

Effort Elicitation, Wage Differentials and Income Distribution in a Wage-led Growth Regime

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JEL Codes: 011, 041, J31.

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1. Introduction

Clearly, a firm's labor productivity is endogenous to the level of effort with which its workers perform their tasks. Even when workers combine their labor with identical units of physical capital, and are homogeneous in all other relevant dimensions, the commitment and effectiveness with which they participate in the production process is rather heterogeneous. Indeed, there is robust empirical and experimental evidence on labor effort being endogenous (especially to wage compensation). Expectedly, wage incentives are found to increase work-related effort, which in turn increases worker productivity.

Meanwhile, following (inter alia) Tarling and Wilkinson (1982), Dickens and Katz (1987), Krueger and Summers (1988) and Katz and Summers (1989), there has always been renewed interest among labor economists in analyzing inter-industry wage differentials. After all, there is overwhelming empirical evidence on the persistence of inter- and intra-industry wage differentials, even after controlling for observable characteristics (schooling or human capital, gender, years of experience, etc). Some robust findings from studies for an extensive number of countries, different time spans, and using different econometric specifications and techniques, are that inter-industry wage differentials do exist, are of a non-negligible size, and are persistent over longer periods of time.¹ Indeed, large and persistent wage differentials have also been found to exist across establishments within industries, even after controlling for standard covariates and individual fixed effects (see, e.g., Groshen, 1991). The reasons explaining the existence and persistence of inter- and intra-industry wage differentials still constitute an unsolved puzzle, and the role played by unobserved workers' characteristics in explaining such wage differentials remains unsettled. However, persistent wage differentials for similar workers and types of jobs are suggestive of the presence of some type of incentivizing wage behavior by many firms. In this context, one of the main contributions of this paper is to propose an evolutionary explanation for persistent wage differentials based on endogenous labor effort (and hence endogenous labor productivity) in macrodynamics driven by aggregate effective demand.

In fact, in the contested-exchange approach elaborated by Bowles and Gintis (1990) it is shown that there needs to be a labor extraction function specifying how much effective labor firms obtains from a given labor input, as the labor contract is not costlessly enforced.

¹ Tarling and Wilkinson (1982), Katz (1986) and Dickens and Katz (1987) review early studies documenting the existence of inter-industry wage differentials, while Katz and Autor (1999), Carruth et al. (2004) and Caju et al. (2005) offer a more updated literature review (along with new evidence) on the same issue.

As the labor contract alone cannot ensure the employer that work will be performed as desired and expected by her, the labor exchange is, as Bowles and Gintis (1990) suitably phrase it, "contested". Meanwhile, the gift-exchange model set forth in Akerlof (1982) portrays the offer of employment by a firm as an offer to "exchange gifts", with the worker's effort level therefore indicating the size of the reciprocal gift. Indeed, there is robust evidence from both laboratory experiments (see, e.g., Fehr et. al., 1998; Fehr and Falk, 1999; Charness, 2004, Charness and Kuhn, 2007; Fehr and Gächter, 2008) and field experiments (see, e.g., Gneezy and List, 2006; Bellemare and Shearer, 2009) that higher wages elicit more effort from workers. There is also survey evidence that firms consider wages as affecting effort (see, e.g., Campbell and Kamlani, 1997), while Cappelli and Chauvin (1991) use plant-level data to find robust evidence that the wage-effort elasticity is positive. Similarly, Goldsmith, Veum and Darity, Jr. (2000) find empirical evidence that being paid a wage premium (a wage that is above the wage paid by other firms for comparable labor) enhances a workers' effort and that workers providing greater effort earn higher wages. Meanwhile, Weisskopf, Bowles and Gordon (1983) connect the value of the wage to the productivity of labor through a partial gift exchange process in an econometric study of the productivity slowdown in the U.S. since the mid-1960s. The authors find that the work intensity and effort, and hence the aggregate labor productivity growth, varies positively with the cost of job loss.

Motivated by all such (experimental and empirical) evidence on wage differentials and endogenous labor effort, this paper sets forth a dynamic model to explore the implications for income distribution, capacity utilization and economic growth of firms following different strategies to elicit labor effort from workers. The frequency distribution of the existing effortelicitation strategies in the population of firms is not parametric, though, being driven by an evolutionary dynamic (the so-called replicator dynamic) that may yield a wage differential as a long-run equilibrium outcome (a result which is in keeping with the empirical evidence reported above on the persistence of wage differentials). Although firms willing to elicit more labor effort have to compensate workers with a higher wage rate, a larger proportion of firms adopting such strategy will not necessarily produce higher rates of capacity utilization and economic growth. The intuition is that, depending on the resulting differential labor effort (and hence the ensuing increase in average labor productivity), the wage share in income (and hence the aggregate effective demand) may not vary positively with the proportion of firms paying higher wages. Consequently, endogenous labor productivity and wage differentials carry relevant theoretical and policy implications for a wage-led growth regime. As it turns out, a broader attribute of this paper is that it blazes a trail for an alternative integration of endogenous labor productivity growth into demand-led models of output growth. In fact, when labor productivity growth has been made endogenous in the literature on demand-led output growth, it is invariably made to depend either on capital accumulation (the Kaldor technical progress function) or output growth (the Kaldor-Verdoorn approach) or the profit rate or market concentration (the latter two due to Schumpeterian effects) or unit labor costs (due to Marx-Hicks effects) or the profit share (see, e.g., Dutt, 1994; Lima, 2000; Palley, 2002; Lima, 2004; von Arnim, 2011; Rezai, 2012). Meanwhile, in the model herein, which features the average intensity of labor effort as endogenously time-varying, the average labor productivity varies with the wage differential and the distribution of the available wage compensation strategies across firms. The latter, in turn, varies endogenously according to an evolutionary dynamic driven by profit differentials. Evidently, our contribution is predicated on the understanding that labor effort elicitation is a strictly finite process and, therefore, it is just one potential source of longer-term variation in the output to labor ratio.

Our contribution is also related to a literature on the existence of wage differences in a multisector economy. In the model set forth in van de Klundert (1989), for instance, there is a primary sector where labor efficiency depends on the intersectoral wage differential, which is endogenous, along with a secondary sector where there is no such dependence. Meanwhile, Julius (2009) explores the wage distributions which emerge from bargaining in a circulating-capital economy. One interesting result is that if many goods are produced by goods and labor, a simple, Nash bargaining mechanism is enough to cause indistinguishable workers to be paid different wages. In the present paper, on the other hand, we model a one-sector, single-good economy which may be dual not in a structural sense, but in a strategic one, as firms may play different wage compensation strategies. Furthermore, the degree of such strategic dualism, as measured by the frequency distribution of wage compensation strategies across firms, varies over time according to an endogenous, evolutionary dynamic driven by profit differentials.

Methodologically speaking, our chosen evolutionary modeling strategy can be seen as quite suitable for the purposes of the paper for several reasons. First, as the paper is intended to explore the integration of microeconomic heterogeneity and macroeconomic outcomes, our evolutionary modeling makes it sufficiently transparent both the structure of the economy and several behavioral equations and forms of interaction among agents. Second, our evolutionary modeling imposes less stringent requisites in terms of rationality and access to information on the part of agents. In fact, agents in the model have bounded rationality and behave according to rules of thumb, and firms revise their choice of wage compensation strategy having limited and localized knowledge concerning the system as a whole. Third, and more importantly, the payoff matrix of strategy-revising firms is endogenously time-varying in a way that can be neatly formalized and analyzed by means of our evolutionary modeling approach. Indeed, the replicator dynamic employed in the paper allows us to conduct a proper formal analysis of the long-run equilibrium configuration of the economy.

The remainder of the paper is organized in the following manner. Section 2 describes the structure of the model, while Section 3 analyses its behavior in the short and long run. The paper closes with a summary of the main conclusions derived along the way.

2. Structure of the model

The economy is a closed one and with no government activities, producing only one good for both investment and consumption purposes. Output production is carried out by imperfectly-competitive firms that combine capital and labor through a fixed-coefficient technology. Firms have some leverage on their price but are small with respect to the overall market. They produce (and hire labor) according to effective demand, which is assumed to be insufficient for any of them to produce at full capacity at prevailing prices. The economy is populated by homogeneous workers whose effort in performing labor tasks is nonetheless endogenous. Firms are also homogeneous except with respect to the strategy for eliciting effort from workers they choose to follow, which determines the wage rate they are willing to pay. At a given short run each firm chooses between paying a lower wage rate $w_l \in \mathbb{R}_{++}$ or paying a higher wage rate $w_h > w_l$. A firm that decides to pay a higher wage rate is termed h firm, while a firm that decides to pay a lower wage rate is referred to as l-firm. An h-firm is willing to pay a higher wage rate because it allows it to elicit more effort from workers. As a result, in a given short run there is a proportion $\lambda \in [0,1] \subset \mathbb{R}$ of h-firms, while the remaining proportion, $1-\lambda$, is composed by *l*-firms. Workers hired by a given type of firm, and hence receiving the same wage rate, behave alike as regard effort provision. However, while labor effort (and hence productivity) is homogeneous across firms of a certain type, labor effort (and productivity) is heterogeneous across the two types of firms. Moreover, the resulting labor productivity differential is endogenous, continuously varying over time along with the frequency distribution of effort-elicitation strategies played by firms.

Having chosen a given wage compensation strategy, a firm makes a take-it-or-leave-it offer to available workers to hire as many workers it needs to produce its demand-determined level of output. These workers, who are always in excess supply, not only take the received offer, but also deliver the labor effort ensuring that their actual productivity is in line with the expected one by firms when they decide what wage compensation to offer.² Consequently, workers actually have a higher productivity if the hiring firm pay they them a higher wage.

To keep focus on the dynamics of the distribution of employee wage compensation strategies and their implications for wage differences, income distribution, capacity utilization and economic growth, we simplify matters by assuming that the wage rates w_h and w_l remain constant over time. The distribution of effort-elicitation strategies across firms, $(\lambda, 1-\lambda)$, which is given in both the ultra-short run and the short run as a result from previous dynamics of the economy, changes beyond the short run according to an evolutionary dynamic (the so-called replicator dynamic). In the ultra-short run, for given values of wage rate differential, effort differential (as proxied by the labor productivity differential) and frequency distribution of effort-elicitation strategies, individual markups vary so as to ensure that individual prices are equalized. Therefore, over time changes in the frequency distribution of effort-elicitation strategies in the average markup and average labor productivity (and hence in the average wage rate and the wage share in income), generate changes in aggregate effective demand and hence in the short-run equilibrium values of capital capacity utilization and economic growth.

In line with the suggestive theoretical and experimental evidence on the endogenous nature of labor effort evoked in the preceding section, the process of labor effort elicitation on the part of firms is conceptualized as a "contested" game, with effort depending both on wage levels and differences. In such "contested" exchange of labor power, the average wage rate can be perceived by workers as either a conventional measure of their outside opportunities or a conventional reference point to which a given wage offer is to be compared as it embodies workers' wage expectations.

² According to Sawyer (1989), the supply of labor to the capitalist economies (and within capitalist economies, supply to the capitalist sectors) can, at least over some range, be readily increased when it is necessary. Within a country, the capitalist sectors can pull workers from the non-capitalist sectors when demand for labor is relatively high and push workers back when demand is low. Indeed, other mechanisms include migration of labor from one country to another and changes in the age of entry into and departure from the labor force as demand for labor varies.

Formally, let $\overline{w} \equiv \lambda w_h + (1-\lambda)w_l$ be the average wage rate, so that the differential between the higher wage rate and the average wage rate is therefore given by $w_h - \overline{w} = (1-\lambda)(w_h - w_l) \ge 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$. As intimated above, we assume that the extent to which labor productivity in *h*-firms is greater than labor productivity in *l*-firms varies positively with the relative wage rate differential given by $w_h - \overline{w}$. Formally, we consider the following effort elicitation function:

(1)
$$\alpha \equiv \frac{a_h}{a_l} = f\left((1-\lambda)(w_h - w_l)\right),$$

where $a_i = X_i / L_i$ denotes labor productivity in firms i = h, l, X_i is total output of firms i = h, l, and L_i is total employment of firms i = h, l. In the above expression, $f'(\cdot) > 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$. Hence, the greater the proportion of h-firms, the smaller the labor productivity differential between the two types of firms: in fact, given that $w_h > w_l$ and $f'(\cdot) > 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$, it then follows that $d\alpha/d\lambda = -(w_h - w_l)f'((1-\lambda)(w_h - w_l)) < 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$. Moreover, if all firms follow the higher wage compensation strategy $(\lambda = 1)$, the relative wage rate differential given by $w_h - \overline{w} = (1-\lambda)(w_h - w_l)$ vanishes. In this case, given that the labor productivity is uniform across all firms, and should actually be higher than the labor productivity when all firms pay the lower wage, we assume that f(0) > 1. Meanwhile, if all firms are of the l-type $(\lambda = 0)$, the potential relative wage rate differential given by $w_h - \overline{w} = (1-\lambda)(w_h - w_l)$ for all given by $w_h - \overline{w} = (1-\lambda)(w_h - w_l)$ for all firms are of the l-type $(\lambda = 0)$, the potential relative wage rate differential given by $w_h - \overline{w} = (1-\lambda)(w_h - w_l)$ takes its maximum value. In this case, an l-firm which decides to switch effort-elicitation strategy to become an h-firm is therefore capable of eliciting the largest possible additional labor effort, since $f(w_h - w_l) > f((1-\lambda)(w_h - w_l))$ for all $\lambda \in (0,1] \subset \mathbb{R}$. In order to better fix ideas, all these properties of the extra effort elicitation function in (1) are represented in Figure 1.

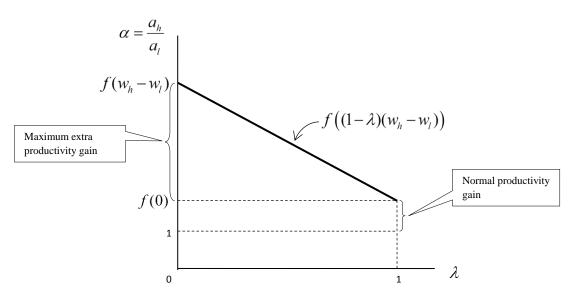


Figure 1. Extra effort elicitation function

The following intuitive rationales can be offered for the specification of the extra labor productivity gain given by (1). First, the average wage rate, \overline{w} , can be seen by a worker as a conventional measure of her perceived outside opportunities. Therefore, a worker who is offered a higher wage rate performs her labor tasks with an additional effort (relatively to the level of effort she would provide if offered a lower wage rate) that varies positively with the excess of the higher wage rate over her perceived outside opportunities. Second, the average wage rate can be seen by a worker as the conventional reference point against which a higher wage rate offer should be compared when choosing how much above-normal effort to provide in return. Therefore, an above-average wage rate is interpreted by a worker as justifying the provision of an above-normal effort level. Alternatively, such conventional reference point can be taken as embodying the worker's wage expectation under uncertainty, so that a higher wage offer is greeted as a pleasant surprise justifying the provision of an above-normal effort. Indeed, expectation-based reference-dependent models of workers' performance predict that labor effort provision varies with the worker's wage expectation, and Abeler et al. (2011) offer robust experimental evidence that wage expectations can indeed act as a reference point and thereby affect effort provision.

As regards pricing behavior, we assume that the population of established firms faces a binding limit-price constraint, \overline{P} , arising from the purpose of forestalling entry by potential competitors.³ This exogenously fixed common price, which is herein normalized to one, is set as a markup over unit labor costs:

(2)
$$1 = (1 + z_h) \frac{w_h}{a_h} = (1 + z_l) \frac{w_l}{a_l},$$

where $z_l \in \mathbb{R}_{++}$ and $z_h \in \mathbb{R}_{++}$ are, respectively, the markups applied by *l*-firms and *h*-firms. For further simplicity, we also normalize labor productivity in firms following the lower wage strategy, a_l , to one, which further requires that $w_l < a_l = 1$ and implies that $\alpha = a_h$ in (1). Moreover, given the properties of (1), we have to assume that $w_h < f(0)$, so that $w_h < \alpha = a_h$ for all $\lambda \in [0,1] \subset \mathbb{R}$. These assumptions about the components in (2) ensure that the income shares going to capital and labor remain each of them in the open interval $(0,1) \subset \mathbb{R}$.

In this context, firms following the higher wage strategy can be intuitively described as firms willing to bet on the possibility of obtaining a productivity differential, α , which is high enough to permit them to set a markup, z_h , which is higher than the markup set by *l*firms, z_l , while charging the same price (and therefore without harming their ability to sell as much output as *l*-firms).⁴ As explored in the next section, however, the resulting labor productivity differential, which is given by the effort elicitation function (1), may fall short of the level required for the higher wage bet to prove successful. Moreover, given that such productivity differential varies over time with the frequency distribution of effort-elicitation strategies, the average markup given by $z \equiv \lambda z_h + (1-\lambda)z_l$ varies over time as well. The ultrashort-run equilibrium values of the individual markups can be obtained by combining (1) and (2):

(3)
$$z_h^* = \frac{f((1-\lambda)(w_h - w_l))}{w_h} - 1$$

and:

³ The prolific approach to entry and limiting pricing was pioneered by Bain (1948) and Harrod (1952), although the possibility of such behavior was raised much earlier by Kaldor (1935).

⁴ Another alternative would be for *h*-firms to use their productivity differential to charge a lower price than *l*-firms while applying the same markup. We abstract from this possibility by assuming that firms face a kinked demand curve (Hall and Hitch (1939), Sweezy (1939)), the market price for which is stable. The uniform price is sustained over time by each firm's fear that, if it undercuts, all the other firms will do the same. Therefore, the wage differential $(w_h - w_l)$ in (1) and throughout the paper can be interpreted either in nominal or in real terms.

(4)
$$z_l^* = \frac{1}{w_l} - 1.$$

Indeed, note that $z_h^* > z_l^*$ if $f((1-\lambda)(w_h - w_l)) > w_h / w_l$, a condition that may not be satisfied even if λ is sufficiently low (recall from (1) that the greater the proportion of *h*-firms, the smaller the labor productivity differential between the two types of firms).

The total real profits of h - and l -firms are given, respectively, by:

(5)
$$R_h \equiv X_h - w_h L_h = \left(1 - \frac{w_h}{a_h}\right) X_h$$

and:

(6)
$$R_l \equiv X_l - w_l L_l = (1 - w_l) X_l$$

Using (1), (5), and (6), the shares of real profit in the total real output generated by h - and l - firms in the ultra-short-run equilibrium are given, respectively, by:

(7)
$$\pi_h^*(\lambda) \equiv \frac{R_h}{X_h} = 1 - \frac{w_h}{f\left((1-\lambda)(w_h - w_l)\right)}$$

and:

(8)
$$\pi_l^* \equiv \frac{R_l}{X_l} = 1 - w_l$$

Therefore, although the share of profits in the output generated by l-firms is constant, the ultra-short-run equilibrium value of the share of profits in the output generated by h-firms varies with the proportion of these firms:

(9)
$$\frac{\partial \pi_h^*(\lambda)}{\partial \lambda} = -\frac{w_h(w_h - w_l)f'(\cdot)}{\left[f(\cdot)\right]^2} = -\frac{w_h}{f(\cdot)}\frac{\eta(\lambda)}{1 - \lambda} < 0,$$

for all $\lambda \in [0,1] \subset \mathbb{R}$, where $\eta(\lambda) \equiv f'(\cdot) \frac{(1-\lambda)(w_h - w_l)}{f(\cdot)} = f'(\cdot) \frac{(w_h - \overline{w})}{f(\cdot)}$ is the elasticity of the labor productivity differential with respect to the relative wage rate differential given by $w_h - \overline{w}$, with $\eta(\lambda) > 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$. Note that $\lim_{\lambda \to 1^-} \frac{\eta(\lambda)}{1-\lambda} = f'(0) \frac{(w_h - w_l)}{f(0)} > 0$, so that

(9) is well defined at $\lambda = 1$. The intuition behind (9) is that the greater the proportion of *h* - firms, the smaller the differential between the higher wage rate and the average wage rate and

hence the smaller the elicitation of additional labor effort. Moreover, we assume herein that the second derivative of (7) is strictly negative for all $\lambda \in [0,1] \subset \mathbb{R}$, so that *h*-firm's profit share function (7) is strictly concave.⁵

As shown formally later, firms accumulate capital at the same rate, which implies that the aggregate capital stock, K, remains uniformly distributed across firms. It then follows that:⁶

(10)
$$\frac{K_h}{\lambda} = \frac{K_l}{1-\lambda} = K,$$

where K_i is the total capital stock of firms i = h, l. Given (10), it follows that the proportion of the aggregate capital stock that is available to the firms of each type is proportional to the share of each type in the population of firms, that is, $K_h / K = \lambda$ and $K_l / K = 1 - \lambda$.

Meanwhile, given that prices are equalized across firms, aggregate effective demand is uniformly distributed not only across firms, but across wage compensation strategies as well. Therefore, individual nominal demand (or individual nominal revenue), which is the same for all firms playing a given wage compensation strategy, is equalized across wage compensation strategies as well. As a result, (capital) capacity utilization is likewise equalized across wage compensation strategies:

(11)
$$u_h = u_l = u = \frac{X}{K},$$

where $u_i \equiv X_i / K_i$ is (capital) capacity utilization of firms i = h, l, while *u* denotes average capacity utilization and *X* average output.

Using (5), (6), (7), (8), (10) and (11), the profit rates of h - and l-firms in the ultra-short-run equilibrium can then be expressed as follows:

⁵ This assumption requires that $f''((1-\lambda)(w_h - w_l)) > -\eta(\lambda)/(1-\lambda)$ for all $\lambda \in [0,1] \subset \mathbb{R}$. Hence, the function $f(\cdot)$ can be either convex or even strictly concave, since the last inequality holds for all $\lambda \in [0,1] \subset \mathbb{R}$.

⁶ The meaning of the implied assumption (10) can be explained as follows. Let *F* be the total measure of firms in the economy and F_h the measure of *h*-firms. As the aggregate capital stock is uniformly distributed across firms, it follows that $\frac{K_h}{F_h} = \frac{K_l}{F - F_h} = \frac{K}{F}$. By definition, $\lambda = \frac{F_h}{F}$, so we obtain (10) by multiplying both sides of these equalities by *F*.

(12)
$$r_h \equiv \frac{R_h}{K_h} = \left(1 - \frac{w_h}{a_h}\right) \frac{X_h}{K_h} = \pi_h^*(\lambda) u ,$$

and:

(13)
$$r_l \equiv \frac{R_l}{K_l} = (1 - w_l) \frac{X_l}{K_l} = \pi_l^* u \,.$$

While the frequency distribution of wage compensation strategies is given in both the ultra-short run and the short run, it varies beyond the short run according to an evolutionary dynamic based on strategy payoffs. More precisely, an individual firm revises periodically its strategy for eliciting labor effort in a manner described by the following replicator dynamic:⁷

(14)
$$\dot{\lambda} = \lambda(r_h - r) = \lambda(1 - \lambda)(r_h - r_l) = \lambda(1 - \lambda)[\pi_h^*(\lambda) - \pi_l^*]u.$$

where $r \equiv \lambda r_h + (1 - \lambda)r_l$ is the average profit rate, and the latter equality is obtained using (12) and (13). Under the replicator dynamic in (14), therefore, the frequency distribution of the higher-wage effort-elicitation strategy in the population of firms increases (decreases) exactly when it has above-average (below-average) payoff.

Using (7) and (8), the replicator dynamic (14) then becomes:

(15)
$$\dot{\lambda} = \lambda (1 - \lambda) \left[w_l - \frac{w_h}{f\left((1 - \lambda)(w_h - w_l)\right)} \right] u$$

3. The behavior of the model in the short and long run

The short run is defined as the time frame along which the population of firms, F, the capital stock, K, the labor supply, N, the wages rates, w_h and w_l (and the corresponding productivity levels, $a_h = \alpha$ and $a_l = 1$), and the (limit) price level, $\overline{P} = 1$, can all be taken as given. The short run is also defined as the time frame along which the proportion of firms playing the higher-wage effort-elicitation strategy, λ , is predetermined and capital capacity utilization and economic growth adjust to bring about equilibrium in the goods market. The ultra-short-run equilibrium values of the individual markups, z_h and z_l , are given by (3) and (4), which we assume to be achieved fast enough for them (and therefore income distribution) to be taken as predetermined in the short-run income-generating process driven by aggregate

⁷ The replicator dynamic can be derived from a model of (social or individual) learning as in Weibull (1995, sec. 4.4).

effective demand. The existence of excess aggregate (and individual) capital capacity implies that aggregate (and individual) output adjusts in the short run to remove any excess aggregate (and individual) demand or supply in the economy (and for any individual firm). In the short-run equilibrium, therefore, aggregate savings, S, are equal to aggregate investment, I.

As regards saving and consumption behavior, we assume that workers spend all of their wage income on consumption, while firm-owner capitalists save a constant proportion $s \in (0,1) \subset \mathbb{R}$ of the corresponding real profits. Therefore, using (10)-(13), aggregate savings normalized by the aggregate stock of capital can be expressed as follows:

(16)
$$\frac{S}{K} = s \left(\frac{R_h}{K} + \frac{R_l}{K} \right) = s \left[\lambda \pi_h^*(\lambda) u_h + (1 - \lambda) \pi_l^* u_l \right] = s \left[\lambda \pi_h^*(\lambda) + (1 - \lambda) \pi_l^* \right] u.$$

Let us now turn to the derivation of the aggregate investment function. For simplicity and to keep focus on the issue of wage differentials, we have assumed above that workers consume all of their income regardless of what type of firm for which they work, while firmowner capitalists have a homogeneous saving behavior no matter what wage compensation strategy they adopt. In the same spirit of simplicity and focus, we assume that firms behave alike as far as desired investment, I_i^d , is concerned:

(17)
$$\frac{I^d}{K} = \frac{I_h^d}{K} = \frac{I_l^d}{K} = \beta + \gamma r^e + \delta u^e,$$

where r^e and u^e denote, respectively, the (common) expected rates of profit and capacity utilization by *h*-firms and *l*-firms, while $\beta \in \mathbb{R}_{++}$, $\gamma \in \mathbb{R}_{++}$ and $\delta \in \mathbb{R}_{++}$ are parametric constants. We follow Kalecki (1935) and Robinson (1962) in assuming that the (average) rate of capital accumulation depends on the (average) expected profit rate, which we then proxy (as Kalecki and Robinson themselves often do) by the (average) current rate of profit. The rationale is that the current profit rate is an index of expected future earnings. Meanwhile, we follow Rowthorn (1981) and Dutt (1984), who in turn follow Steindl (1952), in making the (average) desired rate of capital accumulation to depend positively on the (average) rate of capacity utilization due to accelerator-type effects (recall that $u_h = u_l = u$). Therefore, as we assume that the expected rates of capacity utilization and profit are proxied by their current average values, it follows from (10)-(13) that:

(18)
$$u^e = \lambda u_h + (1 - \lambda)u_l = u,$$

and:

(19)
$$r^{e} = \frac{R_{h} + R_{l}}{K} = \lambda r_{h} + (1 - \lambda) r_{l} = [\lambda \pi_{h}^{*}(\lambda) + (1 - \lambda) \pi_{l}^{*}]u.$$

Substituting (18)-(19) in (17) we get the aggregate desired investment function:

(20)
$$\frac{I^d}{K} = \beta + \{\gamma [\lambda \pi_h^*(\lambda) + (1-\lambda)\pi_l^*] + \delta\} u.$$

Finally, by substituting (16) and (20) in the goods market equilibrium condition given by $S / K = I^d / K$, we can solve for the short-run equilibrium capacity utilization to obtain:

(21)
$$u^{*}(\lambda) = \frac{\beta}{(s-\gamma) \left[\lambda \pi_{h}^{*}(\lambda) + (1-\lambda)\pi_{l}^{*}\right] - \delta}.$$

Using (7) and (8), the preceding expression can be re-written as follows:⁸

(21-a)
$$u^*(\lambda) = \frac{\beta}{(s-\gamma) \left\{ \lambda \left[1 - \frac{w_h}{f\left((1-\lambda)(w_h - w_l)\right)} \right] + (1-\lambda)(1-w_l) \right\} - \delta}.$$

Moreover, we can substitute (21) in (16) to obtain the short-run equilibrium growth rate:

(22)
$$g^*(\lambda) = s \Big[\lambda \pi_h^*(\lambda) + (1-\lambda) \pi_l^* \Big] u^*(\lambda).$$

In the long run, we assume that the ultra- and short-run equilibrium values of all the corresponding variables are always attained. While the growth rate of the aggregate capital stock is therefore given by (22), the number of potential workers, N (which are always in excess supply), grows at the exogenously given rate n. Meanwhile, the frequency distribution of effort-elicitation strategies in the (constant) population of firms (and hence the average levels of labor productivity and markup) varies over time according to the replicator dynamic (15). Substituting (21) in (15) we obtain:

(23)
$$\dot{\lambda} = \lambda(1-\lambda) \left[w_l - \frac{w_h}{f\left((1-\lambda)(w_h - w_l)\right)} \right] u^*(\lambda) \, .$$

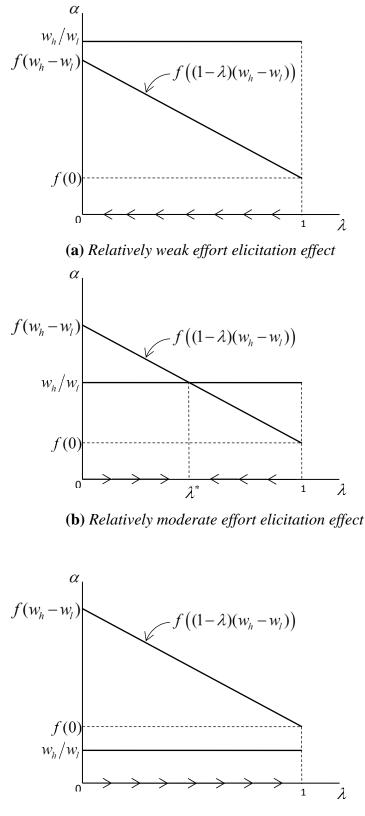
⁸ We are assuming that $(s-\gamma) \left[\lambda \pi_h^*(\lambda) + (1-\lambda)\pi_l^* \right] - \delta > 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$, which is the standard Keynesian stability condition in effective demand-driven models ours. This means that $u^*(\lambda)$ is positive and stable if average saving (as a proportion of the capital stock) is more responsive than average desired capital accumulation to changes in average capacity utilization, which in turn requires that the denominator of the expression in (21-a) is positive.

Solving for the long-run equilibrium solution by imposing $\dot{\lambda} = 0$, we then find that there are two pure-strategy long-run equilibria, $\lambda^* = 0$ and $\lambda^* = 1$. Meanwhile, the stability properties of such equilibria, as well as whether there is also a mixed-strategy long-run equilibrium (in which both wage compensation strategies are followed across firms), turn out to depend on the properties of the effort elicitation function (1). More precisely, a mixed-strategy long-run equilibrium, $\lambda^* \in (0,1) \subset \mathbb{R}$, is implicitly defined as follows:

(24)
$$\pi_h^*(\lambda) - \pi_l^* = 0 \Leftrightarrow w_l = \frac{w_h}{f\left((1-\lambda)(w_h - w_l)\right)} \Leftrightarrow \phi(\lambda) \equiv f\left((1-\lambda)(w_h - w_l)\right) - \frac{w_h}{w_l} = 0.$$

We can show that there is one, and only one, $\lambda^* \in (0,1) \subset \mathbb{R}$ such that (24) is satisfied, that is, such that $\phi(\lambda^*) = 0$, if $\phi(0) = f(w_h - w_l) - (w_h / w_l) > 0$ and $\phi(1) = f(0) - (w_h / w_l) < 0$. Since $\phi(0) > 0$, $\phi(1) < 0$ and ϕ is continuous in the closed interval $[0,1] \subset \mathbb{R}$, we can then apply the intermediate value theorem to conclude that there is some $\lambda^* \in (0,1) \subset \mathbb{R}$ such that $\phi(\lambda^*) = 0$. Moreover, given that $f'(\cdot) > 0$ and $w_h - w_l > 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$, we have $\phi'(\lambda) \equiv -f'(\cdot)(w_h - w_l) < 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$. As a result, since $\phi'(\lambda)$ is continuous in the closed interval $[0,1] \subset \mathbb{R}$, there is only one $\lambda^* \in (0,1) \subset \mathbb{R}$ such that $\phi(\lambda^*) = 0$.

Referring back to Figure 1, the conditions for the existence of a mixed-strategy longrun equilibrium derived above (that is, $\phi(0) \equiv f(w_h - w_l) - (w_h / w_l) > 0$ and $\phi(1) \equiv f(0) - (w_h / w_l) < 0$) imply that the relative strength of the effort elicitation effect for each level of λ , which is given by α , is such that an additional horizontal line defined by w_h / w_l would cross the existing downward sloping line at some $\lambda \in (0,1) \subset \mathbb{R}$. However, this is just one possible configuration for the strength of the effort elicitation effect relatively to the wage differential defined by w_h / w_l . Indeed, there are three cases to be considered, as represented in Figure 2, and we explore the implications of each of them in what follows.



(c) Relatively strong effort elicitation effect

Figure 2. Relative strength of the effort elicitation (labor productivity) effect

Case (a): $f(0) < f(w_h - w_l) \le w_h / w_l$ (relatively weak effort elicitation effect)

This case is represented in panel (a) in Figure 2. It typifies a configuration in which the effort elicitation effect is relatively weak, as the labor productivity differential given by α is lower than the wage differential given by w_h / w_l for all levels of the frequency distribution of wage compensation strategies. In this case, (23) shows that there are only two pure-strategy long-run equilibria, $\lambda^* = 0$ and $\lambda^* = 1$. Given that $f(w_h - w_l) \le w_h / w_l$ and $f(w_h - w_l) \ge f((1 - \lambda)(w_h - w_l))$ for all $\lambda \in (0,1] \subset \mathbb{R}$, it follows that:

(25)
$$\frac{w_h}{f\left((1-\lambda)(w_h-w_l)\right)} > w_l$$

for all $\lambda \in (0,1] \subset \mathbb{R}$. Therefore, it follows that:

(26)
$$\pi_l^* = 1 - w_l > 1 - \frac{w_h}{f((1 - \lambda)(w_h - w_l))} = \pi_h^*(\lambda)$$

for all $\lambda \in (0,1] \subset \Re$. Meanwhile, (14) implies that $\dot{\lambda} < 0$ for all $\lambda \in (0,1) \subset \mathbb{R}$, so that $\lambda^* = 0$ is asymptotically stable, while $\lambda^* = 1$ is unstable.

Case (b): $f(0) < w_h / w_l < f(w_h - w_l)$ (relatively moderate effort elicitation effect)

This case is represented in panel (b) in Figure 2. It typifies a configuration in which the effort elicitation effect is relatively moderate, as the labor productivity differential given by α is higher (lower) than the wage differential given by w_h / w_l for lower (higher) levels of the frequency distribution of wage compensation strategies (recall from (1) that the greater the proportion of *h*-firms, the smaller the labor productivity differential between the two types of firms). In this case, (23) and (24) show that there are two pure-strategy long-run equilibria, $\lambda^* = 0$ and $\lambda^* = 1$, and one mixed-strategy long-run equilibrium, $\lambda^* \in (0,1) \subset \mathbb{R}$.

As regards stability, we can show that the equilibria given by $\lambda^* = 0$ and $\lambda^* = 1$ are both unstable, while the equilibrium given by $\lambda^* \in (0,1) \subset \mathbb{R}$ is asymptotically stable. Indeed, given that in (23) we obtain $\lambda(1-\lambda)u^*(\lambda) > 0$ for all $\lambda \in (0,1) \subset \mathbb{R}$, the behavior of the state variable λ depends on $\phi(\lambda)$, as defined in (24). Given that $\phi'(\lambda) \equiv -f'(\cdot)(w_h - w_l) < 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$ and $\phi(\lambda^*) = 0$, if $\lambda \in (0,\lambda^*) \subset \mathbb{R}$, then $\phi(\lambda) > 0$ and if $\lambda \in (\lambda^*,1) \subset \mathbb{R}$, then $\phi(\lambda) < 0$. Consequently, given (23) and (24), we can then deduce that for any initial condition $\lambda(0) \in (0,1) \subset \mathbb{R}$ the frequency distribution of effort-elicitation strategies converges to the mixed-strategy long-run equilibrium solution given by $\lambda^* \in (0,1) \subset \mathbb{R}$, which is characterized by the existence of a wage differential given by $w_h/w_l > 1$ across employed workers. Interestingly, therefore, such (asymptotically) stable mixed-strategy long-run equilibrium is in line with the empirical evidence reported in section 1, according to which wage differentials do exist and are persistent over longer periods of time.

Moreover, we can use (24) to compute the impact of a change in each one of the wage rates on the mixed-strategy equilibrium value of the distribution of compensation strategies as follows:

(27)
$$\frac{\partial \lambda^*}{\partial w_l} = \frac{1 - \partial \pi_h^* / \partial w_l}{\partial \pi_h^* / \partial \lambda},$$

and:

(28)
$$\frac{\partial \lambda^*}{\partial w_h} = -\frac{\partial \pi_h^* / \partial w_h}{\partial \pi_h^* / \partial \lambda}.$$

Given that $\partial \pi_h^* / \partial \lambda < 0$ (per (9)), the signs in (27) and (28) ultimately depend on the impact of a change in each one of the wage rates on the ultra-short run equilibrium value of the profit share of *h*-firms, which (using (7)) are given by:

(29)
$$\frac{\partial \pi_h^*}{\partial w_l} = -\frac{w_h(1-\lambda)f'(\cdot)}{[f(\cdot)]^2} < 0,$$

and:

(30)
$$\frac{\partial \pi_h^*}{\partial w_h} = \frac{w_h (1 - \lambda) f'(\cdot) - f(\cdot)}{[f(\cdot)]^2} = \frac{w_h}{(w_h - w_l) f(\cdot)} \left[\eta(\lambda) - \left(\frac{w_h - w_l}{w_h}\right) \right].$$

Consequently, it follows from (27) and (29) that $\partial \lambda^* / \partial w_l < 0$. Intuitively, a rise in the lower wage rate reduces the (exogenously given) profit share of l-firms to which the profit share of h-firms must converge back so as to generate a new equilibrium value of the mixed-strategy evolutionary equilibrium given by $\lambda^* \in (0,1) \subset \mathbb{R}$. Moreover, the same rise in the lower wage, by reducing the wage differential given by $w_h - w_l$ at the current level of the distribution of wage compensation strategies, lowers the labor productivity differential given by (1) to a relatively moderate extent, so that the fall in the profit share of h-firms is larger than the fall

in the profit share of l-firms. As a result, the replicator dynamic (27) takes the economy to a new mixed-strategy evolutionary equilibrium characterized by a lower proportion of firms paying the higher wage.

Meanwhile, the impact of a change in the higher wage on the profit share of h-firms is ambiguous, as shown in (30), where $\eta(\lambda)$ is the elasticity of the productivity differential with respect to the relative wage differential given by $w_h - \overline{w}$ featuring in (9). If such elasticity is so relatively low (high) in absolute value that (30) is negative (positive), it follows that a rise in the higher wage leads to a fall (rise) in the profit share of h-firms and hence (per (28)) to a fall (rise) in the mixed-strategy long-run equilibrium value of the proportion of h-firms. Intuitively, if such elasticity is low (high), a rise in the higher wage, by raising the wage differential given by $w_h - w_l$ at the current distribution of wage compensation strategies, leads to a less (more) than proportional increase in the productivity differential and hence lowers (raises) the profit share of h-firms. Given that the profit share of h-firms converges back to the (exogenously given) profit share of l-firms, the replicator dynamic (27) takes the economy to a new mixed-strategy long-run equilibrium characterized by a lower (higher) proportion of h-firms if the elasticity in question is low (high) and hence the profit share of h-firms initially falls (rises) below (above) the (exogenously given) profit share of l-firms.

Case (c): $w_h / w_l \le f(0) < f(w_h - w_l)$ (relatively strong effort elicitation effect)

This case is described in panel (c) in Figure 2. It characterizes a situation in which the effort elicitation effect is relatively strong, as the labor productivity differential given by α is higher than the wage differential given by w_h / w_l for all levels of the frequency distribution of wage compensation strategies. In this case, (23) shows that there are only two pure-strategy long-run equilibria, $\lambda^* = 0$ and $\lambda^* = 1$. Given that $f(0) \ge w_h / w_l$ and $f((1-\lambda)(w_h - w_l)) > f(0)$ for all $\lambda \in [0,1) \subset \mathbb{R}$, it follows that:

(31)
$$w_l > \frac{w_h}{f\left((1-\lambda)(w_h - w_l)\right)}$$

for all $\lambda \in [0,1) \subset \mathbb{R}$. Therefore, it follows that:

(32)
$$\pi_h^*(\lambda) = 1 - \frac{w_h}{f((1-\lambda)(w_h - w_l))} > 1 - w_l = \pi_l^*$$

for all $\lambda \in [0,1] \subset \mathbb{R}$. Meanwhile, (14) implies that $\dot{\lambda} > 0$ for all $\lambda \in (0,1] \subset \mathbb{R}$, so that $\lambda^* = 0$ is unstable, whereas $\lambda^* = 1$ is asymptotically stable. As a result, and intuitively, it is only when the effort elicitation effect is strong enough to dominate the wage differential effect that all firms play the higher wage strategy in the long run.

It is also worth investigating the behavior of income distribution, capacity utilization, and economic growth in each one of these three configurations regarding the relative strength of the effort elicitation effect. Using (7) and (8), the ultra-short run equilibrium average profit share is given by:

(33)
$$\pi^*(\lambda) \equiv \lambda \pi_h^*(\lambda) + (1-\lambda)\pi_l^*.$$

Therefore, the response of the ultra-short run equilibrium average profit share to a change in the frequency distribution of wage compensation strategies is given by:

(34)
$$\frac{\partial \pi^*(\lambda)}{\partial \lambda} = [\pi_h^*(\lambda) - \pi_l^*] + \lambda \frac{\partial \pi_h^*(\lambda)}{\partial \lambda}$$

When the effort elicitation effect is relatively weak, as in case (a) above, (26) shows that $\pi_l^* > \pi_h^*(\lambda)$ for all $\lambda \in [0,1] \subset \mathbb{R}$. Hence, it follows from (9) and (34) that $\partial \pi^*(\lambda) / \partial \lambda < 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$. Moreover, it follows from (21) that $u^*(1) > u^*(0)$, and from (22) that $g^*(1) > g^*(0)$. In fact, noting that in this case the effort elicitation effect is so relatively weak that $w_h / w_l > f(0)$, we have:

(35)
$$u^{*}(1) = \frac{\beta}{(s-\gamma)\left(1-\frac{w_{h}}{f(0)}\right)-\delta} > \frac{\beta}{(s-\gamma)(1-w_{l})-\delta} = u^{*}(0),$$

and:

(36)
$$g^{*}(1) = \frac{s\left(1 - \frac{w_{h}}{f(0)}\right)\beta}{(s - \gamma)\left(1 - \frac{w_{h}}{f(0)}\right) - \delta} > \frac{s(1 - w_{l})\beta}{(s - \gamma)(1 - w_{l}) - \delta} = g^{*}(0).$$

Therefore, as depicted in Figure 3, convergence to the long-run equilibrium given by $\lambda^* = 0$ from any $\lambda \in (0,1) \subset \mathbb{R}$ is accompanied by a (monotonically) rising average profit share and a (monotonically) falling rate of economic growth. The intuition for this result is that capacity utilization and economic growth are both wage led, meaning that they vary negatively with

the (average) profit share in income. In fact, by substituting (33) in (21) and (22) we find that $\partial u^*(\lambda)/\partial \pi^*(\lambda) < 0$ and $\partial g^*(\lambda)/\partial \pi^*(\lambda) < 0.9$ As a result, given any initial heterogeneity in wage compensation across firms, the replicator dynamic (14) does take the economy to a long-run equilibrium configuration in which all firms pay the lower wage (and hence there is no wage differential), while the wage share and the rate of economic growth are both at their lowest possible level.

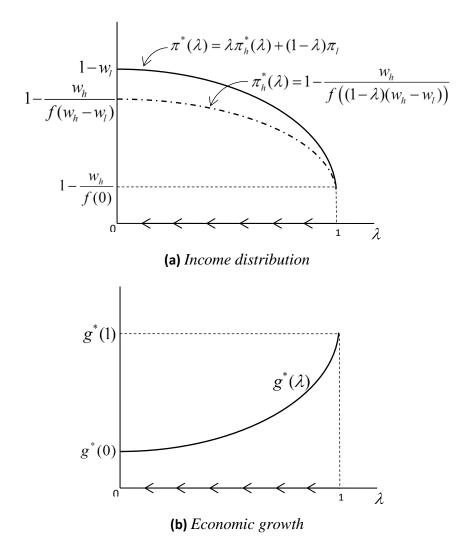


Figure 3. Income distribution and economic growth along the convergence path to long-run equilibrium under a relatively weak effort elicitation effect

⁹ Although the behavior of capacity utilization along the convergence to long-run equilibrium is not depicted either in Figure 3 or in the following figures, it is qualitatively the same as the behavior of the growth rate depicted in such figures. However, it follows from (21)-(22) that $u^*(\lambda) > g^*(\lambda)$ for all $\lambda \in [0,1] \subset \mathbb{R}$.

Recall that when the effort elicitation effect is relatively moderate, as explored in case (b) above, the labor productivity differential is higher (lower) than the wage differential for lower (higher) levels of the frequency distribution of wage compensation strategies. Hence, there are two pure-strategy long-run equilibria, $\lambda^* = 0$ and $\lambda^* = 1$, and one mixed-strategy long-run equilibrium, $\lambda^* \in (0,1) \subset \mathbb{R}$. Concerning the mixed-strategy long-run equilibrium given by λ^* , which is defined implicitly by the condition given by $\pi_h^*(\lambda) - \pi_l^* = 0$, we can use (21) and (22) to compute the corresponding long-run equilibrium capacity utilization and economic growth:

(37)
$$u^{*}(\lambda^{*}) = \frac{\beta}{(s-\gamma)(1-w_{l})-\delta} = u^{*}(0),$$

and:

(38)
$$g^{*}(\lambda^{*}) = \frac{s(1-w_{l})\beta}{(s-\gamma)(1-w_{l})-\delta} = g^{*}(0)$$

Thus, when the effort elicitation effect is relatively moderate, the rates of capacity utilization and economic growth in the mixed-strategy long-run equilibrium given by $\lambda^* \in (0,1) \subset \mathbb{R}$ are the very same as in the pure-strategy long-run equilibrium given by $\lambda^* = 0$. In fact, given that $\lambda^* \in (0,1) \subset \mathbb{R}$ is defined implicitly by the condition given by $\pi_h^*(\lambda) = \pi_l^*$, it then follows that $\lambda^* \in (0,1) \subset \mathbb{R}$ satisfies the condition given by $\pi_h^*(\lambda^*) = \pi_l^*$, from which it follows that $\pi^*(\lambda^*) = \pi^*(0) = \pi_l^*$. The intuition for this commonality of rates of capacity utilization and economic growth is clear: when compared to an equilibrium in which all firms are paying the lower wage, $\lambda^* = 0$, in the mixed-strategy long-run equilibrium given by $\lambda^* \in (0,1) \subset \mathbb{R}$ the productivity differential is as much higher as the wage differential, so that the average profit share is the same. Moreover, since in this case the effort elicitation effect is so relatively moderate that the condition given by $w_h / w_l > f(0)$ is still satisfied, it then follows from (35) and (36) that the common rates of capacity utilization and economic growth in such equilibria $(\lambda^* = 0 \text{ and } \lambda^* \in (0,1) \subset \mathbb{R})$ are lower than in the pure-strategy long-run equilibrium given by $\lambda^* = 1$. Hence, as in this case the two pure-strategy long-run equilibria are unstable, while the mixed-strategy long-run equilibrium is stable, starting from any initial heterogeneity in wage compensation across firms, the evolutionary dynamics take the economy to a long-run equilibrium in which there is a wage differential across employed workers, while both the wage share and the rate of economic growth are not at their highest possible level.¹⁰ As depicted in Figure 4, however, in this case of a relatively moderate effort elicitation effect, unlike in the case of a relatively weak such effect (as represented in Figure 3), whether the dynamics of income distribution and economic growth along the path of convergence to the long-run equilibrium is monotonic depends on the initial disequilibrium position.

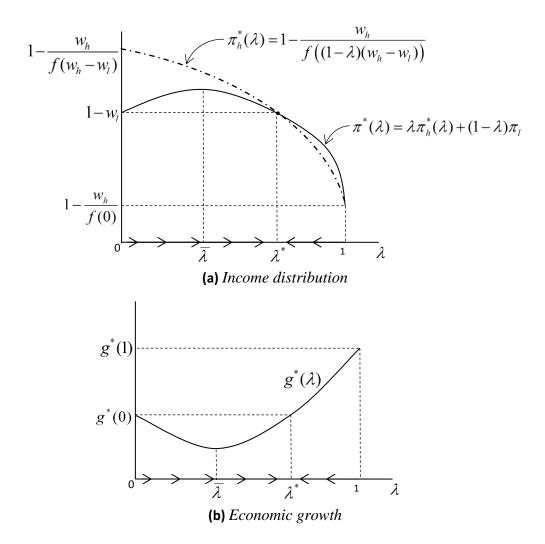


Figure 4. Income distribution and economic growth along the convergence path to long-run equilibrium under a relatively moderate effort elicitation effect

In the situation represented in Figure 4, note that for all $\lambda \in (\lambda^*, 1] \subset \mathbb{R}$ we have $\pi_h^*(\lambda) < \pi^*$. The behavior of the economy, therefore, is qualitatively the same as in the

¹⁰ Moreover, it follows from (35)-(38) that each one of such long-run equilibrium values of capacity utilization and economic growth are positively related to the parameters of the investment function and the respective wage rate (the paradox of costs), and negatively related to the saving rate (the paradox of thrift). The intuition is that capacity utilization and economic growth vary positively with aggregate effective demand.

situation depicted in Figure 3 (or, equivalently, referring back to Figure 2, for all $\lambda \in (\lambda^*, 1] \subset \mathbb{R}$ in panel (b) the behavior of the economy is qualitatively the same as in the situation depicted in panel (a)). Hence, as depicted in Figure 4, convergence to the long-run, evolutionary equilibrium given by $\lambda^* \in (0,1) \subset \mathbb{R}$ from all $\lambda \in (\lambda^*,1) \subset \mathbb{R}$ is accompanied by a (monotonically) rising average profit share and a (monotonically) falling economic growth. Now, since $f(w_h - w_l) > w_h / w_l$ and $f(w_h - w_l) > f((1 - \lambda)(w_h - w_l))$ for all $\lambda \in [0, \lambda^*) \subset \mathbb{R}$, it follows that:

(39)
$$\frac{w_h}{f\left((1-\lambda)(w_h-w_l)\right)} < w_l$$

for all $\lambda \in [0, \lambda^*) \subset \mathbb{R}$. Therefore, it follows that:

(40)
$$\pi_h^*(\lambda) = 1 - \frac{w_h}{f((1-\lambda)(w_h - w_l))} > 1 - w_l = \pi_l^*$$

for all $\lambda \in [0, \lambda^*) \subset \mathbb{R}$, so that it follows from (9) and (34) that the sign of $\partial \pi^*(\lambda) / \partial \lambda$ in such interval requires further investigation. Note that we can use (7)-(9) to conveniently re-write (34) as follows:

(41)
$$\frac{\partial \pi^*(\lambda)}{\partial \lambda} = w_l - \left[1 + \frac{\lambda \eta(\lambda)}{1 - \lambda}\right] \frac{w_h}{f\left((1 - \lambda)(w_h - w_l)\right)}$$

for all $\lambda \in [0,1] \subset \mathbb{R}$. Therefore, it follows that:

(42)
$$\frac{\partial \pi^*(0)}{\partial \lambda} = w_l - \frac{w_h}{f(w_h - w_l)} > 0$$

and

(43)
$$\frac{\partial \pi^*(\lambda^*)}{\partial \lambda} = -\frac{\lambda^* \eta(\lambda^*)}{1 - \lambda^*} \frac{w_h}{f\left((1 - \lambda^*)(w_h - w_l)\right)} < 0,$$

where the positive sign in (42) is implied by (39), while (43) is obtained using the mixedstrategy equilibrium condition given by $\pi_h^*(\lambda) - \pi_l^* = w_l - w_h / f((1 - \lambda^*)(w_h - w_l)) = 0$. Thus, it follows from (42)-(43) and the continuity of (41) that there is a frequency distribution of effort elicitation strategies $\overline{\lambda} \in (0, \lambda^*) \subset \mathbb{R}$ at which the following condition holds:

(44)
$$\frac{\partial \pi^*(\bar{\lambda})}{\partial \lambda} = [\pi_h^*(\bar{\lambda}) - \pi_l^*] + \bar{\lambda} \frac{\partial \pi_h^*(\bar{\lambda})}{\partial \lambda} = 0.$$

Moreover, given (9) and the assumption that $\partial^2 \pi_h^*(\lambda) / \partial \lambda^2 < 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$, it follows from (34) that:

(45)
$$\frac{\partial^2 \pi^*(\lambda)}{\partial \lambda^2} = 2 \frac{\partial \pi_h^*(\lambda)}{\partial \lambda} + \lambda \frac{\partial^2 \pi_h^*(\lambda)}{\partial \lambda^2} < 0$$

for all $\lambda \in [0,1] \subset \mathbb{R}$. Note that it follows from (45) that $\overline{\lambda} \in (0,\lambda^*) \subset \mathbb{R}$ is unique, with the straightforward implication that $\partial \pi^*(\lambda) / \partial \lambda$ is positive for all $\lambda \in (0,\overline{\lambda}) \subset \mathbb{R}$ and negative for all $\lambda \in (\overline{\lambda},1) \subset \mathbb{R}$.

Therefore, as represented in Figure 4, convergence to the mixed-strategy long-run equilibrium given by $\lambda^* \in (0,1) \subset \mathbb{R}$ from all $\lambda \in (0,\overline{\lambda}) \subset \mathbb{R}$ is accompanied by a rising average profit share and a falling economic growth rate. Meanwhile, convergence to such long-run equilibrium from all $\lambda \in [\overline{\lambda}, \lambda^*) \subset \mathbb{R}$ is accompanied by a falling average profit share and a rising rate of economic growth. As it turns out, given any initial heterogeneity in wage compensation across firms, the economy converges to an evolutionary equilibrium characterized by a wage differential across employed workers and the rate of economic growth, though it is not at its lowest possible level (which is $g^*(\overline{\lambda})$), it is not at its highest possible level (which is $g^*(1)$) either. Or, to say it from the viewpoint of income distribution, there is convergence to a long-run equilibrium in which the wage share, though it is not at its highest possible (economically meaningful) level (which is $1-\pi^*(\overline{\lambda})$), it is not at its highest possible (economically meaningful) level (which is $1-\pi^*(1)$) either.

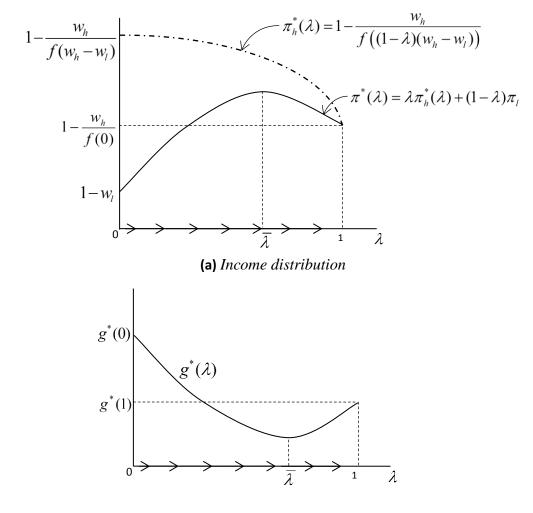
Finally, let us investigate the behavior of income distribution, capacity utilization and economic growth when the strength of the effort elicitation effect is relatively strong, as in case (c) above. Since the labor productivity differential is higher than the wage differential for all levels of the distribution of wage compensation strategies, it follows that $\pi_h^*(\lambda) > \pi_l^*$ for all $\lambda \in [0,1) \subset \mathbb{R}$. Given that in this case we have $f(0) \ge w_h / w_l$, it follows that the inequalities in (35) and (36) are reversed, so that we now have $u^*(0) \ge u^*(1)$ and $g^*(0) \ge g^*(1)$. As in the preceding situation in which the effort elicitation effect is relatively moderate, the sign of $\partial \pi^*(\lambda) / \partial \lambda$ in the closed interval given by $[0,1] \subset \mathbb{R}$ requires further investigation.

Initially, note that (42) also holds when the effort elicitation effect is relatively strong. Moreover, recalling (9), it follows from (41) that:

(46)
$$\frac{\partial \pi^*(1)}{\partial \lambda} = w_l - \left[1 + \frac{f'(0)}{f(0)}(w_h - w_l)\right] \frac{w_h}{f(0)},$$

whose sign is ambiguous, given the further assumptions made after (1) that f'(.) > 0 for all $\lambda \in [0,1] \subset \mathbb{R}$ and f(0) > 1. As a result, there are two subcases to be considered, one in which $\partial \pi^*(1)/\partial \lambda < 0$, and the other in which $\partial \pi^*(1)/\partial \lambda \ge 0$. Intuitively, given the wage rates, the former subcase materializes if the average labor productivity when all firms follow the higher wage strategy, given by f(0), is sufficiently close to the wage ratio, w_h/w_l , or alternatively if the proportionate rate of change the extra productivity gain with respect to the relative wage differential given by $w_h - \overline{w}$ when all firms follow the higher wage strategy, the latter subcase arises if such conditions are reversed. Therefore, given the wage rates, the former subcase materializes when the effort elicitation effect is relatively very strong, while the latter subcase materializes when the effort elicitation effect is relatively very strong.

Given (42) and the continuity of (41), if $\partial \pi^*(1)/\partial \lambda < 0$, it then follows that there is a $\overline{\lambda} \in (0, \lambda^*) \subset \mathbb{R}$ at which the condition (44) holds. Moreover, given (45), it follows that $\overline{\lambda}$ is unique. In this case (c) (in which $f(0) \ge w_h / w_l$), however, there are only two pure-strategy long-run equilibria, viz. $\lambda^* = 0$, which is unstable, and $\lambda^* = 1$, which is asymptotically stable. As depicted in Figure 5, therefore, starting from any $\lambda \in (0,1) \subset \mathbb{R}$, the behavior of the average profit share and the rate of economic growth along the convergence path to the asymptotically stable equilibrium is non-monotonic. In fact, convergence to such evolutionary equilibrium from all $\lambda \in (0, \overline{\lambda}) \subset \mathbb{R}$ is accompanied by a rising average profit share and hence a falling rate of economic growth. Meanwhile, convergence to such evolutionary equilibrium from all $\lambda \in [\overline{\lambda}, 1) \subset \mathbb{R}$ is accompanied by a falling average profit share and therefore a rising rate of economic growth.



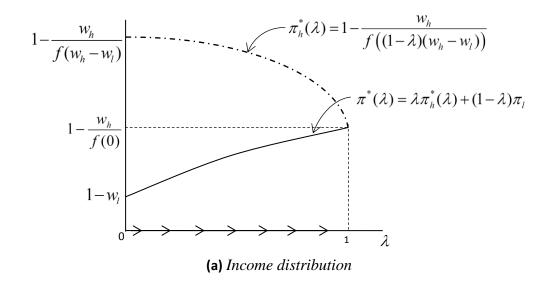
(b) Economic growth

Figure 5. Income distribution and economic growth along the convergence path to long-run equilibrium under a relatively strong effort elicitation effect

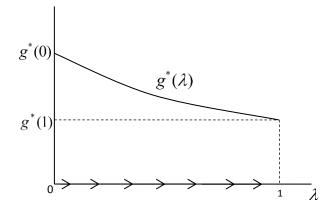
Consequently, given any initial heterogeneity in wage compensation across firms, the economy converges to an evolutionary equilibrium in which all firms play the higher wage strategy. The rate of economic growth, however, although it is not at its lowest possible level (which is $g^*(\bar{\lambda})$), it is not at its highest possible level (which is $g^*(0)$) either. Or, to state it from the viewpoint of income distribution, there is convergence to a long-run equilibrium in which the wage share, although it is not at its lowest possible (economically meaningful) level

(which is $1 - \pi^*(\overline{\lambda})$), it is not at its highest possible (economically meaningful) level (which is $1 - \pi^*(1)$) either.¹¹

Meanwhile, if $\partial \pi^*(1)/\partial \lambda \ge 0$, and using (41) and (42), it follows that the existence of a turning point given by $\overline{\lambda} \in (0, \lambda^*) \subset \mathbb{R}$ is excluded. In this subcase, therefore, the average profit share rises monotonically. As depicted in Figure 6, starting from any $\lambda(0) \in (0,1) \subset \mathbb{R}$, the behavior of the average profit share and economic growth along the convergence path to the asymptotically stable equilibrium is monotonic. In fact, convergence to such evolutionary equilibrium from any $\lambda(0) \in (0,1) \subset \mathbb{R}$ is accompanied by a rising average profit share and hence a falling rate of economic growth.



¹¹ Therefore, Figure 5 depicts a situation in which $\partial \pi^*(1)/\partial \lambda < 0$, although $f(0) > w_h / w_l$. In fact, if $f(0) = w_h / w_l$, which also implies that $\partial \pi^*(1)/\partial \lambda < 0$, the inequalities in (35) and (36) turn into equalities, so that $u^*(0) = u^*(1)$ and $g^*(0) = g^*(1)$. Therefore, given an initial very small proportion of firms paying the higher wage, the wage share converges U-shapedly to a long-run equilibrium value which is the highest possible (economically meaningful) one, with the same applying qualitatively to the rates of capacity utilization and economic growth.



(b) *Economic growth*

Figure 6. Income distribution and economic growth along the convergence path to long-run equilibrium under a relatively very strong effort elicitation effect

As it turns out, given any initial heterogeneity in wage compensation across firms, although the evolutionary dynamics take the economy to a long-run equilibrium position in which all firms pay the higher wage (and hence there is no wage differential), the wage share, capacity utilization and economic growth are all at their lowest possible level.

4. Conclusions

Motivated by compelling (experimental and empirical) evidence on wage differentials and endogenous labor effort, this paper sets forth a dynamic model to explore the implications for income distribution, capacity utilization and economic growth of firms following different strategies to elicit effort from workers. The economy is populated by homogeneous workers whose effort (and hence productivity) in carrying out labor tasks is nonetheless endogenous. Firms are also homogeneous except as regards the strategy for eliciting effort from workers they choose to follow, which then determines the wage rate they have to pay and the markup they are able to set without harming their price competitiveness. In fact, our treatment of both workers and firms as intrinsically homogeneous is coherent with the econometric evidence that wage differentials persist even after controlling for observable characteristics of workers and firms. Meanwhile, the frequency distribution of effort-elicitation strategies across firms is not parametric, as it is driven by an evolutionary dynamic centered on profit differentials that may yield a wage differential as a long-run equilibrium outcome (a result which is in keeping with the empirical evidence that wage differentials do exist and are persistent over longer periods of time). Therefore, the model furnishes a potential explanation for the documented existence and persistence of wage differentials as a result of the wage compensation strategies chosen by firms (according to profit differentials) rather than the individual characteristics of workers.

Intuitively, the extent to which labor productivity in firms paying the higher wage is greater than labor productivity in firms paying the lower wage varies positively with the relative wage differential given by the excess of the higher wage over the average wage. One rationale is that the average wage is a conventional measure of the worker's fallback position or outside option. Another rationale is that the average wage rate is the conventional reference point against which the higher wage rate offer is compared by the worker when deciding how much effort beyond the normal level to provide. Alternatively, such conventional reference point can be taken as embodying the worker's wage expectation under uncertainty, so that the higher wage offer is greeted as a pleasant surprise justifying the provision of an above-normal effort.

The behavior of the economy over time depends on the strength of the effort elicitation effect relatively to the wage differential. If the effort elicitation effect is relatively weak, there are two pure-strategy long-run equilibria, one in which all firms pay the higher wage and the other featuring all firms paying the lower wage. However, the former is unstable, whereas the latter is asymptotically stable. Besides, convergence to the long-run equilibrium featuring all firms paying the lower wage is accompanied by a monotonically rising average profit share and therefore by monotonically falling rates of capacity utilization and economic growth. Given an initial heterogeneity in wage compensation across firms, the evolutionary dynamics take the economy to a long-run equilibrium in which although there is no wage differential, the wage share, capacity utilization and economic growth are all at their lowest possible level.

If the effort elicitation effect is relatively moderate, in addition to the same two purestrategy long-run equilibria, there is also a mixed-strategy long-run equilibrium in which both wage compensation strategies are followed across firms. While the two pure-strategy long-run equilibria are unstable, the long-run equilibrium featuring both wage compensation strategies being played in the population of firms is asymptotically stable. Meanwhile, the rates of capacity utilization and economic growth in the mixed-strategy long-run equilibrium are the same as in the pure-strategy long-run equilibrium with all firms paying the lower wage. Moreover, such common rates of capacity utilization and economic growth are lower than in the pure-strategy long-run equilibrium with all firms paying the number of the pure-strategy long-run equilibrium and economic growth are lower than in the pure-strategy long-run equilibrium with all firms paying the higher wage. Given an initial heterogeneity in wage compensation across firms, the evolutionary dynamics take the economy to a long-run equilibrium solution featuring a wage differential across employed workers. Moreover, the behavior of income distribution and the level of economic activity in the convergence to such long-run equilibrium may be non-monotonic. In fact, the economy converges to a mixed-strategy evolutionary equilibrium in which the wage share, though it is not at its lowest possible (strictly positive) level, it is not at its highest possible (strictly less than one) level either. In such equilibrium, therefore, the rate of economic growth, which varies positively with aggregate effective demand, though it is not at its lowest possible level, it is not at its lowest possible level, it is not at its lowest possible level.

Meanwhile, if the effort elicitation effect is relatively strong, there are only the same two pure-strategy long-run equilibria. However, the evolutionary equilibrium with all firms paying the lower wage is unstable, while the one featuring all firms paying the higher wage is asymptotically stable. Intuitively, it is only when the effort elicitation effect is strong enough relatively to the wage differential effect that all firms play the higher wage strategy in the long run. However, the rates of capacity utilization and economic growth in such stable long-run equilibrium are both lower than in the unstable long-run equilibrium with all firms paying the lower wage. Moreover, the coupled dynamics of income distribution and economic activity in the convergence path to the evolutionary equilibrium with all firms paying the higher wage is non-monotonic. In fact, in such pure-strategy long-run equilibrium the wage share, though it is not at its lowest possible (strictly positive) level, it is not necessarily at its highest possible (strictly less than one) level either. Hence, in such long-run equilibrium, the rate of economic growth, which varies positively with aggregate effective demand, though it is not at its lowest possible level, it is not necessarily at its highest possible level either.

Finally, if the effort elicitation effect is very strong relatively to the wage differential effect, the behavior of the economy is similar to the preceding case, except that the coupled dynamics of income distribution and economic activity in the convergence to the evolutionary equilibrium with all firms paying the higher wage is rather monotonic. In fact, convergence to such evolutionary equilibrium is accompanied by a monotonically rising average profit share and hence a monotonically falling rate of economic growth. Given an initial heterogeneity in wage compensation across firms, though the evolutionary dynamics take the economy to an evolutionary equilibrium featuring all firms paying the higher wage, the wage share, capacity utilization and economic growth are all at their lowest possible level.

Though derived in a specific economic setting, the series of analytical results compiled above has broader implications for the design of wage policies in pursuit of higher economic growth. One such public policy implication is that composition effects such as changes in the heterogeneity in wage compensation strategies across firms (and hence in the resulting wage distribution across employed workers) matter for the efficacy and sustainability of a wage-led growth strategy.

References

Abeler, J., Falk, A., Goette, L. and Huffman, D. (2011) Reference points and effort provision, *American Economic Review*, 101(2), pp. 470–92.

Akerlof, G. (1982) Labor contracts as partial gift exchange, *The Quarterly Journal of Economics*, 87, pp. 543-69.

Bain, J. S. (1945) A note on pricing in monopoly and oligopoly, *American Economic Review*, 39, pp. 448–464.

Bellemare, C. and Shearer, B. (2009) Gift giving and worker productivity: Evidence from a firm-level experiment, *Games and Economic Behavior*, 67, pp. 233–244.

Bowles, S. and Gintis, H. (1990) Contested exchange: New microfoundations of the political economy of capitalism, *Politics and Society*, 18, pp. 165-222.

Caju, P. D., Ktay, G., Lamo, A., Nicolitsas, D. and Poelhekke, S. (2010) Inter-industry wage differentials in E.U. countries: What do cross-country time varying data add to the picture? *Journal of the European Economic Association*, 8, pp. 478–86.

Campbell, C. M. and Kamlani, K. S. (1997) The reasons for wage rigidity: evidence from a survey of firms, *The Quarterly Journal of Economics*, 112(3), pp. 759–789.

Cappelli, P. and Chauvin, K. (1991) An interplant test of the efficiency wage hypothesis, *The Quarterly Journal of Economics*, 106(3), pp. 769-787.

Carruth, A., Collier, W. and Dickerson, A. (2004) Inter-industry wage differences and individual heterogeneity, *Oxford Bulletin of Economics and Statistics*, 66, pp. 811–846.

Charness, G. (2004) Attribution and reciprocity in an experimental labor market, *Journal of Labor Economics*, 22, pp. 665–688.

Charness, G. and Kuhn, P. (2007) Does pay inequality affect worker effort? Experimental evidence, *Journal of Labor Economics*, 25, pp. 693–723.

Dickens, W. and Katz, L. (1987) Inter-industry wage differences and industry characteristics. In: Kevin Lang and Jonathan Leonard (eds.) *Unemployment and the Structure of Labor Markets*, Oxford: Basil Blackwell, pp. 28–89.

Dutt, A. K. (1984) Stagnation, income distribution and monopoly power, *Cambridge Journal of Economics*, 8, pp. 25–40.

Dutt, A. K. (1994) On the long-run stability of capitalist economies: implications of a model of growth and distribution. In: A. K. Dutt (ed.) *New Directions in Analytical Political Economy*, Aldershot: Edward Elgar.

Fehr, E. and Falk, A. (1999) Wage rigidity in a competitive incomplete market, *Journal of Political Economy*, 107, pp. 106–34.

Fehr, E. and Gächter, S. (2008) Wage differentials in experimental efficiency wage markets. In: Charles R. Plott and Vernon L. Smith (eds.) *Handbook of Experimental Economics Results*, Volume 1, Elsevier, pp. 120–126.

Fehr, E., Kirchler, E., Weichbold, A. and Gächter, S. (1998) When social norms overpower competition: gift exchange in experimental labor markets, *Journal of Labor Economics*, 16, pp. 324–51.

Gneezy, U. and List, J. (2006) Putting behavioral economics to work: Testing for gift exchange in labor markets using field experiments, *Econometrica*, 74(5), pp. 1365–1384.

Goldsmith, A. H., Veum, J. R. and Darity, Jr., W. (2000) Working hard for the money? Efficiency wages and worker effort, *Journal of Economic Psychology*, 21, pp. 351–385.

Groshen, E. (1991) Sources of intra-industry wage dispersion: How much do employers matter? *The Quarterly Journal of Economics*, 106, pp. 869–884.

Hall, R. and Hitch, C. (1939) Price theory and business behaviour, *Oxford Economic Papers*, 2, pp. 12-45.

Harrod, R. F. (1952) Theory of imperfect competition revised, in *Economic Essays*, London: Macmillan, chapter 8, pp. 139–187.

Julius, A. J. (2009) The wage-wage-...-wage-profit relation in a multisector bargaining economy, *Metroeconomica*, 60, pp. 537-559.

Kaldor, N. (1935) Market imperfection and excess capacity, *Economica*, 2(5), pp. 33–50.

Kalecki, M. (1935) A macrodynamic theory of the business cycle, *Econometrica*, 3, pp. 327–44.

Katz, L. (1986) Efficiency wage theories: A partial evaluation. In: Stanley Fischer (ed.) *NBER Macroeconomics Annual*, Cambridge, MA: MIT Press, pp. 235–276.

Katz, L. and Summers, L. (1989) Industry rents: evidence and implications, *Brookings Papers* on Economic Activity: Microeconomics, 1989, pp. 209–275.

Katz, L. and Autor, D. (1999) Changes in the wage structure and earnings inequality. In: Orley C. Ashenfelter and David Card (eds.) *Handbook of Labor Economics*, Volume 3, Part A, Elsevier , pp. 1463–1555.

Krueger, A. and Summers, L. (1988) Efficiency wages and the inter-industry wage structure, *Econometrica*, 56, pp. 259-94.

Lima, G. T. (2000) Market concentration and technological innovation in a dynamic model of growth and distribution, *Banca Nazionale del Lavoro Quarterly Review*, 215, December, pp. 447-75.

Lima, G. T. (2004) Endogenous technological innovation, capital accumulation and distributional dynamics, *Metroeconomica*, 55, pp. 386-408.

Palley, T. (2012) Growth, unemployment and endogenous technical progress: A Hicksian resolution of Harrod's knife-edge, *Metroeconomica*, 63, pp. 512-541.

Rezai, A. (2012) Goodwin cycles, distributional conflict and productivity growth, *Metroeconomica*, 63, pp. 29-39.

Robinson, J. (1962) Essays in the Theory of Economic Growth, London, UK: Macmillan.

Sawyer, M. (1989) The Challenge of Radical Political Economy, New York: Harvester Wheatsheaf.

Steindl, J. (1952) *Maturity and Stagnation in American Capitalism*, New York: Monthly Review Press.

Sweezy, P. (1939) Demand under conditions of oligopoly, *Journal of Political Economy*, 47, pp. 568-573.

Tarling, R. and Wilkinson, F. (1982) Changes in the inter-industry structure of earnings in the post-war period, *Cambridge Journal of Economics*, 6, pp. 231-148.

van de Klundert, T. (1989) Wage differentials and employment in a two-sector model with a dual labour market, *Metroeconomica*, 40, pp. 235-256.

von Arnim, R. (2011) Wage policy in an open-economy Kalecki-Kaldor model: A simulation study, *Metroeconomica*, 62, pp. 235-264.

Weibull, J. W. (1995) Evolutionary Game Theory, Cambridge, MA: The MIT Press.

Weisskopf, T., Bowles, S. and Gordon, D. (1983) Hearts and minds: a social model of U.S. productivity growth, *Brookings Papers on Economic Activity*, 2, pp. 381-441.