Risk premia estimation in Brazil: wait until 2041

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Abstract:

The estimation results in the literature on Brazilian risk premia are not robust. For instance, among the 133 market risk premium estimates reported in the literature, 41 are positive, 18 are negative, and the remainder are not significant. In this study, we investigate the grounds for this lack of consensus. First, we analyze the sensitivity of the US risk premia estimation to two relevant constraints present in the Brazilian market: the small number of assets (137 eligible stocks) and the short time-series sample available for estimation (14 years). We conclude that the second constraint, small T, has greater impact on the results. Then, we evaluate the two potential causes of problems in risk premia estimations with small T: i) small sample bias on betas, and ii) divergence between *ex-post* and *ex-ante* risk premia. Through Monte Carlo simulations, we conclude that for the T available for Brazil, the beta estimates are no longer a problem. However, it is necessary to wait until 2041 to be able to estimate *ex-ante* risk premia with Brazilian data.

Keywords: Risk premia, Asset pricing, Multi-factor model.

JEL Codes: G12, G17

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Resumo

Os resultados das estimações de prêmios de risco brasileiros não são robustos na literatura. Dentre 133 estimativas de prêmio de risco de mercado documentadas, 41 são positivas, 18 negativas e o restante não é significante. No presente trabalho, investigamos os motivos da falta de consenso. Primeiramente, analisamos a sensibilidade da estimação dos prêmios de risco norte-americanos a duas restrições presentes no mercado brasileiro: o baixo número de ativos (137 ações elegíveis) e a pequena quantidade de meses disponíveis para estimação (14 anos). Concluímos que a segunda restrição, T pequeno, tem maior impacto sobre os resultados. Em seguida, avaliamos as duas potenciais causas de problemas para a estimação de prêmios de risco em amostras com T pequeno: i) viés de pequenas amostras nas estimativas dos betas; e ii) divergência entre prêmio de risco *ex-post* e *ex-ante*. Através de exercícios de Monte Carlo, concluímos que para o T disponível no Brasil, a estimativa dos betas já não é mais um problema. No entanto, ainda precisamos esperar até 2041 para conseguirmos estimar corretamente os prêmios *ex-ante* com os dados brasileiros.

Palavras-chaves: Prêmios de risco, Precificação de ativos, Modelos multifatorias.

Códigos JEL: G12, G17.

Abstract

The estimation results in the literature on Brazilian risk premia are not robust. For instance, among the 133 market risk premium estimates reported in the literature, 41 are positive, 18 are negative, and the remainder are not significant. In this study, we investigate the grounds for this lack of consensus. First, we analyze the sensitivity of the US risk premia estimation to two relevant constraints present in the Brazilian market: the small number of assets (137 eligible stocks) and the short time-series sample available for estimation (14 years). We conclude that the second constraint, small T, has greater impact on the results. Then, we evaluate the two potential causes of problems in risk premia estimations with small T: i) small sample bias on betas, and ii) divergence between *ex-post* and *ex-ante* risk premia. Through Monte Carlo simulations, we conclude that for the T available for Brazil, the beta estimates are no longer a problem. However, it is necessary to wait until 2041 to be able to estimate *ex-ante* risk premia with Brazilian data.

**Keywords:** Risk premia, Asset pricing, Multi-factor model.

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1   Introduction

The estimation results of Brazilian risk premia show no consensus in the literature. Reported estimates not only disagree with the US and international results but also vary among themselves. The Brazilian risk premia estimates collected from several studies[5] and presented in Figure 1 illustrate lack of consensus.

Figure 1: Dispersion in the risk premia estimations for Brazil

The figure shows the dispersions in the estimations of market (*Mkt*), size (*SMB*), value (*HML*), and momentum (*WML*) risk premia in the Brazilian stock market. The figure shows a box-plot for each risk factor, and each circle represents one reported estimation (% p.m.). The box-plot reports the 0th, 25th, 50th, 75th, and 100th percentiles. The numbers within the brackets to the right of the 0th and 100th percentiles are the number of negative and positive estimates of each factor respectively. The higher the t-value reported, the larger the circle. A significant estimate ($|$t-value$| \geq 1.64$) is indicated by a blue circle. The coordinate axis reports the factor's name followed by the number of estimates reported.
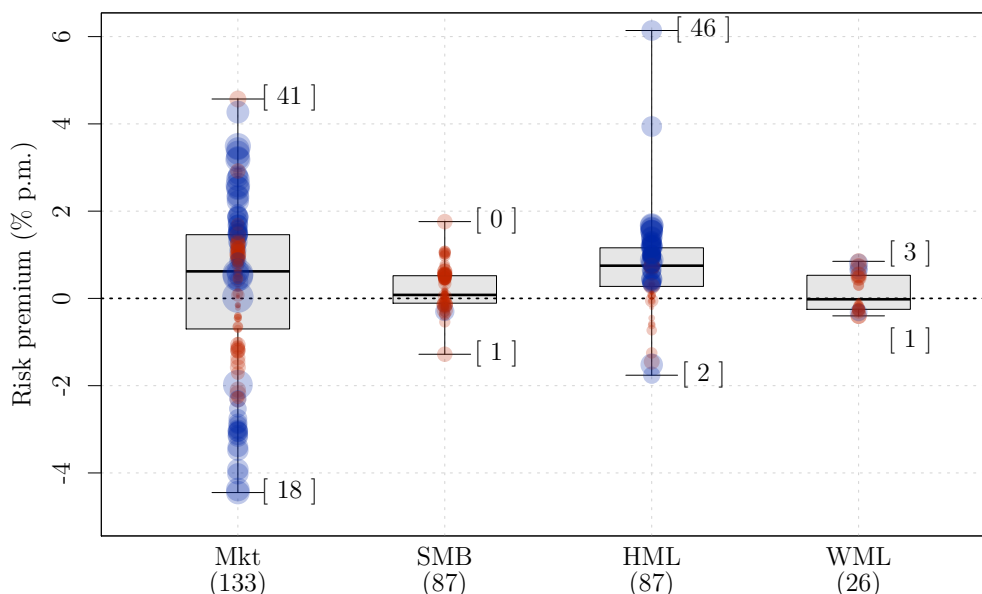
The estimates in Figure 1 are obtained by exploring many sub-periods between 1976 and 2015 and by applying a variety of techniques[6]. The estimates of the market (*Mkt*) risk premium are positive in 41 cases, negative in 18 cases, and not significant in 74 cases. The size (*SMB*) of the risk premium does not present any positive and statistically significant estimates. The value (*HML*) risk premium presents the most robust estimates. Of the 87 estimates, 46 are positive and significant, and only 2 are significant and negative. Finally,

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[5]Fama and French (1998); Rouwenhorst (1999); Bonomo and Garcia (2001); Bonomo et al. (2002); Sampaio (2002); Malaga and Securato (2004); Matos (2006); Chague and Bueno (2007); Bellizia (2009); Mussa et al. (2009); Brito and Murakoshi (2009); Mussa et al. (2011); Bodur (2011); Rizzi (2012); Mussa et al. (2012); Varga and Brito (2015); Eid Jr and Martins (2015), and Piccoli et al. (2015).

[6]Sample means, (Fama and MacBeth, 1973), Generalized Method of Moments (GMM), and Iterative Nonlinear Seemingly Unrelated Regression Estimation (ITNLSUR).

there are 26 estimates for the momentum (*WML*) risk premium, but only a few cases are significant, with three positives and one negative.

This paper investigates the reasons behind the lack of robustness in the estimations of Brazilian risk premia. First, we analyze the sensitivity of the US risk premia estimations to two relevant constraints present in the Brazilian market: the small number of assets (small N) and the short time-series samples (small T).[7]

We conclude that the restriction imposed by the small T is more relevant than that imposed by the small N. While Brazilian data offer values of T over 14 years, our analysis indicates that it is necessary to analyze a time-series sample using data exceeding 40 years to obtain robust risk premia estimates. On the other hand, the Brazilian value of N does not pose an issue.

Given these results, we then investigate the problems caused by the small T. One problem could be the use of poorly estimated betas in the second stage of the estimation. Another problem could be the use of poorly estimated expected returns of stocks. Both would induce errors in the estimation of the risk premia. Poorly estimated betas generate biased risk premia estimates. Poorly estimated expected returns lead to the estimation of *ex-post* instead of *ex-ante* risk premia.[8]

To assess the relative importance of these two issues, we perform Monte Carlo simulations. We conclude that the most important issue is the use of poorly estimated expected returns. Indeed, the difference between *ex-post* and *ex-ante* risk premia proves to be significant when the estimation is performed with the small T available for Brazil. For instance, when data are simulated using a market risk premium of 0.65% p.m., we estimate a positive and significant risk premium in only 17% of the samples.

Our main conclusion is that one needs to wait until 2041 to be able to safely estimate the *ex-ante* risk premia for Brazil. Anyone interested in computing the cost of equity for Brazilian firms should use Brazilian data only to estimate the betas. In turn, the price of risk should be taken from data with longer time-series sample, US data, for instance.

The remainder of the paper is organized as follows. Section 2 describes the dataset, followed by the first analysis of portfolios and risk factors. Section 3 presents the factor model we use in the estimation as well as the methodology supporting it. Section 4 compares the premia results for the US with those found for Brazilian markets (4.1),

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[7]There are few liquid stocks in Brazil. In 2000, only 37 stocks could be considered liquid. In 2014, this number increased to 137. The details on the liquidity criteria can be found at the NEFIN website (`http://nefin.com.br`). Moreover, until 1999, the risk-free rate was used as an instrument of the pegged exchange rate regime, and it was often set to very high levels. Therefore, to estimate the risk premia in Brazil, one commonly uses data beginning in the year 2000.

[8]The *ex-ante* risk premium is the excess return that the investor expects to receive when investing in the asset. This is the value to be estimated. The *ex-post* risk premium is the realized excess return after shocks.

followed by the analysis of the impact of estimating risk premia with a small number of assets (4.2) and with small time-series samples (4.3). This section ends with the assessment of the consequences of estimating risk premia with small time-series samples (4.4). Section 5 concludes the paper by summarizing the findings and its implications.

2 Data

The main paper's datasets consist of monthly portfolio returns from and risk factors of the Brazilian and the US stock markets. The US information is obtained from French's website[9] and covers the period between January 1927 and December 2014. The Brazilian information is taken from the Brazilian Financial Studies Lab (NEFIN[10]) and covers the period from January 2001 to December 2014. The Brazilian stock market was already operational prior to this period. However, until 1999, the risk-free rate was used as an instrument of the pegged exchange rate regime, and it was often set to very high levels. Therefore, to estimate the risk premia in Brazil, one commonly uses data beginning in the year 2000. The US portfolio and risk factor returns are value-weighted, while the Brazilian ones are equally weighted[11].

The 22 portfolios used in this work are presented in Table 1. This table is organized as follows. The first column shows the variables used to arrange the assets into portfolios. The second column names each portfolio, the third and fourth columns contain the percentiles used as breakpoints for the construction of the portfolios, the fifth and sixth columns report the means of the portfolios' returns, and, finally, the last two columns list the autocorrelation consistent standard deviations. The data available at NEFIN's and French's websites have different numbers of portfolios. Therefore, in order to conduct a fair comparison between the Brazilian and the US markets, some portfolios are combined by calculating their value-weighted returns. The first 17 portfolios are defined based on information about Size, Book-to-market, and Momentum, and the other portfolios are organized by grouping assets of the same industry. Note that both countries have the same number of industry portfolios; however, some industries appear only in one of the markets.

Table 1 shows that the US portfolios' returns are negatively correlated with Size and positively correlated with Book-to-market and Momentum, as described in the literature (Fama and French, 1992, 1993; Carhart, 1997). The Brazilian portfolios also generally exhibit the same behavior as those of the US market. However, three points require attention: i) the Book-to-market effect is not observable among the Big High and Big Low

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[9] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[10] Núcleo de Pesquisas em Economia Financeira da Universidade de São Paulo http://nefin.com.br.

[11]We test both measures for the Brazilian market, and they remain mostly unchanged.

Table 1: US and Brazilian portfolios

Portfolios of the the US and Brazilian asset market. The portfolios differ between countries by their breakpoints and the period contemplated. Their means were calculated on the excess of return and the standard deviation are autocorrelation consistent.

| Variables | Labels | Breakpoints | | Mean (% p.m.) | | Sd (% p.m.) | |
		US Jan/1927 - Dec/2014 (1056 months)	BRA Jan/2001 - Dec/2014 (168 months)	US	BR	US	BR
Size	Small	[0,30]	[0,33.3]	1.00***	0.25	8.44	8.01
	Medium size	[30,70]	[33.3,66.6]	0.88***	0.25	6.78	6.99
	Big	[70,100]	[66.6,100]	0.63***	0.12	5.25	6.20
Book-to-market	Low	[0,10]	[0,33.3]	0.58***	0.10	5.77	6.74
	Medium bm	[30,70]	[33.3,66.6]	0.72***	0.13	5.78	6.99
	High	[90,100]	[66.6,100]	1.09***	0.49	9.22	7.17
Momentum	Loser	[0,30]	[0,33.3]	0.37	-0.42	7.71	8.61
	Normal	[30,70]	[33.3,66.6]	0.63***	0.44	5.62	6.29
	Winner	[70,100]	[66.6,100]	0.98***	0.71	5.61	6.36
Size x Book-to-market	Small Low	[0,50 ; 0,20]	[0,50 ; 0,50]	0.65***	0.24	8.09	7.70
	Small High	[0,50 ; 80,100]	[0,50 ; 50,100]	1.27***	0.47	8.77	8.06
	Big Low	[50,100 ; 0,20]	[50,100 ; 0,50]	0.62***	0.15	5.47	6.21
	Big High	[50,100 ; 80,100]	[50,100 ; 50,100]	0.99***	0.08	8.12	6.77
Size x Momentum	Small Loser	[0,50 ; 0,30]	[0,50 ; 0,50]	0.55*	-0.14	9.21	8.88
	Small Winner	[0,50 ; 70,100]	[0,50 ; 50,100]	1.35***	0.84	7.30	7.08
	Big Loser	[50,100 ; 0,30]	[50,100 ; 0,50]	0.38	-0.08	7.63	7.25
	Big Winner	[50,100 ; 70,100]	[50,100 ; 50,100]	0.94***	0.42	5.56	5.79
Industry	Basic Products	-	Basic Products		0.48		7.83
	Consumer	Consumer	Consumer	0.72***	-0.09	5.35	6.81
	Energy	-	Energy		0.26		7.13
	HiTec	HiTec	-	0.67***	-	5.64	-
	Healthcare	Healthcare	-	0.81***	-	5.63	-
	Manufacturing	Manufacturing	Manufacturing	0.69***	0.88	5.59	8.70
	Other	Other	Other + Finance	0.63***	0.40	6.50	7.39

Significance: * 10%; ** 5%; ***1%.

portfolios, since Big High presents an average return of 0.08, which is smaller than the average return of 0.15 for Big Low, ii) the Size effect is also not observed between the Big Loser and Small Loser portfolios, since the Small Loser has lower average return, and iii) none of the Brazilian portfolios' returns have a significant average.

Besides the mentioned portfolios, we use four risk factors and a risk-free rate for each economy. Table 2 presents the mean and standard deviation of those variables. The names used for each factor are provided in the first column. They are *Mkt* for the Market factor, *SMB* for the Size factor, *HML* for Book-to-market, and *WML* for Momentum. The risk-free rate used for the US is the Treasury bill month rate, and for the Brazilian market, we use the 30-day Deposito Interbancário (DI) swap rate.

Again the variables for the US follow the pattern documented in the literature, that is, all factors have positive and statistically significant return averages. On the other hand, the Brazilian data show significance only for *WML*, despite the fact that all factors also have positive averages.

Table 2: Factors and risk-free rate

Factors and risk-free rates of the US and Brazilian markets. The information refers to the period between January 1927 and December 2014 for the American market and between January 2001 and December 2014 for the Brazilian market. The table present the means for each factor and the respective autocorrelation consistent standard deviation.

Factor	Mean (% p.m.)		Sd (% p.m.)	
	US	BR	US	BR
Mkt	0.6502***	0.2411	5.41	6.18
SMB	0.2357**	0.0147	3.23	4.81
HML	0.3973***	0.4460	3.54	4.52
WML	0.6755***	1.2493***	4.74	5.50
risk-free	0.2840***	1.0655***	0.25	0.35

Significance: * 10%; ** 5%; ***1%.

3 Methodology of risk premium estimation

This section explains how the risk premium is estimated. In order to do so, we first present the multi-factor model in subsection 3.1, and then explain the methodology for the model estimation.

3.1 Multi-factor model

Define K as the number of risk factors, N as the number of assets, and T as the number of observed periods. The multi-factor model assumes that excess asset returns are governed by the following linear relation:

$$E(R_i^e) = \alpha + \beta_i'\lambda \tag{1}$$

where R_i^e is the excess return of an asset $i \in \{1, 2, ..., N\}$, α is the model pricing error, λ is a Kx1 vector with the risk premia for the K factors, and β_i is a Kx1 vector with the risk measures of asset i for each factor.

The model also proposes that the β_i vector respect the following relation in the time series:

$$R_{it}^e = a_i + \beta_i'f_t + \epsilon_{it} \tag{2}$$

where R_{it}^e is the excess return of asset i in period $t \in \{1, 2, ..., T\}$, a_i is the expected pricing error of asset i, f_t is a Kx1 vector with the realizations of the factors in period t, and ϵ_{it} is the random error of asset i in period t.

3.2   Risk premium estimation

The model estimation is conducted using the GMM methodology introduced by (Hansen, 1982), with a similar framework proposed by (Cochrane, 2001). This methodology provides a joint estimation of all parameters of the model and easily handles the problems of serial correlation and conditional heteroscedasticity.

The GMM estimation is based on the hypotheses derived from the model's introduction in Section 3.1. We can build the following matrix using the model's assumptions:

$$g_t(a, \beta, \alpha, \lambda) = \begin{bmatrix} (R_t^e - a - \beta f_t) \\ [(R_t^e - a - \beta f_t) \otimes f_t] \\ [\mathbf{1}, \beta'] \left[(R_t^e - \alpha - \beta \lambda) \right] \end{bmatrix}_{([N+NK+1+K]\text{x}1)} \tag{3}$$

where R_t^e is the Nx1 vector of excess returns in period t, f_t is the Kx1 vector of risk factors in period t, such that $t \in \{1, 2, ..., T\}$, $\otimes$ is the Kronecker operator, a is the Nx1 vector of expected pricing errors for each asset, β is the NxK matrix of risk related to each asset and factors of the model, α is the expected pricing error of the model, λ is the Kx1 vector of risk premia for each risk factor, and $\mathbf{1}$ is a 1xN vector of ones.

From the model's assumptions, we know that the expected value of each line of the matrix equals zero. The GMM method estimates the parameters $(\hat{a}, \hat{\beta}, \hat{\alpha}, \hat{\lambda})$ by solving the following optimization:

$$\left\{ \hat{a}, \hat{\beta}, \hat{\alpha}, \hat{\lambda} \right\} = \operatorname*{argmin}_{\{a, \beta, \alpha, \lambda\}} \sum_{t=1}^{T} [g_t(a, \beta, \alpha, \lambda)]' W^{-1} \sum_{t=1}^{T} [g_t(a, \beta, \alpha, \lambda)] \tag{4}$$

where W is a weighting matrix of moments, which is typically set to generate as effective estimates as possible. However, since the number of parameters and equations are the same, the weighting matrix does not affect the estimation. Thus, we use the identity matrix to solve the problem.

Finally, we have the following matrix of variance and covariance parameters:

$$Var(\hat{a}, \hat{\beta}, \hat{\alpha}, \hat{\lambda}) = (d'Wd)^{-1}(d'WSWd)(d'Wd)^{-1} \tag{5}$$

where d is the derivate of g_t with respect to the parameter vector ($d = E\left[\partial g_t / \partial(a, \beta, \alpha, \lambda)\right]$), W is the identity matrix ($[N + NK + 1 + K]$), and $S = E(g_t g_t')$ is estimated using the Parzen kernel with the same band as the entire value of $0.75T^{1/3}$.

4 Risk premium analysis

4.1 Full samples analysis

The Brazilian risk premia diagnostic starts by taking the US stock market as the benchmark and comparing the results between both markets. The *Mkt*, *SMB*, *HML*, and *WML* risk premia are estimated using data from January 1927 to December 2014 for the US stock market, and from January 2001 to December 2014 for the Brazilian one.

Table 3 presents the results, which are organized in two panels. Panel A presents the estimated parameters for the time series regression, with the risk measures of each portfolio. Panel B arranges the cross-section regression results, with the risk premia estimates followed by their p-values and standard error deviations. In both panels, the values on the left refer to the US, and on the right, to Brazil.

The results for the time series in Panel A show some similarities between both markets. As expected, the US market data confirm the patterns widely documented in the literature. Most values of the intercept a are not significant and, despite some significant cases, they do not demonstrate correlation with the variables Size, Book-to-market, and Momentum. The values of b are mostly around 1.00, with low variation between portfolios, and the parameters s, h, and m behave according to the ordering patterns reported for the portfolios returns. In other words, the lower the value of the assets that integrate the portfolios , the higher the estimated values of s; on the other hand, h and m grow positively correlated with portfolios ordered by the variables Book-to-market and Momentum, respectively.

In the Brazilian case, most of the patterns observed in the US time-series regression repeat themselves with a few caveats. First, the estimates of a show a lower frequency of significant cases, which goes in favor to the model's adjustment to the data . In addition, the parameters' estimates follow the same order highlighted in the results of the US data. s is negatively correlated with Size, and h and m are positively correlated with Book-to-market and Momentum respectively, despite the lower frequency of significant estimates than those observed for the US market.

In contrast, the cross-section results in Panel B indicate some divergence between Brazil's and the US' risk premia estimations. The results for the US demonstrate significant risk premia for all factors except the market factor, which supports the capacity of the model to fit the US data. On the other hand, the Brazilian results reject the model's ability to replicate the returns of assets, since only the factor *WML* demonstrates a positive and significant risk premium.

The analysis of this result by itself could lead to the precipitated conclusion that the multi-factor model does not fit Brazilian data. However, this result should be interpreted with caution. Therefore, in order to identify the source of the problem, we conduct a more careful analysis of two points that distinguish both markets: i) the size of N, that is, the number of assets used to build the portfolios in the estimation process, and ii) the size of

Table 3: **Full sample regression**

The table presents the estimated values for the United States (US) and Brazil (BR) of the parameters of the time series regression (equation 2) in Panel A and cross-section regression (equation 1) in Panel B. The periods used for the estimates cover January 1927 to December 2014 to the United States, and January 2001 to December 2014 for Brazil.

Panel A: Time series regression									
$R_{it}^e = a_i + b_i Mkt_t + s_i SMB_t + h_i HML_t + m_i WML_t$									
US					**BR**				
Portfolio	**a**	**b**	**s**	**h**	**m**	**a**	**b**	**s**	**h**	**m**

Portfolio	a	b	s	h	m	a	b	s	h	m
Small	-0.08**	1.04***	1.19***	0.43***	-0.05**	0.17	0.92***	0.86***	0.01	-0.14***
Medium size	-0.01	1.06***	0.56***	0.19***	0.00	0.13	0.93***	0.25***	0.15**	-0.14***
Big	0.02***	0.99***	-0.13***	-0.01	-0.01**	0.05	0.95***	-0.12***	0.00	-0.12***
Low	0.04	1.07***	-0.07***	-0.34***	-0.02	0.18	0.9***	0.33***	-0.4***	-0.09***
Medium b/m	-0.02	0.99***	-0.05***	0.31***	-0.02	0.11	0.95***	0.28***	0.04	-0.18***
High	-0.14	1.15***	0.51***	1.07***	-0.1***	0.12	0.91***	0.34***	0.59***	-0.09**
Loser	0.10***	1.07***	0.08***	0.04**	-0.67***	0.11	0.95***	0.38***	0.08*	-0.64***
Normal	0.06	0.98***	-0.09***	0.10***	-0.12***	0.37*	0.87***	0.19***	0.08*	-0.14***
Winner	-0.01	1.07***	0.06***	0.05***	0.39***	-0.05	0.97***	0.35***	0.09**	0.39***
Small low	-0.18***	1.12***	1.11***	-0.25***	-0.07***	0.37*	0.90***	0.65***	-0.35***	-0.17***
Small high	0.06*	1.05***	0.98***	0.86***	-0.06***	0.17	0.96***	0.71***	0.39***	-0.10***
Big low	0.09***	1.03***	-0.09***	-0.28***	-0.01	0.08	0.92***	0.00	-0.20***	-0.05
Big high	-0.10	1.14***	0.06**	0.94***	-0.06***	-0.08	0.90***	-0.08	0.42***	-0.20***
Small loser	-0.08	1.07***	1.00***	0.25***	-0.58***	0.20	0.95***	0.62***	0.14**	-0.51***
Small winner	0.10**	1.08***	0.94***	0.23***	0.36***	0.25	0.97***	0.57***	0.13**	0.23***
Big loser	0.14***	1.08***	-0.06***	0.03	-0.68***	0.29	0.89***	-0.15***	0.07	-0.49***
Big winner	-0.03	1.07***	0.00	0.05***	0.38***	-0.11	0.91***	0.13***	0.01	0.25***
Basic Products	-	-	-	-	-	0.22	0.76***	0.51***	-0.08	0.08
Consumer	0.13**	0.92***	0.01	-0.01	-0.01	0.09	0.83***	0.22***	-0.09*	-0.27***
Energy	-	-	-	-	-	-0.09	0.84***	0.17**	0.54***	-0.07
HiTec	0.21***	0.97***	0.04	-0.35***	-0.07***	-	-	-	-	-
Healthcare	0.31***	0.89***	-0.10*	-0.17***	0.02	-	-	-	-	-
Manufacturing	-0.03	0.99***	-0.10***	0.19***	0.04**	0.40	1.12***	0.4***	0.27***	0.06
Other	-0.14**	1.04***	0.07**	0.33***	-0.08***	0.27	1.02***	0.25***	-0.01	-0.1**

Panel B: Cross-section regression									
$E(R_i^e) = \alpha + b_i \lambda_{mkt} + s_i \lambda_{smb} + h_i \lambda_{hml} + m_i \lambda_{wml}$									

factor	α	λ_{mkt}	λ_{smb}	λ_{hml}	λ_{wml}	α	λ_{mkt}	λ_{smb}	λ_{hml}	λ_{wml}
estimate (% p.m.)	1.06***	-0.34	0.20**	0.33***	0.62***	-0.47	0.85	0.20	0.26	1.09**
p-value	0.001	0.334	0.046	0.003	0.000	0.636	0.481	0.611	0.495	0.019
se(λ)	(0.31)	(0.35)	(0.10)	(0.11)	(0.15)	(1.00)	(1.20)	(0.40)	(0.39)	(0.46)

Significance: * 10%; ** 5%; ***1%.

T, that is, the length of the time-series sample of the Brazilian data. The next step is to investigate the impact of these divergences.

4.2 Does the size of N restrict the estimation?

There is a huge difference between the number of assets available in the US and Brazilian markets. Table 4 reports the number of eligible assets[12] for both markets and

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[12]In both markets, some eligibility rules are applied to select the assets that compose the portfolios. The US' and Brazil's rules are listed at French's website and NEFIN's website respectively.

organizes this information in sub-periods[13].

Table 4: Number of eligible assets

The table indicates the maximum and minimum number of eligible assets observed for each sub-period and stock market.

Period	US		BR	
	Minimum	Maximum	Minimum	Maximum
[1927 ; 1962]	478	1,097	-	-
[1963 ; 1972]	1,924	2,365	-	-
[1973 ; 2000]	4,353	7,123	-	-
[2001 ; 2014]	3,546	5,874	37	137

While the number of US assets varies, ranging from 478 to 7,123 in a history of 84 years, we observe a maximum of 137 assets over the 14-year period analyzed for Brazil. This result may be attributed to the portfolio behavior, since the fewer the assets used to build a portfolio, the greater the idiosyncratic risk impact on the portfolio's returns.

The small number of assets could affect the risk premium estimation in two ways: i) by generating distortion in the portfolios' returns, which are used as dependent variables in the regressions, and ii) by impacting the estimations of the risk factors. The first concern is discarded by analyzing the standard deviations of the returns on the 22 portfolios for both markets, reported in Table 1. One can see that the standard deviations are quite similar between the markets, and there is no pattern. This means that the standard deviations of the Brazilian data are not always greater than those of the US data. This indicates that no relevant distortions are generated in the Brazilian portfolios, which are used as dependent variables. However, the second highlighted concern demands more attention, considering that, as reported in Table 2, all the risk factors in the Brazilian case show higher standard deviations than those of the US.

The procedure to calculate the risk factors is based on building portfolios using the correlated assets' characteristics with the returns and then defining the returns on those portfolios as realizations of the risk factors Fama and French (1993); Carhart (1997). For example, the factor realization *SMB* is obtained by calculating the return from a portfolio long in small assets and short in big assets. However, considering the low number of assets available in Brazil, risk factor estimation by this procedure can be affected by the asset's idiosyncratic risk. This could explain the pattern observed with regard to the

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[13]The sub-periods are as follows. The first sub-period is the first period built by Center for Research in Security Prices (CRSP) and spans from 1927 to 1962. The second sub-period begins from 1963 and continues until the foundation of the NASDAQ in 1972. The third sub-period begins in 1972 and continues until the starting point of the data period for Brazil, and it ends immediately after the end of the dot-com bubble in 2000. Finally, the last sub-period ranges from 2001 to 2014 and covers the same period as the Brazilian data.

standard deviation of Brazilian factors in Table 2, or the results observed for the risk premia estimations in Panel B of Table 3, in which most parameters have non-significant estimates.

In order to measure the impact of this feature on risk premia estimation, we decrease the number of assets used in the estimation of the US risk factors to a similar number of assets available for the Brazilian market and verify if the risk premia significance frequency is affected. Therefore, we run the following procedure 1,000 times: i) for each year of the 84 years of historical data, we select a sample of 37 or 137 assets, and use them to estimate the risk factor realization for the months of the respective year, and ii) we process the risk premia estimation and verify if the estimated λ_k, such that $k \in \{mkt, smb, hml, wml\}$, are positive and significant, that is, have a t-value ≥ 1.64.

Two methods of sample selection are applied. The first one, denominated "Random," is a random selection from the eligible assets of each year of the 84 years' samples. The second, denominated "Size x Book-to-market", intends to preserve the distribution on the variables Size and Book-to-market among the selected assets[14].

The results obtained from this procedure are presented in Table 5. The first column reports the selection method applied, the second indicates the number of assets selected each year, and the remaining columns show the percentages from the 1,000 estimations that return positive and significant estimates of the parameters α, λ_{mkt}, λ_{smb}, λ_{hml}, and λ_{wml}. We run both selection methods with 37 and 137 assets, the two extreme cases observed in the Brazilian data. The results using 37 assets are presented in Figure 2, where we plot the observed density of the t-values estimated for each risk factor.

The results show that most of the parameters are barely affected. The most affected parameter is the *SMB* risk premium, which has around 85% to 92% of significant estimates when 37 assets are used to build the risk factors, but this result changes to around 99% when 137 assets are used. The second most affected risk premium is *HML*, and its biggest impact is observed with the random selection with 37 assets; 97.8% of the estimates are significant. *WML* has a very small impact and shows more than 98% significant estimates in all cases. *Mkt* shows non-significant results in all cases. Finally, the mean pricing error (α) has more than 93% of significant and positive estimates.

The results show that most of the risk factors calculated using 37 or more assets reach the same outcome as when the whole set of assets is used. In other words, there is no indication that the number of assets available for the risk premium estimation generates a big impact in the Brazilian case.

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[14]The procedure applied to build the new risk factors' time-series is detailed in Appendix A, which can be provided if requested.

Table 5: Percentage of significant cases by the number of assets

Selection Method	Number of assets	percentage $(t \geq 1.64)$				
		α	λ_{mkt}	λ_{smb}	λ_{hml}	λ_{wml}
Random	37	97.4	0.0	85.0	97.8	99.8
	137	99.9	0.0	99.7	100.0	100.0
Size x Bookt-to-market	37	93.5	0.0	91.6	99.4	98.8
	137	100.0	0.0	100.0	100.0	100.0

Figure 2: Density of $t_{\hat{\lambda}}$ estimated with factors for 37 assets
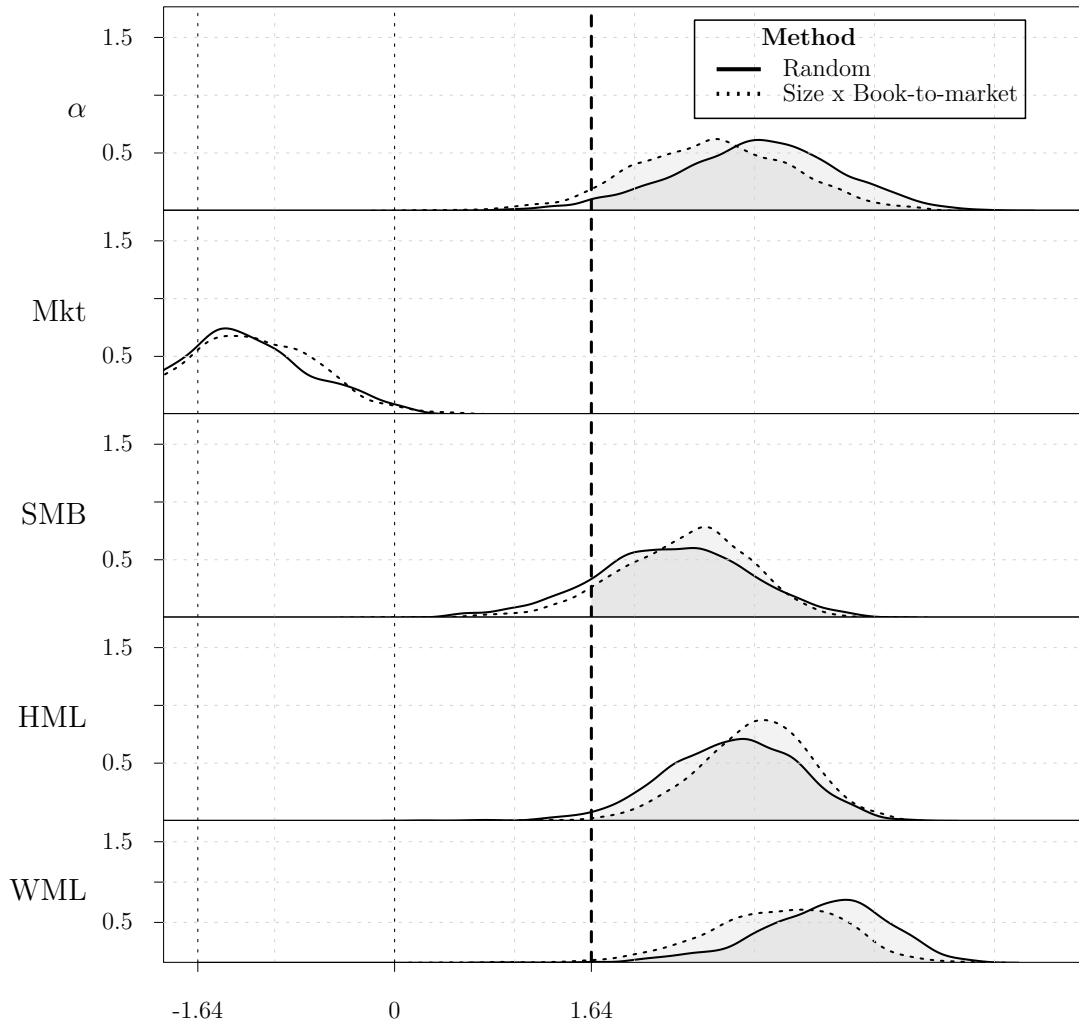
4.3 Does the size of T restrict the estimation?

While the US risk premia results are based on 84 years of historical data, the Brazilian market, for which we conduct the regression, has only 14 years of data. In order to verify

whether this divergence is a constraint for Brazilian risk premia estimations, we use the US data again as a benchmark and verify the impact of time-series length on the risk premia estimations.

The analysis uses the US risk factors and portfolios' returns presented in Section 2, and the procedure is described as follows: i) to define a time window length that has the same number of months as the Brazilian data (168 months), ii) to estimate as many regressions as possible on the US data using only the number of months defined in the previous step, that is, starting with the oldest 168-month window allowed until the most recent one, and always dropping the oldest month in exchange for a more recent one. The results are the risk premia estimates for December 1940 to December 2014, using the data of only the last 168 months available. At the end of this procedure, we have a total of 889 sets of estimates.

Based on the results obtained from the described procedure, we draw Figure 3. Each graphic in the figure is related to one parameter, α, λ_{mkt}, λ_{smb}, λ_{hml}, or λ_{wml}, and each point of each graphic indicates the t-value obtained for a particular parameter from an estimated model with the last 168 months observed at the reference date. Therefore, Figure 3 is nothing but a history of 889 t-values for each risk premium estimated with the US data, using fixed windows of 168 months. In addition, each graph is accompanied by a dotted line at value 1.64, which is the critical value adopted for rejecting the hypothesis that the parameter is less than or equal to zero.

The results indicate the importance of time-series sample size for the factor model estimation and demonstrate that, even with the US data, it is not uncommon to find that risk premia are not significant when short time-series samples are used. It is also notable that *WML* is the most robust factor, just as in the Brazilian case. At the start of the analysis of the market risk premium, we observe that, in most estimates, this parameter is not significant, except for a few points at the beginning of the historical data. *SMB* risk premium shows significance only in a few periods, mostly at the beginning of the historical data, but we can find some significant points in the middle and in very recent periods. The parameters relate to *HML* and *WML*, which, on the other hand, are more robust, since their t-values exceed the critical value on many more occasions. This is especially true for the second factor, which is significant for almost the entire history.

After establishing the importance of time-series sample size to risk premia estimation, we verify how sensitive the estimation is to the time-series sample size. To do so, we apply the following procedure: i) we select several window lengths (48, 72,..., 1056 months), ii) for each option selected in the previous step, we repeat the procedure applied to build Figure 3 and compute the percentage that each risk premium is significant. This analysis is presented in Figure 4. The five graphs in the figure indicate the significance of each parameter according to the window used for the estimations. The dotted line crossing the graphics vertically highlights the percentage obtained with windows of 14 years, the same

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Figure 3: History of t-values for an estimation window of 14 years

Each point in the figure is the t-value of the parameter α, λ_{mkt}, λ_{smb}, λ_{hml}, or λ_{wml} resulting from the estimation of a model with all the factors and 168 months (14 years) of data. The horizontal dotted line crosses the ordinate axis at 1.64, the critical value to reject the hypothesis that the estimated values are equal to or smaller than zero, with a significance level of 5%.
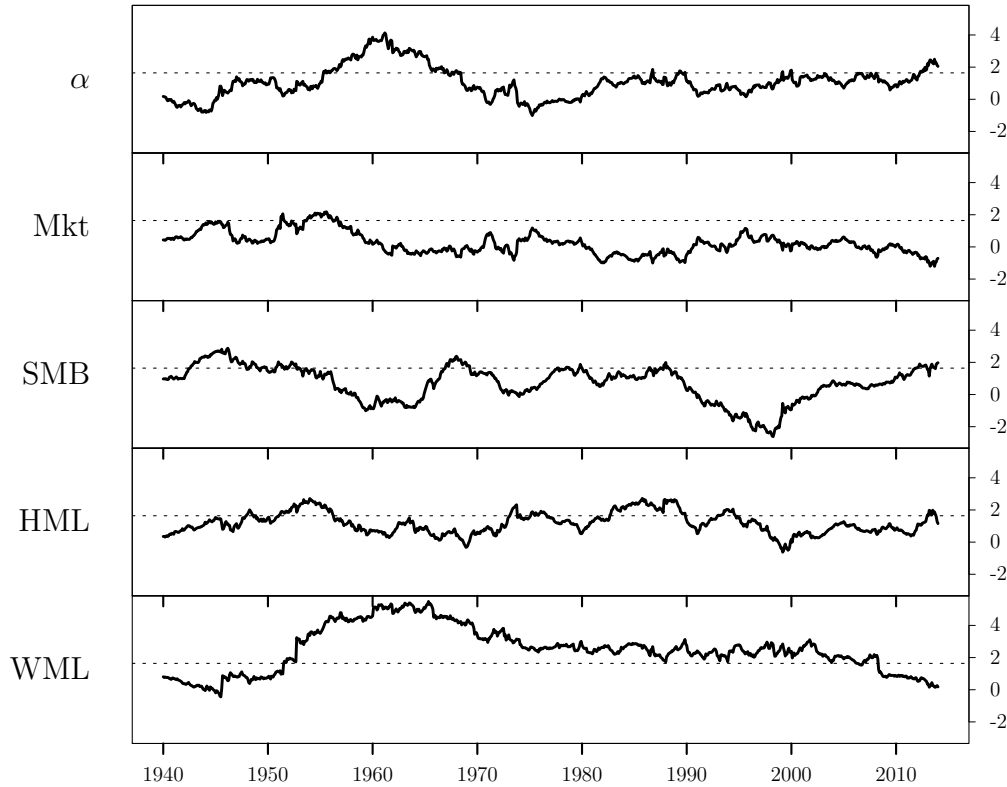

number available for the Brazilian data.

The results indicate that a large time-series sample is required in order to obtain robust estimates. We start with the parameter α, which shows positive and significant results around 20% of the time when 14 years are used for the estimation. However, as the time-series sample becomes larger, it becomes increasingly evident that the data do not support the zero mean error hypothesis. The market risk premium, on the other hand, demonstrates robust results independent of the time-series sample length. Most results for this parameter do not demonstrate positive and significant estimates. The results of the *SMB* risk premium are very sensitive to time-series sample size. For this parameter, less than 40% of the estimates are positive and significant even with 64 years. The other two parameters again demonstrate themselves as more robust. Their estimations seem to be less sensitive to window size. *HML* risk premium has about 20% significant estimations when 14 years are used, and this percentage reaches 80% with 30 years. The *WML* factor seems to be the most robust of all. Its results are positive and significant about 80% of

Figure 4: Percentage of significant cases by the number of years

The graph below shows the percentages for which the t-values of the parameters α, λ_{mkt}, λ_{smb}, λ_{hml}, and λ_{wml} are greater than 1.64, according to the time-series sample size used for the estimations. The dotted line crossing the graphics vertically highlights the percentage obtained with windows of 14 years.
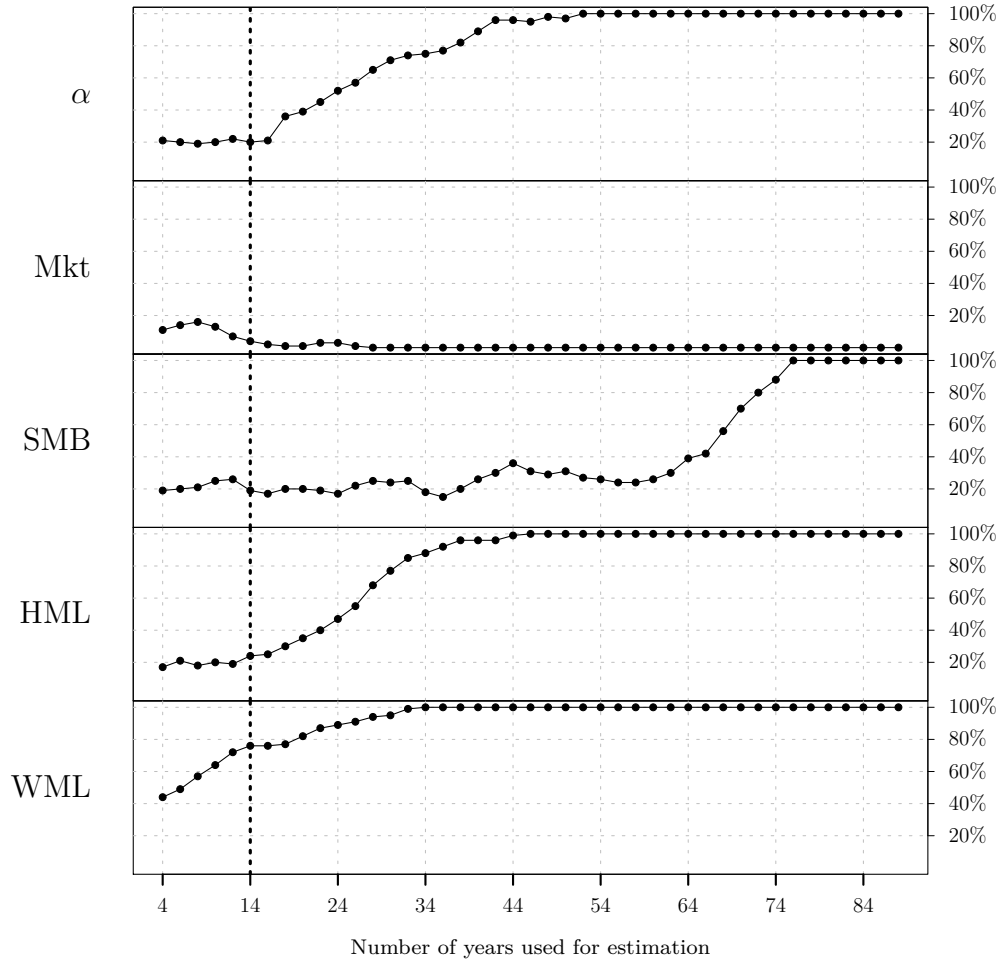
the time when 14 years or more are used for the estimation.

This section demonstrates that the time-series sample size is a relevant restriction on Brazilian risk premia estimation. Based on the US data, we demonstrate how common it is to estimate non-significant risk premia when the number of observed periods is too short. Furthermore, it seems that one should not expect robust results on factor models to which time-series samples shorter than 40 years are applied.

4.4   Why is the impact of T so high?

The analysis of Brazilian risk premia shows that most of the parameters have non-significant estimates. In addition, we demonstrate that the source of the problem is not the small number of assets available or their characteristics, but the short historical data

of the Brazilian market. The next step is to understand why the "size of T" has such a considerable impact and the consequences of applying a factor model to such a short time-series sample.

The literature on risk premiums estimated with a small T began with (Shanken, 1992), followed by (Jegadeesh and Noh, 2013; Kim and Skoulakis, 2014; Raponi et al., 2015), and (Bai and Zhou, 2015). Two points emerge from an analysis of risk premium estimations with a small time-series sample: i) small sample bias on betas, and ii) divergence between *ex-post* and *ex-ante* risk premia.

To understand these consequences, note that as presented in Equation 1, the expected excess return from asset i equals a linear relation between the pricing error (α) and risk compensation ($\beta_i'\lambda$). However, the relation used in the risk premia estimation is

$$\bar{R}_i^e = \alpha^* + \hat{\beta}_i'\lambda^* \tag{6}$$

where $\bar{R}_i^e$ is the average excess return from asset i, $\hat{\beta}_i$ is the estimated risk vector from asset i, and α^* and λ^* are the parameters resulting from this relation. Thus, while Equation 1 has only true parameters, Equation 6 consists only of estimated values.

The first divergence arises from using the estimated beta ($\hat{\beta}_i$) instead of the true beta (β_i). According to (Shanken, 1992), since the independent variable in Equation 6 is measured with error, the estimator is subject to an errors-in-variables problem, making it biased in small samples. However, the measurement error declines as T increases. Hence, (Shanken, 1992) shows how the asymptotic standard errors are influenced by the estimation error in the betas and proposes an adjustment for the standard errors and a bias-adjusted estimator. Simulating studies with the US data show a bias of about -16% and -20% when less than 172 months are used in the risk premium estimation Raponi et al. (2015); Bai and Zhou (2015); Jegadeesh and Noh (2013).

The second divergence is caused by the use of the average excess return instead of its true expected value. Averaging (2) over time, imposing (1), and noting that $E(R_i^e) = a_i + \beta_i'E(f)$ yields

$$\bar{R} = \alpha + \beta\left[\lambda - E(f) + \bar{f}\right]. \tag{7}$$

Equation 7 demonstrates that the relation between the true beta and the average excess return results in the so-called *ex-post* risk premium, $\lambda^P = \lambda - E(f) + \bar{f}$, which is equal to the sum of the *ex-ante* risk premium and the unexpected factor outcomes. Since one cannot hope for $\bar{f}$ to be a good estimation of $E(f)$ unless T is large, as (Shanken, 1992) points out, it is not possible to obtain a consistent estimate of λ when T is fixed.

In order to analyze the distortion that these two divergences may cause in the risk

premia estimation, we perform a Monte Carlo simulation based on the following set-up:

$$f_t \quad = \lambda + \epsilon_t \tag{8}$$

$$R_t \quad = \beta f_t + e_t \tag{9}$$

such that $t \in \{1, 2, ..., T\}$, $\epsilon_t \sim N(0, \sigma^2)$, $e_t \sim N(0, \Sigma)$, and $\epsilon_t \perp e_t$. $\lambda = 0.6502$ and $\sigma = 5.41$, which are the mean and standard deviation respectively with regard to the US market risk factors in Table 2. β is the $1x22$ vector of the market risk measure from the US market, whose values are presented in Panel A (see the first column) of Table 3. Σ is the $22x22$ residual covariance matrix resulting from the US time-series regression in Section 4.1.

Based on this set-up, we select several values of T, and for each of them, we simulate 10,000 draws. We then estimate the risk premium of each draw by two methods: i) the same method presented in Section 3.2 and applied to our paper so far, and ii) a modified estimation in which the risk premium is estimated using the true betas instead of the estimated values. The simulation allows the analysis of several time-series sample sizes, and we can isolate the *ex-post* impact using the true betas in the estimation procedure.

Table 6 presents the results obtained for the risk premium estimation using the estimated beta or the true beta, both for several values of T, varying from 72 to 1056 months. The first column indicates the risk measure used as the independent variable, the true beta ($\hat{\beta}$) or the estimated beta (β). The second column shows the number of months used (T), the third column reports the means of the estimated risk premiums, the fourth column reports the percentage of the 10,000 simulations with a positive estimated value, and the fifth column shows the percentage of positive and significant estimates.

The results confirm that, as expected, the risk premium obtained with the estimated beta is in fact biased, as demonstrated by (Raponi et al., 2015; Bai and Zhou, 2015; Jegadeesh and Noh, 2013). When the time-series sample has only 72 or 168 months of data, estimates are biased by about -20% and -8% respectively , and for the estimations with 1056 months, the bias is around 1%. In contrast, the estimation provided by the true beta shows almost no bias regardless of the time-series sample length.

However, the percentage of positive and significant estimates is almost the same irrespective of the beta used. The percentage of positive and significant estimates is between 9% and 14% for the 72-month time-series sample, about 17% to 20% for the time-series samples of 168 months, and about 64% to 65% for 1056 months.

These results indicate that even though the beta bias is actually a problem in small samples, it does not appear to be a major problem, considering that the magnitude of the bias is small in relation to the *ex-post* distortion on risk premium. The difference between the *ex-post* and the *ex-ante* risk premium lies in the unexpected factor outcomes that have zero mean but high volatility, as the data indicate. Consequently, the *ex-post*

Table 6: Simulation results

The table presents the results obtained for the risk premium estimation using the estimated beta or the true beta for time-series samples varying from 72 to 1056 months. The first column shows which risk measure was used as the dependent variable, the true beta ($\hat{\beta}$) or the estimated beta (β). The second column shows the number of months used. The third column reports the means of the estimated risk premiums. The fourth column indicates the percentage the 10,000 simulations with a positive estimated value, and the fifth column reports the percentage of positive and significant estimates.

Cross-section independent variable	n. of periods (months | years)		$\bar{\hat{\lambda}}$ (% p.m.)	$\hat{\lambda} \geq 0$ (%)	$t_{\hat{\lambda}} \geq 1.64$ (%)
	72	6	0.5224	69.23	9.00
	168	14	0.5953	79.12	17.08
	312	26	0.6192	87.01	27.46
$\hat{\beta}$	456	38	0.6290	90.42	36.15
	600	50	0.6343	93.58	44.14
	744	62	0.6350	95.14	50.89
	888	74	0.6353	96.41	57.37
	1056	88	0.6422	97.71	64.44
	72	6	0.6404	69.64	13.84
	168	14	0.6475	79.27	20.37
	312	26	0.6480	87.10	29.60
β	456	38	0.6472	90.58	37.73
	600	50	0.6498	93.64	45.43
	744	62	0.6459	95.22	52.16
	888	74	0.6453	96.53	58.29
	1056	88	0.6502	97.79	65.25

risk premium, under an estimation scenario of short time-series samples, may have a wide range of potential values and shows large divergence from the *ex-ante* risk premium. Thus, the results obtained from small time-series samples should not be used to draw conclusions about the actual behavior of the stock market.

5   Conclusion

This paper investigates the reasons behind the lack of robustness in the estimations of Brazilian risk premia. We conclude that the source of the problem is not the number of assets or their characteristics. The real problem in the estimations of Brazilian risk premia lies in the fact that the available time-series sample is short.

Accordingly, we investigate what issues the small T causes. We demonstrate that the real problem is due to the high dispersion observed in the risk factors' outcomes, which

induces high divergence between *ex-post* and *ex-ante* risk premia. We also point out that the betas estimated with time-series samples as long as those in the Brazilian case do not generate relevant distortions in the results.

Brazil's data offer a time-series sample of 14 years, while our analysis indicates that is necessary to have a time-series sample greater than 40 years in order to obtain robust results. Therefore, it would not be possible to estimate *ex-ante* risk premia for Brazil before 2041.

The application of the factors model for Brazilian risk premia estimation poses a significant concern. Practitioners often face this problem when estimating cost of capital and usually apply an alternative solution. While they estimate the risk measure (betas) using the Brazilian risk factors, they use a risk premium calculated with longer time-series, such as that of the US Damodaran (1999). The results of this paper are similar to those of this alternative solution, since we demonstrate there is no relevant distortion from the betas. However, the risk premia estimates made using short time-series samples are not valid.

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