimal. Randomization may be used to separate high cash flows from medium cash flow investors. Proposition 1 implies that for values of θ such that the probability of repayment is 1, the amount repaid does not depend on θ. Individuals with very high values for θ have incentives to repay with probability one in order not to loose their future cash flows. But it is possible that some values of θ are not so high as to stimulate repayment at this pooling value, but are high enough to make it worth that a discount is given on some occasions so that there is no liquidation. But these discounts cannot be offered with probability one: a probability of liquidation[7] must be given as a threat for high θ individuals not to pretend to be one of these intermediate types.

[7] Setting a value of p_r that is so high that the choice for liquidation or default is always optimal is equivalent to assigning individuals to liquidation or default (that implies liquidation with a positive probability).

The following lemma determines that, in optimal contracts, defaulters do not liquidate when courts fail to liquidate, or, putting it differently, that l in figure 1 is always equal to zero.

Lemma 2 *Let b, k and w be given. Suppose assumption (e) is valid and $c > 0$. Then, the probability that in the optimal contract $I_d(\theta f(k), p) = 1$ (there is default) and $l = 1$ (defaulters liquidate after courts fail to liquidate) is zero.*

Proof. See appendix 1 ■

The next proposition shows that both an increase in λ and a decrease in c can be interpreted as institutional improvements.

Proposition 3 *Given the conditions of Lemma 2, both an increase in the reliability of courts, λ and a decrease in the cost of courts, c do not decrease welfare.*

Proof. See appendix 1 ■

An increase in λ increase the set of feasible payoffs and a decrease in c is merely a decrease in possibe costs.

The following proposition reveals that both asymmetric information and uncertainty about the outcome of courts are essential for default to be a possibility in optimal contracts. Also, it reveals that without asymmetric information, there is no inefficient liquidation of projects. Projects are liquidated only when the cash flows they produce are lower than their liquidation value.

Proposition 4 *If there is no asymmetric information (constraints (2) and (1) are not required) and courts are costly ($c > 0$), there is no default and voluntary liquidation happens if and only if $ik > \theta f(k)$. Also, even if with asymmetric information, if $\lambda = 1$ and courts are costly, there is no default.*

Proof. I prove both claims by contradiction.

First, suppose that there is no asymmetric information and, when $\theta = \overline{\theta}$, there is a positive probability of default with a pair of transfers $(\overline{p}_{d1}, \overline{p}_{d2})$. In the case where $ik > \overline{\theta} f(k)$, setting λ $(\overline{p}_{d1} - c) + (1 - \lambda)\overline{p}_{d2}$ as the transfers from the borrower to the lender with voluntary liquidation would increase the borrowers utility keeping the lenders revenue. If $ik \leq \overline{\theta} f(k)$, setting $\lambda(\overline{p}_{d1} - c) + (1 - \lambda)\overline{p}_{d2}$ as the repayment value (p_r) would allow an increase in utility keeping revenues unchanged. Notice that if there were asymmetric information, this last arrangement might give incentives for higher cash flow individuals to misreport their cash flow and pretend to have $\theta = \overline{\theta}$, thus repaying a lower amount. So the argument above is only valid without asymmetric information.

Second, suppose there is asymmetric information, $\lambda = 1$ and , when $\theta = \overline{\theta}$ there is a positive probability of default with the amount of transfers $\overline{p}_{d1}$ ($\overline{p}_{d2}$ is irrelevant since failure by courts have probability zero). Replacing this by voluntary liquidation with transfers $\overline{p}_{d1}$ would keep utility constant, and therefore not affect incentives. But revenues of the lender would increase since the cost of courts would not be paid. ■

3 Linear Programming Solution

As the solution to the optimal contract may involve randomization of transfers between borrowers and lenders, a reasonable approach to solve the model numerically is the discretization of the choice space and the solution by linear programming. This approach has the disadvantage that, in order to be computationally feasible, it requires sometimes coarse grids. But it has the advantage of being general and allowing for randomization. I present below an example of numerical solution of Program 1 (so the scale of projects k and borrowing b are taken as given). The functional form used for utility in this numerical exercise is:

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

I used a grid with 12 values for θ, 50 values for p_r, 30 values for p_v and 5 values for p_{d1}. The parameters used are $c = 0.5$, $\sigma = 0.1$, $i = 0.6$, $\beta = 0.95$, $r = 0.02$, $\lambda = 0.4$, $b = 9$, $k = 17$. Five possible values of s_1 were used in constraint (2). The distribution of θ used in the computations is presented in figure 2[8]:

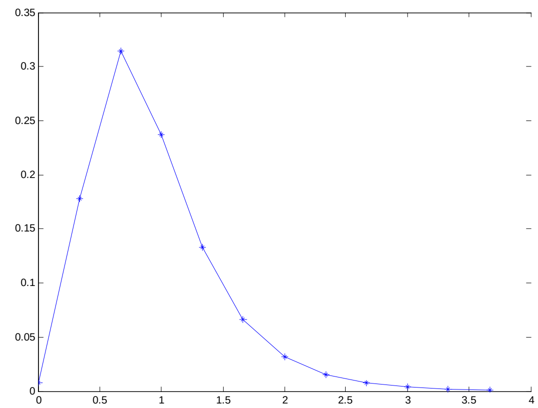


Figure 2 - Distribution of the Shock θ.

The solutions are expressed in figures 3 to 6. Figures 3 and 4 show that, as stated in Proposition 1, the repayment schedule resembles a debt contract, in the sense that individuals with cash flows above a certain level repay a common amount with certainty. Randomization is present for 2 values of θ. Borrowers with these intermediary cash flows may have a discount in the repayment value, but they also have a positive probability of being assigned to default, so the

[8]This distribution was generated from a histogram of a lognormal distribution with mean 1.1, variance 0.3 and median 1.

probability of repayment is positive and smaller than 1 for these values of θ.

Figures 3 to 6 - θ is in the horizontal axis

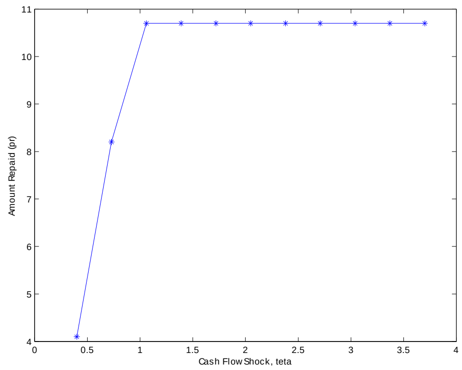


Fig 3 - Amount Repaid

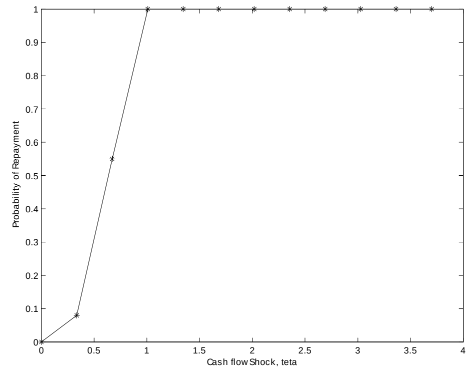


Fig. 4 - Prob. of Repayment

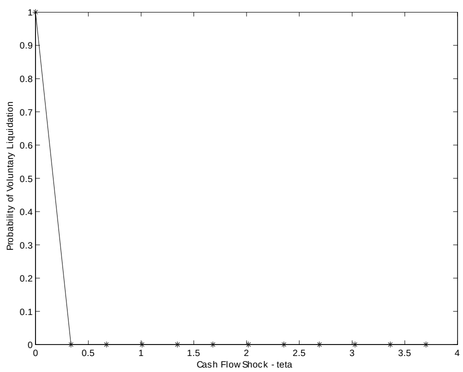


Fig.5 - Prob. of Voluntary Liq.

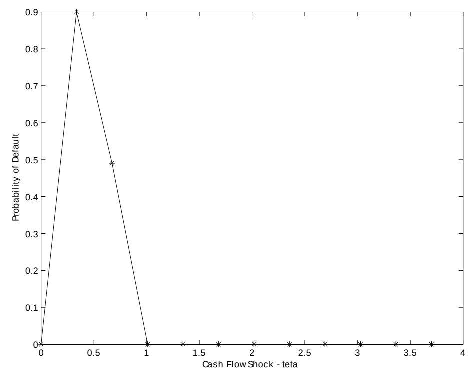


Fig.6 - Prob. of Default

Figure 5 shows the probability of voluntary liquidation for each realization of θ. Figure 6 shows the probability of default for echo θ. Notice that despite the fact that there is a positive cost ($c = 0.5$) of activating courts, default happens with positive probability in optimal contracts. Individuals with high cash flows tend to be repayers. Individuals with very low cash flows tend to voluntarily liquidate. Those with a intermediary levels of cash flows have a positive probability of default. This result (repayment by high values of θ, voluntary liquidation for low values of θ and default for intermediary values of θ), and also resemblance of debt contracts and randomization for a few values of θ were found in all the examples computed[9].

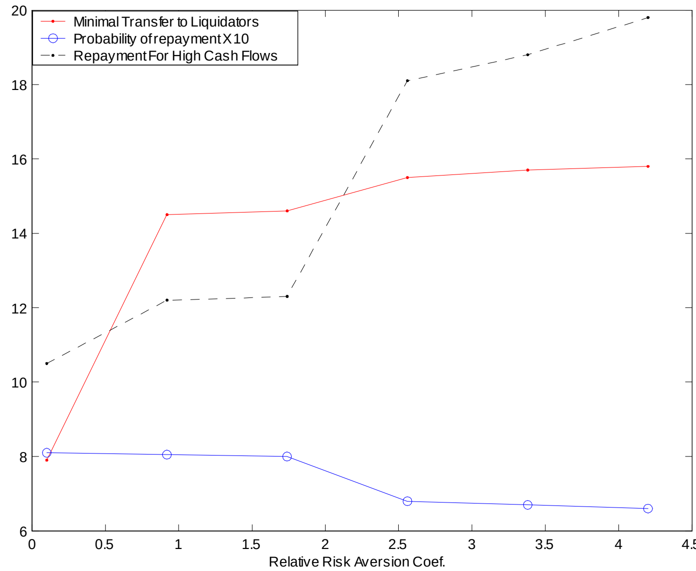


Figure - 7

Another feature that is present in the solution of program 1, is that when borrowers are risk averse, voluntary liquidators normally receive positive transfers from the lender. These positive transfers work as an insurance device. Voluntary liquidators tend to be individuals with low cash flows. Positive transfers to these individuals provide some level of risk protection. In the simulations, both the amount received by voluntary liquidators and the amount paid by successful projects increase with risk aversion. The amount paid by successful investors reflects not only repayment of initial borrowing, but also payments to compensate the transfers received in the contingency of low cash flows. Figure 7, shows, for the parameters used in the example above, how the pooling amount of repayment for high cash flows, the minimal amount of transfers received by voluntary liquidators and the probability of repayment evolve with risk aversion. Notice that the fact that the probability of repayment decreases with risk aversion also contributes for the values of repayments to be increasing with risk aversion.

4 Risk Neutrality and Continuum of Shocks

The use of linear programming has the advantage that it is precise (conditional on the grid), admits lotteries and allows the computation of the general model. However, especially with a

[9] Although I found some examples where default is not observed.

fine grid of θ, there is significant curse of dimensionality, and solutions demand large amounts of time and computational capacity. At this section, I specialize the analysis to risk neutrality, with $U(x) = x$. For expositional convenience, I assume, without loss of generality, that $\beta = (1+r) = 1$. Imposing risk neutrality greatly simplifies the analysis. Constraint (2) is not necessary (hidden savings does not provide any advantage to mitigate incentives, since it contributes equally to utility in all branches of the tree in figure 1). Also, numerical solutions of the model with risk neutrality show that, as the number of elements in the grid of possible values of θ increases, the fraction of values of θ with randomization tends to vanish. In other words, as the support of θ approaches a continuum, the solution seems to converge to one in which there is no randomization. So, I solve the problem assuming that there is no randomization when there is a continuum of θ and risk neutrality[10].

With risk neutrality and no randomization, the characterization of the solution is considerably simpler. First, let us consider the trivial case where $b < ik\lambda$. There is full commitment power by the borrower in this case, since an expected loss of at least $ik\lambda$ will always be faced by the borrower whenever he is taken to court. This leads to a trivial solution: whenever $\theta < ik/f(k)$ there is voluntary liquidation and $p_v = b - ik$ (so that the revenue of the lender with each voluntary liquidator is equal to $ik + p_v = b$). If $\theta \geq ik/f(k)$ there is repayment, with $p_r = b$.

Let us now consider the less trivial case with $b \geq \lambda ik$. Whenever there is voluntary liquidation or default, the utility of the borrower is equal to the utility of default with zero transfers. This follows since, with risk neutrality, there are no gains from making transfers to individuals with low cash flows, unless these transfers are used as incentives for individuals not to default. Higher transfers from the lender to the borrower in the case of default and voluntary liquidation require higher transfers from repayers to the lender to keep zero profits. But a higher amount to

[10]This could be seen as a solution of the problem with the additional constraint that randomization is not allowed. However, based on numerical exercises, this analysis is intended to be at least a good approximation to the solution with randomization allowed. In general as I increase the number of points in the θ grid, with its distribution as an approximation of a lognormal, there is randomization for a maximum of 3 values of θ, between the area in which there is repayment with probability 1 and the area where voluntary liquidation or default are chosen with probability 1. The probability of these few points decreases as the grid becomes finer. So I depart from the conjecture that it tends to zero as the support of θ tends to a continuum. A natural extension would be a theoretical proof of this result.

be repaid imposes less incentive for repayment, and thus a higher probability that projects are inefficiently liquidated. So, the utility of defaulters and liquidators is always equal to the utility of default with zero transfers. And the decision between default and voluntary liquidation depends on the revenue that is obtained with default and $p_{d1} = p_{d2} = 0$ and voluntary liquidation with $p_v = -(1-\lambda)\theta f(k)$ (which gives the liquidator an utility equal to the default level). The θ profile of transfers from the borrower to the lender in the second period is described in Figure8.

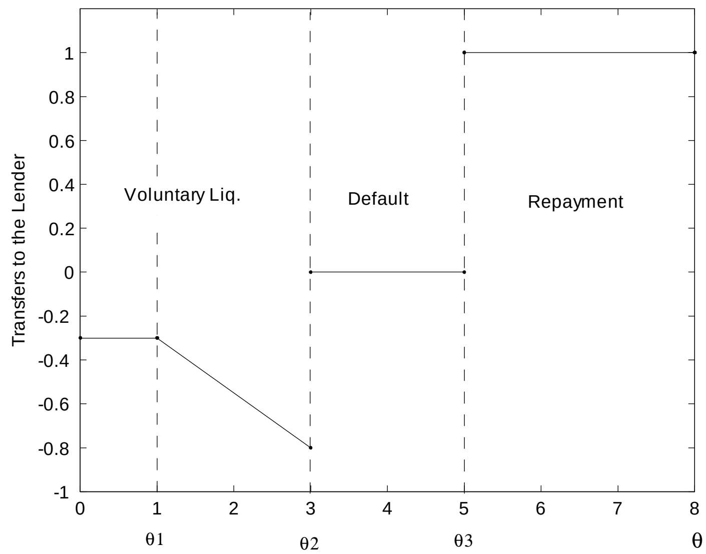


Figure 8- Second period transfers from the Borrower to the lender

For very low values of θ ($\theta \leq \theta_1 \equiv \frac{ik}{f(k)}$), liquidation is efficient (or $ik \geq \theta f(k)$). In order to make borrowers agree to voluntarily liquidate and transfer their liquidated assets to the lender, the lenders must transfer an amount equal to the expected gain that the borrower would have if he was taken to court with zero transfers, which is λik (and does not depend on θ). With θ between θ_1 and $\theta_2 \equiv (ik + \frac{c}{1-\lambda})/f(k)$, voluntary liquidation is less costly than default (that requires the payment of the court cost c), and borrowers would not liquidate if they were taken to courts. In order to make them agree to voluntarily liquidate, they must make a transfer to the borrowers of $(1-\lambda)\theta f(k)$, so $p_v = -(1-\lambda)\theta f(k)$,which is decreasing with θ. Between θ_2 and $\theta_3 \equiv \overline{p}/\lambda f(k)$ (where $\overline{p}$ is the pooling value of repayment for the realizations of θ that imply repayment), default is preferable to repayment of an amount $\overline{p}$, and is less costly than stimulating voluntary liquidation. So there is default and zero transfers from the borrower to the lender. With $\theta > \theta_3$, there is repayment of an amount $\overline{p}$. Those characteristics of the solution are stated formally in Proposition 5.

Proposition 5 *Suppose borrowers are risk neutral, with utility given by* $U(x) = x$*, and* $\beta = (1 + r) = 1$,*and no randomization conditional on* θ *is allowed. Then, whenever* $b > \lambda ik$*, there exists some optimal solution for the optimal transfer policy (Program 1) with the following properties:*

a- There *exists a repayment value* $\overline{p}$ *such that whenever* $\lambda\theta f(k) > \overline{p}$*, or* $\theta > \overline{\theta}_3(\overline{p}) \equiv \overline{p}/\lambda f(k)$*, there is repayment of an amount* $\overline{p}$*.*

b- Whenever $ik > \theta f(k)$*, or* $\theta < \overline{\theta}_1 \equiv ik/f(k)$*, there is voluntary liquidation, and* $p_v = -(1 - \lambda)ik$.

c- Whenever $\overline{\theta}_1 < \theta < \min(\overline{\theta}_2, \overline{\theta}_3)$, where $\overline{\theta}_2 \equiv (ik + \frac{c}{1-\lambda})/f(k)$*, there is voluntary liquidation with* $p_v = -(1 - \lambda)\theta f(k)$

d- Whenever $\overline{\theta}_2 < \theta < \overline{\theta}_3$,there is default, with $p_{d1} = p_{d2} = 0$.

Proof. See Appendix 1. ■

Notice from item c of proposition 5 that, as λ tends to 1, the probability of default converges to zero, as stated in proposition 4. From the properties presented in proposition 5, the characterization of the optimal contract is straightforward. I characterize it for a continuum of θ, which can be interpreted as an approximation of the discreet case with a very fine grid.

For the non-trivial case where $b < \lambda ik$ (where, as argued above, $p_r = b$), the repayment amount for individuals that repay, conditional on the size of loans, b and the scale of project, k is such that the expected second period revenue of the borrower is equal to the amount of borrowing in the first period. Therefore, if $b < \lambda ik$, p_r must solve:

$$\begin{aligned} b &= \lambda ikH(\theta_1) + \int_{\theta_1}^{\min(\theta_2,\theta_3(p_r))} (ik - (1-\lambda)\,\theta f(k))h(\theta)d\theta \\ &\quad + \daleth\,(\theta_2 < \theta_3(p_r)) \int_{\theta_2}^{\theta_3(p_r)} (\lambda ik - c)h(\theta)d\theta + (1 - H(\theta_3(p_r))p_r \end{aligned} \tag{7}$$

where $\daleth\,(\theta_2 < \theta_3(p_r))$ is an indicator function that has value 1 when $\theta_2 < \theta_3(p_r)$ and zero otherwise. The right hand side of equation (7) is the revenue from the borrower given repayment amount p_r. The determination of amount of repayment, p_r, given the amount borrowed, is shown in figure 9[11]:

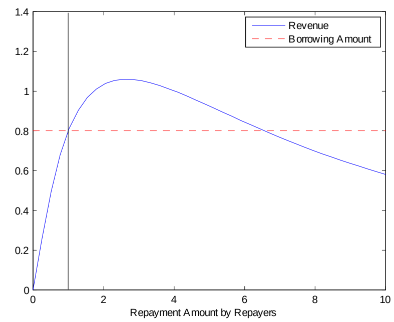


Figure 9

The vertical line shows the value of p_r determined by (7). Notice that there is another higher value of p_r that solves (7), but this implies a higher probability of liquidation of good projects so it produces a lower utility for the borrower. The choice of p_r will always be the smallest value that satisfies (7).

The utility conditional on borrowing and amount of capital is given by:

$$\begin{aligned} U(k,b) &= (w-k+b) + f(k)E(\theta) + (1-\lambda)ik(H(\theta_1)) + \\ &+ \int_{\theta_1}^{\theta_3(p_r(k,b))} (1-\lambda)\theta f(k)h(\theta)d\theta + \int_{\theta_3(p_r(k,b))}^{\infty} (\theta f(k) - p_r(k,b))h(\theta)d\theta. \end{aligned} \tag{8}$$

The problem specialized for the risk neutral case with $\beta = (1+r) = 1$ and no randomization becomes:

$$\begin{aligned} &\max_{(k,b)} U(k,b) \\ &s.t. w-k+b \geq 0 \text{ and } (1.7). \end{aligned} \tag{9}$$

The constraint $w-k+b \geq 0$ will always hold with equality. Indeed, if first period savings are positive, diminishing the amount of borrowing and keeping capital constant will not decrease utility. It will reduce the amount to be paid and thus the probability of states in which

[11] This picture was generated with $k=1$, $b=0.8, i=0.5, c=0.2, \lambda=0.7, \theta$ has lognormal distribution with $\mu=1$ and $\sigma=1$ and $f(k)=k^{0.5}$.

inneficient liquidation happen. On the other hand, a negative value of b could be interpreted as savings. Therefore, given w, b can be written as a function of the scale:

$$b = k - w \tag{10}$$

Substituting (1.10) in (1.8) and using the value of p_r implicitly defined in (1.7) (or $p_r = b$ for the trivial case with $\lambda ik > b$), it is possible to obtain utility as a function of k given w. For the parametric specification used in figure 9, and $w = 1$, the utility of the borrower as a function of the scale of the project is shown in figure 10. The optimal scale of the project, the one that maximizes utility, is determined by the horizontal line in figure 10.

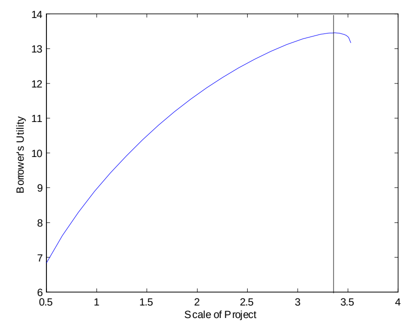


Figure 10

The next proposition rationalizes the common idea that better enforcement decrease the risk of no repayment and thus decrease interest rates.

Proposition 6 *Suppose projects have a fixed scale $\overline{k}$ (meaning that $f(\overline{k}) = K > 0$ and $f(\widetilde{k}) = 0$ for any $\widetilde{k} = \overline{k}$). Then, given an optimal solution to (9) with $b > \lambda ik$, an increase in the reliability of courts λ decrease interest rates.*

Proof. Define the $\Delta(\lambda, p_r)$ as the right hand side of (1.7). Since scale is fixed, b and k are both fixed. Therefore, $\frac{\partial r_b}{\partial \lambda} = -\frac{1}{b}\frac{\partial \Delta(\lambda,p_r)/\partial\lambda}{\partial \Delta(\lambda,p_r)/\partial p_r}$. It is clear from (1.7) that $\partial\Delta(\lambda, p_r)/\partial\lambda > 0$. Also, it must be the case that $\partial\Delta(\lambda, p_r)/\partial p_r > 0$, otherwise a decrease in p_r would increase revenue without decreasing utility, implying that p_r is not optimal. Therefore, $\frac{\partial r_b}{\partial \lambda} < 0$. otherwise a decrease in p_r would increase revenue without decreasing utility, implying that p_r is not optimal. Therefore, $\frac{\partial r_b}{\partial \lambda} < 0$. ■

Note that an essential condition for Proposition 6 to be valid is that the scale of projects is fixed. It is straightforward to see from (1.7) that an increase in λ will always increase the scale of projects, or decrease the interest rates or both. However, it is possible to find examples where interest rates decrease with the reliability of courts. I present in the next section numerical solutions for several parametric specifications, and in all the results found, the scale of projects increase with the reliability of courts. The effects of λon interest rates, on the other hand, are subtler. Several examples where interest rates increase with the reliability of courts are presented.

Notice also that a condition required in Proposition 6 is that $b \geq ik\lambda$. When $b < ik\lambda$, there is full commitment power and the borrowing interest rate, as defined by (1.6), is always equal to zero. In general, very wealthy individuals (with $w > 1 - i\lambda\overline{k}$, where $\overline{k}$ is the optimal stock of capital when there is full commitment) will be unafected by changes in the reliability of courts.

4.1 Numerical Results for the Risk Neutral Case

This section presents numerical solutions for the risk neutral version of the model just described. The model is solved for two specifications of the production function, $f(k)$. The first specification is $f(k) = k^{0.5}$. Then, another production function with higher curvature is employed. The comparison between these two specification reveals that the curvature of the production function is a key ingredient in the determination of the characteristics of the solution. For the first production function I depart from a baseline specification where $c = 0.3$, $i = 0.5$ and the distribution of θ is lognormal with parameters $\mu = 1.375$ and $\sigma = 0.5$. I compute the solutions for several levels of initial wealth and λ. I also check how the solution respond to different values of c and i and for different specifications of the distribution of θ, $h(\theta)$[12].

The first remarkable result concerns credit rationing. Borrowers with low levels of wealth need high amounts of borrowing in order to be able to finance big projects. But the amount of borrowing they are able to obtain is limited: for very large loans, the maximum revenue that can be obtained by the lender after the first period is lower than the amount of borrowing. Figure 11 shows, for the baseline specification and $\lambda = 0.7$, the optimal scale of projects and the maximum possible scale achievable by agents as a function of their initial wealth. Notice that there is some credit rationing. The maximum scale available for low wealth individuals is lower than optimal scale for individuals that have very high wealth and therefore are unconstrained in

[12] The interest rates results presented in this section are borrowing interest rates, as defined in (6).

their choice of scale. This type of credit rationing was also found by Evans and Jovanovic (1989). But differently from them, even for very low wealth agents, the optimal scale is lower them the maximal scale available. Constrained investors do not choose the maximum amount of credit that would be available in feasible contracts, since a higher scale requires high payments in the second period, and the incapacity of borrowers to commit to these high payments would lead to inefficient liquidation of projects. Notice that this result holds only when there is asymmetric information since, as stated in Proposition 4, full information rules out inefficient liquidation.

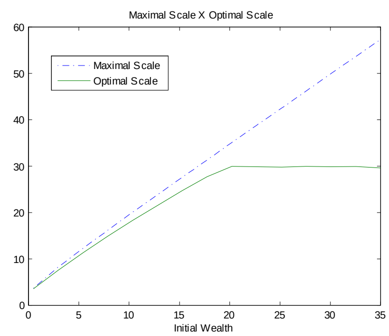


Figure 11

Figure 13, in appendix 2, shows the wealth profile of optimal project scale, size of loans (amount of borrowing), probabilities of default, repayment and voluntary liquidation, and borrowing interest rates, as defined in equation (6). These profiles are shown for 4 different values of the parameter of court reliability, λ : 0.3, 0.5, 0.7 and 0.9. The results show that, for all levels of λ investigated, the optimal scale of projects increase with the initial wealth, up to a point where an optimal scale is achieved. The optimal size of loans profile has an inverted U shape: it is increasing for very low levels of wealth, but after a some point it becomes decreasing. For low levels of wealth, icreases in wealth increase collateral and thus borrowing, but after some level of wealth, individuals start to self finance their projects. Both the probability of default and the interest rates are decreasing with initial wealth. Wealthier individuals not only have access to bigger projects, but they also have access to lower interest rates.

Notice that higher reliability of courts implies higher values of loans and scale. But the effect of λ on interest rates and the probability of default is not clear. This is clearer in figure 14, that shows, for 4 levels of initial wealth, how these variables depend on λ (in the horizontal axis). Both the scale of projects and the size of loans (b) are increasing with the reliability of courts, λ. The effect of λ on the probability of default and the borrowing interest rates is undetermined. For very high values of λ, the probability of default is zero, as stated in proposition 4. But for very low levels of λ the amount of borrowing is extremely low, but these small loans have very low probabilities of default. The intermediary values of λ are those that produce high probability of default. Interest rates also have a non monotonic behavior. They tend to be low with very low levels of court reliability and higher for intermediary levels of λ.

Another remarkable feature of figure 13 is that, as the initial wealth increase, the interest rates and the probability of repayment both go down. This implies that interest rates are not only determined by the probability of repayment. Notice that the as wealth increases, the probability of voluntary liquidation also increases. When there is voluntary liquidation, the value of liquidation is transferred to the lender or, putting it differently, there is collateral seizing by the lender. More collateral transfers make it possible that the repayment generates a smaller fraction of the lenders revenue after the first period. Figures 15 and 16, in appendix 2, show a case in with no collateral value ($i = 0$). In this case, all revenue of the borrower comes from repayment, and non repayment rates explain almost perfectly the interest rates profile.

Figures 20 and 21 in Appendix 2 show the numerical solutions for the optimal scale, interest rates and the probability of default computed for alternative specifications for c, i and h[13]. Some remarkable results from this analysis are that the probability of default and the interest rates increase with the variance of θ, and decreases with the cost of courts c. Also, the scale of projects tend to be higher as the liquidation value of projects increase.

The result that the interest rates may increase as the reliability of courts, λ, increases, contrasts with proposition 6, valid for fixed scale. I recompute the problem using another production function that has a higher curvature, and thus is closer to the case of fixed scale. This production function is:

$$f(k) = (1 + (1 - k)^{-2}).$$

This is a CRRA function with a higher degree of curvature than $f(k) = k^{0.5}$ moved one unit to the left and summed by one so that it is always positive and is zero valued at $k = 0$. Figure 12 plots both this production function (production function 2) and the one chosen before

[13] In the case of the different specifications of $h(\theta)$, the parameters μ and σ of the lognormal distribution are chosen so that the expected value of θ is constant.

(production function 1). This new production function, with higher curvature is closer to a fixed scale case. The gains of scale are initially high, but eventually become very low.

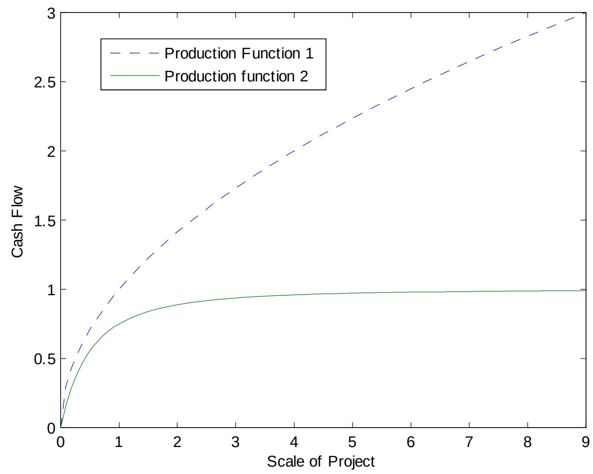


Figure 12

The baseline specification used in the analysis with this production function has $c = 0.3$, $i = 0.5$ and the distribution of θ lognormal with parameters $\mu = 1.5$ and $\sigma = 0.375$. The solutions for a baseline case with this second specification are expressed in figures 17 and 18, in appendix 2. Notice that, as in the previous specification, both interest rates and the probability of default decrease with wealth. Also, the scale of projects tend to increase both with wealth and λ, although the variation on scale is proportionally smaller than in the first specification. However, a remarkable difference that comes from this specification is that the probability of repayment is tends to be increasing and interest rates are consistently decreasing with λ. Better legal enforcement not only increases the scale of projects, but also decreases interest rates. Further, both effects are higher for low wealth individuals. These numerical results, combined with Proposition 6, indicate that the curvature of the production function is a key ingredient to define how better enforcement affects interest rates.

Another ingredient that is affected by the curvature of the production function is the relation between initial wealth and amount of borrowing. In the high curvature case (as well as in the fixed scale case) the amount of borrowing is always decreasing with wealth. This differs from the solution with low curvature, that has borrowing amounts initially increasing with wealth. This is potentially useful for empirical work: the response of amount of borrowing to initial wealth contains information about the curvature of the production function, and this curvature is a key element to determine wether interest rates respond to quality of enforcement or not.

Figures 21 and 22 in Appendix 2 show the solutions for the optimal scale, interest rates and the probability of default for this second production function and different specifications for i, c and the distribution of θ, $h(\theta)$. The general features of the solution are similar to those presented in Figure 17. A remarkable result, not present in the results for the first specification of the production function is that, not only the interest rates are higher as the variance of θ (risk of project) increases, but also the optimal scale of projects is significantly smaller.

5 Discussion and Concluding Remarks

Departing from an environment where debt contracts are optimal, this paper investigates theoretically how legal enforcement affects project financing contracts in a broad sense, looking both at interest rates and scale of projects for heterogeneous agents. The solutions reveal that the interest rate and the scale margins interact in a non trivial manner. When the scale of projects is allowed to vary, borrowing interest rates for each borrower are not necessarily decreasing with the reliability of courts, as happens when scale is fixed. Despite this indeterminacy, the model produces several regularities that are potentially testable. For all of the several parametrization adopted, scale of projects always increase with the parameter of reliability of courts. Also, the reliability of courts affects mostly low wealth investors, and poorer individuals face higher interest rates. A realistic feature that emerges from the model is the possibility that individuals that do not repay are actually punished by costly courts, although the possibility of outside court agreements is also allowed. In practice, some defaulters are legally sued while others make informal agreements with lenders, and these two scenarios (which in the current paper are called default and voluntary liquidation, respectively) can be used to confront the model with the data.

Some theoretical results supplement these potentially testable comparative statics results. Asymmetric information is a necessary condition for punishments with inefficient liquidation ever to take place. Also, and paradoxically, some level of imperfectness (some probability of failure) of courts is necessary for legal enforcement ever to be applied in equilibrium. Finally, increases in the reliability of courts increase welfare, implying that they can really be interpreted as institutional improvements.

The empirical and policy implications of such results are discussed in more detail below.

5.1 Evidence from empirical studies and empirical potential

A first result that is related with empirical studies is that the scale of projects increase with the reliability of courts, λ. Higher scale of projects produce higher outputs, so the model generates a theoretical link between development and the quality of institutions. This relation have been explored in empirical studies such as Knack and Keefer(1995) and Mauro (1999) that present evidence that confidence in institutions, including the judicial system, is a predictor of growth and Acemoglu and Johnson (2005), that present evidence linking property rights institutions and economic growth.

There is also an empirical literature that discusses the impact of institutions on the form of financial intermediation. This includes Acemoglu and Johnson (2005) and LaPorta *et. al.*(1998). Leaven and Madjnoni (2003), that uses cross country data, and Costa and Mello (2006) and Visaria (2005), both of which employ a natural experiment approach relate the quality of judicial system and interest rates. These papers find evidence that bad legal enforcement is connected with high interest rates. This is not generated by all specifications of the model, but is consistent with the results from the second specification of the production function, with high curvature.

Another prediction of the model that have support in empirical studies is that interest rates are higher to low wealth individuals. Karlan and Ziemann (2006) report high interest rates in loans for poor individuals in South Africa. This relation is also found by Araújo and Rodrigues (2003), that use data from credit contracts recorded by the Brazilian Central Bank. They show that the very high interest rates that are prevalent in Brazil affect most strongly small firms. The average interest for firms that are classified by banks as micro-firms is 57%, for those that are classified as small firms it is 44.78%, for medium size firms it is 33.66% and for big firms it is 29.5%. In the model presented, in any parametrization, wealthier individuals have lower borrowing interest rates, so the model is consistent with these findings.

Another finding by Araújo and Rodrigues that is related with the results of the model is that the average interest rates for large loans are smaller than that for big loans. Again, this is not obtained in any specification of the model, but is consistent with at least two particular cases. First, when the production function has low curvature, in the lower part of the wealth distribution the size of loans increases and the interest rates decrease as the initial wealth increases. If most of the credit contracts have borrowers in this lower part of the wealth distribution, it is possible that higher loans have, on average, lower interest rates. Notice however, that with low curvature of the production function, bad enforcement does not explain high interest rates. Such specification is consistent with the Brazilian cross-sectional stylized facts but do not to explain why interest rates in Brazil are higher than in most countries. A second possible explanation for interest rates to be decreasing with size of loans comes from difference in risk of projects. For the second specification of the production function, low variance of θ (or low risk on projects) generates simultaneously high scale of projects, and thus large amounts of borrowing, and low interest rates (see Fig. 21, in appendix 2). The variability on the risk of projects could generate a negative correlation between size of loans and interest rates, and also help to explain the negative correlation between interest rates and scale of firms. This second case departs from a specification of the production function that is consistent with imperfect legal enforcement as an explanation for high interest rates.

An important result that comes from numerical analysis, is that the relation between wealth and the amount of borrowing depends strongly on the curvature of the production function. Therefore, cross-sectional estimates on how the amount of borrowing relates to initial wealth could provide some information about the curvature of the production function. This is important since the curvature is a key ingredient to determine the effect of legal enforcement on interest rates.

In order to evaluate how different specifications of the model fit real credit markets, it is necessary to make a careful empirical analysis with adequate data. But the results just presented illustrate that the model produces several testable implications, that could be useful to confront the model with real credit markets. Also, with adequate data, further research could define the specification of the model that better fit the data through structural estimation of the model or an identifiable version of it. Estimates from different locations and periods, with different legal environments could be compared. An estimated version of the model could also generate forecasts about the effects of improvements in legal enforcement.

5.2 Policy Implications

High interest rates and credit rationing, especially for poor individuals, are commonly regarded as a problem in developing countries, and policies suggested to deal with them include subsidy to credit, public provision of credit and interest rate controls. The model discussed presents high interest rates and low amount of credits for low wealth individuals, but those are optimal given the environment. If the model provides a good characterization of credit markets, such policies would have no advantage over the mere redistribution of initial wealth. On the other hand, proposition 3 and the fact that, in simulations, the contracts respond to the court quality variables, indicate that improvements in legal enforcement could bring welfare gains. This reinforces the importance of evaluating how well the model fits real credit markets data. A research agenda in this direction could include a comparison of this model with other possible explanations for high interest rates and low credit for poor individuals.

The negative result that interest rates may increase even with improvement in legal enforcement also have consequences for policy evaluation. An increase in interest rates does not necessarily imply welfare loss. Proposition 4 determines that increases in the reliability of courts never produce welfare losses. However, sometimes interest rates increase with an improvement in enforcement. In those cases, the gain from the possibility of larger investments more than compensates the losses that may come from higher interest rates. In principle, it is possible that legal improvements expand the amount to credit and simultaneously produce an increase in average interest rates. This may help to explain why countries like Brazil, that have an intermediary level of development, have higher borrowing interest rates than less developed and institutionally more unstable countries (as in Leaven and Majnoni (2003)). In those countries, interest rates are lower, but the amounts of formal borrowing are small.

5.3 Theoretical extensions

The model employed is particularly simple in the dynamic structure. Repayment amounts are limited to one period cash flows and the enforcement power of liquidation is greatly limited by the fact that it affects only one cash flow. Therefore, an extension of the model, either to more periods or infinite periods would increase realism. Such an extension could also provide insights about the effect of legal enforcements on the term structure of borrowing contracts.

Another possible theoretical extension would be to include ex-ante asymmetric information. Individuals would have better information about their projects than the lender before borrowing contracts are firmed up. Stiglitz and Weiss (1981) show that credit rationing can emerge in credit markets as a result of adverse selection. But this result is criticized by Bester (1985) that shows that collateral can be used to screen borrowers with different risks and overcome this problem. A natural extension of the current paper would be to evaluate if the notion of collateral employed here (scale of projects) would be able to produce such screening and, if so, under which conditions.

The current model takes the legal system as exogenous. But, in principle, it is possible to make the parameters related to courts endogenous. One simple way to do so, would be to make the reliability of courts an increasing function of amount invested in legal institutions. This could possibly define the determinants of optimal investment on legal institutions, and serve as a guideline to cross-country comparisons of quality of legal systems.

6 References

Albuquerque, R. and Hopenhayn, H. (2004), "Optimal Lending Contracts and Firm Dynamics ". *Review of Economic Studies, 77, pp. 285-315.*

Aghion, Phillipe and Patrick Bolton (1996). "A Trickle-down Theory of Growth and Development with Debt Overhang". *Review of Economic Studies, 64, pp.151-172.*

Bolton, P. and Sharfstein, D. (1990), "A Theory of Predation Based on Agency Problems in Financial Contracting", American Economic Review, 80, 93-106.

Araújo, A. and Eduardo A. Rodrigues (2004),"Taxas de Juros Bancárias e Garantias Reais: uma Avaliação Preliminar com Base nos Dados da Nova Central de Risco ", Central Bank of Brazil, Mimeo.

Buera, F. (2006), "Persistency of Poverty, Financial Frictions and Entrepreneurship", mimeo, Northwestern University.

Chang, C. (1990), "The Dynamic Strucure of Optimal Debt Contracts", Journal of Economic Theory, 52, 68-86.

Costa, A.C. and Joao M.P. De Mello, (2006). "Judicial Risk and Credit Market Performance: Micro Evidence from Brazilian Payroll Loans," NBER Working Papers 12252, National Bureau of Economic Research, Inc.

DeMarzo, P. M. and M. J. Fishman (2003), "Optimal Long-Term Financial Contracting with Privetely Observed Cash Flows", Mimeo.

Evans, David S. and Boyan Jovanovic. "An Estimated Model of Entrepreneurial Choice Under Liquidity Constraints". *Journal of Political Economy* 97 (1989): 808 - 827.

Gale, D. and M. Hellwig (1985), "Incentive Compatible Debt Contracts : The One Period Problem", Review of Economic Studies, 52, 647-663.

Greenwood, Jeremy and Boyan Jovanovic, "Financial Development, Growth, and the Distribution of Income," *Journal of Political Economics* v. 98 (1990), pp. 1076-1107.

Hart, O.(1995), "Firms, Contracts and Financial Structure", Oxford University Press.

Hart, O. and Moore, J. (1998), "Default and Renegotiation: a Dynamic Model of Debt", Quarterly Journal of Economics, 113, 1-41.

Holtz-Eakin, Douglas, Joulfian, David and Harvey S. Rosen.(1994) "Sticking It Out: Entrepreneurial Survival and Liquidity Constraints." Journal of Political Economy 102: 53 - 75

.Karlan, D. and J. Zinman (2006), "Observing Unobservables: Identifying Information Asymmetries with a Consumer Credit Field Experiment", mimeo, Yale University.

Krassa, S. and Anne Villamil (2000), "Optimal Contracts When Enforcement is a Decision Variable", Econometrica, 68, 119-134.

Krasa, S., T. Sharma and A. Villamil (2004), "Bankruptcy and Firm Finance", University of Illinois, Urbana-Champaign, Mimeo.

Laeven, L and Giovanni Madjnoni (2003), "Does Juditial Efficiency Lower the Cost of Credit?", World Bank Policy Research Working Paper 3159.

La Porta, Rafael; Lopes-de-Silanes, F; Shleifer, A.; and Vishny, R. (1998), "Law and Finance", Journal of Political Economy, 106(6), 1113-1155.

Lloyd-Ellis, Hew and Dan Bernhardtn , (2000) "Enterprise, Inequality, and Economic Development," *Review of Economic Studies*, v.67: 147-68.

Magnac, Thierry and Jean-Marc Robin, (1996)"Occupational Choice and Liquidity Constraints." Richerche Economiche 50: 105 - 133.

Townsend, R. M. (1979), "Optimal Contracts and Competitive Markets with Costly State Verification," Journal of Economic Theory, 21, 1- 29.

Visaria, S. (2006), "Legal Reform and Loan Repayment: The Microeconomic Impact of Debt Recovery Tribunals in India ", IED Working Paper Number 157, Boston University. (Under Review).

Williamson, S. D. (1987), "Costly Monitoring, Loan Contracts and Equilibrium Credit Rationing", Quarterly Journal of Economics, 102, 135-45.

7 Appendix 1

7.1 Proofs of the propositions presented

Proposition 1 *Let b, k and w be given. Suppose that assumptions (a) to (d) are valid and that Θ is finite. Then, if an optimal contract implies that types θ_1 and θ_2 chose repayment with probability one, there exists an optimal contract in which both types repay the same amount $\widehat{p}_r$ with probability 1. If (a') and (d') (which follows from risk aversion) are valid, this is a necessary result: an optimal contract where types θ_1 and θ_2 repay with certainty must have both types repaying the same amount with certainty.*

Proof. First, it is always possible in an optimal contract to have individuals of each type with a deterministic value of repayments. Indeed, suppose there is randomization in the repayment amount for some type. By assumption (a), there is a nonrandom amount of repayment that could bring the same utility to this type without a lower revenue for the lender. By assumption (b) this would not give any extra incentives for individuals with higher values of θ to misreport their type. By assumption (c), this would also not give any extra incentive for hidden savings.

Now, suppose that there are different values of repayment for types that repay with certainty $p_{r1} < p_{r2} < ... < p_{rn}$. (since Θ is finite, there is a finite amount of values). Let Θ_1 be the set of values of θ that pay p_{r1}, and Θ_2 the set of values of θ that pay p_{r2}. By truth telling, all elements in Θ_1 must be bigger than the elements in Θ_2. By the same reason, individuals with repayment values bigger than p_{r2} have θ lower than those of Θ_1 and Θ_2, and thus cannot report having a type in Θ_1 or Θ_2. There is an intermediate level of repayment, p_r' between p_{r1} and p_{r2} that makes the expected utility conditional on being a type in either in Θ_1 or in Θ_2 unchanged. By condition (d), this fixed value of repayment would not decrease the revenue of the lender. By condition (b), this would also not increase the gain of receiving transfers in the second period. Therefore, it would not give additional incentives for hidden savings. Extending this procedure to the other levels of repayment we can find a unique value of repayment for all types that repay with probability one.

Notice that if (a') is valid, moving from randomization to a unique payment value for each type that repays with certainty increases the revenue of the lender, and thus it must be the case that each type repay one value with certainty (The extra revenue could be transferred for the higher θ individual, increasing ex ante expected utility without generating extra incentives for savings by condition (a)). If (d') is valid, substituting p_{r1} and p_{r2} for a unique value p_r' increases the lenders revenue. Thus, it must be the case that types that repay with certainty repay the same amount. ■

Lemma 2 *Let b, k and w be given. Suppose assumption (e) is valid and $c > 0$. Then, the probability that in the optimal contract $I_d(\theta f(k), p)$ (there is default) and $l = 1$ (defaulters liquidate after courts fail to liquidate) is zero.*

Proof. Suppose an investor with ex-post shock $\overline{\theta}$ defaults and choose $l = 1$. Her utility will be $\widehat{V} = \lambda V_2^v(A\overline{\theta} f(k), p_{d1} + ik) + (1 - \lambda)V_2^v(A\overline{\theta} f(k), p_{d2})$[14]. If the lender offers a value p_v for voluntary liquidation such that $V_2^v(A\overline{\theta} f(k), p_v) = \widehat{V}$, borrowers would be willing to voluntarily liquidate. As V_2^v is concave, it is clear that $p_v \geq \lambda(p_{d1} + ik) + (1 - \lambda)p_{d2}$,and thus the revenue of the lender is higher than in the contract with default (notice that the cost of default c will not have to be paid). Furthermore, from assumption (e) investors with $\theta > \overline{\theta}$ will not have additional incentives to pretend to be of type $\overline{\theta}$, as the absolute risk aversion for liquidators is nonicreasing with wealth. Also, from (e), additional savings in the first period would not make this new offer more valuable than the previous one: there is no additional incentives for hidden savings. So, the new contract produces the same outcome for all types of investors and increases the amount of resources obtained by the borrower. ■

Proposition 3 *Given the condition on Lemma (2), both an increase in λ and a decrease in c do not decrease welfare.*

Proof. By Lemma (2), we only have to consider the case with $l = 0$ (no choice of liquidation when courts fail to liquidate). The fact that lower c does not decrease welfare is easily seen: c affects only constraint (3), and lower c relaxes it. Now suppose λ increases from λ' to λ''. Let C_1 be a contract with default and $l = 0$ that is optimal given λ'. I show that there is another contract C_2 , that is feasible given λ'', such that, when $\lambda = \lambda''$, the utility of all types under C_2,is equal to the utility they have when the contract is C_1 and $\lambda = \lambda'$.

[14] Notice from the structure presented in figure 1 that $V_2^{d1}(y, p) = V_2^v(y, p + ik)$, and when $l = 1$, $V_2^{d2}(y, p) = V_2^v(y, p)$.

The new contract C_2 is defined as follows. The values of b and k in C_2 are equal to their values in C_1. All the probabilities of (θ, p) pairs specified in C_1 that resulted in no default under λ' and are kept unchanged. Suppose that before this change in λ, C_1 implies that an individual with cash flow $\overline{\theta}$ have a probability π of defaulting and facing a transfers vector under default $(\overline{p}_{d1}, \overline{p}_{d2})$. In the new contract this is substituted by a randomization between two scenarios for individuals with type $\overline{\theta}$.With a probability $\pi' = \pi\lambda'/\lambda''$, they are assigned to default and a transfer vector of $(\overline{p}_{d1}, \overline{p}_{d2})$. With probability $\pi'' = \pi(1 - \lambda'/\lambda'')$, p is defined as $p_r = p_v = \overline{p}_{d2}$, $p_{d1} = p_{d2} = 0$. Given the choice for $l = 0$ in the first contract, it is clear that individuals with $\theta = \overline{\theta}$ will decide for repayment in this last scenario. Also, individuals with $\theta > \overline{\theta}$ have no additional incentives to pretend to have a type $\overline{\theta}$. Indeed, by the fact that $p_{d1} = p_{d2} = 0$ and $\overline{p}_{d2} \leq 0$, any individual would prefer either voluntary liquidation or repayment to default in this scenario. So, with this new contract, individuals that declare a type $\overline{\theta}$ have a probability $\pi(1 - \lambda'/\lambda'') + (1 - \lambda'')(\pi\lambda'/\lambda'') = \pi\lambda'$ of facing the choice between liquidation or not with transfers $\overline{p}_{d2}$and a probability $\pi(1 - \lambda)$ of facing liquidation and seizing of collateral with transfers $\overline{p}_{d1}$. In terms of utility, the choices available to individuals are unaffected by the change of λ from λ' to λ'' and the change of contract from C_1 to C_2. And the revenue of the lender for these scenarios under λ'' is $\pi(\lambda(ik + p_{d1}) + (1 - \lambda)p_{d2} - \lambda'/\lambda'' c)$, which is bigger than $\pi(\lambda(ik + p_{d1}) + (1 - \lambda)p_{d2} - c)$, the revenue under λ and C_1 for this case. Therefore, the new contract increases welfare. ■

Proposition 5 *Suppose borrowers are risk neutral, with utility given by $U(x) = x$, and $\beta = (1 + r) = 1$,and no randomization conditional on θ is allowed. Then whenever $b > \lambda ik$ (or equivalently, $k - \lambda ik < w$), there exists some optimal solution for the optimal transfer policy (Program 1) with the following properties:*

a- There *exists a repayment value $\overline{p}$ such that whenever $\lambda\theta f(k) > \overline{p}$, or $\theta > \overline{\theta}_3(\overline{p}) \equiv \overline{p}/\lambda f(k)$, there is repayment of an amount $\overline{p}$.*

b- Whenever *$ik > \theta f(k)$, or $\theta < \overline{\theta}_1 \equiv ik/f(k)$, there is voluntary liquidation, and $p_v = -(1 - \lambda)ik$.*

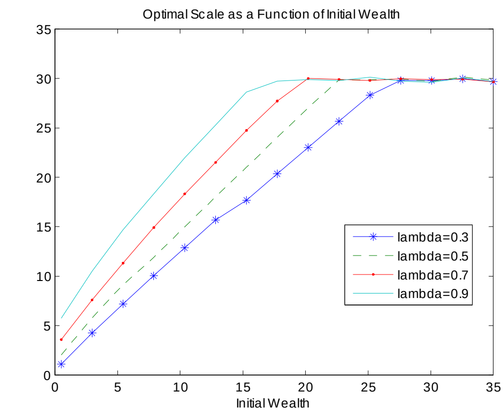
c- Whenever $\overline{\theta}_1 < \theta < \min(\overline{\theta}_2, \overline{\theta}_3)$, where $\overline{\theta}_2 \equiv (ik + \frac{c}{1-\lambda})/f(k)$, *there is voluntary liquidation with $p_v = -(1 - \lambda)\theta f(k)$*

*d- Whenever $\overline{\theta}_2 < \theta < \overline{\theta}_3$,*there is default, with $p_{d1} = p_{d2} = 0$.

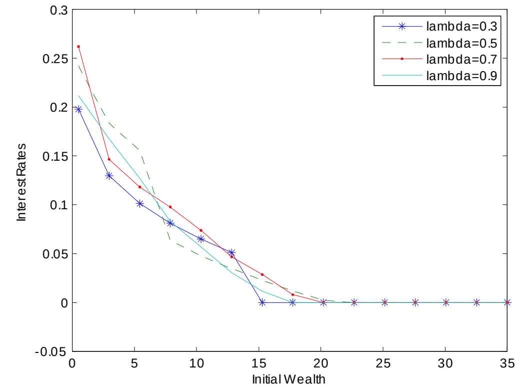
Proof. Suppose $\overline{\theta}$ is the maximum possible value of θ. It must be the case that $\overline{\theta}f(k) > ik$, oterwise there would be no investment up to the scale k.But no other type can pretend to be $\overline{\theta}$, so any arrangement with default or voluntary liquidation can be replaced by one with the same revenue and higher utility with repayment and therefore no risk of liquidation. So type $\overline{\theta}$ will be a repayer of some amount $\overline{p}$. For the reasoning presented in proposition 1, all types that repay, repay the same amount. The next step is to show that there is an optimal contract in which whenever there is voluntary liquidation or default the utility of the borrower is equal to that of default with zero transfers in the second period. Let us first order the values of θ as $\theta_1 < \theta_2 < \theta_3 <$ .....I start with the case where $\theta_1 < \overline{\theta}_1$ (as in statement (b)). Clearly, if $\theta_1 \leq \overline{\theta}_1$there must be voluntary liquidation with probability 1. Indeed replacing any event with no liquidation by liquidation with additional transfers of ik to the borrower would increase the utility of the borrower without changing the revenue of the lender. If $p_v(\theta_1)$ is higher than $-(1-\lambda)ik$ the lender would prefer to default and afterwards liquidate by his own. If $p_v(\theta_1) < -(1-\lambda)ik$, it is possible to write another contract with liquidation and transfers $p_v'(\theta_1) = -(1-\lambda)ik$ and the repayment for those that repay with certain the amount $\overline{p}$ is reduced by $(p_v'(\theta_1) - p_v(\theta_1))$ $(h(\theta_1)/\Pr(rep)$, where $\Pr(rep)$ is the probability of the high θ types with probability 1. The change in the objective function is $(p_v'(\theta_1) - p_v(\theta_1))$ $h(\theta_1) + \Pr(rep)(p_v'(\theta_1) - p_v(\theta_1))$ $(h(\theta_1)/\Pr(rep) = 0$. So, if the original contract was optimal, the new one is also optimal. We can make such changes successively for θ_2, θ_3 until we reach the point in which $\theta_n > \overline{\theta}_1$. For θ_n, it is possible that there is default or voluntary liquidation. If there is voluntary liquidation, it must be the case that $p_v(\theta_n) \leq -(1-\lambda)\theta_n f(k)$, otherwise the borrower would choose default. If $p_v(\theta_n) < -(1-\lambda)\theta_n f(k)$ it is possible to change $p_v(\theta_n)$ to $p_v'(\theta_n) = -(1-\lambda)\theta_n f(k)$. The resulting gain in revenues could be transferred to those high $\theta's$ that repay with probability one. As in the case of $\theta_{1,}$this would not decrease the expected value of the objective function and would keep revenues constant. Also, it would keep the utility of θ_n higher than reporting a lower θ. And it would not give extra incentives for misreporting in the contingency of higher values of θ. If, on the other hand, the solution is default with $\lambda p_{d1}(\theta_n) + (1-\lambda)p_{d2} \equiv p_d < 0$, we could replace p_d by zero and transfer the expected gains from this to the contingency of high $\theta's$ with repayment. As in the case of voluntary liquidation, this would keep the contract optimal. Notice that with these reformulations the utility under default and voluntary liquidation would be the same. So, the key ingredient to determine if it is optimal to voluntarily liquidate or default is the if θ is smaller or bigger than $\overline{\theta}_2$. In the first case, the revenue from voluntary liquidation is higher, and in the second case the revenue from default is higher.

We can proceed with similar reformulations for $\theta_{n+1}, \theta_{n+1}$ and so on until $\overline{\theta}_3$ is reached, after which there is repayment and an utility level that is (except for the treshold case $\overline{\theta}_3$) higher than the utility of liquidation with no transfers. ■

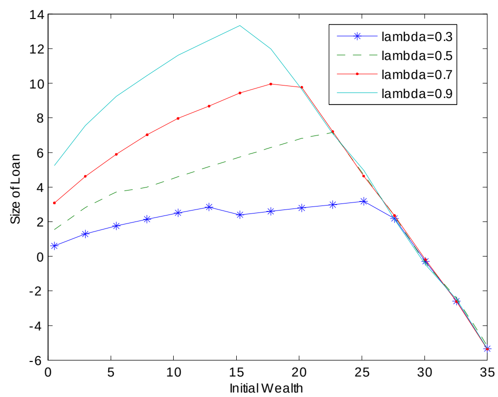
8 Appendix 2 - Numerical solutions



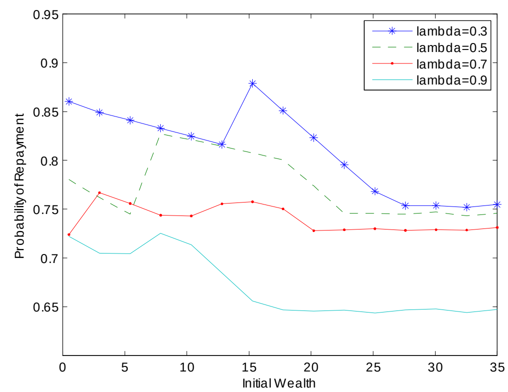
Optimal Scale Given Initial Wealth



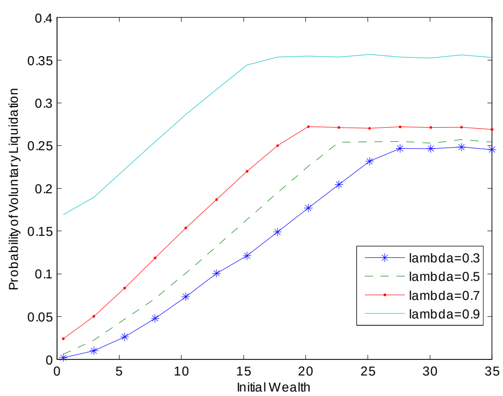
Interest rates Given Initial Wealth



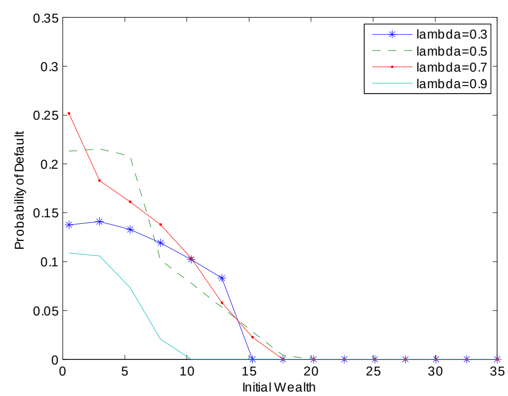
Size of Loan Given Initial Wealth



Repayment Probability Given Initial Wealth

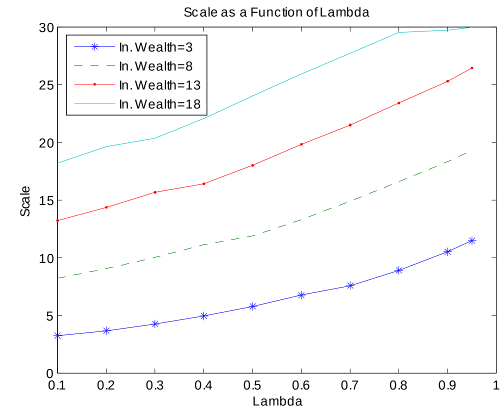


Voluntary Liquidation Probability Given Initial Wealth

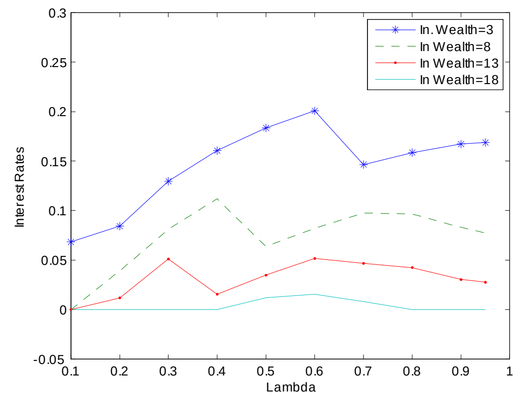


Default Probability Given Initial Wealth

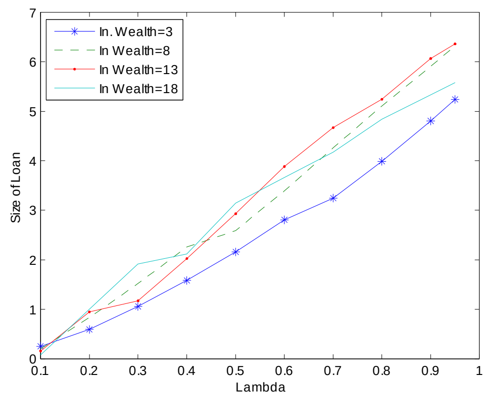
Figure 13 - $f(k) = k^{0.5},$ Baseline Case



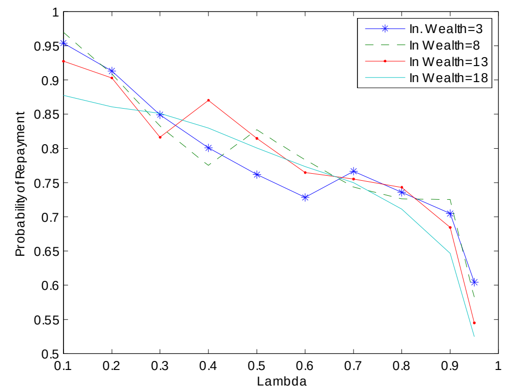
Optimal Scale Given λ



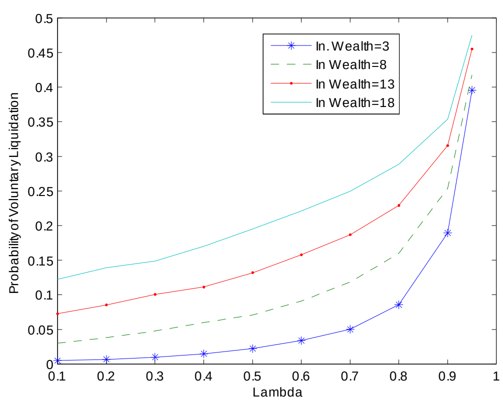
Interest rates Given λ



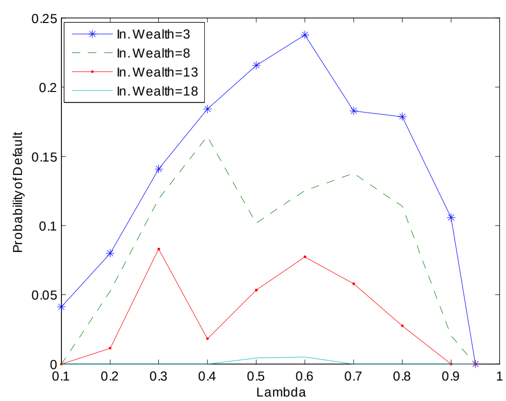
Size of Loan Given λ



Repayment Probability Given λ

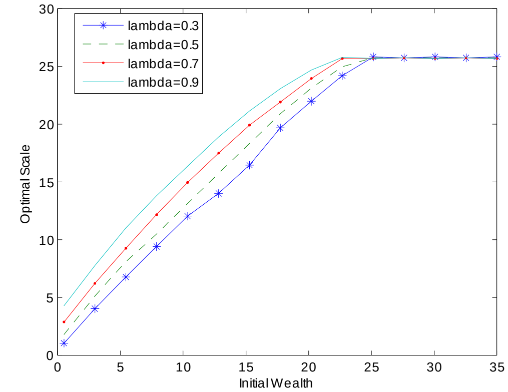


Voluntary Liquidation Probability Given λ

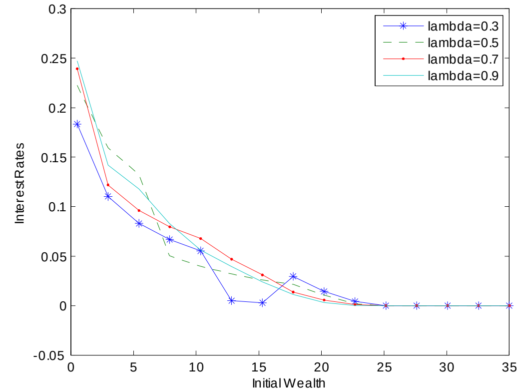


Default Probability Given λ

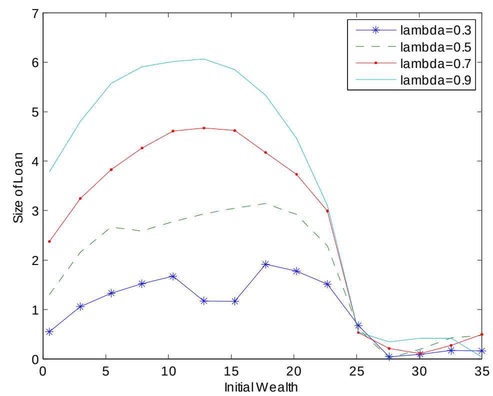
Figure 14 - $f(k) = k^{0.5}$, Baseline Case



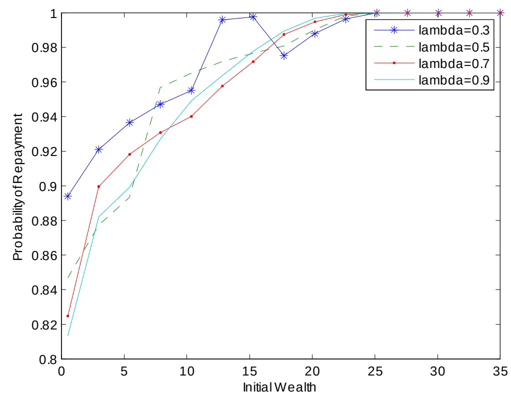
Optimal Scale Given Initial Wealth



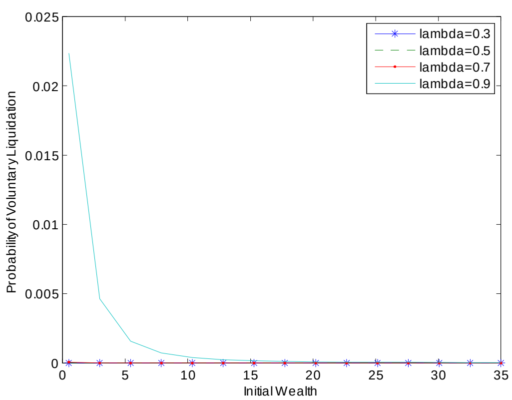
Interest rates Given Initial Wealth



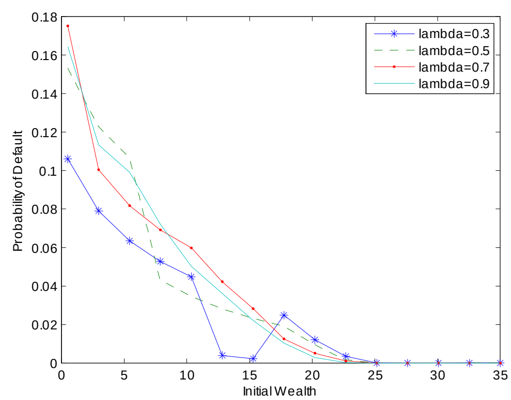
Size of Loan Given Initial Wealth



Repayment Probability Given Initial Wealth

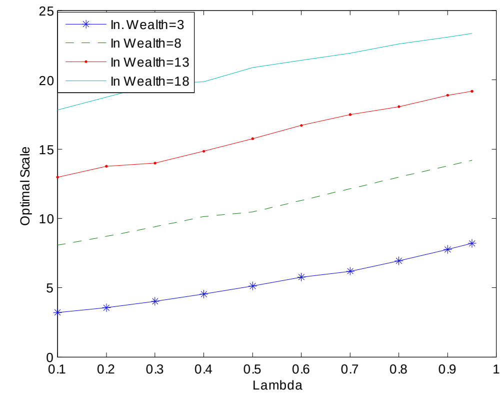


Voluntary Liquidation Probability Given Initial Wealth

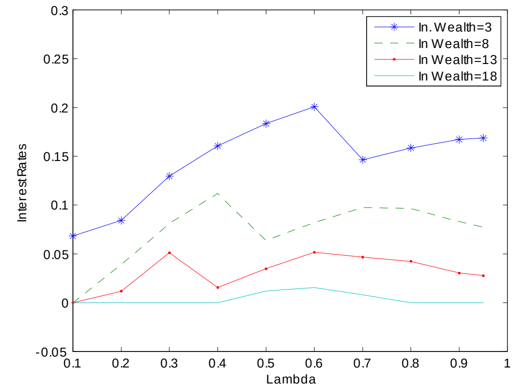


Default Probability Given Initial Wealth

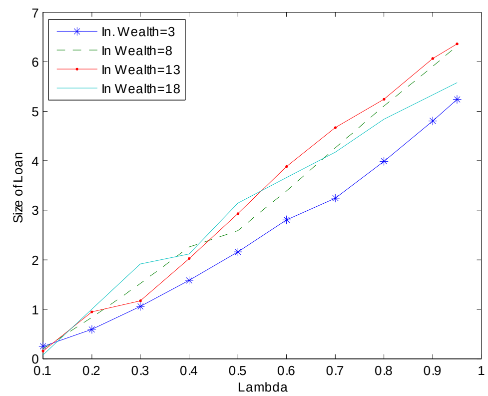
Figure 15 - $f(k) = k^{0.5},\ i = 0$



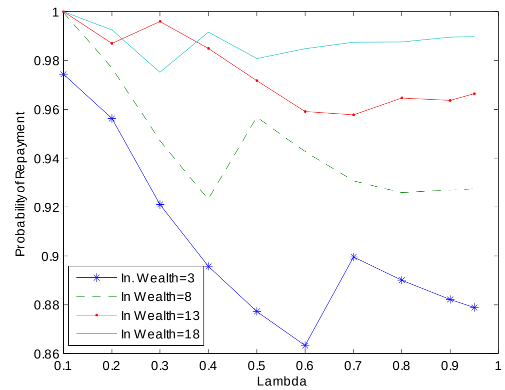
Optimal Scale Given λ



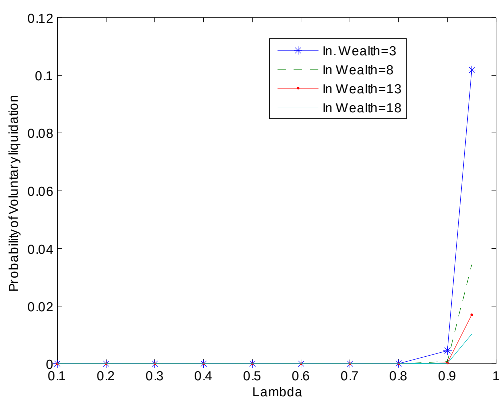
Interest rates Given λ



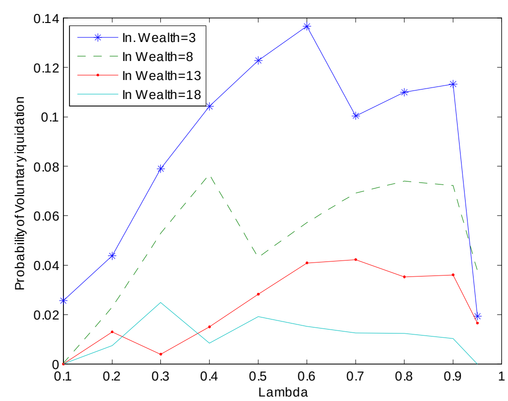
Size of Loan Given λ



Repayment Probability Given λ

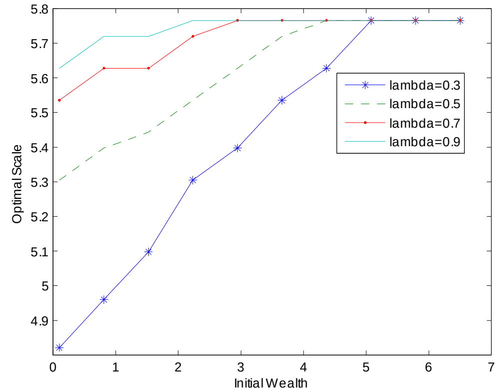


Voluntary Liquidation Probability Given λ

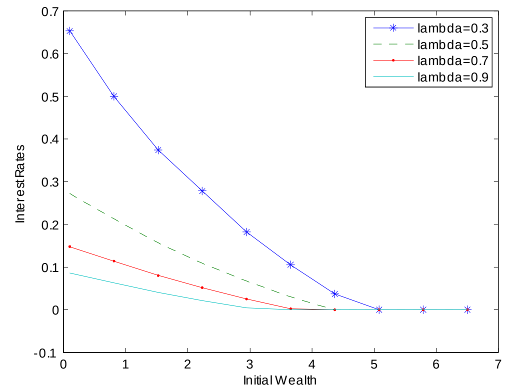


Default Probability Given λ

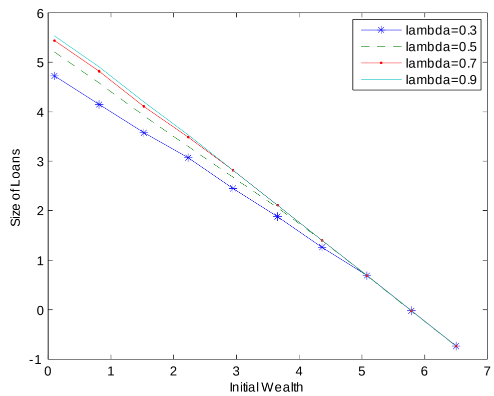
Figure 16 - $f(k) = k^{0.5}, \ i = 0$



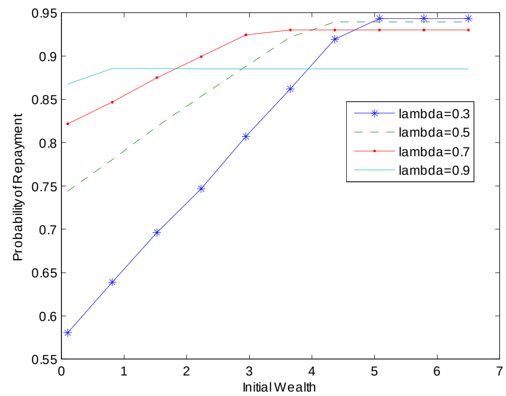
Optimal Scale Given Initial Wealth



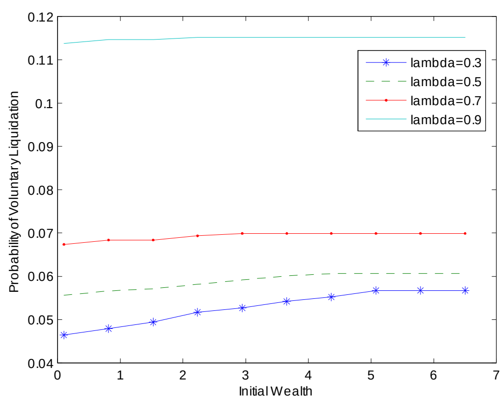
Interest rates Given Initial Wealth



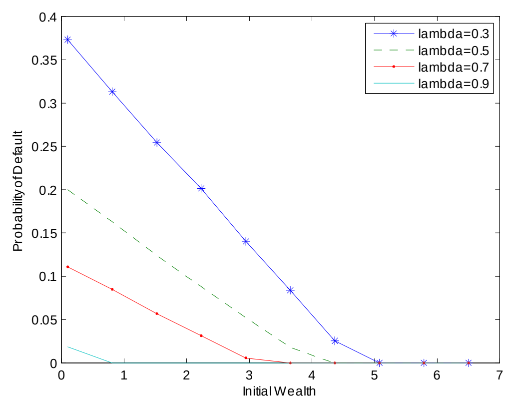
Size of Loan Given Initial Wealth



Repayment Probability Given Initial Wealth

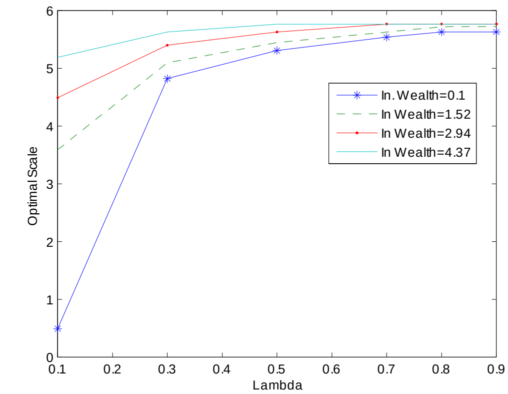


Voluntary Liquidation Probability Given Initial Wealth

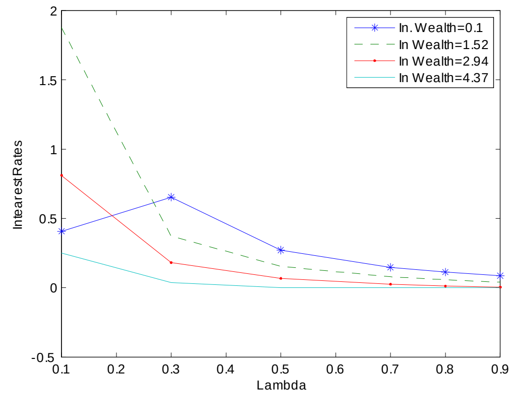


Default Probability Given Initial Wealth

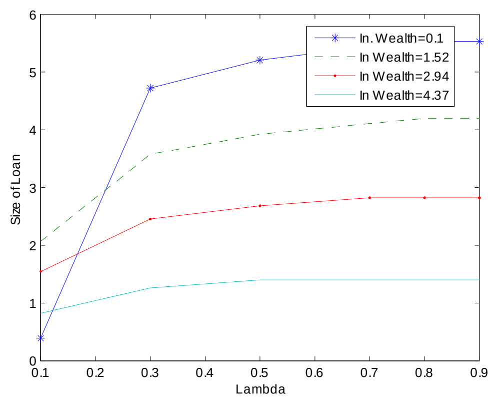
Figure 17 - $f(k) = (1 + (1 - k)^{-2})$ - baseline case



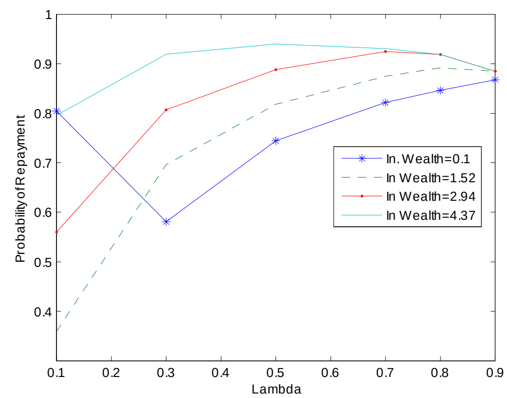
Optimal Scale Given λ



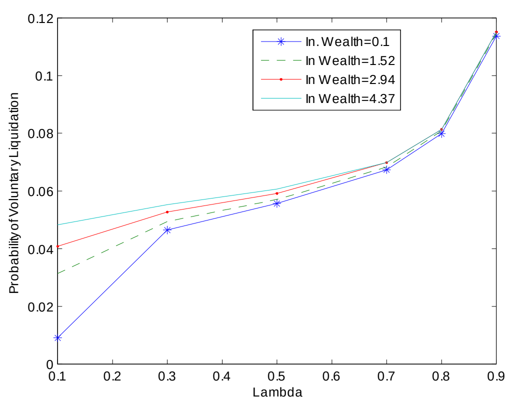
Interest rates Given λ



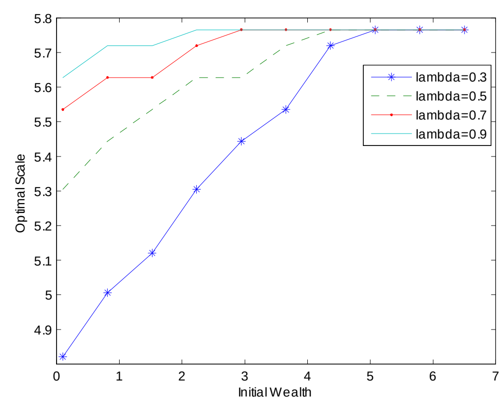
Size of Loan Given λ



Repayment Probability Given λ

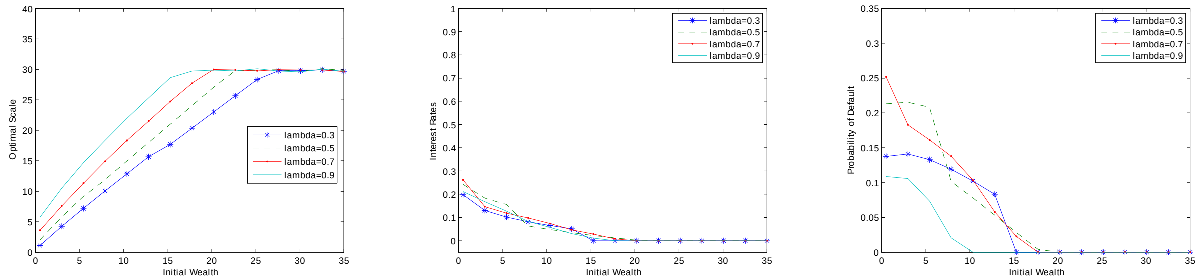


Voluntary Liquidation Probability λ

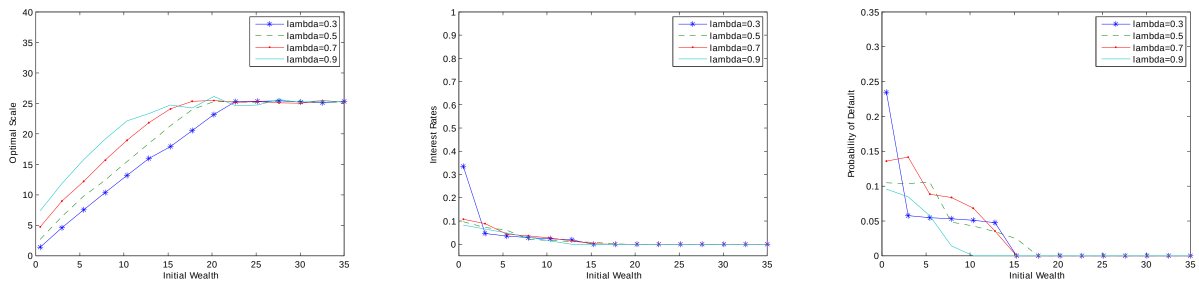


Default Probability Given λ

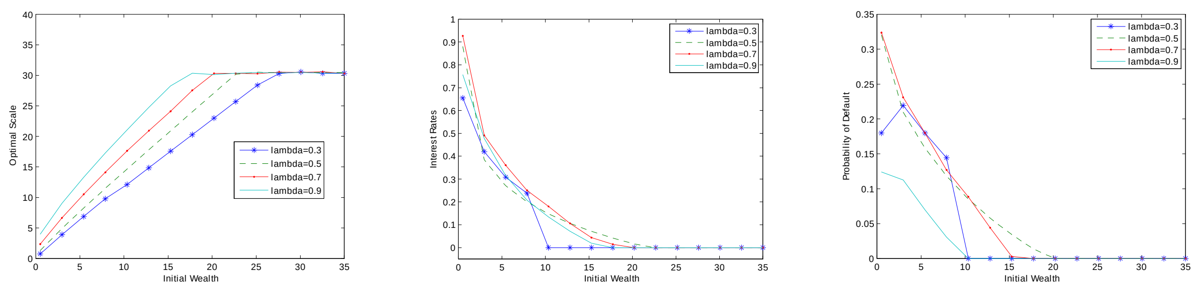
Figure 18 - $f(k) = (1 + (1 - k)^{-2})$ - baseline case



Baseline Case

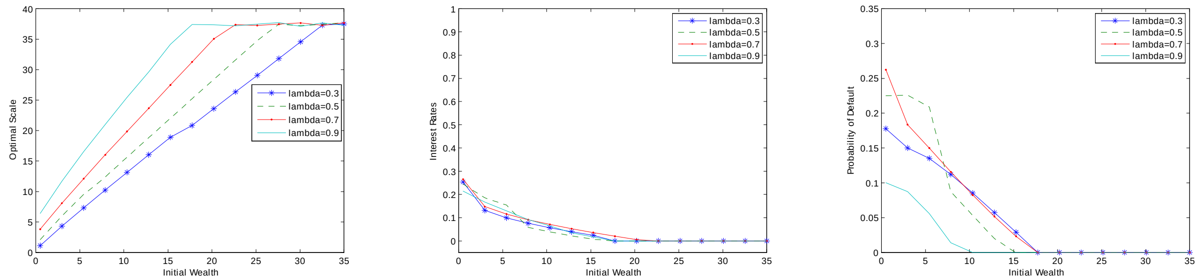


Lower Variance of θ - $\sigma = 1, \mu = 1$

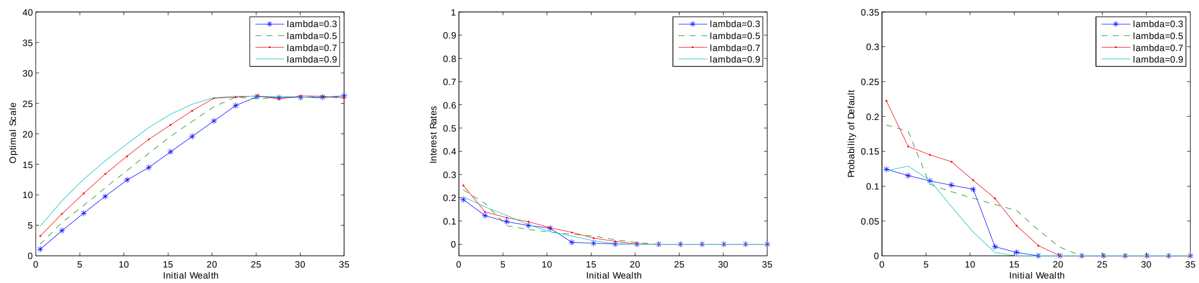


Higher Variance of θ- $\sigma = 0.3, \mu = 1.455$

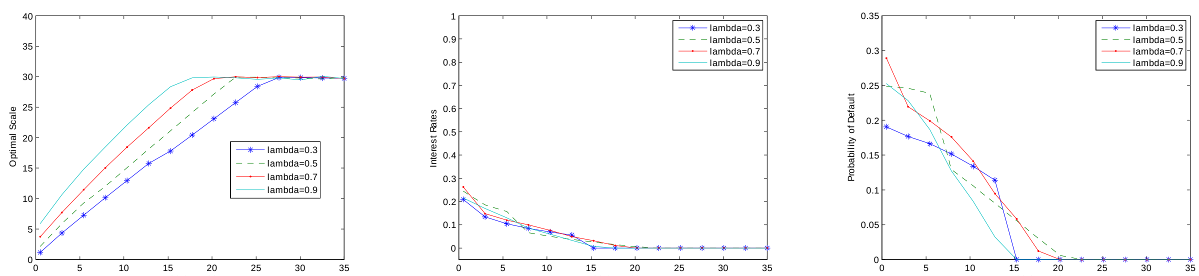
Figure 19 - Baseline case - $f(k) = k^{0.5}$



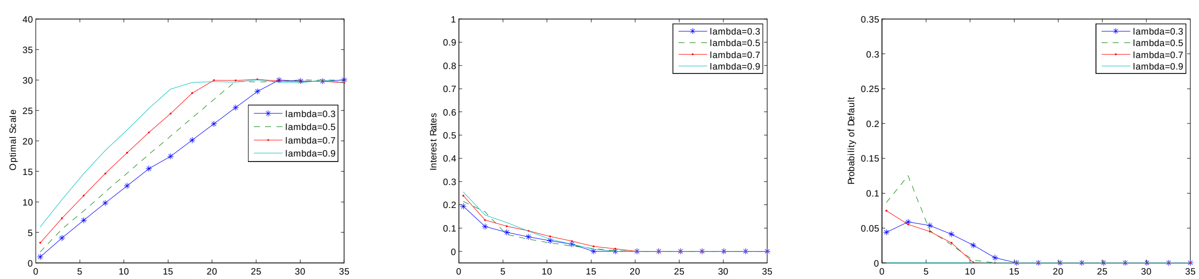
Higher liquidation value - $i = 0.6$



Lower liquidation value - $i = 0.3$

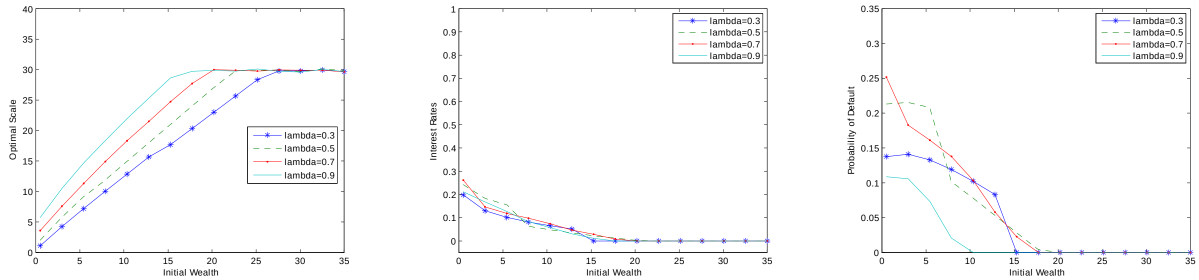


Lower cost of courts - $c = 0.1$

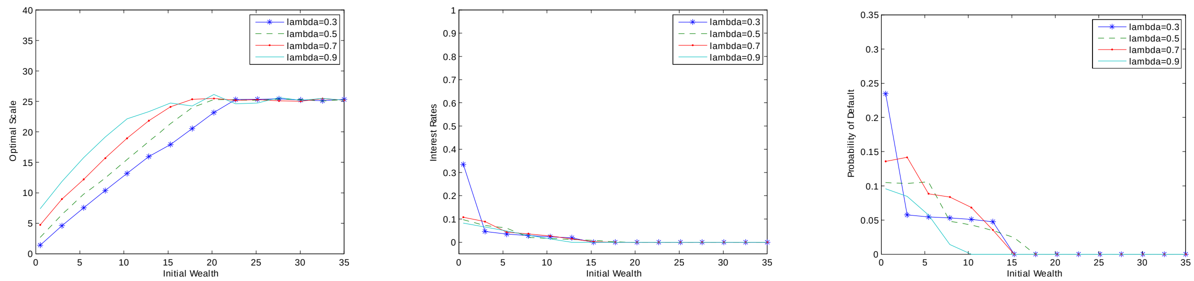


Higher cost of courts - $c = 0.8$

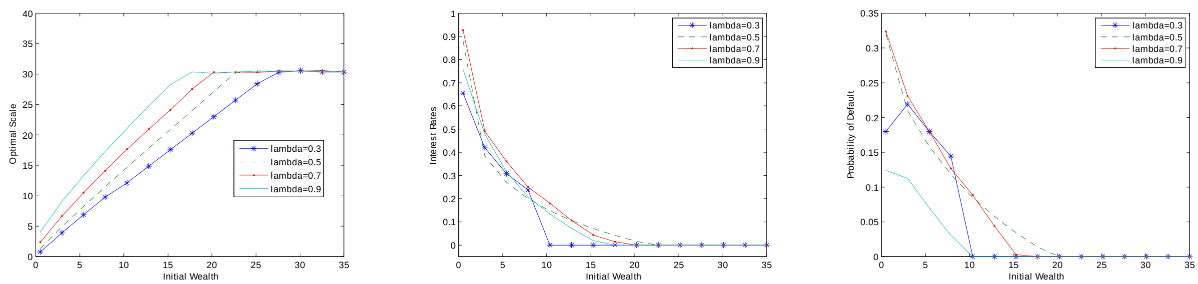
Figure 20 - Baseline case - $f(k) = k^{0.5}$



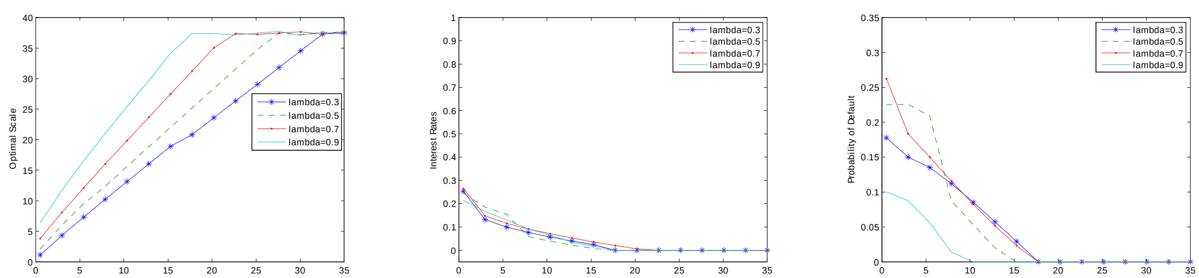
Baseline Case



Lower Variance - $\sigma = 1, \mu = 1$

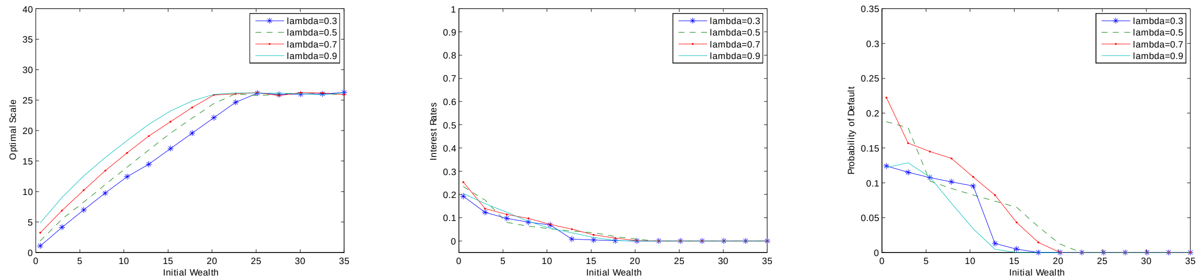


Higher Variance - $\sigma = 2, \mu = -0.5$

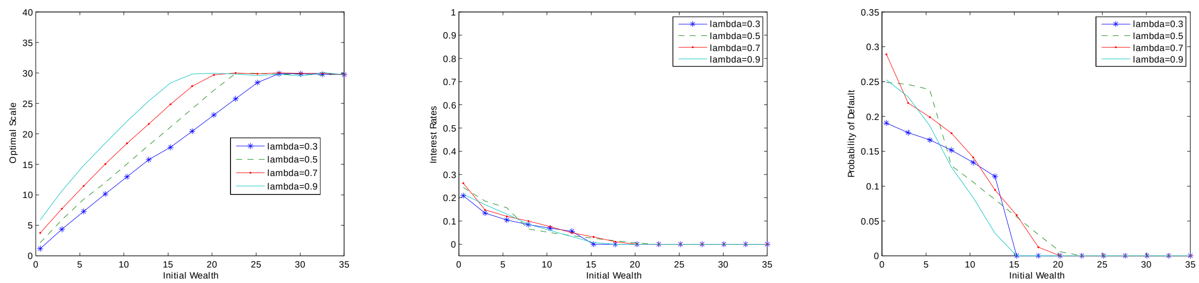


Lower Liquidation Value - $i = 0.2$

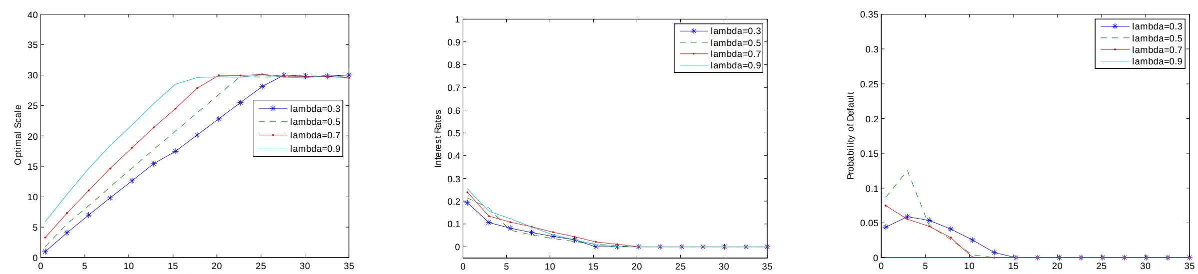
Figure 21 - $f(k) = (1 + (1 - k)^{-2})$ - baseline case



Higher liquidation value - $i = 0.8$



Higher cost of courts - $c = 1$



Lower cost of courts - $c = 0.1$

Figure 22- $f(k) = (1 + (1-k)^{-2})$ - baseline case