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1. Introduction

In terms of general consumption decisions, the anecdotal evidence indicates that, in many cases, the acquisition of goods and services requires performing a set of (costly) actions aiming to obtaining contracted conditions, in addition to the announced price payment. Simple examples of these phenomena are service providers, who require monitoring by customers, and malfunctioning goods, which demand the use of warranties.

However, consumers differ substantially from each other in terms of the cost of performing such tasks. A client buying from a shop next door finds it much easier to go there once more and complain when she finds a defective product, than a client residing in a foreign country (that's why we get double-crossed more often while vacationing!). The result of that heterogeneity is that, for the same monetary price, customers get substantially different bundles. Consumers that are less prone to spending that sort of effort get the lower quality goods.

Another form of the same phenomenon are undue charges. In that case, instead of receiving a good of lower quality, the consumer is charged more for virtually the same purchase, facing the need to complain in order to pay the original price. This sort of practice generally occurs when purchases are charged through a bill[1]. When we look at bank provided services, the context is particularly fit for that sort of practice, given that bank fees are charged directly from the clients' balance. The consequence is, therefore, that the final price is higher for consumers that do not take actions to reverse the extra fees, generating, as a result, situations of different prices for the same service.

In any case, the important characteristic of this sort of price discrimination is that it creates inefficient use of scarce resources without the benefit of a product, and

[1] The literature has focused on the small salience feature of this mechanism, i.e. the possibility that they stay unnoticed by the consumer. See, for a good example, Stango and Zinman (2014) who show that individuals respond to shocks on the salience of overdraft fees. The literature, however, has studied the case of services that are actually used, although without the client noticing that he will have to pay for them.

this inefficiency comes from at least two sources. Firstly, some agents spend effort to avoid a higher price or a smaller quality than the ones contracted. Secondly, suppliers incur additional operational costs associated with the policy, including those to reverse charges of complaining customers or substituting low quality goods.

We analyze, specifically, the case of a mischarge. In our model, the mischarge will be represented simply by an extra charge to the client's demand deposit account. However, it may also represent the supply of a not ordered add-on (cards, extra statements, etc.) or, with a greater degree of abstraction, a case in which a bank creates operational barriers to obstruct the exit of a client (thus impeding her from obtaining the same services from a competitor in better conditions). In these cases, most of the time, the loss imposed on a client may be reversed by a complaint, which may require a series of interactions between the client and the financial institution, possibly involving the bank supervision authority or even a lawsuit[2].

Needless to say, we do not argue that this sort of discrimination is part of the policy of a financial institution. So both authorities and financial institutions should be interested in foreclosing these mechanism, which may result from decisions of lower rank staff (say, bank managers), subject to imperfect incentives. This brings in the central issue of our paper: generally, bank managers know their clients better than anyone else does in the institution (let alone public authorities). This enables them to target these policies to clients who are less likely to complain, thus making the activity harder to detect. In this paper, we have two main aims. First, we propose a simple theoretical model illustrating this kind of price discrimination; and, second, we propose a test to answer the question of how can this sort of discriminating behavior be detected, in a context where the authority has less information about the client than the supplier of the services. Such a test can be useful, for example, as a device to target more costly monitoring activities, like inspections.

[2] In Brazil consumer rights are enforced by law. They are protected by the Consumer Defense National System, which is composed by several institutions including Procons. Additionally, for issues related with financial institutions, consumers can reach to the Central Bank, although, before that, consumers usually try to solve the issue directly with the involved bank first. The first level channel is a direct contact (through a bank agency, correspondent or phone consumer line) and a second level channel is through the bank ombudsman. When a complaint is filled in one of these two levels, we may say that the financial institution has all the relevant information about it. One could then ask: "What changes in the situation when the complaint is taken to the authorities?" Excluding the possibility of dispute regarding the complaint's legitimacy, the answer is that the financial institution learns something about the client: that he is willing to make trouble.

We do not provide an empirical application of the proposed test, given the lack of adequate datasets, but the paper can motivate their construction and give guidance about the information these datasets should contain. Although, detailed information on complaints and complaining clients is scarce, we try to provide some brief stylized facts regarding the issue.

Between January and June 2015 the Central Bank of Brazil received more than 105 thousand complaints concerning mischarges. Although in this case the size of the mischarge is not available, it is possible to find most of the clients in the Brazilian Credit Bureau data, SCR, and obtain some information regarding income. Financial institutions are required to report to SCR individual loan information for all clients who owe more than R$1000. Unmatched complainers correspond to only 6.5% of the total.

Table 1.1 – Stylized facts on complaints

Income level	Complainers % (1)	SCR % (2)	(1)/(2) (3)
No income	.5	3.3	0.14
Up to 3 minimum wages	22.6	60.6	0.37
From 3 to 5 minimum wages	17.2	15.1	1.14
From 5 to 10 minimum wages	26.1	12.3	2.12
Above 10 minimum wages	33.6	8.7	3.86

Table 1.1 reports the frequency distribution of complainers along income levels in column (1). Column (2) reports our proxy for how the total of clients distributes along the same categories. As we may see in column 3, the participation of complainers increases with income. Obviously, this probably results from several causes. For example, clients in higher income groups tend to be more educated, and thus can have smaller costs to complain.

We draw attention to the fact that this pattern is adhering to the model we present ahead, in which higher income clients are targeted with higher mischarges, since other things equal such mischarges should be less worthwhile of complaint (and less salient) to clients who earn more. Thus, the first contribution of this paper is to provide microfundations for price discrimination based on mischarges. We build a game played by a bank manager and potential bank clients. The bank manager must lure the clients to open an account, and then he use an optimal mischarge in a later period to increase his payoff. There is asymmetric information between the clients and the bank manager, thus the optimal mischarge varies with the information he observes and, sometimes, results in a complaint.

The second contribution of this paper is the development of a statistical test designed to compare false mistakes with genuine ones, which are assumed to be randomly distributed to clients. With simulations, we obtain the distribution of a likelihood function statistic of mischarges, under the hypothesis that mischarges are unintentional.

In section 2 we review the literature about mechanisms related to the one we propose. In section 3, we lay out our model of price discrimination using false mistakes. In section 4 we build a statistical test that uses the information of a set of complaints to reject the hypothesis that mistakes are genuine. We evaluate the test performance by simulation in section 5. Finally, section 6 concludes and proposes a research agenda.

2. Literature

According to Borenstein (1985) self-selection sorting mechanism uses a cost that a client faces to qualify for a lower price. Contrary to what happens in third degree price discrimination, it is price differential that determines the size of groups of clients getting low or high price. The examples the author offers are flight and stage performance tickets, which offer lower prices for advance purchases, and warehouse sales and coupons, which demand consumers to spend time (and effort) to take advantage of more attractive prices.

The (scarce) literature on coupons is particularly interesting, since in the case we analyze in our paper the consumer will spend scarce resources to have an undue charge reversed. The final picture is similar to the use of coupons: consumers who take a proposed set of actions end up paying less. The main difference is the order of events, i.e., in the case of coupons, the consumers who are willing to go through the trouble, search for discounts before they buy a good, while in the case of mischarge, the clients start paying more and have to make an effort afterwards, if they want their money back.

The closest reference in coupon literature to our paper is Narasimhan (1984). The author postulates that consumers equate the marginal cost of using a coupon with the discount it offers. The opportunity cost of time is measured by the salary, like in the model we develop ahead. Other interesting references are Shor and Oliver (2006) and Ben-Zion, Hibshoosh and Spiegel (2000)[3]. The first paper focuses on coupons used in internet purchases, portraying the consumer's technical competence to use the internet to find the discount coupons as the relevant dimension for segmentation. The second paper draws attention to the fact that a coupon policy will be more effective if it is possible to target more price-elastic clients with them. That approach is interestingly explored by Bester and Petrakis (1996), who set-up a model with two locations and consumers incur in heterogeneous dislocation costs if they choose the provider from a locality different from theirs.

We may also find similarities between the use of coupons as a mechanism of price discrimination and other forms of discrimination that employ the heterogeneity of consumer effort costs. In his model, Salop (1977) studies the problem of a monopolist who owns several stores. The price in each store is not advertised, but its distribution is known to consumers, who face different costs to visit a store and find its price out. Once they see a price, they must decide whether to visit one additional store, and so on. Thus, by making prices vary along stores, the monopolist can partially separate consumers with high and low dislocation costs. Just like in the case of coupons, there is an extra cost created for consumers that cannot be appropriated by the supplier, which is used merely to segment demand. According to the author, the main difference between price dispersion and the traditional mechanisms of price discrimination, like two-part tariffs and quantity discounts, is that it spends resources. That also happens in the case of

[3] In that paper, the cost for a consumer to use a coupon is ignored and a discount for the price of the first unit purchased is offered and compared with the alternative of third degree price discrimination.

coupons. On the other hand, just like in the case of traditional price discrimination, a group of consumers (the one with low dislocation cost) prefers the result of price dispersion to a single price, profiting from being separated from another group (the one with high cost).

Finally, we acknowledge that another way to understand undue fees as a price discrimination mechanism would be from the bounded rationality literature standpoint. Then, we would argue that some clients are not able to notice those charges or simply that they choose to ignore variations of their balances that they regard as small when compared to the cost of paying attention[4]. For example, the agent proposed by Gabaix (2012) ignores variations in interest rate that would generate an optimal consumption variation below a certain threshold.

Several authors have approached the financial services consumption relationship in ways that draw attention to the insufficiency of the perfect rationality paradigm. The reason for that is that these services may be quite complex to the layman, not only because of the financial knowledge that is necessary, but also for the sizable amount of details and contingencies associated to them. Another issue is that, on many occasions, these services require monitoring or learning by doing, e.g. checking accounts or credit cards (See, for example, Gabaix and Laibson (2006), Stango and Zinman (2014), Agarwal et al. (2008) and Ferman (2011))

Although we will not use a limited rationality framework in our model, it is important to say that it may be reinterpreted from that viewpoint. Yet, if we believe the problem is actually of that nature, policy recommendations may differ substantially, tending towards making information more salient or access to it less expensive. For example, the United States Congress limited the set of fees that banks may charge for credit card services since 2010 and obliged banks to clearly disclose on statements the consequences of paying minimum amounts[5]. Agarwal et al. (2015) evaluate the effects of these policies and find the loss in bank revenue coming from fees limitation was not recomposed with the rise of other charges and did not result in credit restriction.

[4] Simon (1978) points out that attention is a scarce resource, as one of the forms of implementation of procedural rationality. In that case, in environments with a large amount of information available, it may become necessary to learn to ignore part of it.

[5] In particular, banks were required to inform the reduction in interest payments obtained by passing from a situation in which the cardholders choose minimum payment to another in which they pay enough to pay off their debt in 36 months.

Additionally, the improvement in information regarding revolving credit brought about significant, although small, increase in the payments made by debtors. In Brazil, the National Monetary Council standardized checking account fees in 2008 and credit card in 2011. It also imposed some mandatory information in credit card statements.

3. A model of price discrimination using false mistakes

How can you tell if a cashier from a neighboring store is short-changing you or if he is simply bad at calculations? You can never be sure, but probably the pattern of "mistakes" would differ. It would take quite a sophisticated – apart from dishonest – cashier to give you more change than what is owed sometimes in order to camouflage the more frequent subtractions to the value.

To evaluate a question of that nature, we need a model that replicates false mistakes behavior and another one to produce genuine mistakes. In this section we build a simplified model that captures the action of a bank manager that uses undue charges as a price discrimination strategy. It will be employed in the next section to build a statistical test. Its main characteristic is the difference between the bank manager´s information set and the regulator's[6]. The rest of the model is intended to provide a simple setting for the test.

Players

The game is played by a bank manager and a set of potential clients. The bank manager maximizes his payoff by maximizing the bank's current profits. We model him as a residual claimant on them. He is not concerned, however, with long run consequences for the bank, like damaging its public image.

The potential client pool is a continuous population of individuals characterized by two features: time cost of complaining (t) and salary (w). Variable t is consumer's private information. It varies along consumers and is distributed according to probability density function $f(t)$, and independently of other variables in the model.

[6] The regulator in question can be either a public authority or higher rank staff inside the bank, such as an ombudsman.

This specification is intended to capture the fact that some individuals are more efficient than others in making a complaint, so they get a result in less time.

Additionally, the consumer is characterized by a reservation price R_w for checking account services. That value varies with the salary but is constant within a group w. This reservation price represents the idea that the presence of the account makes a consumer more efficient in performing transactions[7]. For that reason, it does not generate utility per se, but enters the consumer optimization problem as a factor that relaxes the budget constraint. Variable w is observed by a bank manager working for a monopolist bank that supplies checking accounts. The regulator, who does not partake in the game, does not observe either t or w. The central feature is that there are variables with information about the client, known to the bank manager and unknown to the agent performing the test. This asymmetry will be only relevant in the next section, when we develop a test for the model.

Stages of the game

Stage 1: Bank manager chooses p_w.

The bank manager chooses the price of having a checking account at the bank (p_w) for potential clients, depending on their observable w. We assume $p_w \geq 0$ and that accounts are "produced" with zero marginal cost. We name the set of clients Λ.

Stage 2: Individuals choose to acquire an account or not.

Individuals observe p_w and choose whether to become clients or not.

Stage 3: Bank manager decides about the mistake policy, choosing to implement it or not and, if it does, its size d_w. If he chooses not to use it, the game ends.

After individuals choose to acquire the bank´s service, the manager decides, for each group w, whether to charge an undue fee and its value, d_w. In case it

[7] Since R_w is a reservation price varying only with w, it is flexible to represent different assumptions. We may assume, for example, that wealthier people transact more, thus obtaining larger benefits from using electronic payments instead of cash. It may also reflect, in a very simplified fashion, access to different sets of assets for investment.

decides to charge it, the bank incurs two types of cost, within each w group: o_1, proportional to participation, represents the implementation cost, and o_2, proportional to the amount of complaining customers, stems from the actions required for charge reversion and may include punishment (a fine or being fired) and monetary reparations. Modeling the bank manager as a residual claimant on profits implicitly allows us the simplicity of including in o_2 a penalty suffered directly by him or a negative impact on the bank´s profits. It also allows us to include in o_1 terms pertaining to the manager (like effort) and to the bank´s cost (like computing resources). For simplicity, we assume these costs are given for a mistake policy, which is under analysis of the manager (i.e. we avoid the complexity of having the manager choosing from a menu of such policies, in which higher costs might be related with mistakes that are more complex for the client to complain about)

Stage 4: If $d_w > 0$, clients decide whether to complain or not.

Individuals who choose to participate, i.e. have an account, observe d_w and decide if it is worth for them to complain or not.

In the case all the information was available for the potential client at the time of deciding to acquire the account, his problem might be written as:

$$Max_{x,l,i,r} U(x,l)$$

$$s.t. x + wl + i[p_w + rtw + (1-r)d_w] = w + iR_w$$

In this formulation[8], we normalize total available time to a unit. Thus, l indicates the proportion of time dedicated to leisure and w represents the opportunity cost of that unit of time. It also indexes the account price (p_w) and reservation price (R_w). The consumption good, with unit price, is represented by x. Finally, we have binary variables i, which has value one in case of participation, and r, which has value one if the consumer decides to complain. By complaining, the consumer obtains a repayment of d_w at the cost tw.

[8] We offer alternative formulations in appendix 1.

Although R_w, p_w and t are known by the individual when deciding to acquire the account, d_w is not. The client´s decision that takes place after the revelation of d_w is time allocation between complaining and leisure (together with consumption).

3.1. Game solution

We solve the game by backward induction, considering a generic salary level w. Strictly speaking, for each w there is a different game between a bank manager and bank clients.

Stage 4

Given the undue charge, d_w, participating clients choose to complain or not. Their objective is to make budget constraint as loose as possible. Therefore, a consumer complains if $tw < d_w$.

Stage 3

Given participation, indicated by the values of t for which clients acquire the account, the bank manager chooses to implement the undue charge (and its size) or not.

For each possible participation set Λ, considering the distribution of t conditioned in participation, it would be necessary to indicate (at least) one optimal action to be chosen by the bank manager. Yet, we may significantly simplify the problem by arguing that if there is any participation it will be total within a w group. For that claim, shown in the solution of stage 2, we employ a restriction in the possible bank manager's strategies.

Since providing the account is costless, the bank manager will be interested, in the first stage, to set a price that guarantees some participation. Therefore, in the third stage we may take full participation as given. If the bank manager chooses to charge an undue fee, its problem may be stated as:

$$\pi^{d_w^*} = \max_{d_w} \int_{-\infty}^{d_w/w} f(t)dt\,(-o_2) + \int_{d_w/w}^{\infty} f(t)dt\, d_w - o_1$$

In that expression, integrals aggregate clients, separating them into two groups. The first term refers to clients whose t is smaller than d_w/w, i.e., those for whom complaining increases utility. This fraction is multiplied by reversion cost o_2. The second integral, which contains individuals who are not interested in complaining is multiplied by the undue charge, d_w, generating the revenue of the policy. In the end of the expression, implementation cost o_1 appears. In case the bank manager decides not to discriminate with the undue fee, the whole expression is set to zero. The expression represents both the profit obtained in a continuous portfolio of clients with mass 1 and the expected profit for a client.

Consequently, the first order condition is given by:

FOC: $\frac{\partial \pi^{d_w}}{\partial d_w} = 0 \therefore$

$$\int_{d_w/w}^{\infty} f(t)dt = \frac{1}{w} f(d_w/w)(o_2 + d_w)$$

That is to say, in order to increase profit by rising d_w, the proportion of the clients who do not complain needs to cover the marginal loss brought about by clients who switch from not complaining to doing so.

The first order condition may also be rewritten as:

$$\frac{[1 - F(d_w/w)]}{f(d_w/w)} = \frac{o_2 + d_w}{w}$$

The second order condition is:

SOC:

$$\frac{\partial^2 \pi^{d_w}}{\partial d_w^2} = -\frac{2}{w} f(d_w/w) - \frac{(o_2 + d_w)}{w^2} f^{'}(d_w/w) < 0$$

Hence, the only possibility of the second order condition failure is for a point where $f'(d_w/w) < 0$. Substituting FOC, we may rewrite the expression as:

$$\frac{\partial^2 \pi^{d_w}}{\partial d_w^2} = -\frac{2}{w} f(d_w/w) - \frac{1}{w}\frac{[1 - F(d_w/w)]}{f(d_w/w)} f'(d_w/w) < 0$$

Which is equivalent to:

$$2f(d_w/w)^2 + [1 - F(d_w/w)]f'(d_w/w) > 0$$

A sufficient condition for this inequality is the property of non-decreasing hazard rate[9].

Still, it is necessary to check condition $\pi^{d_w} \geq 0$, since it is always possible for the bank manager to forego the undue charge policy. Finally, for distributions with a bounded t support, e.g. $[\underline{t}, \bar{t}]$, notice that it will never be optimal to set $d_w/w \geq \bar{t}$ or $0 < d_w/w < \underline{t}$, since in the first case all consumers would complain, while in the second the undue charge might be increased without generating complaints.

Stage 2

We consider a candidate for pure strategy equilibrium. Assume that it accommodates allocation $\sigma = \{p_w; \Lambda; d_w\}$, i.e., that the chosen participation, given p_w is Λ and that the optimal undue charge is d_w. We omit the decision of complaining for simplicity. Then we would like to characterize $\sigma \in \Theta$, the set of possible allocations in equilibrium.

Consider the following restriction to the choice of d_w: if a strategy indicates the choice of $\hat{d}_w/\widehat{\Lambda}$, it also indicates the choice of $\widehat{d_w}/(\widehat{\Lambda} \cup t')$ where $t' \in \widehat{\Lambda}^c$ has zero mass.

[9] Naming the hazard rate $q(d_w/w) = f(d_w/w)/[1 - F(d_w/w)]$, this condition is equivalent to $q'(d_w/w) = \frac{f'(d_w/w)[1-F(d_w/w)]+f^2(d_w/w)}{[1-F(d_w/w)]^2} \geq 0$. Given that $f^2(d_w/w) > 0$ is guaranteed by FOC, we have that the SOC is also guaranteed.

The intuition of this hypothesis is that the bank manager would not review his policies if he was to obtain one single additional client, of infinitesimal size[10].

Consider cases in which the undue charge policy is implemented. Suppose p_w is such that, at the end of the game, consumers who opt in are not regretful, which is a requirement of an equilibrium.

In the first place, we know that if $d_w \neq 0$, there must be some consumer who does not complain, otherwise deviating to $d_w = 0$ would improve bank's profit. Hence, Λ must contain some consumer for whom it is the case that $\hat{t}w \geq d_w$. But that implies Λ must contain all consumers with $tw \geq d_w$, since if some of them were excluded, they would find that they could individually have benefited from participation. By participating, they would get the same payoff as individual $\hat{t}$, because p_w and R_w are equal for all individuals in w and individual $\hat{t}$ would also abstain from complaining.

At the same time, Λ must contain all consumers with $tw < d_w$. These are the consumers who complain, and they get a higher final utility by acquiring an account than the clients who do not complain. If some of them were excluded, deviation to participation would benefit them. As a result, if there is participation, it must be total.

Stage 1

For there to be participation, it is necessary that the bank manager defines p_w such that individuals acquire the account. Since we know participation must be total, it is necessary to guarantee a non-negative payoff for not-complaining clients. Thus, $p_w^* = R_w - d_w^*$.

It seems natural to imagine a situation in which this calculation results in a positive p_w, reflecting that the gain that may obtained from undue charges is relatively small, when compared to clients reservation price[11].

[10] On the other hand, when a client considers deviation from an initial situation, he may take the situations as constant. This may mean there is a mass of clients who individually prefer deviation.

It is interesting to point out that in the case when charging the undue fee is not possible or if there is some binding agreement available that it will not be used, the result would be $p_w^* = R_w$. This results in a situation that is strictly worse for clients who complain (and it is indifferent to all others)[12]. This result is in line with Gabaix e Laibson (2006)[13] and Armstrong (2006)[14]. On the other hand, the bank manager would always prefer this possibility, since he would extract all surplus from the clients.

Example: exponential distribution of t

The part of the game that is of interest for the construction of the statistical test we propose in the following section is the generation of complaint. For this reason, we concentrate on stages 3 and 4. For variable t, we assume an exponential distribution[15], i.e.,$t \sim Exp(\lambda)$:

$$f(t) = \begin{cases} \lambda e^{-\lambda t}, & se\ x \geq 0 \\ 0, & se\ x < 0 \end{cases}$$

Therefore, average time to meet the solution of a complaint is $1/\lambda$.

We use this distribution because we regard as more plausible the situation in which complaint times are concentrated on shorter values, with decreasing probability as they increase. Additionally, this is a common distribution for processes related to timing of events[16].

With this assumption, the bank manager's problem to decide about the undue charge becomes:

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[11] Cases with $d_w^* > R_w$ might imply a negative participation price or, if that is not possible, that the w group is left without service. That is because a promise not to charge undue fee d_w^*, given participation, would not be credible.

[12] To make welfare considerations, however, it would be necessary to analyze the possibilities in the previous footnote.

[13] In that article, myopic consumers subsidize sophisticated ones.

[14] When a monopolist chooses prices for two periods for fully rational consumers, the possibility of commitment benefits the firm at the consumers' expense.

[15] In appendix 2, we present the solution of these parts of the game with a uniform distribution for t.

[16] In particular, when the number of events in a time interval is generated by a Poisson distribution, the time between them is exponentially distributed. In our analysis, this would be equivalent to say that, knowing the expected number of solutions a customer would get if she used a certain amount of time to complain, she would also know the expected time it would take to solve the next issue.

$$\pi^{d_w^*} = \max_{d_w} \int_0^{d_w/w} \lambda e^{-\lambda t} dt\,(-o_2) + \int_{d_w/w}^{\infty} \lambda e^{-\lambda t} dt\, d_w - o_1$$

FOC:

$$\frac{\partial \pi^{d_w}}{\partial d_w} = \frac{1}{w}\lambda e^{-\lambda d_w/w}(-o_2) + \int_{d_w/w}^{\infty} \lambda e^{-\lambda t} dt - \frac{1}{w}\lambda e^{-\lambda d_w/w} d_w = 0$$

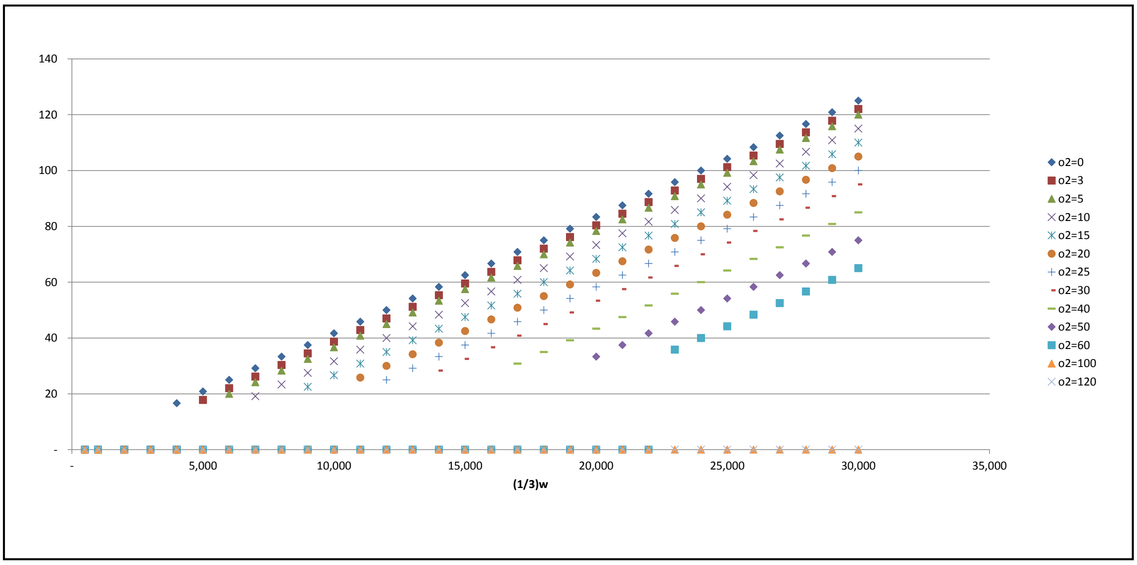
Hence, $d_w^* = \frac{w}{\lambda} - o_2$ in case the expression is positive, and zero otherwise[17].

Expected profit corresponding to d_w^* is given by:

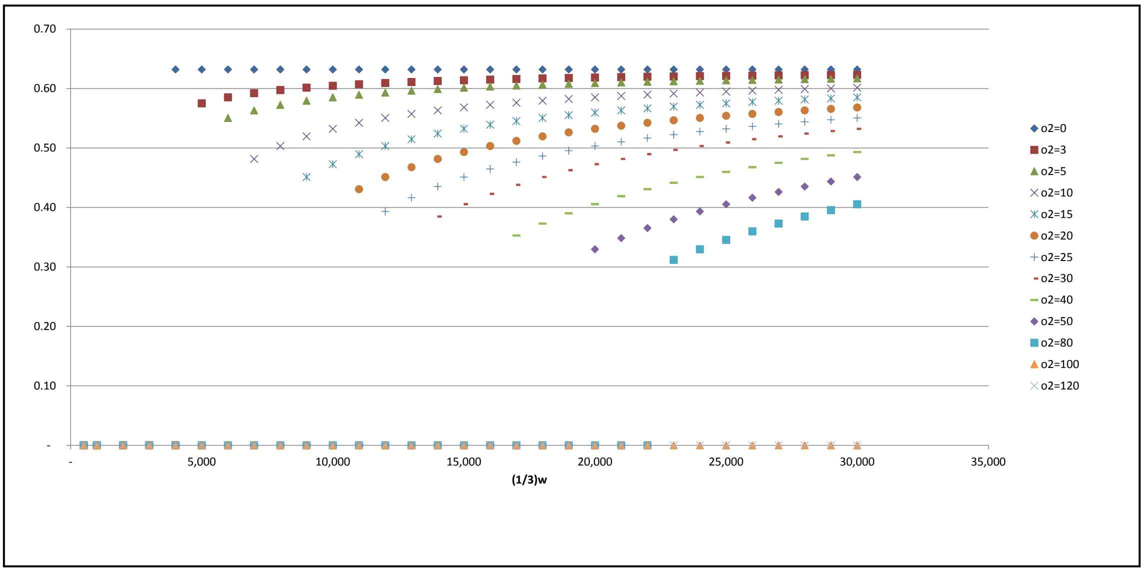
$$\pi^{d_w^*} = -o_2\left[1 - e^{-\lambda d_w^*/w}\right] + e^{-\lambda d_w^*/w} d_w^* - o_1$$

As a consequence of the exponential distribution, expected t corresponds to $1/\lambda$. Using for calibration the average time of one hour, and considering a month as time unit, $\left(t = \frac{1}{24\times 30} = 0.0014\right)$, and $o_1 = 5$, we obtain the optimal undue fees and the proportion of complainers shown in Graphs 3.1 and 3.2, respectively, for varying levels of reversion cost o_2.

[17] The SOC is guaranteed, so it suffices to compare the first order condition solution to the possibility of not charging an undue fee.



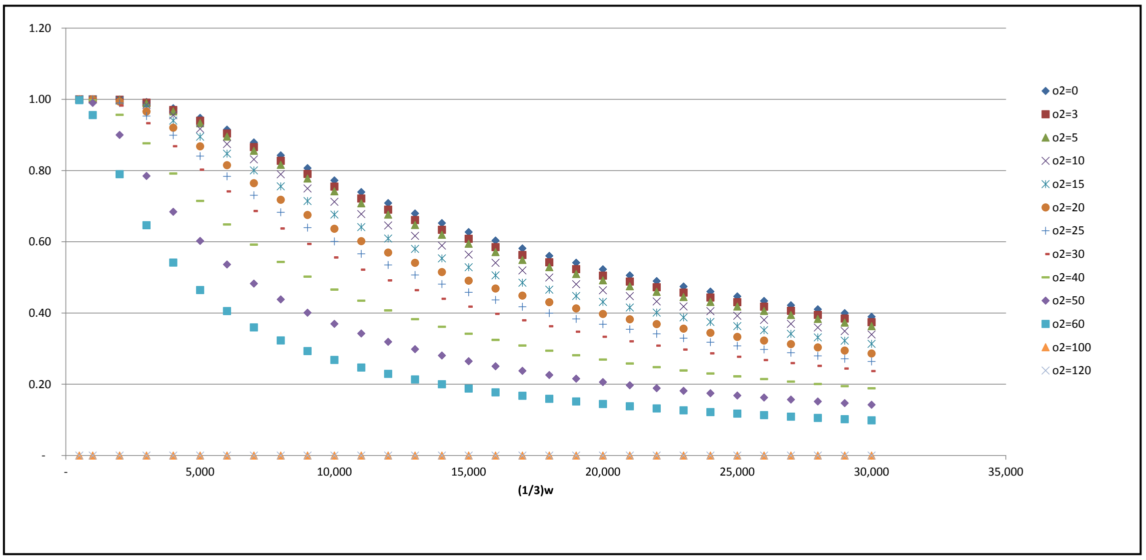
Graph 3.1 – Optimal undue fees



Graph 3.2 – Proportion of complainers with optimal undue fee

Graph 3.1 depicts optimal undue fees. It increases with income and falls with the rise of the reversion cost o_2. The lower bound for the income levels for which applying the policy is profitable increases with o_2. In Graph 3.2 we notice that the proportion of complainers increases with income at decreasing rates while it falls with the increase of o_2. Points that display zero complaints mean that the combination of salary and reversion cost o_2 makes it unprofitable for the bank manager to employ an undue charge policy.

Alternatively, we might ask how would the pattern of complaints be if undue charges were generated by candid mistake. Just to give a flavor of it, we compute the same cases with an average undue fee (assuming initially that the participation of income groups is the same). We present the outcome in Graph 3.3, where we find that the proportion of complainers decreases with income.



Graph 3.3 – Proportion of complainers with an average undue fee

The difference in patterns between graph 3.2 and 3.3 illustrates the essence of the statistical test we propose in the next section. What we expect when undue charges are generated by genuine mistakes should be closer to what we see in Graph 3.3 rather than to Graph 3.2.

4. A test for the discrimination model

In this section, we present a statistical test based on our model. In order to do that, we build a likelihood function implied by the discrimination model and discuss an alternative model, which represents randomly assigned undue fees while maintaining the same general appearance.

The proposed test statistic is based on computing the expected value of the likelihood function originated by the discrimination model, under the null hypothesis that undue charges are mistakes.

The hypotheses about the distribution of t, given the observable variable (in our example, w) were already stated. Now assume that a regulator, interested in detecting and foreclosing the use of the undue fee as a discrimination device[18], does not observe any of these variables. He only knows their distributions. Let $g(w)$ represent the probability density funtion of w. The regulator observes only whether each client has made a complaint and, if he did, the size of the undue charge.

Then, a client portfolio (from some relevant agent perspective[19]) is an extraction from the population with the joint distribution of t and w. We assume that, when the complaint is not made, no variable related to the client is observed, although it is known that he is part of the portfolio and that he did not complain. On the other hand, when the client complains, the value of the undue charge d_w is observed.

Consider a portfolio with n clients, constituted of n_1 complainers and n_2 who do not complain[20].

4.1. Likelihood function for the discrimination model

First let us think of the group composed of n_1 clients who were charged an undue fee large enough to make them complain. Indexing individuals with i, using the results of the discrimination model, we may define the (inverse) function $w_i(d^*_{w,i}, o_2, \Gamma)$, where Γ represents the set of relevant parameters for the distribution of t condicional on w. So, when we observe d_w, knowing the reversion cost o_2 and Γ[21], it

[18] The test we build might also be used inside a financial institution. For instance, the ombudsman might be interested in applying it to detect principal-agent problems.

[19] We are agnostic with regard to who is the relevant agent to take the decision to implement the undue fee. At one extremity it may be an institutional policy, while, at the other, it may come from a single employee, trying to meet internal individual goals.

[20] Where it does not compromise understanding, we refer to the amounts of individuals or to the groups themselves by n_1 and n_2.

[21] For this group it is not necessary to use the implementation cost o_1, given that those hit by an undue charge are necessarily on a salary group sufficiently high for o_1 not to make the discrimination unprofitable.

is possible to obtain the salary w. The likelihood contribution representing the occurrence of this observation in the client portfolio under scrutiny, for $i \in n_1$, is given by:

$$l_{i \in n_1}\left(d_{w,i}^*, o_2, \Phi\right) = g\left(w\left(d_{w,i}^*, o_2, \Gamma\right)\right) Prob\left(tw < d_{w,i} \middle| w\left(d_{w,i}^*, o_2, \Gamma\right)\right)$$

Taking the exponential distribution of t, we know that, for individuals who face an undue fee, $d_{w,i}^* = \frac{w_i}{\lambda} - o_2$ or $w_i = \lambda\left(d_{w,i}^* + o_2\right)$, then:

$$l_{i \in n_1}\left(d_{w,i}^*, o_2, \lambda\right) = g\left(\lambda\left(d_{w,i}^* + o_2\right)\right) Prob\left(tw < d_{w,i}^* \middle| w\right)$$

$\therefore$

$$l_{i \in n_1}\left(d_{w,i}^*, o_2, \lambda\right) = g\left(\lambda\left(d_{w,i}^* + o_2\right)\right)\left[1 - e^{-d_{w,i}^*/\left(d_{w,i}^* + o_2\right)}\right]$$

On the other hand, n_2 contains clients not subject to undue charges as well as those who are, but prefer not to complain. From the regulator's viewpoint, it will not be possible to distinguish between these two situations. Therefore, the probability of observing a client who does not complain, i.e. $i \in n_2$, is:

$$l_{i \in n_2}(o_1, o_2, \Gamma) = Prob(w \notin \boldsymbol{W}) + Prob(w \in \boldsymbol{W}) Prob\left(tw \geq d_{w,i}^* \middle| w \in \boldsymbol{W}\right)$$

Here we define $\boldsymbol{W}$ as the set of salary values for which it is profitable to implement an undue fee discrimination policy, given its costs. For cases in which the complaint takes place, w always belongs to that set. Given the choices of variables that characterize the individual in our formulation, the condition of belonging to $\boldsymbol{W}$ may be reduced to w being higher than a certain threshold $\underline{w}(o_1, o_2, \Gamma)$[22]. Let $G(w)$ be the salary accumulated density function. We then have:

---

[22] Notice that $\pi^{d_w} = \int_0^{\frac{d_w}{w}} \lambda e^{-\lambda t} dt\,(-o_2) + \int_{\frac{d_w}{w}}^{\infty} \lambda e^{-\lambda t} dt\, d_w - o_1 = -o_2\left[1 - e^{-\frac{\lambda d_w}{w}}\right] + \left[e^{-\frac{\lambda d_w}{w}}\right] d_w - o_1 \;\; \therefore$ $\pi^{d_w} = \left[e^{-\lambda d_w/w}\right](d_w + o_2) - o_1 - o_2$. Hence, $\frac{\partial \pi^{d_w}}{\partial w} = (\lambda d_w/w^2)\left[e^{-\lambda d_w/w}\right](d_w + o_2)>0$, so that an increase in w from a level with $\pi^{d_w^*} > 0$, may not result in a salary for which $d_w^* = 0$.

$$l_{i \in n_2}(o_1, o_2, \Gamma) = G(\underline{w}) + [1 - G(\underline{w})] Prob(tw \geq d_w^* | w_i \in \boldsymbol{W})$$

with

$$Prob(tw \geq d_w^* | w \in \boldsymbol{W}) = \frac{1}{[1 - G(\underline{w})]} \int_{\underline{w}}^{\infty} g(w)\, Prob(tw \geq d_w^* | w) dw$$

Thus, using the exponential distribution:

$$Prob(tw \geq d_w^* | w_i \in \boldsymbol{W}) = \frac{1}{[1 - G(\underline{w})]} \int_{\underline{w}}^{\infty} g(w) \int_{d_w^*/w}^{\infty} \lambda e^{-\lambda t} dt\, dw$$

$$= \frac{1}{[1 - G(\underline{w})]} \int_{\underline{w}}^{\infty} g(w)\, e^{-\lambda d_w^*/w} dw$$

$$= \frac{1}{[1 - G(\underline{w})]} \int_{\underline{w}}^{\infty} g(w)\, e^{-\lambda\left(\frac{w}{\lambda} - o_2\right)/w} dw$$

∴

$$Prob(tw \geq d_w^* | w \in \boldsymbol{W}) = \frac{1}{[1 - G(\underline{w})]} \int_{\underline{w}}^{\infty} g(w)\, e^{(\lambda o_2/w - 1)} dw$$

Consequently, the total probability of absence of complaint is:

$$l_{i \in n_2}(o_1, o_2, \lambda) = P = G(\underline{w}) + \int_{\underline{w}}^{\infty} g(w)\, e^{(\lambda o_2/w - 1)} dw$$

For the n observations, the likelihood function may be written as:

$$L = \prod_i l_{i\in n_2}(o_2, \lambda) \prod_i l_{i\in n_1}\left(d^*_{w,i}, o_2, \lambda\right)$$
$$= P^{n_2} \prod_{i\in n_1} g\left(\lambda\left(d^*_{w,i} + o_2\right)\right)\left[1 - e^{-d^*_{w,i}/\left(d^*_{w,i}+o_2\right)}\right]$$

For the main example we study, we consider salary uniformly distributed: $w \sim U[w_l, w_h]$. This choice stems from the objective of showing the methodology and analyzing its performance, even in a context where the distribution of the variable observed by the bank manager is the least informative. We show also another possibility in an appendix[23]. Define $\delta = 1/(w_h - w_l)$

Consider that $w_l < \underline{w}$ (In case $w_l = \underline{w}$, all clients will get an undue charge and the problem is simplified).

$$P = \delta\left(\underline{w} - w_l\right) + \delta \int_{\underline{w}}^{w_h} e^{(\lambda o_2/w - 1)}\, dw$$

$$L = P^{n_2} \prod_{i\in n_1} \delta \left[1 - e^{-d^*_{w,i}/\left(d^*_{w,i}+o_2\right)}\right]$$

4.2. Baseline model – null hypothesis

In order to test if the mischarge is employed as a form of discrimination, we need a baseline setup to compute the distribution of the test statistic under the case with no discrimination. That is how we will be able to tell when a calculated value of the statistic is too unlikely to have come from candid mistakes.

[23] In appendix 4, we analyze what happens if we adopt a more plausible log-normal distribution for salaries.

We use the null hypothesis we believe would be the hardest to distinguish from the alternative. Therefore, we use a model where the unconditional distribution of undue charges is equal the one in the discrimination model. The difference is that here they will not be targeted to clients with a certain profile, but randomly assigned. We maintain the same rationale for the client's choice of whether to complain or not.

The discrimination model implies a distribution of d_w^* conditional on w. In our particular specification, given w, d_w^* is known. From that, we compute the unconditional distribution of d_w^* and use it in the baseline model. We call this distribution $h(d_w)$. In this case we do not use * in our notation, since undue charges in this context are not resulting from an optimization process.

Assume $w \sim U[w_l, w_h]$ and define $\delta = 1/(w_h - w_l)$. Assume further that $w_l < \underline{w}$, so we have all cases (no undue fee, undue fee/not complain, undue fee/complain). We know that $d_w^* = \frac{w}{\lambda} - o_2$ for $w \geq \underline{w}$ and $d_w^* = 0$ otherwise. The way we will compute $\underline{w}$ implies $\frac{\underline{w}}{\lambda} - o_2 \geq 0$. Therefore, $h(d_w)$ is given by:

$Prob[d_w = 0] = \delta(\underline{w} - w_l)$;

$$Prob\left[d_w = \hat{d}_w \in \left[\frac{\underline{w}}{\lambda} - o_2, \frac{w_h}{\lambda} - o_2\right]\right]$$

$$= Prob\left[d_w \in \left[\frac{\underline{w}}{\lambda} - o_2, \frac{w_h}{\lambda} - o_2\right]\right] Prob\left[d_w = \hat{d}_w / d_w \in \left[\frac{\underline{w}}{\lambda} - o_2, \frac{w_h}{\lambda} - o_2\right]\right]$$

$$= Prob\left[w \in [\underline{w}, w_h]\right]\left(\frac{1}{\frac{w_h}{\lambda} - o_2 - \left(\frac{\underline{w}}{\lambda} - o_2\right)}\right) = [1 - \delta(\underline{w} - w_l)]\frac{\lambda}{(w_h - \underline{w})}$$

$$= \left[1 - \frac{(\underline{w} - w_l)}{w_h - w_l}\right]\frac{\lambda}{(w_h - \underline{w})} = \left[\frac{w_h - w_l - (\underline{w} - w_l)}{w_h - w_l}\right]\frac{\lambda}{(w_h - \underline{w})}$$

$$= \left[\frac{w_h - \underline{w}}{w_h - w_l}\right]\frac{\lambda}{(w_h - \underline{w})} = \lambda\delta$$

With this framework, it is as if the probability of observing a certain set of complains depended on the occurrence that the $d_{w,i}$ we observe were drawn for clients who find it worthwhile to complain.

Since t, w e d_w are independently distributed, the probability that a client will not complain is:

$$l_{i\in n_2}^{B}(o_2,\Gamma) = P_B = \int_0^{\infty} h(d_w) \int_0^{\infty} g(w) \int_{d_w/w}^{\infty} f(t)dt\, dw\, dd_w$$

Where B stands for baseline. Using the exponential distribution for t:

$$l_{i\in n_2}^{B}(o_2,\lambda) = P_B = \int_0^{\infty} h(d_w) \int_0^{\infty} g(w) \int_{d_w/w}^{\infty} \lambda e^{-\lambda t} dt\, dw\, dd_w$$

$$l_{i\in n_2}^{B}(o_2,\lambda) = P_B = \int_0^{\infty} h(d_w) \int_0^{\infty} g(w) e^{-\lambda d_w/w} dw\, dd_w$$

On the other hand, the likelihood contribution attached to the observation of a complaint with the respective d_w is:

$$l_{i\in n_1}^{B}\left(d_{w,i},o_2,\lambda\right) = h\left(d_{w,i}\right) \int_0^{\infty} g(w) \int_0^{d_{w,i}/w} f(t)dt\, dw$$

Using the exponential distribution for t:

$$l_{i\in n_1}^{B}\left(d_{w,i},o_2,\lambda\right) = h\left(d_{w,i}\right) \int_0^{\infty} g(w) \int_0^{d_{w,i}/w} \lambda e^{-\lambda t} dt\, dw$$

$\therefore$

$$l_{i\in n_1}^{B}\left(d_{w,i},o_2,\lambda\right) = h\left(d_{w,i}\right) \int_0^{\infty} g(w)\left[1 - e^{-\lambda d_{w,i}/w}\right] dw$$

Consequently, we obtain the following likelihood function:

$$L_B = \prod_i l_{i\in n_2}(o_2,\lambda) \prod_i l_{i\in n_1}\left(d^*_{w,i}, o_2, \lambda\right)$$

$$= P_B^{n_2} \prod_{i\in n_1} h\left(d_{w,i}\right) \int_0^\infty g(w)\left[1 - e^{-\lambda d^*_{w,i}/w}\right] dw$$

With uniform distribution of salaries, it becomes:

$$l^B_{i\in n_2}(o_2,\lambda) = P_B = \int_0^\infty h(d_w) \int_{w_l}^{w_h} \delta e^{-\lambda d_w/w} dw\, dd_w$$

$$\therefore\ l^B_{i\in n_2}(o_2,\lambda) = P_B = \delta \int_{w_l}^{w_h} \int_0^\infty h(d_w) e^{-\lambda d_w/w} dd_w\, dw$$

Splitting $h(d_w)$ into $d_w = 0$ and $d_w > 0$:

$$\therefore$$

$$l^B_{i\in n_2}(o_2,\lambda) = P_B = \delta \int_{w_l}^{w_h} \delta\left(\underline{w} - w_l\right) e^{-\lambda 0/w}\, dw + \delta \int_{w_l}^{w_h} \int_{\frac{\underline{w}}{\lambda}-o_2}^{\frac{w_h}{\lambda}-o_2} \lambda \delta e^{-\lambda d_w/w} dd_w\, dw$$

$$= \delta^2 \int_{w_l}^{w_h} \left(\underline{w} - w_l\right) dw + \delta^2 \int_{w_l}^{w_h} \int_{\frac{\underline{w}}{\lambda}-o_2}^{\frac{w_h}{\lambda}-o_2} \lambda e^{-\lambda d_w/w} dd_w\, dw$$

$$= \delta^2 \left(\underline{w} - w_l\right)(w_h - w_l) + \delta^2 \int_{w_l}^{w_h} w \int_{\frac{\underline{w}}{\lambda}-o_2}^{\frac{w_h}{\lambda}-o_2} \frac{\lambda}{w} e^{-\lambda d_w/w} dd_w\, dw$$

$$= \delta^2 \left(\underline{w} - w_l\right)(w_h - w_l) + \delta^2 \int_{w_l}^{w_h} w\left[1 - e^{-\lambda d_w/w}\right]_{\frac{\underline{w}}{\lambda}-o_2}^{\frac{w_h}{\lambda}-o_2} dw$$

$$= \delta\left(\underline{w} - w_l\right) + \delta^2 \int_{w_l}^{w_h} w\left[1 - e^{-\lambda\left(\frac{w_h}{\lambda}-o_2\right)/w} - 1 + e^{-\lambda\left(\frac{\underline{w}}{\lambda}-o_2\right)/w}\right] dw$$

$$= \delta\left(\underline{w} - w_l\right) + \delta^2 \int_{w_l}^{w_h} w\left[e^{-\lambda\left(\frac{\underline{w}}{\lambda}-o_2\right)/w} - e^{-\lambda\left(\frac{w_h}{\lambda}-o_2\right)/w}\right] dw$$

$$\therefore\ P_B = \delta\left(\underline{w} - w_l\right) + \delta^2 \int_{w_l}^{w_h} w\left[e^{(\lambda o_2 - \underline{w})/w} - e^{(\lambda o_2 - w_h)/w}\right] dw$$

For complaint observations:

$$l^B_{i\in n_1}\left(d_{w,i}, o_2, \lambda\right) = \lambda\delta \int_0^\infty g(w)\left[1 - e^{-\lambda d_{w,i}/w}\right] dw$$

$$\therefore\ l^B_{i\in n_1}\left(d_{w,i}, o_2, \lambda\right) = \lambda\delta\left\{1 - \int_0^\infty g(w) e^{-\lambda d_{w,i}/w} dw\right\}$$

With uniform w:

$$l^B_{i\in n_1}(d_{w,i}, o_2, \lambda) = \lambda\delta\left\{1 - \int_{w_l}^{w_h} \delta e^{-\lambda d_{w,i}/w} dw\right\}$$

∴

$$l^B_{i\in n_1}(d_{w,i}, o_2, \lambda) = \lambda\delta\left\{1 - \delta\int_{w_l}^{w_h} e^{-\lambda d_{w,i}/w} dw\right\}$$

Therefore:

$$L_B = P_B^{n_2} \prod_{i\in n_1} \lambda\delta\left\{1 - \delta\int_{w_l}^{w_h} e^{-\lambda d_{w,i}/w} dw\right\}$$

∴

$$L_B = P_B^{n_2} (\lambda\delta)^{n_1} \prod\nolimits_{i\in n_1} \left\{1 - \delta \int_{w_l}^{w_h} e^{-\lambda d_{w,i}/w} dw\right\}$$

The expressions calculated for the baseline model are useful to compute the mean and variance of the test statistic.

4.3. A test to detect the use of undue fees as a discrimination mechanism

The test we propose has the advantage of not requiring knowledge from the regulator of all the client information the bank manager has. On the other hand, its execution requires a high level of information about the discrimination mechanism itself. In other words, the regulator needs precise knowledge about the "accusation" that will be tested.

As we explained before, the baseline model assumes that the undue charges are not targeted to specific groups of clients, so it is possible to believe they are simple mistakes. That is our H_0, which we would like to reject in case there is enough evidence that, actually, the discrimination mechanism is in action.

Thus, the statistic of interest is $E_{H_0}(lnL)$. We opt, however, for the version $S = E_{H_0}\left(\frac{lnL}{n}\right)$, because normalizing by n, given the size of the portfolio, does not affect the comparison between models and because, in that form, it suffices to show than $ln(l_i)$ has finite mean and variance under H_0 to apply the central limit theorem. Additionally to using the theorem, we obtain simulated distributions for finite sample examples.

For the calculation of $E_{H_0}\left(ln(l_i)\right)$, we use the likelihood contributions from the discrimination model, weighted by the likelihood contributions of the baseline model:

$$E_{H_0}\left(ln(l_i)\right) = Pr_{H0}(i \in n_1)E\left(ln\left(l_{i\in n_1}\right)/i \in n_1\right) + Pr_{H0}(i \in n_2)E\left(ln\left(l_{i\in n_2}\right)/i \in n_2\right)$$

$$= [1 - P_B]E_{H0}\left(ln\left(l_{i\in n_1}\right)/i \in n_1\right) + P_A ln(P)$$

$$= [1 - P_B]\frac{1}{[1 - P_B]}\int_{\frac{w}{\lambda}-o_2}^{\infty} h(d_w)\int_0^{\infty} g(w)\left[1 - e^{-\lambda d_w/w}\right]dw\, ln\left(l_{i\in n_1}(d_w)\right)dd_w + P_B ln(P)$$

$$= \int_{\frac{w}{\lambda}-o_2}^{\infty} h(d_w)\int_0^{\infty} g(w)\left[1 - e^{-\lambda d_w/w}\right]dw\, ln\left(l_{i\in n_1}(d_w)\right)dd_w + P_B ln(P)$$

$$= \int_{\frac{w}{\lambda}-o_2}^{\infty} h(d_w)\left(1 - \int_0^{\infty} g(w)e^{-\lambda d_w/w}dw\right)ln\left(l_{i\in n_1}(d_w)\right)dd_w + P_B ln(P)$$

$\therefore$

$E_{H_0}\left(ln(l_i)\right) =$

$$\int_{\frac{w}{\lambda}-o_2}^{\infty} h(d_w)\left(1 - \int_0^{\infty} g(w)e^{-\lambda d_w/w}dw\right)ln\left(g(w)\left[1 - e^{-d_w/(d_w+o_2)}\right]\right)dd_w + P_B ln(P)$$

Using the uniform distribution for salaries and $h(d_w)=\lambda\delta$ for $d_w \in \left[\frac{\underline{w}}{\lambda}-o_2, \frac{w_h}{\lambda}-o_2\right]$:

$E_{H_0}\left(\ln(l_i)\right) =$

$$\lambda\delta \int_{\frac{\underline{w}}{\lambda}-o_2}^{\frac{w_h}{\lambda}-o_2} \left(1-\delta\int_{w_l}^{w_h} e^{-\lambda d_w/w} dw\right) \ln\left(\delta\left[1-e^{-d_w/(d_w+o_2)}\right]\right) dd_w + \mathrm{P_B}\ln(\mathrm{P})$$

$=$

$$\lambda\delta \int_{\frac{\underline{w}}{\lambda}-o_2}^{\frac{w_h}{\lambda}-o_2} \left(ln\left(\delta\left[1-e^{-d_w/(d_w+o_2)}\right]\right) - \delta\int_{w_l}^{w_h} e^{-\lambda d_{w,i}/w} ln\left(\delta\left[1-e^{-d_w/(d_w+o_2)}\right]\right) dw\right) dd_w + P_B ln(P)$$

$=$

$$\lambda\delta \int_{\frac{\underline{w}}{\lambda}-o_2}^{\frac{w_h}{\lambda}-o_2} \left(ln\left(\delta\left[1-e^{-d_w/(d_w+o_2)}\right]\right)\right) dd_w - \lambda\delta^2 \int_{\frac{\underline{w}}{\lambda}-o_2}^{\frac{w_h}{\lambda}-o_2} \int_{w_l}^{w_h} e^{-\lambda d_{w,i}/w} ln\left(\delta\left[1-e^{-d_w/(d_w+o_2)}\right]\right) dw dd_w + P_B ln(P)$$

From that expression, it is easy to obtain the variance $var_{H_0}\left(ln(l_i)\right)$=$E_{H_0}(ln(l_i)^2)$-$E^2_{H_0}\left(ln(l_i)\right)$, where:

$$E_{H_0}(ln(l_i)^2) = \lambda\delta \int_{\frac{\underline{w}}{\lambda}-o_2}^{\frac{w_h}{\lambda}-o_2} \left(ln^2\left(\delta\left[1-e^{-d^*_w/(d^*_w+o_2)}\right]\right)\right) dd_w - \lambda\delta^2 \int_{\frac{\underline{w}}{\lambda}-o_2}^{\frac{w_h}{\lambda}-o_2} \int_{w_l}^{w_h} e^{-\lambda d_{w,i}/w} ln^2\left(\delta\left[1-e^{-d^*_w/(d^*_w+o_2)}\right]\right) dw dd_w + P_B ln^2(P)$$

5. Results of Simulation

5.1. Simulation algorithm

In order to obtain finite sample test statistics for S, we use the following procedure. We generate R random portfolios of n clients, each one with a t drawn from the exponential distribution with mean $1/\lambda$, a w drawn from the uniform distribution between w_l e w_h, and d_w drawn from $h(d_w)$. The latter variable was generated by obtaining, for each individual, an additional draw of w, with a distribution equal to the original one and independent from it. With that additional w, we compute d_w as the d_w^* resulting from the discrimination model. This second draw of w will only be used again at the end of the simulation, to estimate the power of the test.

Next, d_w is compared with tw, to compute the set of complaints observed by the regulator. For each portfolio we calculate $\frac{lnL}{n}$, thus obtaining its simulated distribution.

The value of $\underline{w}$ is also obtained from the simulation, instead of solving the equation for zero profit. For the $n \times R$ draws of t and additional w, we compute d_w^* from the first order condition and then calculate expected profit. Then, for individuals with a negative expected profit, d_w^* is set to zero. The procedure implies the separation of the drawn values of w in two sets, one where it is profitable to apply the discrimination policy and the other where it is not. Therefore, $\underline{w}$ must lay between the maximum w of the unprofitable set and the minimum w of the profitable one[24]. For a large enough number of draws it is possible to approximate $\underline{w}$ with the desired degree of precision.

5.2. A simulation

[24] We use the average of these two values.

The values used for this simulation are not originated from any real world data, since there are no existing datasets of our knowledge that might be employed. Actually, one of the goals of this paper is to inspire the constructions of such datasets. Consequently, the values presented are merely for illustration.

We choose values picturing a monthly periodicity, so that total time corresponds approximately to $30 \times 24 = 720$ hours. Just for approximate comparison with a regular work journey of 8 daily hours, we consider a third of the value w. Table 5.1 contains the values used in the simulation.

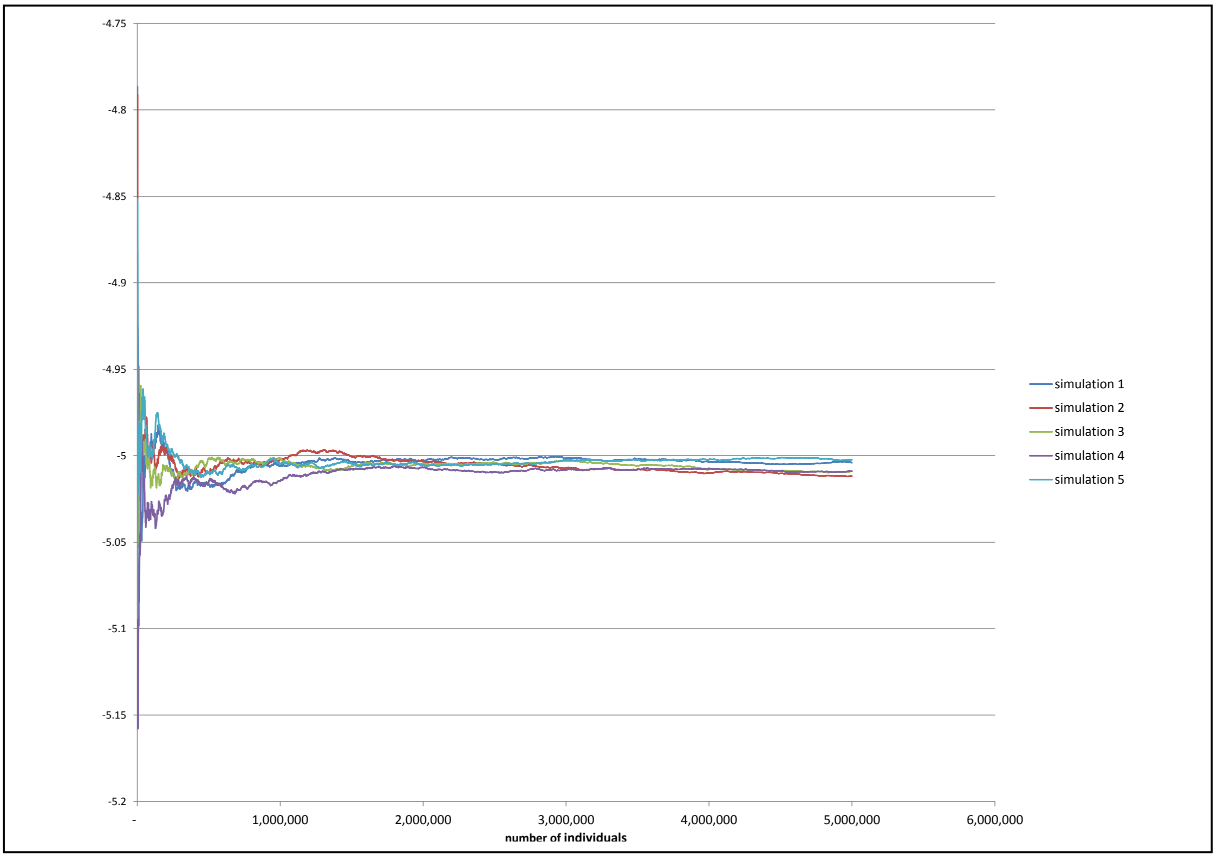
Table 5.1 – Values used in the simulation

Variable	Meaning	Value	Unit	Clarification
$1/\lambda$	Average complaining time	0.0014	month	1 hour
w_h	Maximum monthly salary	90,000	monetary units	30.000 for 8 daily hours
w_l	Minimum monthly salary	0	monetary units	-
o_1	Implementation cost	5	monetary units	Refers to each individual
o_2	Reversion cost	25	monetary units	Refers to each complaint
n	Size of portfolio	250	individuals	-
R	Number of simulations	20,000	Simulated portfolios	-

The simulations presented here were executed with MATLAB 7.10.0[25]. The values of R and n are due to computational capacity limitations.

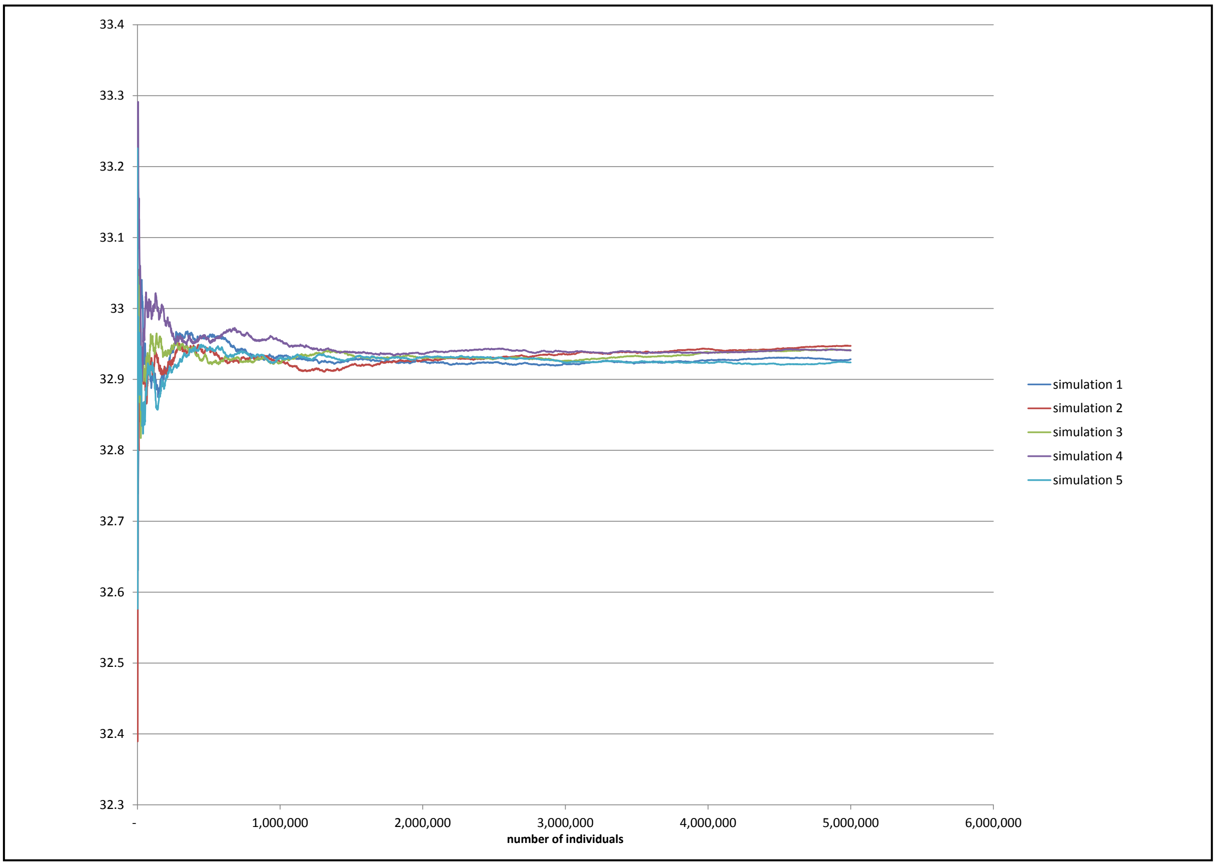
Mean and variance of $ln(l_i)$

In order to show convergence of the mean and variance of $ln(l_i)$, we calculate sample average and variance of an increasing set of individuals, which goes from size 1,000 to 5,000,000 with a step equal to 1,000. We do that five times. The outcomes are shown in Graphs 5.1 and 5.2.



Graph 5.1 – Simulated average of $ln(l_i)$

[25] In case the reader wishes to replicate results, it is in order to say that for each simulation the session was restarted. The exceptions are cases in which we were interested in executing the same routine without any changes to evaluate the sensibility of the results to values randomly drawn (like Graphs 5.1 and 5.2). In those cases, the same session was used.



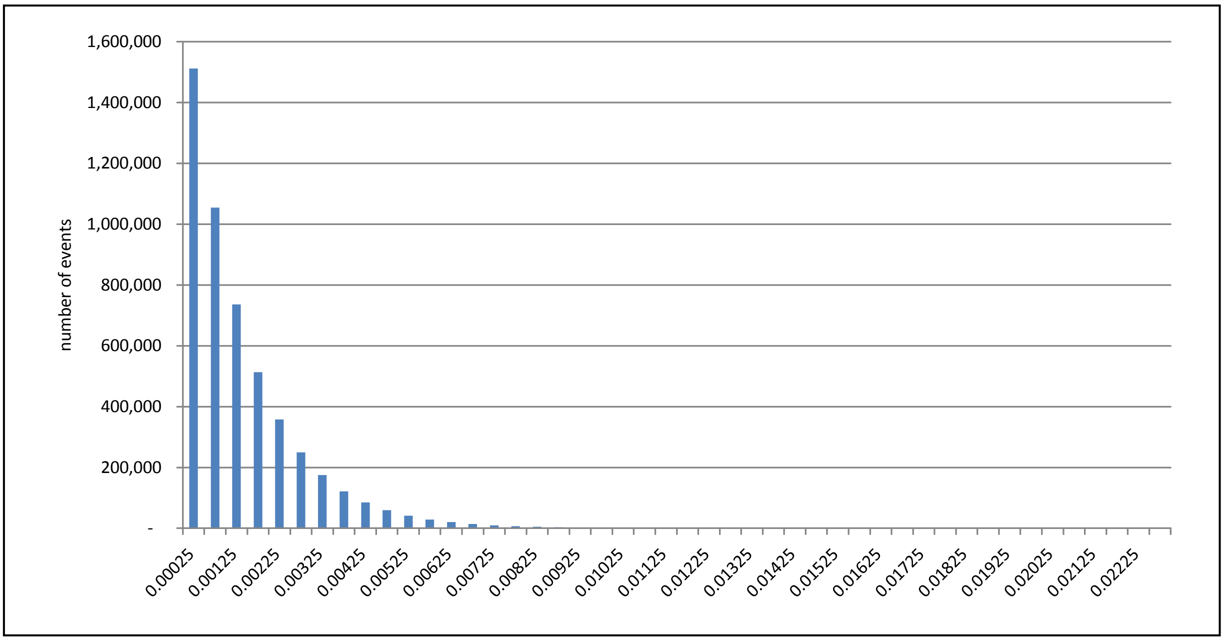
Graph 5.2 – Simulated variance of $\boldsymbol{ln(l_i)}$

The average values obtained including the 5×10^6 individuals were: -5.004041; -5.011831;-5.008962; -5.009006 and -5.002341. The algebraic calculation using the formula obtained previously results in -5.007659.

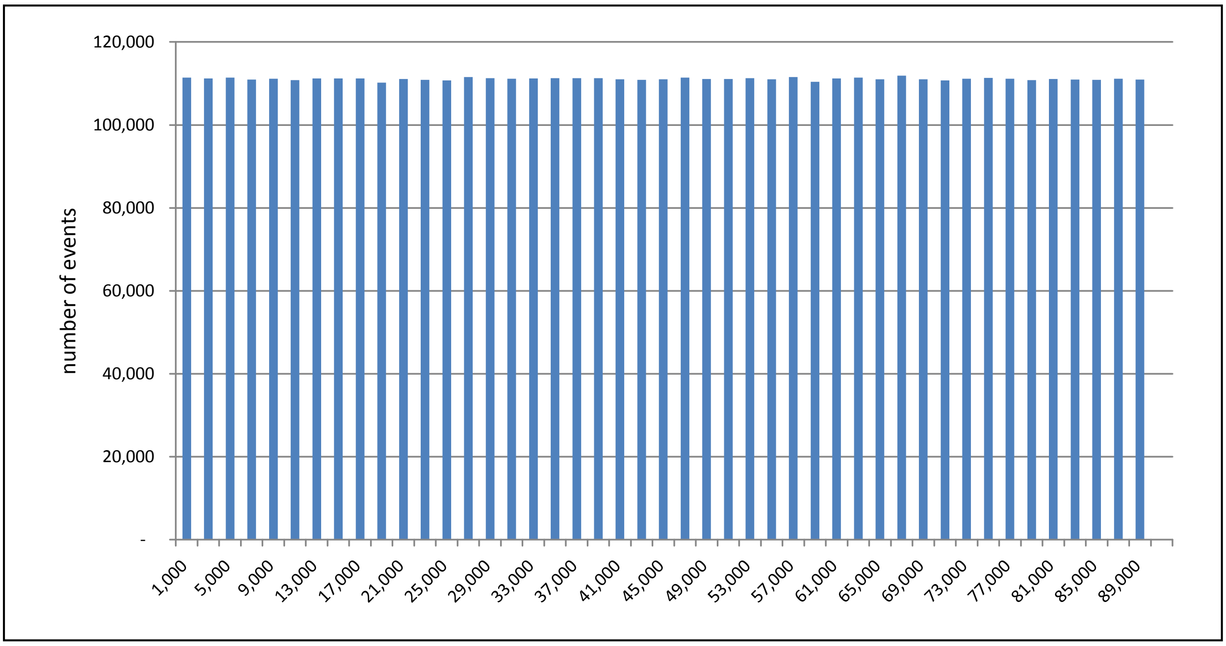
As for the variance, the simulated values were: 32.928323; 32.947205; 32.940491; 32.940941; 32.923879. Algebraically, we obtain 32.937265. Thus, there is a close match between the simulated means and variances and the algebraic expressions for them.

Simulated distributions of t,w and d_w

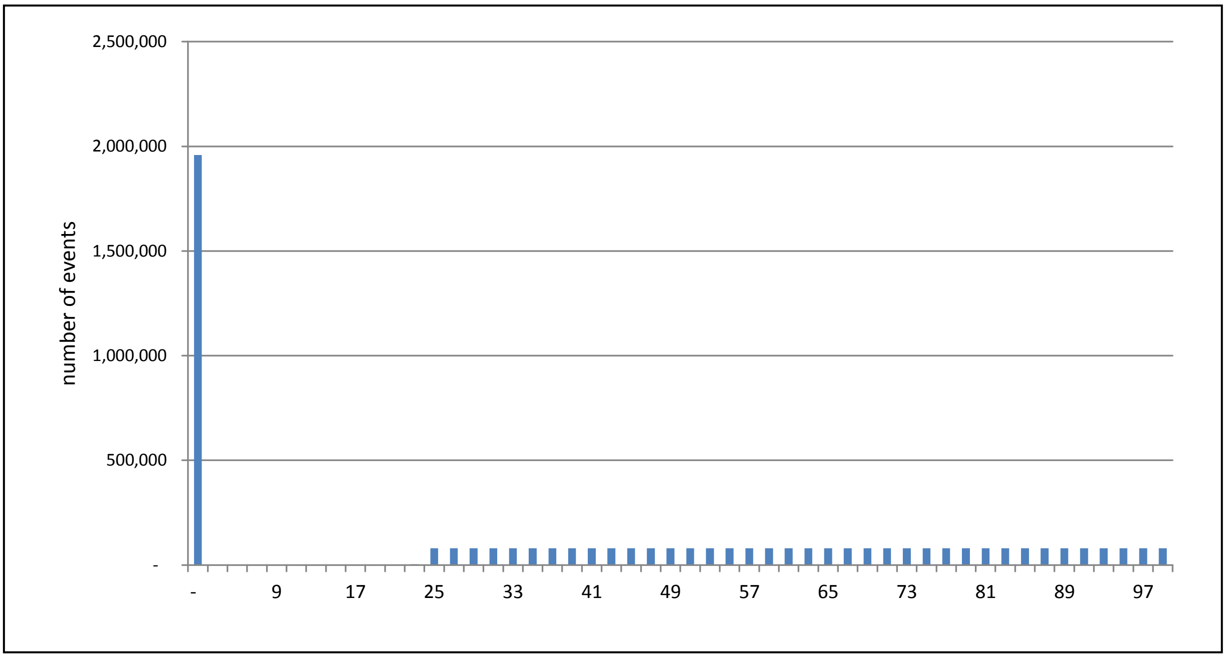
The distributions for the simulated values for t, w and d_w are depicted in Graphs 5.3 to 5.5.



Graph 5.3 – Histogram of simulated values of *t*



Graph 5.4 – Histogram for simulated values of *w*



Graph 5.5 – Histogram of simulated values of d_w

Approximately 39% of the individuals are not subject to an undue fee. This corresponds to the value obtained for $\underline{w}$, of 35,220[26], which may be easily checked, given the uniform distribution of w.

Simulated distributions of $\frac{logL}{n}$

The expected value of $ln(l_i)$, μ=-5.007659 (obtained algebraically[27]), was employed in the calculation of $\sqrt{n}\left(\frac{lnL}{n} - \mu\right)$, which we use to compare results with CLT values.

Graph 5.6 displays, in black, the simulated distribution of the test statistic under H_0. In grey, we show the simulations under the alternative, H_A, which was computed by pairing d_w values with w drawn in the additional sampling, and using the same values

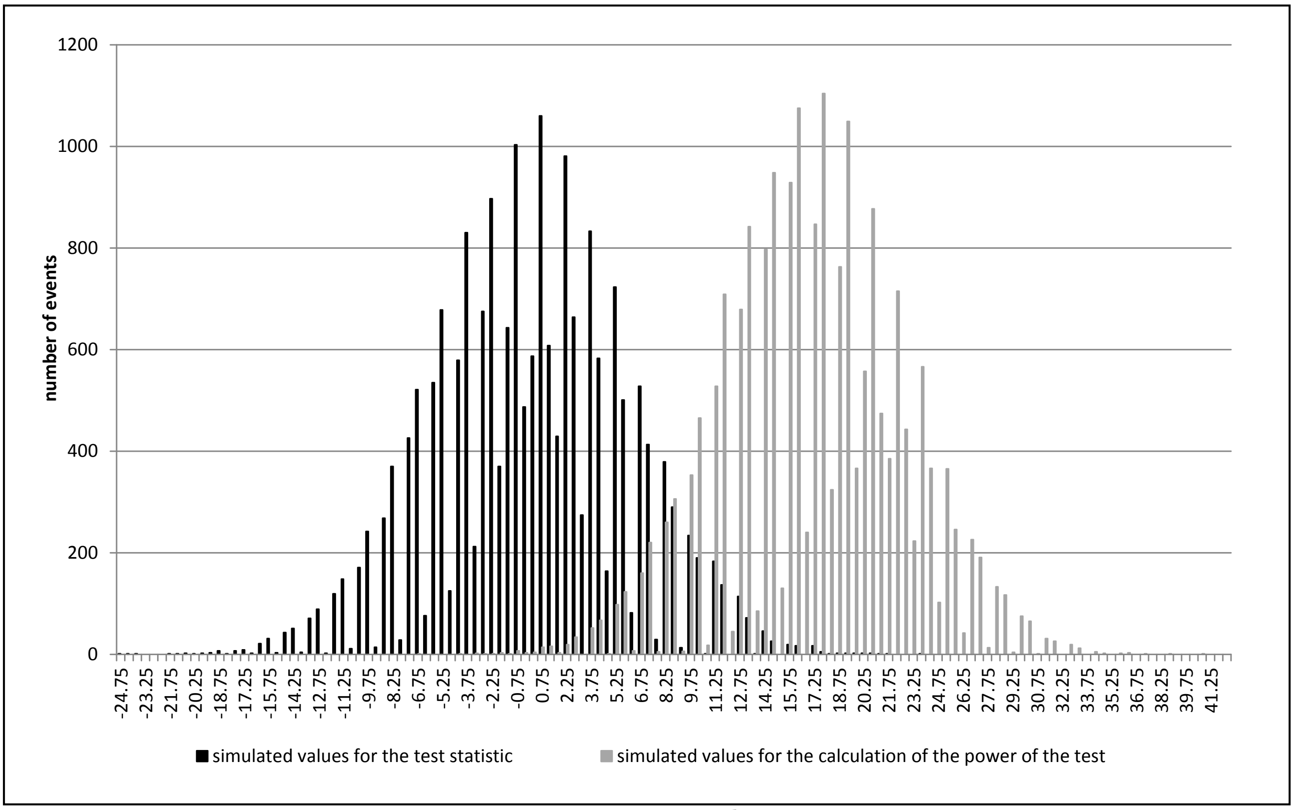
[26] To grasp the precision of the approximated calculation of $\underline{w}$ from the simulations, we repeat the exercise five times, sequentially, obtaining values: 35,220.3893; 35,220.4219; 35,220.4206; 35,220.4109 and 35,220.3984.

[27] For more complex formulations, it may be interesting to use the average of l_i along all simulations ($\hat{\mu}$) as an estimator of the mean of $\frac{lnL}{n}$. In that case, we would compute expression $\sqrt{n}\left(\frac{lnL}{n} - \hat{\mu}\right)$. This approach will be employed for the sensitivity analysis presented in the appendix, since it is more computationally efficient.

of t to determine the complaints. Table 5.2 contains the main results, in terms of critical values for the test of discrimination with undue fees.

Table 5.2 – Test statistic

α	Simulated critical value	CLT critical value	1-β
0.10	7.4191	7.3550	0.9613
0.05	9.6320	9.4400	0.9177
0.01	13.3226	13.3511	0.7697



Graph 5.6 – Distribution of simulated values of $\frac{lnL}{n} - \mu$ under H_0 and H_A

Table 2 enables us to compare the simulated critical values with those implied by the CLT. This is useful to grasp how much precision we would lose by simply using the normal distribution. In the particular case presented, values seem close to one another, although this is obviously a subjective consideration. In the last column the power of the test is displayed. Its calculation is associated to the superposition of both distributions, which is shown in Graph 5.6.

5.3 Sensitivity analysis

In appendix 3[28], we show the results obtained when we vary the values of the parameters chosen for the simulation. Some results are intuitive, like the fact that $\underline{w}$ increases with o_1 and o_2 and falls with $1/\lambda$. However, the effects of such variation on test statistics or in the performance of the test are difficult to anticipate because some of the relationships are not monotonic.

6. Conclusion and research agenda

In this paper we model in a very simple way the use of false mistakes by bank managers to segment clients with different degrees of difficulty to produce a complaint. Using the model, we show how the knowledge of population parameters of some key features of the population may be combined with the content of complaints to statistically test if this sort of price discrimination is taking place.

The test is based on the central limit theorem, applied on a likelihood function. Its usefulness lays on the fact that there are substantial costs to monitor that sort of practice directly.

The construction of a detailed dataset of complaints, containing the relevant features for this discrimination mechanism would enable the design of the blocks of the model with characteristics relevant to reality, given that the microeconomic model we set up is simplified to allow us to show the statistic technique. In the particular case we

[28] As mentioned in the previous footnote, in this appendix we use $\hat{\mu}$ as the estimator for the expected value of l_i. The loss of precision due to the use of such method can be inferred by comparing the critical values shown in the table, which correspond to the simulation presented in the text. In this case, the deviation was $\sqrt{n}(\hat{\mu}-\mu) \cong \sqrt{250}(-5.004041+5.007659) \cong 0.057206$, which approximately corresponds to 0.01 standard deviation.

develop, it is also possible to evaluate how the power of the test varies with environment characteristics, like the average time it takes to perform a complaint and with the technology used for discrimination, like implementation and charge reversion costs.

We see two research agendas that would benefit from actual data. The first is to consider a more complex game framework. In particular, it could be relevant to extend the number of periods and take into account elements like reputation and switching costs, additional to clients' investments to learn the complaining technology. In the dynamic model we proposed, once the stage in which the discrimination takes place is reached, the bank becomes a *de facto* monopolist and the possibility of losing the client in posterior periods is not taken into account. In addition, in a repeated game, the information about which clients complain would be used to optimize future undue fees[29]. It is necessary to evaluate how much these features are relevant, keeping in mind the market niche under analysis. Another related issue is that bank managers could take the test into account when they choose their discrimination policies. While this is an interesting point to model, implementation of a test ignoring this feature would already impose restrictions on bank manager discrimination strategies if they wish to avoid being caught. This would certainly reduce profitability of bank managers discrimination policies, thus making them less attractive.

Secondly, a dataset would make the information requirements about the population distribution more flexible. For instance, we may be interested in estimating the average complaining time, given its distribution. In that case, the statistic test could be replaced by something in line with a Cox[30] test. The main difference is that the formulation of the likelihood needs to be maximized for both the discrimination and the baseline model, under the null hypothesis. The requirement of intensive use of numeric methods, e.g. simulated maximum likelihood, and the possible specificity of the solutions to the specification of the models, makes it unattractive to solve without real data.

[29] The discussion along these lines might be inspired by the literature on behavior based price discrimination. Some examples of that sort of discrimination are surveyed in Armstrong (2006).
[30] See, for example, Pesaran and Pesaran (1993).

Finally, we remind the readers that the framework we proposed may be easily adapted to complaints in other consumption contexts and to the cost of using product warranties.

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Appendixes – Alternative formulations and sensitivity analysis

Appendix 1 – Alternative formulations for the bank customer's utility

In this appendix, we present some observations about ways to model the consumer that could also be used instead of the particular form chosen in the text.

Alternative i: no leisure choice

Suppose an exogenous individual income, *B*. Consumption utility is given by *U(x)=f(x)*, where x represents the consumption expenditure. Assume $f'(y) > 0$ and $f''(y) < 0 \ \forall y$.

Consumer's problem is:

$$Max_{x,i,r}\ U(x) + iR_B - rc$$

$$s.t.: x + + i[p_B + \ (1 - r)d_B] \ = \ B$$

Variable i indicates the possession of a checking account, assuming value 1 in the positive case and zero otherwise. The utility gain of having an account is R_B. The cost of the undue charge is given by d_B subtracted from consumer's income, in case he does not complain ($r = 0$) and by c, subtracted from his utility, if he complains ($r = 1$). Variable c, representing complaining disutility is private information and heterogeneous among consumers.

Therefore, if a consumer participates and faces an undue charge, he has to choose between utility $U = f(w - p_B - d_B) + R_B$, with $r = 0$, or $U = f(B - p_B) + R_B - c$, with $r = 1$.

Consumer complains if: $f(B - p_B) + R_B - c > f(B - p_B - d_B) + R_B$

∴

$$c < f(B - p_B) - f(B - p_B - d_B)$$

We define the upper limit of c for complaining as $g(B, p_B, d_B) \ = \ f(B - p_B) - f(B - p_B - d_B)$.

Therefore:

$$g_{d_B}(B, p_B, d_B) = f^{'}(B - p_B - d_B) > 0$$

$$g_B(B, p_B, d_B) = f'(B - p_B) - f'(B - p_B - d_B)$$

Using $f''(x) < 0$ we obtain $g_B(B, p_B, d_B) < 0$.

We conclude that if the distribution of c is independent from the distribution of B, the probability of someone complaining, given the undue charge, falls with the increase of income, given that the upper limit of c for complaining become smaller. An increase in d_B, given B, increases the probability of complaining.

Alternative ii: leisure choice and quasilinear utility

Consumer problem is given by:

$$Max_{x,l,i,r}\ \ x + lnl + iR_w - rc$$

$$s.t.:\ x + wl + i[p_w + (1 - r)d_w] = w$$

In this formulation, leisure choice does not substantially alter the conclusions from alternative (i), if the solution there is not a corner solution, i.e. pure leisure. That condition is guaranteed for $w > 1$.

Alternative iii: leisure choice and Cobb-Douglas utility

Consumer solves:

$$Max_{x,l,i,r}\ U(x, l) = x^{\alpha} l^{1-\alpha} + iR_w - rc$$

$$s.t.:\ x + wl + i[p_w + (1 - r)d_w] = w$$

where $\alpha \in (0,1)$

Tangency condition is:

FOC: $$\frac{\alpha x^{\alpha-1} l^{1-\alpha}}{(1-\alpha) x^{\alpha} l^{-\alpha}} = \frac{1}{w}$$

$$\therefore \frac{\alpha l}{(1-\alpha)x} = \frac{1}{w} \therefore l = \frac{1-\alpha}{\alpha}\frac{x}{w}$$

Assuming participation ($i = 1$) and not complaining ($r = 0$):

$$x + wl + p_w + d_w = w$$

$$\therefore x + w\frac{1-\alpha}{\alpha}\frac{x}{w} = w - p_w - d_w \therefore x\left(1 + \frac{1-\alpha}{\alpha}\right) = w - p_w - d_w \therefore x\left(\frac{\alpha+1-\alpha}{\alpha}\right) = w - p_w - d_w$$

$$\therefore x^*_{i=1,r=0}(w, p_w, d_w) = \alpha(w - p_w - d_w)$$

$$\therefore l^*_{i=1,r=0}(w, p_w, d_w) = (1-\alpha)\frac{(w-p_w-d_w)}{w}$$

Utility is:

$$[\alpha(w - p_w - d_w)]^\alpha \left[(1-\alpha)\frac{(w - p_w - d_w)}{w}\right]^{(1-\alpha)} + R_w$$

$$\therefore \alpha^\alpha(1-\alpha)^{(1-\alpha)}\frac{(w-p_w-d_w)}{w^{(1-\alpha)}} + R_w$$

Assuming participation ($i = 1$) and complaining ($r = 1$):

$$x + wl + p_w = w$$

$$\therefore x^*_{i=1,r=1}(w, p_w) = \alpha(w - p_w)$$

$$\therefore l^*_{i=1,r=1}(w, p_w) = (1-\alpha)\frac{(w-p_w)}{w}$$

Utility is $\alpha^\alpha(1-\alpha)^{(1-\alpha)}\frac{(w-p_w)}{w^{(1-\alpha)}} + R_w - c$

Thus, consumer complains if:

$$\alpha^\alpha(1-\alpha)^{(1-\alpha)}\frac{(w - p_w)}{w^{(1-\alpha)}} + R_w - c > \alpha^\alpha(1-\alpha)^{(1-\alpha)}\frac{(w - p_w - d_w)}{w^{(1-\alpha)}} + R_w$$

$\therefore$

$$-c > \alpha^\alpha(1-\alpha)^{(1-\alpha)}\frac{(-d_w)}{w^{(1-\alpha)}}$$

$\therefore$

$$c < \alpha^{\alpha}(1-\alpha)^{(1-\alpha)} \frac{d_w}{w^{(1-\alpha)}}$$

Therefore, the higher w, given d_w and c, the smaller the probability that this condition will be met and that there will be a complaint.

Appendix 2 – Solution of stages 3 and 4 of the game with a uniform distribution of t

With the uniform distribution U[$\underline{t}, \bar{t}$], the problem of the bank manager in the third stage becomes:

$$\pi^{d_w^*} = \max_{d_w} \left(\frac{\frac{d_w}{w} - \underline{t}}{\bar{t} - \underline{t}} \right) (-o_2) + \left(\frac{\bar{t} - \frac{d_w}{w}}{\bar{t} - \underline{t}} \right) d_w - o_1$$

Assuming an interior solution:

FOC:

$$\frac{\partial \pi^{d_w}}{\partial d_w} = \left(\frac{1}{w} \frac{1}{\bar{t} - \underline{t}} \right) (-o_2) + \left(\frac{\bar{t} - \frac{d_w}{w}}{\bar{t} - \underline{t}} \right) + \left(-\frac{1}{w} \frac{1}{\bar{t} - \underline{t}} \right) d_w = 0$$

$$\therefore (-o_2) + \left(\bar{t}w - d_w \right) + -d_w = 0$$

$$\therefore d_w^* = \frac{\bar{t}w - o_2}{2}$$

SOC: $$\frac{\partial^2 \pi^{d_w}}{\partial {d_w}^2} = -2 \frac{1}{w} \left(\frac{1}{\bar{t} - \underline{t}} \right) < 0$$

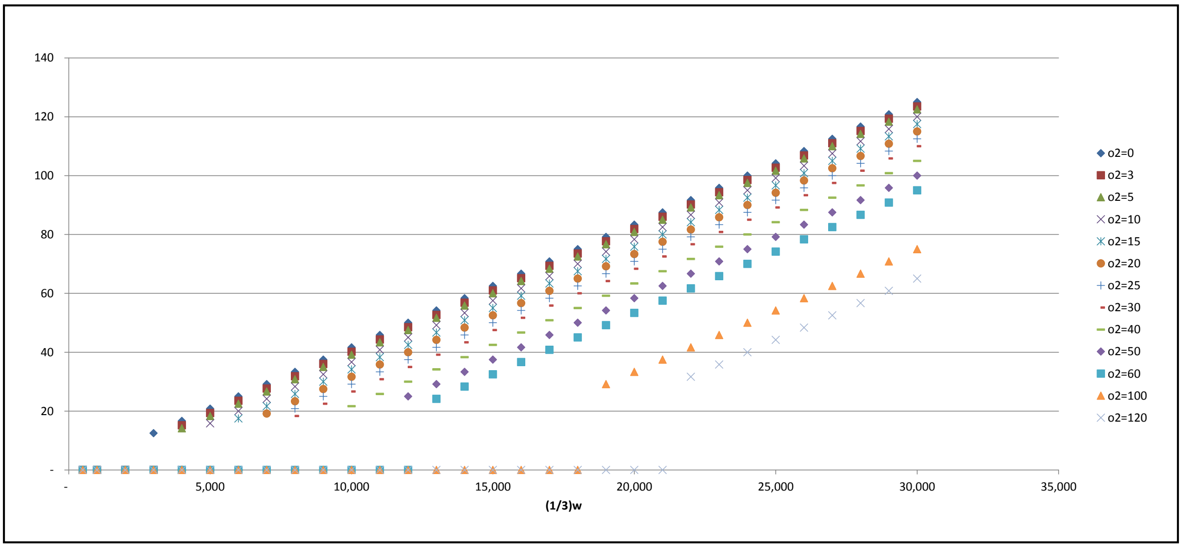
The second order condition is guaranteed by the distribution. The solution implies $\frac{d_w^*}{w} < \bar{t}$, therefore, we need not worry about the superior limit of the support. However, it may happen that the expression results in $\frac{d_w^*}{w} < \underline{t}$, which is suboptimal.

Consequently, if a bank manager implements the undue charge:

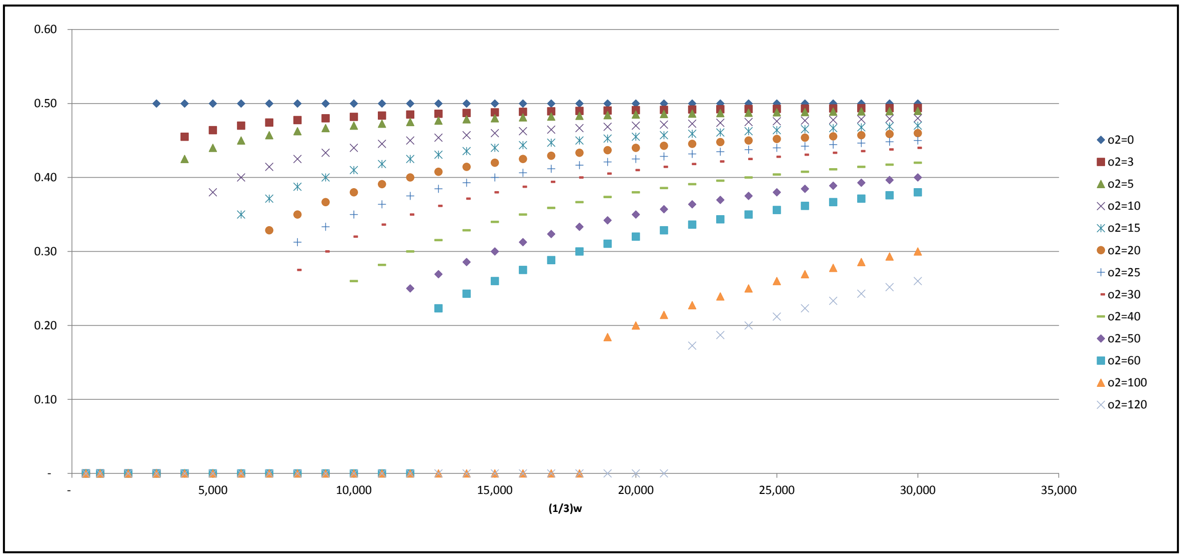
$$d_w^* = Max\left\{\frac{\bar{t}w - o_2}{2}; \underline{t}w\right\}$$

It is necessary to check whether the value obtained does not imply π^{d_w}<0, i.e. the resulting profit must be compared with the option of $d_w = 0$.

Graphs A2.1 and A2.2 show results for a population with t~U[0; 0.0028], corresponding to a complaint time between 0 and 4 hours. We set $o_1 = 5$.

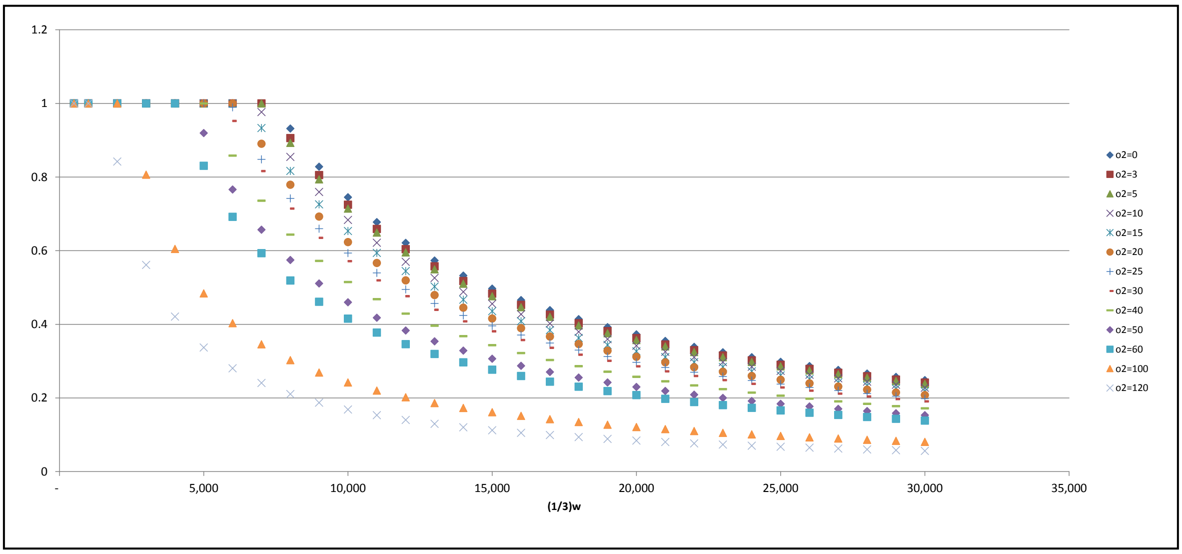


Graph A2.1 – Optimal undue fees



Graph A2.2 – Proportion of complainers with optimal undue fee

Assuming equal participation of all groups of income, and computing average undue charges for each o_2, we simulate what would happen if mischarges were not targeted to groups. The result, with a very different pattern from the one in the previous graph, is depicted in Graph A2.3.



Graph A2.3 – Proportion of complainers with an average undue fee

The patterns are very similar to those in Graphs 3.1 to 3.3. Probably the most evident difference is that in Graph A2.3, with the uniform distribution of t, we find a more abrupt change when, with the increase of salaries, from the situation in which everyone complains (horizontal portion at 1), to the one where the proportion of complainers is decreasing.

Appendix 3 – Sensitivity analysis

In this appendix, we present results of simulations of our test, obtained with the variations of the underlying model parameters. Variables which are not explicitly mentioned in the forthcoming tables, keep their values as stated in table 5.1. Tables A3.1 to A3.8, show the effects of variations in o_1 and o_2. Table A3.1 reports the level of client income up to which mischarge discrimination is expected to be unprofitable ($\underline{w}$). It increases with both o_1 and o_2. The shaded area indicates that the maximum income considered in the simulations is not enough to make this strategy attractive. Table A3.2 shows the corresponding probability of no complaint (P). Tables A3.3 to A3.5 show the critical values for the statistical tests for different significance levels. Tables A3.6 to A3.8 show the power of the test for different significance levels.

A3.1 - $\underline{w}$

		o_1										
		0	5	10	15	20	25	30	35	40	45	50
	0	0	9,786	19,572	29,357	39,143	48,929	58,715	68,501	78,287	88,072	90,000
	5	3,600	15,520	25,491	35,354	45,182	54,994	64,799	74,598	84,394	90,000	90,000
	10	7,200	20,750	31,040	41,059	50,982	60,857	70,707	80,541	90,000	90,000	90,000
	15	10,800	25,722	36,353	46,560	56,602	66,561	76,472	86,354	90,000	90,000	90,000
	20	14,400	30,530	41,501	51,905	62,079	72,134	82,118	90,000	90,000	90,000	90,000
o_2	25	18,000	35,220	46,522	57,128	67,441	77,599	87,663	90,000	90,000	90,000	90,000
	30	21,600	39,823	51,445	62,251	72,707	82,972	90,000	90,000	90,000	90,000	90,000
	35	25,200	44,356	56,286	67,291	77,890	88,264	90,000	90,000	90,000	90,000	90,000
	40	28,800	48,831	61,059	72,260	83,001	90,000	90,000	90,000	90,000	90,000	90,000
	45	32,400	53,260	65,775	77,167	88,051	90,000	90,000	90,000	90,000	90,000	90,000
	50	36,000	57,647	70,441	82,020	90,000	90,000	90,000	90,000	90,000	90,000	90,000

A3.2 - P

		o_1										
		0	5	10	15	20	25	30	35	40	45	50
	0	0.3679	0.4366	0.5053	0.5741	0.6428	0.7115	0.7803	0.8490	0.9177	0.9865	1.0000
	5	0.4491	0.5042	0.5662	0.6304	0.6957	0.7616	0.8279	0.8946	0.9616	1.0000	1.0000
	10	0.5092	0.5610	0.6196	0.6808	0.7436	0.8074	0.8719	0.9370	1.0000	1.0000	1.0000
	15	0.5611	0.6111	0.6674	0.7266	0.7875	0.8497	0.9127	0.9763	1.0000	1.0000	1.0000
	20	0.6073	0.6562	0.7109	0.7686	0.8280	0.8888	0.9505	1.0000	1.0000	1.0000	1.0000
o_2	25	0.6492	0.6972	0.7508	0.8072	0.8655	0.9251	0.9857	1.0000	1.0000	1.0000	1.0000
	30	0.6875	0.7349	0.7875	0.8429	0.9001	0.9588	1.0000	1.0000	1.0000	1.0000	1.0000
	35	0.7226	0.7695	0.8213	0.8759	0.9323	0.9901	1.0000	1.0000	1.0000	1.0000	1.0000
	40	0.7550	0.8014	0.8526	0.9065	0.9622	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	45	0.7848	0.8309	0.8816	0.9348	0.9898	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	50	0.8123	0.8581	0.9083	0.9609	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

A3.3 – Critical value for α=0.01

		o_1										
		0	5	10	15	20	25	30	35	40	45	50
	0	12.6357	13.0362	13.0583	13.4346	13.4228	12.7602	12.3726	10.4001	8.3332	3.4047	
	5	12.6732	13.0359	13.6125	13.4186	12.9555	11.9108	11.2754	9.0085	5.2680		
	10	13.2641	13.2451	13.7616	12.9744	12.7868	11.3463	9.6840	7.1364			
	15	13.6748	13.5709	13.3850	12.7560	12.1060	10.3242	8.2956	4.7430			
	20	13.4154	13.4073	13.2065	12.7397	11.6476	9.5075	6.3970				
o_2	25	13.4227	13.2654	12.4840	12.1798	10.6765	8.0882	3.1494				
	30	13.4801	12.7299	12.5649	11.0675	9.0955	6.1413					
	35	13.2229	12.7651	12.1129	10.0984	7.6679	2.8172					
	40	13.3991	12.3345	11.0772	9.2840	5.6773						
	45	13.1270	12.0005	10.1552	7.8451	3.0559						
	50	12.9667	11.7675	9.3990	6.6068							

A3.4 – Critical value for α=0.05

		o_1									
		0	5	10	15	20	25	30	35	40	45	50
	0	8.5122	8.8477	9.5217	9.1422	9.0876	9.1154	8.6986	7.4395	6.0981	2.6551	
	5	9.1042	9.4544	9.3350	9.0852	9.3030	8.9641	7.5765	6.7725	3.7688		
	10	9.5522	9.6018	9.4344	9.3155	9.1059	8.3796	7.4448	4.8879			
	15	9.5483	9.2381	9.0463	9.0666	8.4039	7.3400	6.0452	3.2353			
	20	9.5781	9.6668	9.4751	9.0388	7.9323	6.5086	4.8876				
o_2	25	9.6299	9.5748	8.7594	8.4486	7.6676	5.8232	2.3910				
	30	9.5783	9.5476	8.8397	8.0422	6.8228	4.6227					
	35	9.4466	8.9926	8.3662	7.7943	5.3891	2.0541					
	40	9.5394	8.6222	8.0150	6.9871	4.1480						
	45	9.3706	8.2424	7.8301	5.5471	2.2875						
	50	9.1770	7.9940	7.0727	4.3015							

A3.5 – Critical value for α=0.10

		o_1									
		0	5	10	15	20	25	30	35	40	45	50
	0	7.1378	6.7535	7.3997	6.9961	6.9200	6.9285	6.4942	5.9593	4.6080	1.9055	
	5	6.9959	7.3325	7.1868	6.9177	7.1180	6.7622	6.0946	5.2825	3.0192		
	10	7.3635	7.4439	7.2594	7.1273	6.9029	6.1623	5.9531	4.1375			
	15	7.3858	7.0753	7.5047	6.8611	6.9087	5.8460	4.5462	2.4814			
	20	7.3704	7.4605	7.2852	6.8260	6.4321	5.7482	4.1321				
o_2	25	7.4279	7.3618	7.2257	6.9394	6.1644	4.3159	2.3907				
	30	7.4854	7.3423	6.6270	6.5253	5.3122	3.8626					
	35	7.2479	7.3976	6.8313	5.5604	4.6219	2.0539					
	40	7.3967	6.9934	6.4767	5.4697	3.3828						
	45	7.1951	6.6513	6.2778	4.7702	1.5193						
	50	7.0160	6.4068	5.5383	3.5279							

A3.6 – 1-β for α=0,01

		o_1										
		0	5	10	15	20	25	30	35	40	45	50
	0	0.0010	0.0670	0.2645	0.4395	0.4966	0.4976	0.4165	0.3142	0.1637	0.0000	
	5	0.0600	0.2470	0.4448	0.5261	0.5440	0.5003	0.3913	0.2395	0.0793		
	10	0.2155	0.4386	0.5518	0.6019	0.5580	0.4760	0.3410	0.1547			
	15	0.4226	0.5801	0.6559	0.6362	0.5543	0.4392	0.2506	0.0626			
	20	0.6195	0.6853	0.6920	0.6542	0.5401	0.3645	0.1439				
o_2	25	0.7437	0.7697	0.7402	0.6293	0.4735	0.2583	0.0399				
	30	0.8260	0.8171	0.7440	0.5982	0.4135	0.1502					
	35	0.8771	0.8230	0.7218	0.5611	0.2924	0.0000					
	40	0.9022	0.8312	0.6941	0.4858	0.1683						
	45	0.9220	0.8242	0.6704	0.3559	0.0000						
	50	0.9311	0.8032	0.5879	0.2189							

A3.7 – 1-β for α=0,05

		o_1										
		0	5	10	15	20	25	30	35	40	45	50
	0	0.0085	0.2085	0.5311	0.7081	0.7557	0.7572	0.6966	0.5803	0.3992	0.1246	
	5	0.1764	0.4850	0.7023	0.7904	0.7967	0.7619	0.6689	0.5037	0.2560		
	10	0.4474	0.6941	0.8029	0.8283	0.8083	0.7340	0.6068	0.3902			
	15	0.6921	0.8289	0.8692	0.8584	0.8049	0.7067	0.5135	0.2044			
	20	0.8310	0.8825	0.8970	0.8642	0.7847	0.6450	0.3796				
o_2	25	0.9086	0.9177	0.9118	0.8565	0.7494	0.5360	0.1638				
	30	0.9492	0.9374	0.9127	0.8360	0.6831	0.3697					
	35	0.9680	0.9505	0.9055	0.8061	0.5754	0.1425					
	40	0.9789	0.9559	0.8926	0.7486	0.4127						
	45	0.9827	0.9541	0.8669	0.6466	0.1598						
	50	0.9863	0.9473	0.8275	0.4914							

A3.8 – 1-β for α=0,10

		o1										
		0	5	10	15	20	25	30	35	40	45	50
	0	0.0217	0.3299	0.6751	0.8179	0.8629	0.8610	0.8172	0.7225	0.5502	0.2203	
	5	0.2974	0.6273	0.8191	0.8838	0.8828	0.8558	0.7894	0.6507	0.3938		
	10	0.6033	0.8144	0.8920	0.9091	0.8958	0.8477	0.7442	0.5306			
	15	0.8110	0.9099	0.9318	0.9283	0.8932	0.8183	0.6629	0.3288			
	20	0.9109	0.9416	0.9453	0.9317	0.8770	0.7740	0.5307				
o2	25	0.9560	0.9613	0.9552	0.9240	0.8500	0.6789	0.2875				
	30	0.9760	0.9711	0.9596	0.9136	0.8016	0.5277					
	35	0.9876	0.9782	0.9542	0.8921	0.7121	0.2564					
	40	0.9915	0.9808	0.9500	0.8477	0.5608						
	45	0.9942	0.9786	0.9328	0.7807	0.2936						
	50	0.9955	0.9769	0.9084	0.6501							

Tables 3.9 to 3.16 show the effects of variations in λ and o_2. Table A3.9 reports the level of client income up to which mischarge discrimination is expected to be unprofitable ($\underline{w}$). It falls with average complaint time $(1/\lambda)$ and increases with o_2. The shaded area indicates that the maximum income considered in the simulations is not enough to make this strategy attractive. Table A3.10 shows the corresponding probability of no complaint (P). Tables A3.11 to A3.13 show the critical values for the statistical tests for different significance levels. Tables A3.14 to A3.16 show the power of the test for different significance levels.

A3.9 – $\underline{w}$

Average complaining time in hours		0.5	1	2	3	4	5	6	7	8	9	10
λ		1440	720	360	240	180	144	120	102.86	90	80	72
	0	19,572	9,786	4,893	3,262	2,446	1,957	1,631	1,398	1,223	1,087	979
	5	31,040	15,520	7,760	5,173	3,880	3,104	2,587	2,217	1,940	1,724	1,552
	10	41,501	20,750	10,375	6,917	5,188	4,150	3,458	2,964	2,594	2,306	2,075
	15	51,445	25,722	12,861	8,574	6,431	5,144	4,287	3,675	3,215	2,858	2,572
	20	61,059	30,530	15,265	10,177	7,632	6,106	5,088	4,361	3,816	3,392	3,053
o2	25	70,441	35,220	17,610	11,740	8,805	7,044	5,870	5,031	4,403	3,913	3,522
	30	79,646	39,823	19,911	13,274	9,956	7,965	6,637	5,689	4,978	4,425	3,982
	35	88,711	44,356	22,178	14,785	11,089	8,871	7,393	6,337	5,544	4,928	4,436
	40	90,000	48,831	24,416	16,277	12,208	9,766	8,139	6,976	6,104	5,426	4,883
	45	90,000	53,260	26,630	17,753	13,315	10,652	8,877	7,608	6,657	5,918	5,326
	50	90,000	57,647	28,824	19,216	14,412	11,529	9,608	8,235	7,206	6,405	5,765

A3.10 – P

Average complaining time in hours		0.5	1	2	3	4	5	6	7	8	9	10
λ		1440	720	360	240	180	144	120	102,86	90	80	72
	0	0.5053	0.4366	0.4022	0.3908	0.3851	0.3816	0.3793	0.3777	0.3765	0.3755	0.3748
	5	0.6196	0.5042	0.4412	0.4188	0.4071	0.3999	0.3950	0.3915	0.3888	0.3867	0.3849
	10	0.7109	0.5610	0.4750	0.4433	0.4266	0.4162	0.4090	0.4038	0.3998	0.3966	0.3941
	15	0.7875	0.6111	0.5055	0.4657	0.4445	0.4312	0.4220	0.4152	0.4101	0.4059	0.4026
	20	0.8526	0.6562	0.5336	0.4866	0.4613	0.4453	0.4342	0.4260	0.4197	0.4148	0.4107
o_2	25	0.9083	0.6972	0.5600	0.5063	0.4772	0.4587	0.4458	0.4363	0.4290	0.4232	0.4184
	30	0.9554	0.7349	0.5848	0.5250	0.4923	0.4715	0.4570	0.4462	0.4379	0.4313	0.4259
	35	0.9949	0.7695	0.6083	0.5429	0.5069	0.4838	0.4677	0.4558	0.4465	0.4392	0.4331
	40	1.0000	0.8014	0.6306	0.5600	0.5208	0.4957	0.4781	0.4650	0.4549	0.4468	0.4401
	45	1.0000	0.8309	0.6518	0.5765	0.5344	0.5072	0.4882	0.4740	0.4630	0.4542	0.4470
	50	1.0000	0.8581	0.6721	0.5923	0.5474	0.5184	0.4979	0.4827	0.4709	0.4614	0.4536

A3.11 – Critical value for α=0,01

Average complaining time in hours		0.5	1	2	3	4	5	6	7	8	9	10
λ		1440	720	360	240	180	144	120	102,86	90	80	72
	0	13.0583	13.0362	12.4484	12.9127	12.3965	12.4841	12.5303	12.5585	12.5771	12.5918	12.6011
	5	13.7616	13.0359	12.7106	12.9282	12.9581	12.6704	12.9158	12.4212	12.5470	12.6382	12.7113
	10	13.2065	13.2451	13.0071	12.9533	12.8497	12.8704	12.6514	12.9599	12.5172	12.6903	12.8200
	15	12.5649	13.5709	13.3294	13.0002	12.7441	13.0416	13.0453	12.8274	12.4960	12.7448	12.9382
	20	11.0772	13.4073	13.0555	13.0828	12.6627	12.6116	12.7749	12.7004	13.1004	12.7950	13.0375
o_2	25	9.3990	13.2654	13.4100	13.1713	13.1776	12.7925	13.1477	12.5887	13.0799	12.8418	13.1295
	30	6.7770	12.7299	13.1734	13.2309	13.1250	12.9869	12.8876	13.0736	13.0257	12.9024	12.6336
	35	1.7775	12.7651	13.5494	13.3168	13.0749	13.1916	13.2047	12.9447	13.0224	12.9351	12.7434
	40		12.3345	13.4247	13.4155	13.5744	13.3313	13.0306	12.8294	12.9873	12.9847	12.8530
	45		12.0005	13.2146	13.5113	13.4836	13.5759	12.8309	13.3290	12.9680	13.0193	12.9508
	50		11.7675	13.6345	13.5960	13.4506	13.2083	13.1472	13.2100	12.9688	13.0652	13.0471

A3.12 – Critical value for α=0,05

Average complaining time in hours		0.5	1	2	3	4	5	6	7	8	9	10
λ		1440	720	360	240	180	144	120	102,86	90	80	72
	0	9.5217	8.8477	8.9839	8.7663	8.9459	9.0363	9.0844	9.1139	9.1336	9.1491	9.1591
	5	9.4344	9.4544	9.1867	8.7674	8.8053	9.1916	8.7777	8.9573	9.0879	9.1798	9.2494
	10	9.4751	9.6018	9.4443	9.4214	9.3276	8.7278	9.1601	8.8092	9.0372	9.2105	8.6898
	15	8.8397	9.2381	9.1083	9.4505	9.2098	8.9111	8.8905	9.3103	9.0025	9.2423	8.7972
	20	8.0150	9.6668	9.4141	9.4685	9.1049	9.0737	9.2566	9.1897	8.9700	9.2832	8.9075
o_2	25	7.0727	9.5748	9.1334	9.5240	9.5820	9.2505	8.9825	9.0566	8.9413	9.3293	9.0101
	30	5.2139	9.5476	9.4614	9.5798	9.4880	9.4132	9.3476	8.9122	8.9084	9.3582	9.1235
	35	1.7775	8.9926	9.2557	9.5911	9.4350	9.0238	9.0856	9.3935	8.8724	9.3736	9.2158
	40		8.6222	9.6177	9.1349	9.3465	9.1765	9.4336	9.2740	9.4303	9.3849	9.3109
	45		8.2424	9.4360	9.1983	9.2546	9.3526	9.1954	9.1458	9.3959	9.4450	9.3885
	50		7.9940	9.2921	9.3125	9.1700	9.5299	9.4912	9.0448	9.3695	8.9319	9.4636

A3.13 – Critical value for α=0,10

Average complaining time in hours		0.5	1	2	3	4	5	6	7	8	9	10
λ		1440	720	360	240	180	144	120	102,86	90	80	72
	0	7.3997	6.7535	6.9052	6.6931	6.8755	6.9676	7.0168	7.0472	7.0675	7.0834	7.0938
	5	7.2594	7.3325	7.0838	6.6799	6.7240	7.1137	6.7020	6.8828	7.0138	7.1082	7.1804
	10	7.2852	7.4439	7.3213	7.3053	7.2306	7.2626	7.0770	6.7296	6.9594	7.1347	6.6155
	15	6.6270	7.0753	6.9788	7.3248	7.1095	6.8076	6.7949	7.2285	6.9196	7.1717	6.7207
	20	6.4767	7.4605	7.2725	7.3624	6.9922	6.9747	7.1630	7.0945	6.8815	7.1978	6.8247
o_2	25	5.5383	7.3618	7.5396	7.4099	6.8973	7.1399	6.8830	6.9568	6.8466	7.2341	6.9248
	30	3.6713	7.3423	7.3049	7.4332	7.3790	7.2994	7.2307	6.8173	6.8054	7.2693	7.0272
	35	1.7775	7.3976	7.0797	7.4854	7.2985	7.4179	6.9654	7.2865	6.7758	7.2844	7.1256
	40		6.9934	7.4180	7.5201	7.2051	7.0582	7.3109	7.1607	7.3123	7.2978	7.2107
	45		6.6513	7.2482	7.1425	7.1154	7.2246	7.0774	7.0368	7.2932	7.3283	7.2769
	50		6.4068	7.1655	7.1187	7.5373	7.3959	7.4123	6.9374	7.2611	7.3787	6.8274

A3.14 –1-β for α=0,01

Average complaining time in hours		0.5	1	2	3	4	5	6	7	8	9	10
λ		1440	720	360	240	180	144	120	102,86	90	80	72
	0	0.2645	0.0670	0.0168	0.0082	0.0058	0.0045	0.0032	0.0023	0.0027	0.0022	0.0022
	5	0.5518	0.2470	0.0594	0.0232	0.0131	0.0103	0.0066	0.0062	0.0048	0.0039	0.0035
	10	0.6920	0.4386	0.1333	0.0534	0.0304	0.0183	0.0151	0.0100	0.0097	0.0071	0.0058
	15	0.7440	0.5801	0.2200	0.0999	0.0572	0.0321	0.0219	0.0180	0.0166	0.0117	0.0086
	20	0.6941	0.6853	0.3475	0.1599	0.0970	0.0591	0.0383	0.0287	0.0191	0.0178	0.0126
o_2	25	0.5879	0.7697	0.4246	0.2260	0.1235	0.0838	0.0480	0.0446	0.0270	0.0239	0.0180
	30	0.3424	0.8171	0.5407	0.2941	0.1730	0.1109	0.0757	0.0495	0.0383	0.0319	0.0286
	35	0.0000	0.8230	0.5922	0.3614	0.2307	0.1402	0.0931	0.0699	0.0503	0.0406	0.0363
	40		0.8312	0.6621	0.4245	0.2548	0.1736	0.1278	0.0956	0.0663	0.0509	0.0433
	45		0.8242	0.7248	0.4866	0.3120	0.2026	0.1711	0.1023	0.0847	0.0632	0.0515
	50		0.8032	0.7465	0.5387	0.3689	0.2657	0.1855	0.1323	0.1064	0.0780	0.0615

A3.15 – 1-β for α=0,05

Average complaining time in hours		0.5	1	2	3	4	5	6	7	8	9	10
λ		1440	720	360	240	180	144	120	102,86	90	80	72
	0	0.5311	0.2085	0.0727	0.0380	0.0294	0.0238	0.0219	0.0175	0.0179	0.0171	0.0156
	5	0.8029	0.4850	0.1808	0.1025	0.0660	0.0425	0.0390	0.0309	0.0256	0.0216	0.0191
	10	0.8970	0.6941	0.3160	0.1675	0.1024	0.0885	0.0589	0.0544	0.0412	0.0336	0.0364
	15	0.9127	0.8289	0.4909	0.2605	0.1734	0.1315	0.0980	0.0664	0.0627	0.0481	0.0500
	20	0.8926	0.8825	0.5946	0.3599	0.2561	0.1820	0.1252	0.0989	0.0876	0.0643	0.0651
o_2	25	0.8275	0.9177	0.7187	0.4553	0.3044	0.2316	0.1844	0.1407	0.1176	0.0846	0.0805
	30	0.6347	0.9374	0.7740	0.5403	0.3817	0.2779	0.2138	0.1878	0.1520	0.1086	0.0983
	35	0.0000	0.9505	0.8395	0.6258	0.4582	0.3707	0.2826	0.2029	0.1903	0.1345	0.1199
	40		0.9559	0.8640	0.7211	0.5312	0.4185	0.3051	0.2523	0.1952	0.1666	0.1388
	45		0.9541	0.8968	0.7660	0.6010	0.4604	0.3763	0.3061	0.2324	0.1921	0.1614
	50		0.9473	0.9201	0.8062	0.6679	0.5044	0.4083	0.3579	0.2707	0.2554	0.1895

A3.16 – 1-β for α=0,10

Average complaining time in hours		0.5	1	2	3	4	5	6	7	8	9	10
λ		1440	720	360	240	180	144	120	102,86	90	80	72
	0	0.6751	0.3299	0.1371	0.0818	0.0626	0.0508	0.0437	0.0386	0.0367	0.0340	0.0332
	5	0.8920	0.6273	0.2979	0.1897	0.1283	0.0864	0.0815	0.0670	0.0565	0.0486	0.0431
	10	0.9453	0.8144	0.4614	0.2846	0.1906	0.1406	0.1155	0.1078	0.0858	0.0714	0.0777
	15	0.9596	0.9099	0.6402	0.3971	0.2887	0.2331	0.1841	0.1303	0.1211	0.0973	0.1006
	20	0.9500	0.9416	0.7347	0.5055	0.3911	0.2975	0.2233	0.1858	0.1630	0.1259	0.1265
o_2	25	0.9084	0.9613	0.8133	0.6035	0.4862	0.3593	0.3021	0.2443	0.2121	0.1617	0.1534
	30	0.7740	0.9711	0.8689	0.6845	0.5281	0.4170	0.3406	0.3063	0.2608	0.1961	0.1840
	35	0.0000	0.9782	0.9150	0.7521	0.6123	0.4869	0.4208	0.3257	0.3095	0.2358	0.2121
	40		0.9808	0.9317	0.8077	0.6784	0.5714	0.4492	0.3873	0.3173	0.2741	0.2435
	45		0.9786	0.9517	0.8659	0.7416	0.6158	0.5247	0.4481	0.3620	0.3128	0.2755
	50		0.9769	0.9650	0.8919	0.7741	0.6517	0.5512	0.5076	0.4087	0.3456	0.3455

Tables 3.17 to 3.24 show the effects of variations in w_l and w_h. Table A3.17 reports the level of client income up to which mischarge discrimination is expected to be unprofitable ($\underline{w}$). In some cases the extremes of the income distribution make every level profitable or unprofitable. The shaded area indicates the latter case. Table A3.18 shows the corresponding probability of no complaint (P). Tables A3.19 to A3.21 show the critical values for the statistical tests for different significance levels. Tables A3.22 to A3.24 show the power of the test for different significance levels.

A3.17–$\underline{w}$

		w_h										
		0	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
	0		10,000	20,000	30,000	35,220	35,220	35,220	35,220	35,220	35,220	35,220
	10,000			20,000	30,000	35,220	35,220	35,220	35,220	35,220	35,220	35,220
	20,000				30,000	35,220	35,220	35,220	35,220	35,220	35,220	35,220
	30,000					35,220	35,220	35,220	35,220	35,220	35,220	35,220
	40,000						40,000	40,000	40,000	40,000	40,000	40,000
w_l	50,000							50,000	50,000	50,000	50,000	50,000
	60,000								60,000	60,000	60,000	60,000
	70,000									70,000	70,000	70,000
	80,000										80,000	80,000
	90,000											90,000
	100,000											

A3.18 – P

		w_h										
		0	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
	0		1.0000	1.0000	1.0000	0.9515	0.8712	0.8111	0.7646	0.7275	0.6972	0.6720
	10,000			1.0000	1.0000	0.9353	0.8390	0.7734	0.7254	0.6886	0.6594	0.6355
	20,000				1.0000	0.9030	0.7853	0.7167	0.6705	0.6367	0.6107	0.5900
	30,000					0.8060	0.6780	0.6223	0.5881	0.5640	0.5458	0.5314
	40,000						0.5499	0.5304	0.5154	0.5035	0.4938	0.4856
w_l	50,000							0.5109	0.4982	0.4881	0.4798	0.4727
	60,000								0.4856	0.4767	0.4694	0.4632
	70,000									0.4679	0.4613	0.4558
	80,000										0.4548	0.4497
	90,000											0.4447
	100,000											

A3.19 – Critical value for α=0,01

		w_h										
		0	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
	0					6.4526	10.2512	11.8381	12.1883	12.8552	13.2654	13.3544
	10,000					6.8096	10.7513	12.0152	12.4261	12.9178	13.1577	13.0210
	20,000					7.5291	10.6356	12.2416	12.6921	12.7208	13.1779	13.2654
	30,000					8.9821	11.2284	11.9935	12.3001	12.6190	12.6180	13.1198
	40,000						10.5833	11.4469	11.9210	12.2584	12.4958	12.8075
w_l	50,000							10.7631	11.1712	11.9987	11.9230	12.1496
	60,000								10.2473	11.1950	11.6230	11.7524
	70,000									10.3805	10.9800	11.7234
	80,000										10.4662	11.4306
	90,000											10.7004
	100,000											

A3.20 – Critical value for α=0,05

		w_h										
		0	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
	0					4.9963	7.3423	8.2022	9.1447	9.1661	9.5748	9.5947
	10,000					4.7022	7.2557	8.4454	8.8032	9.2641	9.4641	9.3033
	20,000					5.4969	7.8478	8.7468	9.1563	9.1254	9.5119	9.5959
	30,000					6.4814	7.9657	8.6309	8.8240	9.1435	9.0700	9.5329
	40,000						7.5999	8.2577	8.6337	8.4873	9.0442	8.7784
w_l	50,000							7.2903	8.0298	8.2032	8.5980	8.7432
	60,000								7.3532	8.0902	8.3940	8.4414
	70,000									7.5080	7.8904	8.5015
	80,000										7.6066	7.7687
	90,000											7.2959
	100,000											

A3.21 – Critical value for α=0,10

		w_h										
		0	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
	0					3.5595	5.8740	6.7034	6.9980	6.9891	7.3618	7.3618
	10,000					3.9874	5.7970	6.3394	7.2481	7.1043	7.2793	7.1199
	20,000					4.1505	5.8251	6.6903	7.0712	7.0129	7.3826	7.4521
	30,000					5.2233	6.0268	6.6413	6.7989	7.0690	6.9922	7.4278
	40,000						5.8269	6.3765	6.6871	7.0329	7.0132	7.2705
w_l	50,000							6.0606	6.1617	6.2926	6.6182	6.7243
	60,000								5.6179	6.2342	6.4703	6.4740
	70,000									5.7845	6.0455	6.5848
	80,000										5.8954	5.9332
	90,000											5.5908
	100,000											

A3.22 –1-β for α=0,01

		w_h										
		0	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
	0					0.3385	0.6982	0.7758	0.8079	0.7923	0.7697	0.7309
	10,000					0.1918	0.4360	0.5252	0.5556	0.5455	0.5259	0.5247
	20,000					0.0864	0.1768	0.2145	0.2369	0.2448	0.2266	0.2208
	30,000					0.0304	0.0422	0.0486	0.0530	0.0534	0.0597	0.0531
	40,000						0.0107	0.0113	0.0134	0.0152	0.0139	0.0149
w_l	50,000							0.0102	0.0105	0.0121	0.0124	0.0133
	60,000								0.0106	0.0096	0.0104	0.0119
	70,000									0.0101	0.0107	0.0097
	80,000										0.0103	0.0107
	90,000											0.0096
	100,000											

A3.23 – 1-β for α=0,05

	w_h										
w_l	0	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
0					0.6259	0.8913	0.9348	0.9405	0.9307	0.9177	0.9020
10,000					0.4525	0.7162	0.7837	0.8008	0.7898	0.7742	0.7617
20,000					0.2520	0.4078	0.4679	0.4889	0.4933	0.4783	0.4622
30,000					0.1135	0.1444	0.1603	0.1697	0.1710	0.1854	0.1709
40,000						0.0502	0.0531	0.0571	0.0684	0.0657	0.0787
50,000							0.0510	0.0506	0.0558	0.0575	0.0599
60,000								0.0487	0.0502	0.0520	0.0545
70,000									0.0493	0.0506	0.0498
80,000										0.0484	0.0483
90,000											0.0471
100,000											

A3.24 – 1-β for α=0,10

	w_h										
w_l	0	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
0					0.7698	0.9458	0.9700	0.9730	0.9694	0.9613	0.9531
10,000					0.5960	0.8259	0.8838	0.8883	0.8835	0.8710	0.8650
20,000					0.3789	0.5601	0.6114	0.6311	0.6431	0.6253	0.6115
30,000					0.2007	0.2537	0.2694	0.2825	0.2856	0.3051	0.2839
40,000						0.1012	0.1044	0.1126	0.1183	0.1282	0.1286
50,000							0.1033	0.1030	0.1131	0.1125	0.1169
60,000								0.1001	0.0977	0.0999	0.1073
70,000									0.0991	0.0974	0.0983
80,000										0.0967	0.0983
90,000											0.0986
100,000											

Appendix 4 – Model with a lognormal distribution of salaries

In this appendix, we consider a distribution of salaries given by:

$$g(w) = \frac{1}{w\sigma_w\sqrt{2\pi}} e^{-\left[\frac{(lnw-\mu_w)^2}{2\sigma_w^2}\right]}$$

where μ_w and σ_w represent, respectively, the mean and standard deviation of $ln(w)$.

For individuals who are charged an undue fee, thus belonging to n_1, we have:

$$l_{i \in n_1}\left(d_{w,i}^*, o_2, \lambda\right) = g\left(\lambda(d_w^* + o_2)\right)\left[1 - e^{-d_w^*/(d_w^*+o_2)}\right]$$

Substituting the salaries distribution:

$$l_{i \in n_1}\left(d_{w,i}^*, o_2, \lambda\right) = \frac{1}{\lambda(d_w^* + o_2)\sigma_w\sqrt{2\pi}} e^{-\left[\frac{\left(ln\left(\lambda(d_w^*+o_2)\right)-\mu_w\right)^2}{2\sigma_w^2}\right]}\left[1 - e^{-d_w^*/(d_w^*+o_2)}\right]$$

For individuals belonging to n_2:

$$l_{i \in n_2}(o_1, o_2, \lambda) = P = G(\underline{w}) + \int_{\underline{w}}^{\infty} g(w)\, e^{(\lambda o_2/w-1)} dw$$

∴

$$P = \int_0^{\underline{w}} \frac{1}{w\sigma_w\sqrt{2\pi}} e^{-\left[\frac{(lnw-\mu_w)^2}{\sigma_w^2}\right]} dw + \int_{\underline{w}}^{\infty} \frac{1}{w\sigma_w\sqrt{2\pi}} e^{-\left[\frac{(lnw-\mu_w)^2}{2\sigma_w^2}\right]} e^{(\lambda o_2/w-1)} dw$$

∴

$$P = \int_0^{\underline{w}} \frac{1}{w\sigma_w\sqrt{2\pi}} e^{-\left[\frac{(lnw-\mu_w)^2}{\sigma_w^2}\right]} dw + \int_{\underline{w}}^{\infty} \frac{1}{w\sigma_w\sqrt{2\pi}} e^{\left(\lambda o_2/w-1-\frac{(lnw-\mu_w)^2}{2\sigma_w^2}\right)} dw$$

In this expression, we may substitute the normal accumulated density function from minus infinity to $\frac{ln\underline{w}-\mu_w}{\sigma_w}$ for the first term.

Baseline model, with lognormal distribution of salaries

$$g(w) = \frac{1}{w\sigma_w\sqrt{2\pi}} e^{-\left[\frac{(lnw-\mu_w)^2}{2\sigma_w^2}\right]}$$

$d_w^* = \frac{w}{\lambda} - o_2$ for $w \geq \underline{w}$ and $d_w^* = 0$ otherwise. The computation procedure of $\underline{w}$ implies
$\frac{\underline{w}}{\lambda} - o_2 \geq 0.$

Therefore, $h(d_w)$ is given by:

$$Prob[d_w = 0] = Prob[w < \underline{w}] = \int_0^{\underline{w}} g(w)dw = \int_0^{\underline{w}} \frac{1}{w\sigma_w\sqrt{2\pi}} e^{-\left[\frac{(lnw-\mu_w)^2}{\sigma_w^2}\right]} dw$$

We may substitute the normal accumulated density function from minus infinity to $\frac{ln\underline{w}-\mu_w}{\sigma_w}$ for the first term.

As for $d_w > 0$, we can make a transformation in probability density function:

$$g(w) = \frac{1}{w\sigma_w\sqrt{2\pi}} e^{-\left[\frac{(lnw-\mu_w)^2}{2\sigma_w^2}\right]}$$

$$h(d_w) = g(w(d_w)) \frac{\partial w(d_w)}{\partial d_w} = \frac{1}{w(d_w)\sigma_w\sqrt{2\pi}} e^{-\left[\frac{(lnw(d_w)-\mu_w)^2}{2\sigma_w^2}\right]} \frac{\partial w(d_w)}{\partial d_w}$$

$$= \frac{1}{\lambda(d_w^* + o_2)\sigma_w\sqrt{2\pi}} e^{-\left[\frac{(ln(\lambda(d_w^*+o_2))-\mu_w)^2}{2\sigma_w^2}\right]} \lambda$$

$$= \frac{1}{(d_w + o_2)\sigma_w\sqrt{2\pi}} e^{-\left[\frac{(ln(\lambda(d_w+o_2))-\mu_w)^2}{2\sigma_w^2}\right]}$$

Likelihood function:

Using:

$$l_{i\in n_2}^{A}(o_2,\mu_w,\sigma_w) = P_A = \int_0^\infty h(d_w)\int_0^\infty g(w)e^{-\lambda d_w/w}dw\, dd_w$$

$\therefore$

$$l_{i\in n_2}^{A}(o_2,\mu_w,\sigma_w) = P_A = \int_0^\infty h(d_w)\int_0^\infty \frac{1}{w\sigma_w\sqrt{2\pi}}e^{-\left[\frac{(lnw-\mu_w)^2}{2\sigma_w^2}\right]}e^{-\lambda d_w/w}dw\, dd_w$$

$\therefore$ splitting the expression into $d_w = 0$ and $d_w > 0$:

$$l_{i\in n_2}^{A}(o_2,\lambda,\mu_w,\sigma_w) = P_A$$

$$= Prob[d_w = 0]\int_0^\infty \frac{1}{w\sigma_w\sqrt{2\pi}}e^{-\left[\frac{(lnw-\mu_w)^2}{2\sigma_w^2}\right]}e^{-\lambda 0/w}dw$$

$$+ \int_{limd_w\to 0}^\infty \frac{1}{(d_w+o_2)\sigma_w\sqrt{2\pi}}e^{-\left[\frac{\left(ln\left(\lambda(d_w+o_2)\right)-\mu_w\right)^2}{2\sigma_w^2}\right]}\int_0^\infty \frac{1}{w\sigma_w\sqrt{2\pi}}e^{-\left[\frac{(lnw-\mu_w)^2}{2\sigma_w^2}\right]}e^{-\lambda d_w/w}dw\, dd_w$$

$\therefore$

$$P_A = Prob[d_w = 0]1 + \int_{\frac{w}{\lambda}-o_2}^\infty \frac{1}{(d_w+o_2)\sigma_w\sqrt{2\pi}}e^{-\left[\frac{\left(ln\left(\lambda(d_w+o_2)\right)-\mu_w\right)^2}{2\sigma_w^2}\right]}\int_0^\infty \frac{1}{w\sigma_w\sqrt{2\pi}}e^{-\left[\frac{(lnw-\mu_w)^2}{2\sigma_w^2}\right]}e^{-\lambda d_w/w}dw\, dd_w$$

$$P_A = Prob[d_w = 0] + \int_{\frac{w}{\lambda}-o_2}^\infty \int_0^\infty \frac{1}{(d_w+o_2)w\sigma_w^2 2\pi}e^{-\left[\frac{\left(ln\left(\lambda(d_w+o_2)\right)-\mu_w\right)^2}{2\sigma_w^2}\right]-\left[\frac{(lnw-\mu_w)^2}{2\sigma_w^2}\right]-\lambda d_w/w}dw\, dd_w$$

$$l_{i\in n_1}^{A}\left(d_{w,i},o_2,\mu_w,\sigma_w\right) = h\left(d_{w,i}\right)\int_0^\infty g(w)\left[1-e^{-\lambda d_{w,i}/w}\right]dw$$

In that case, we are only interested in $d_w > 0$:

$$l_{i\in n_1}^{A}\left(d_{w,i},o_2,\lambda,\mu_w,\sigma_w\right)$$

$$= \frac{1}{(d_w+o_2)\sigma_w\sqrt{2\pi}}e^{-\left[\frac{\left(ln\left(\lambda\left(d_{w,i}+o_2\right)\right)-\mu_w\right)^2}{2\sigma_w^2}\right]}\int_0^\infty g(w)\left[1-e^{-\frac{\lambda d_{w,i}}{w}}\right]dw$$

$\therefore$

$$l^{A}_{i\in n_1}\left(d_{w,i},o_2,\mu_w,\sigma_w\right)=$$

$$\frac{1}{(d_w+o_2)\sigma_w\sqrt{2\pi}}e^{-\left[\frac{\left(ln\left(\lambda(d_{w,i}+o_2)\right)-\mu_w\right)^2}{2\sigma_w^2}\right]}\int_0^\infty \frac{1}{w\sigma_w\sqrt{2\pi}}e^{-\left[\frac{(lnw-\mu_w)^2}{2\sigma_w^2}\right]}\left[1-e^{-\lambda d_{w,i}/w}\right]dw$$

$$=\frac{1}{(d_w+o_2)\sigma_w^2 2\pi}e^{-\left[\frac{\left(ln\left(\lambda(d_{w,i}+o_2)\right)-\mu_w\right)^2}{2\sigma_w^2}\right]}\int_0^\infty \frac{1}{w}e^{-\left[\frac{(lnw-\mu_w)^2}{2\sigma_w^2}\right]}\left[1-e^{-\lambda d_{w,i}/w}\right]dw$$

$$=\int_0^\infty \frac{1}{(d_w+o_2)w\sigma_w^2 2\pi}e^{-\left[\frac{\left(ln\left(\lambda(d_{w,i}+o_2)\right)-\mu_w\right)^2}{2\sigma_w^2}\right]-\left[\frac{(lnw-\mu_w)^2}{2\sigma_w^2}\right]}\left[1-e^{-\lambda d_{w,i}/w}\right]dw$$

Mean of l_i under H_0:

$$E_{H_0}\left(ln(l_i)\right)=\int_{\frac{w}{\lambda}-o_2}^{\infty} l^{A}_{i\in n_1}(d_w,o_2,\mu_w,\sigma_w)\,ln\left(l_{i\in n_1}(d_w)\right)dd_w+P_A ln(P)$$

$$E_{H_0}\left(ln(l_i)\right)=\int_{\frac{w}{\lambda}-o_2}^{\infty} l^{A}_{i\in n_1}(d_w,o_2,\mu_w,\sigma_w)\,ln\left(\frac{1}{\lambda(d_w^*+o_2)\sigma_w\sqrt{2\pi}}e^{-\left[\frac{\left(ln\left(\lambda(d_w^*+o_2)\right)-\mu_w\right)^2}{2\sigma_w^2}\right]}\left[1-e^{-d_w^*/(d_w^*+o_2)}\right]\right)dd_w+P_A ln(P)$$

Variance of l_i under H_0:

$$var_{H_0}\left(ln(l_i)\right)=E_{H_0}\left(ln(l_i)^2\right)-E^2_{H_0}\left(ln(l_i)\right)$$

$$E_{H_0}(ln(l_i)^2) = \int_{\frac{w}{\lambda}-o_2}^{\infty} l_{i\in n_1}^{A}(d_w, o_2, \mu_w, \sigma_w)\, ln^2\left(\frac{1}{\lambda(d_w^* + o_2)\sigma_w\sqrt{2\pi}} e^{-\left[\frac{\left(ln\left(\lambda(d_w^*+o_2)\right)-\mu_w\right)^2}{2\sigma_w^2}\right]}\left[1 - e^{-d_w^*/(d_w^*+o_2)}\right]\right) dd_w + \mathrm{P_A} ln^2(\mathrm{P})$$

In order to preserve the possibility of comparison with the results of the main text, we use the following calculation, where w_l and w_h are the limits of the uniform distribution used there:

- Equal means: $e^{\mu_w+\frac{\sigma_w^2}{2}} = \frac{w_h+w_l}{2}$;
- Equal variances: $\left(e^{\sigma_w^2} - 1\right)e^{2\mu_w+\sigma_w^2} = \frac{(w_h-w_l)^2}{12}$.

Thus:

$$\left(e^{\sigma_w^2} - 1\right)\left(e^{\mu_w+\frac{\sigma_w^2}{2}}\right)^2 = \frac{(w_h - w_l)^2}{12}$$

$$\therefore \left(e^{\sigma_w^2} - 1\right)\left(\frac{w_h+w_l}{2}\right)^2 = \frac{(w_h-w_l)^2}{12}$$

$$\therefore \left(e^{\sigma_w^2} - 1\right) = \frac{1}{3}\frac{(w_h-w_l)^2}{(w_h+w_l)^2}$$

$$\therefore \sigma_w^2 = ln\left(1 + \frac{1}{3}\frac{(w_h-w_l)^2}{(w_h+w_l)^2}\right)$$

So that μ may be obtained from:

$$\mu_w + \frac{\sigma_w^2}{2} = ln\left(\frac{w_h + w_l}{2}\right)$$

$$\therefore \mu_w = ln\left(\frac{w_h+w_l}{2}\right) - \frac{\sigma_w^2}{2}$$

$$\therefore \mu_w = ln\left(\frac{w_h+w_l}{2}\right) - \frac{1}{2} ln\left(1 + \frac{1}{3}\frac{(w_h-w_l)^2}{(w_h+w_l)^2}\right)$$

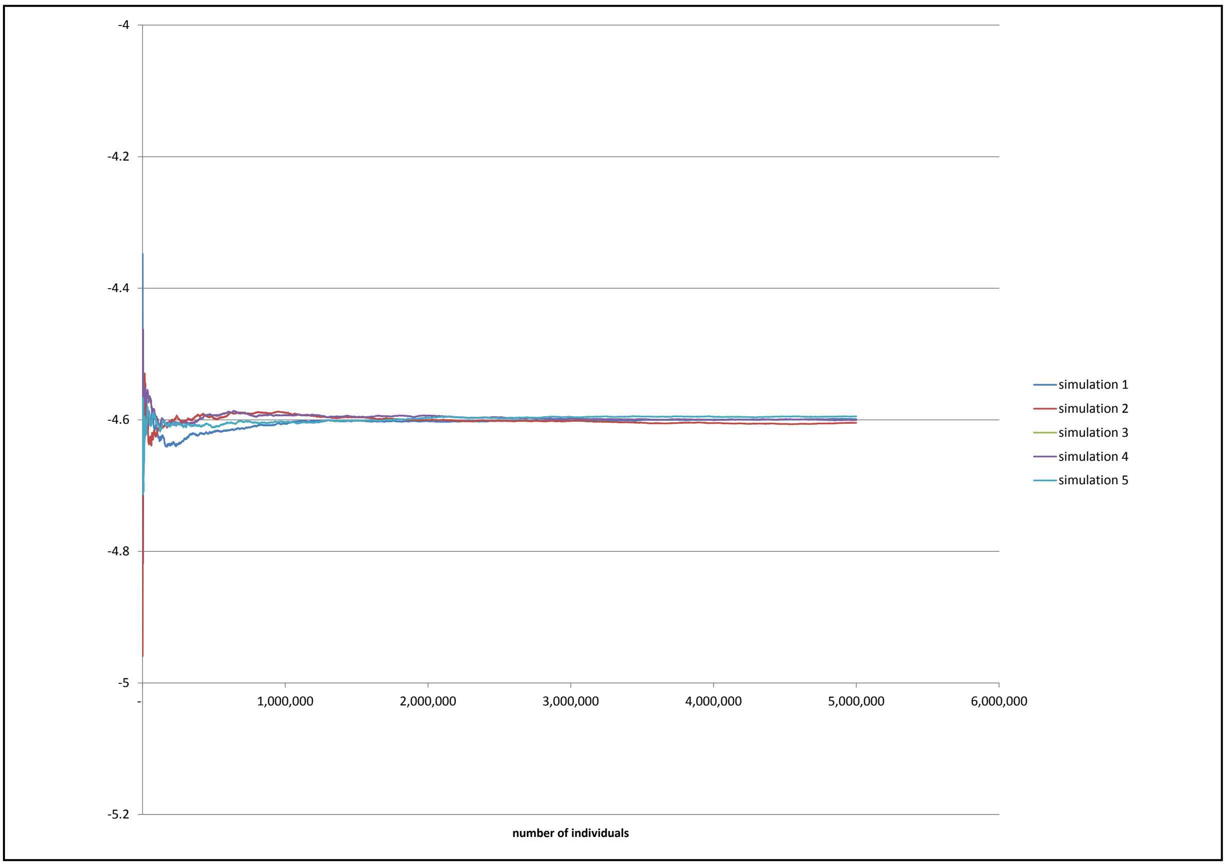
For $w_h = 90{,}000$ and $w_l = 0$:

$$\sigma_w^2 = ln\left(1 + \frac{1}{3}\right) = \ln\left(\frac{4}{3}\right) \cong 0.2877$$

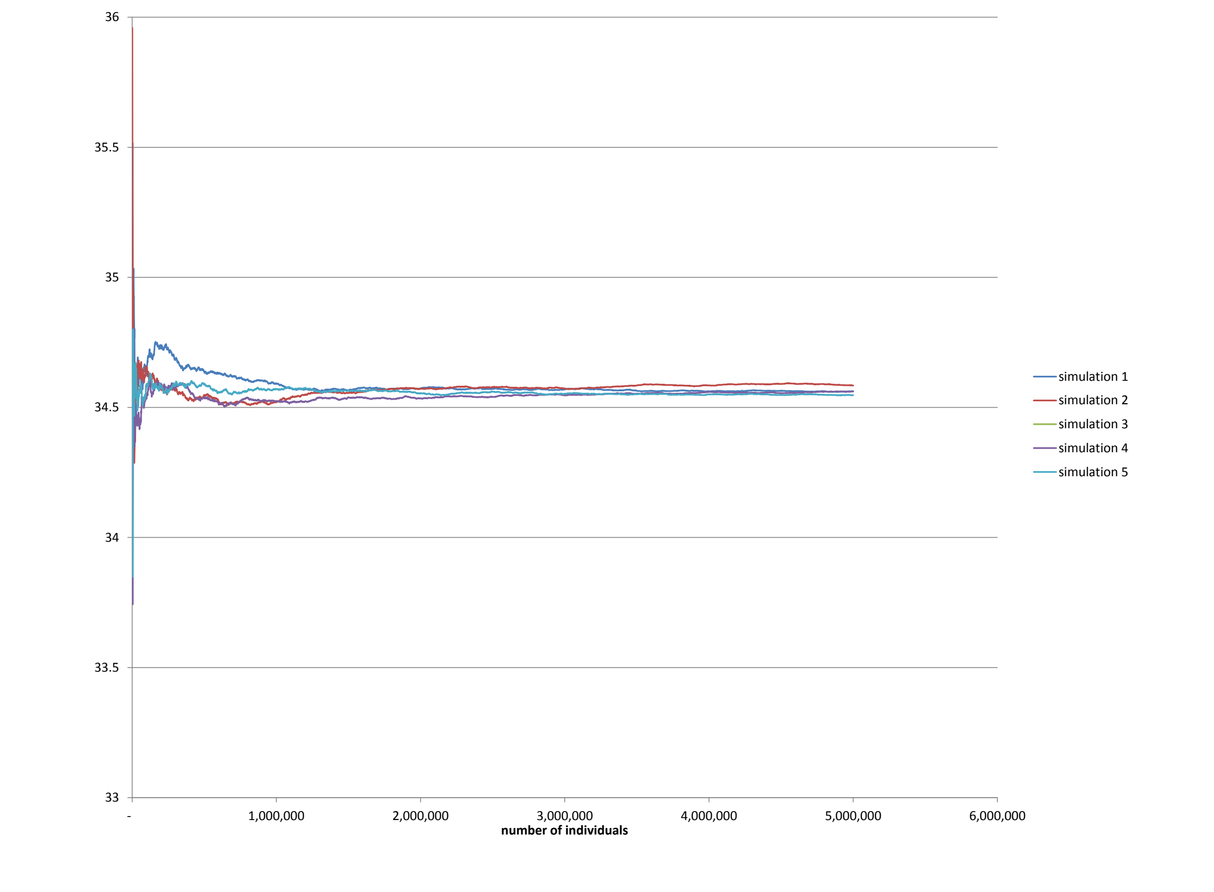
$$\mu_w = ln(45{,}000) - \frac{\ln\left(\frac{4}{3}\right)}{2} \cong 10.5706$$

Results

In Table A4.1, we show that the performance of the test was inferior in this case, considering the parameters for the lognormal distribution used.



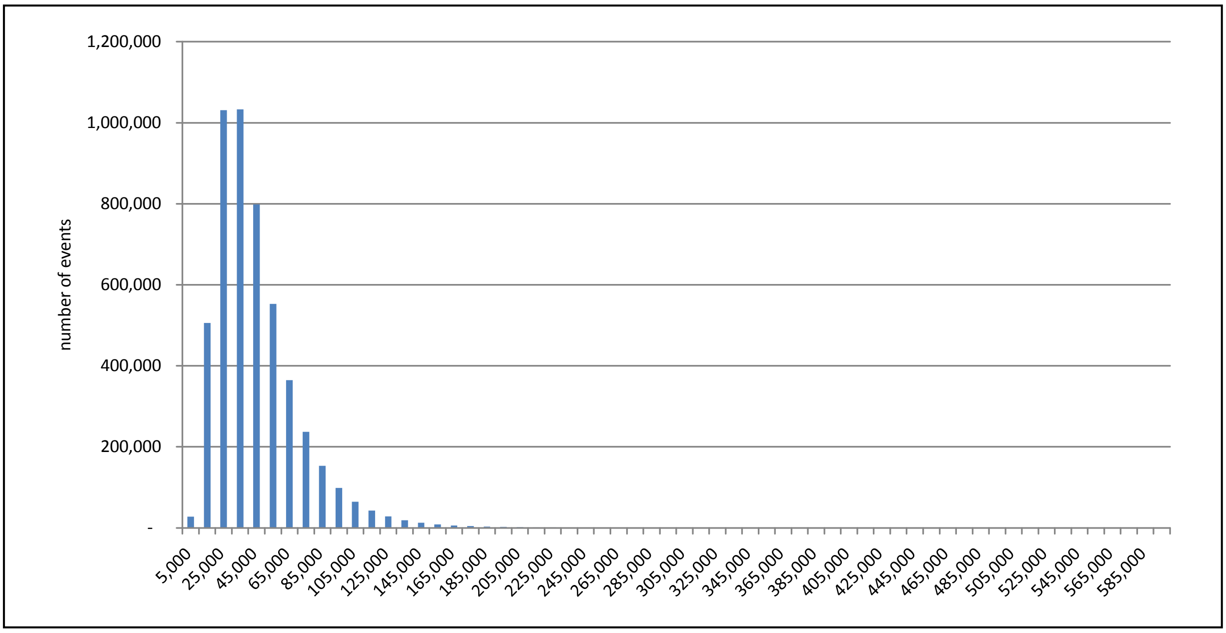
Graph A4.1 – Simulated average of $ln(l_i)$



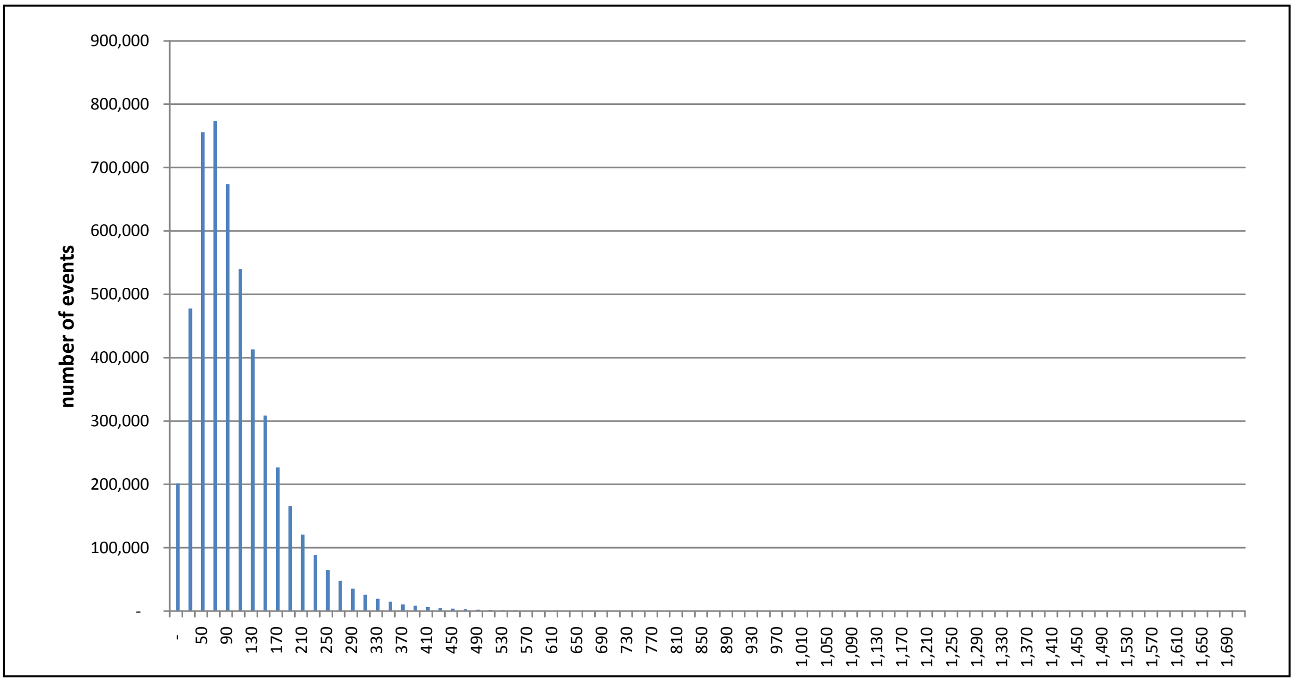
Graph A4.2 – Simulated variance of $ln(l_i)$

The simulated averages including 5×10^6 individuals where:-4.599058; -4.604500; -4.604259; -4.600514 and -4.594811. Algebraically we obtain -4.608873. As for the variance, simulated values were 34.56231; 34.584243; 34.574991; 34.560997 and 34.547123. The algebraic value was 34.568114.

The comparison of the distributions of w and d_w, depicted in Graphs A4.3 and A4.4, shows that the latter is more concentrated at zero, resulting from the individuals which are not chosen for an undue charge.



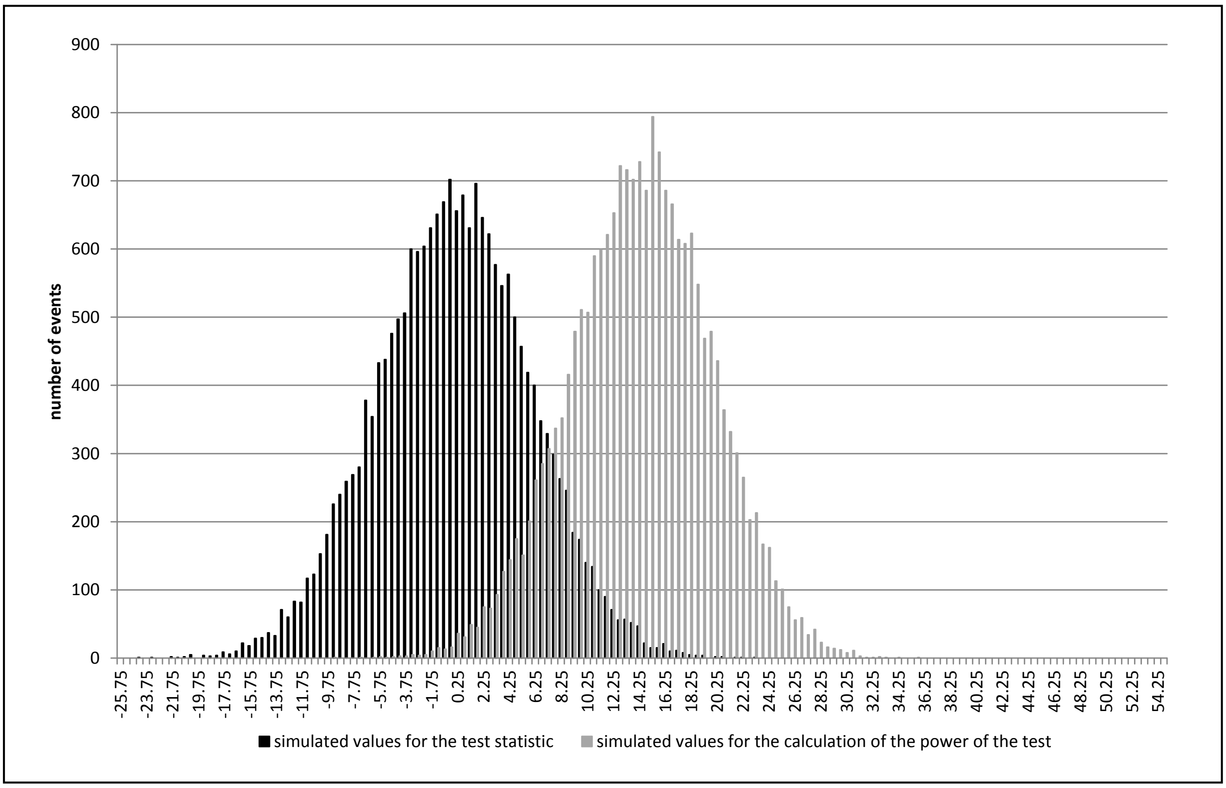
Graph A4.3 – Distribution of simulated values for *w*



Graph A4.4 – Distribution of simulated values for d_w

Table A4.1 – Test statistics

α	Critical value obtained by simulation	CLT critical value	1-β
0.10	7.5648	7.5348	0.8911
0.05	9.6167	9.6709	0.8087
0.01	13.6913	13.6777	0.5541



Graph A4.5 Distribution of simulated values for $\frac{lnL}{n}$ under H_0 and H_A

Like the case analyzed in the main text, critical values seem close to those coming from the CLT and the power of the test seems satisfactory, although these statements are subjective.