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Working Paper Series Nº 2019-54

DEPARTMENT OF ECONOMICS, FEA-USP
WORKING PAPER Nº 2019-54

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Labor Supply and the No-Growth Trap

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December 2019

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†We thank Mauro Rodrigues Jr., Eduardo Correia de Souza, Vladimir Kühl Teles and José Raymundo Novaes Chiappin for comments and suggestions. The usual disclaimer applies.

1 Introduction

A common feature of equilibrium allocations in the R&D-based growth literature is their suboptimality. The seminal paper of Aghion and Howitt (1992) argues that the prospect of future research discourages current research, since it threatens to destroy the rents created by current entrepreneurship. An extreme manifestation of suboptimality would be given by what they called the no-growth trap: a condition on the model's parameters that not only would lead to a number of researchers short of what a planner would choose, but in fact to zero researchers.

In order to tackle such suboptimality, different policies may be considered. We will be looking at models that generally do not generate enough research effort (that is, number of researchers times the number of hours they choose to work). For this reason, price caps should not help. However, a class of policies than in fact can help in this scenario, as argued by Sinn (1996) and García-Peñalosa and Wen (2008), is that of redistributive taxation.

An intrinsic factor to vertical innovation models with occupational choice such as those mentioned above is the high risk to which researchers are exposed. Empirical evidence supports the commonsense view that entrepreneurs face larger risks than paid employees. Hamilton (2000) investigates earnings differentials between self-employment and paid employment and verifies that the cross-sectional standard deviation of self-employment is substantially higher. Dunne et al. (1988) studies the US manufacturing industry and observes that 61.5% of all firms exit during the five years following their first census, while 79.5% of all firms exit within ten years.

According to Mayshar (1977), in most countries private insurance is insufficient to cover these risks due to incompleteness of the capital market structure, and, therefore, government intervention can benefit the economy. Sinn (1996) defends the importance of redistributive taxation as social insurance, not only on the account that it generates safety but also that it stimulates income-generating risky activities.

García-Peñalosa and Wen (2008) add risk-averse agents to Aghion and Howitt (1992) and analyze the equilibrium effects of redistributive taxation. They conclude that redistribution provides insurance to entrepreneurs, diluting their risk, and encourages agents to engage in research, therefore stimulating growth.

They also study the effects of redistribution on inequality and welfare. They show that low tax rates increase inequality relative to laissez-faire, while high tax rates can simultaneously promote growth and reduce inequality. In terms of welfare, García-Peñalosa and Wen (2008) suggest that extremely high income tax rates (99.8% in their numerical exercise) not only maximize but generate near first best welfare.

Meghir and Phillips (2010) examine the relationship between labor supply and taxation and emphasize the importance, when performing policy analysis, of considering the incentive effects of taxation – essentially, how hours worked respond to changes in taxes and transfers.

The present study incorporates elastic labor supply to García-Peñalosa and Wen's (2008) model. We will discuss how redistributive taxation may, in some situations, provide an escape route from the no-growth trap of Aghion and Howitt (1992), and we will then analyze this policy's effects on growth, inequality and welfare. We will also show that large income tax rates do not bring about a higher expected growth rate of the economy as in García-Peñalosa and Wen (2008), but moderate redistribution can encourage entrepreneurship and growth.

Increasing the leisure elasticity parameter reduces the welfare maximizing tax rate as well as increases the gap between the optimal redistribution and the first-best welfare levels. Thus, according to our analysis, although redistributive taxation can help to address the market failures present in Aghion and Howitt's (1992) Schumpeterian economy, this is only to a much more limited extent than that suggested by García-Peñalosa and Wen's (2008) analysis.

The next section presents the model. In section 3, we define its equilibria and determine some of its basic characteristics, such as prevailing or not of the R&D sector and of growth in the economy, as well as discuss the no-growth trap. Section 4 presents analytical and graphical comparative statics analysis, while the fifth section concludes the paper.

2 The model

2.1 Individuals

The economy is populated by a continuum of size $N > 0$ of infinitely-lived individuals. They are split into $L \in [0, N)$ unskilled agents, who can only be employed for the production of the final good Y, and H $(= N - L > 0)$ skilled agents, who choose between working as researchers or as manufacturers producing intermediate good x.

Researchers, or entrepreneurs, engage in R&D with the intention of making an innovation and, thus, obtaining a patent and licensing it to the intermediate goods sector. At any given period $t \in \mathbb{N}$ $(= \{0, 1, 2, \dots\})$, there exist $R_t \in [0, H]$ researchers,

$M_t = H - R_t$ manufacturers, and L unskilled workers. Manufacturers and unskilled workers earn, respectively, w_t and v_t for each unit of labor they supply.

All individuals have identical utility functions, that depend on consumption $C \in \mathbb{R}_+$ and leisure $l \in [0,1]$. The instantaneous utility at t is given by

$$U(C_t, l_t) = (C_t l_t^\eta)^\alpha, \tag{1}$$

where $\alpha \in (0,1]$ is a risk tolerance parameter, and $\eta \in [0, 1/\alpha]$ is the elasticity of leisure. The framework of García-Peñalosa and Wen (2008) corresponds to the $\eta = 0$ case, while that of Aghion and Howitt (1992) can be obtained by additionally assuming $\alpha = 1$.

2.2 Firms

The economy produces a single homogeneous final good Y, to be treated as numeraire, and a single intermediate good x, priced at p. At any given period $t \in \mathbb{N}$, the final good is produced in a competitive sector according to the Cobb-Douglas technology

$$Y_t = A_t x_t^\theta O_{u,t}^{1-\theta}, \tag{2}$$

where $\theta \in (0,1)$, A is the productivity index, and O_u is the amount demanded of unskilled individuals' work (which, in equilibrium, must equal $(1 - l_u^a) L$, with l_u^a standing for average leisure of unskilled individuals). As in García-Peñalosa and Wen (2008), in order to ensure that wage rates available to skilled individuals dominate those available to unskilled ones, we shall actually impose on θ a certain lower bound larger than 0 – more precisely, we assume $(1-\theta)/\theta^2 < L/H$.

Each innovation increases the value of the productivity index A_t by a factor $\gamma > 1$, so that $A_{t+1} = \gamma A_t$ if an innovation occurs at t and $A_{t+1} = A_t$ otherwise.

There is a large number of risk-neutral firms willing to produce intermediate goods. When an innovation occurs, one of these firms buys the exclusive right to produce the intermediate good according to the new blueprint, thus becoming a temporary monopolist until a new innovation takes place. Production of the intermediate good depends solely on skilled labor, according to the linear technology

$$x_t = O_{m,t}, \tag{3}$$

where O_m is the amount demanded of manufacturers' work (which, in equilibrium, must equal $(1-l_m^a) M$, with l_m^a standing for average leisure of manufacturers).

2.3 Research

At any given period $t \in \mathbb{N}$, R_t individuals engage in research with the intent of developing the next innovation (which will define the new production design for intermediate goods) and receiving the corresponding dividends. Each researcher makes a discovery in period t at the Poisson rate $\lambda(1-l_{r,t})$, where $\lambda > 0$ is an exogenous parameter and l_r is the leisure level of the researcher. For ease of exposition, we shall assume $\lambda < 1$ (it is typically calibrated to be in the thousandths or ten-thousandths). Assuming independence among researchers, the aggregate number of innovations is a random variable which will also follow a Poisson distribution, with parameter $\lambda\left(1-l_{r,t}^{a}\right)R_t$, where l_r^a stands for the average researchers' leisure (and $\left(1-l_{r,t}^{a}\right)R_t$ is the total research effort in the economy). Let ξ_t denote a realization of this variable, and χ_t, the indicator function of the event "at least one innovation happened at t", that is,

$$\chi_t=\begin{cases}1, & \text{if } \xi_t>0\\ 0, & \text{if } \xi_t=0\end{cases}. \tag{4}$$

As usual in the literature, we ignore the possibility that two or more researchers come up with an invention at the same time. Thus, when a discovery is made, a patent is granted to one researcher (i.e., a unit measure of researchers). The patent will then be licensed to an intermediary firm, which will start the new production of intermediate goods and collection of profits thereof in the next period. Thus, the productivity index at any period $t\in\mathbb{N}$ is given by

$$A_t=A_0\gamma^{\sum_{i=0}^{t-1}\chi_i}, \tag{5}$$

where $A_0>0$ is given. We assume complete bargaining power on the R&D sector, whence, at the very time of invention/licensing (say, t), that period's successful researcher receives from its licensee V_{t+1}, the expected value of his/her innovation.

2.4 Government

Given both the deadweight stemming from monopoly power and the researchers' uncovered risks, equilibrium will be suboptimal. Among different forms of government intervention that could be studied, as mentioned in the introduction, price caps will not work in this economy, for discouraging R&D.

As in García-Peñalosa and Wen (2008), we shall then investigate how far a simple redistributive policy can go in terms of promoting growth and welfare. We thus consider a linear income tax schedule, in which individuals pay a tax rate $\tau \in [0, 1)$ on their income and receive a lump-sum transfer $B \geq 0$.

We shall impose as an equilibrium constraint that government runs a balanced budget at every period $t \in \mathbb{N}$. That is, total tax revenue should equal the value of an individual transfer multiplied by the number of agents in the economy:

$$\tau Y_t = B_t N. \tag{6}$$

2.5 Firms' maximization problem

The final goods sector is competitive. At any given period $t \in \mathbb{N}$, firms solve

$$\max_{x_t, O_{ut}} A_t x_t^{\theta} O_{u,t}^{1-\theta} - p_t x_t - v_t O_{u,t}. \tag{7}$$

It may be noted that there is no information asymmetry in the model, whence firms only pay for hours effectively worked.

By differentiating (7) with respect to x_t, one obtains the inverse demand function for intermediate goods, to be plugged into the monopolistically competitive intermediate firm's problem:

$$\max_{x_t} \quad p_t x_t - w_t x_t \text{ s.t.} \tag{8}$$

$$p_t = A_t \theta x_t^{\theta-1} O_{u,t}^{1-\theta}. \tag{9}$$

The profit maximizing production of x_t is then

$$x_t = \left(\frac{\theta^2 A_t}{w_t}\right)^{\frac{1}{1-\theta}} O_{u,t},$$

which together with (9) leads to the following expressions for wages and the intermediate firm's profit π:

$$p_t = \frac{w_t}{\theta}, \tag{10}$$

$$\pi_t = \theta \left(1-\theta\right) Y_t, \tag{11}$$

$$\left(1 - l_{u,t}^a\right) v_t L = \left(1-\theta\right) Y_t, \tag{12}$$

$$\left(1 - l_{m,t}^a\right) w_t \left(H - R_t\right) = \theta^2 Y_t. \tag{13}$$

Equations (12) and (13) show that the wage rates v_t and w_t, as proportions of total income, are decreasing in the total work effort in their respective sectors, that is, $\left(1 - l_{u,t}^a\right) L$ and $\left(1 - l_{m,t}^a\right) \left(H - R_t\right)$. It may also be noted from equation (13) that the wage paid to a worker employed in the manufacturing sector, as a proportion of total income, will be higher the more inflated is the R&D sector, which competes with it for the same workers.

Assuming there was an innovation at t, the value of its patent, V_{t+1}, can be determined by the asset condition

$$V_{t+1} = \frac{\pi_{t+1}}{r + \lambda\left(1 - l^{a}_{r,t+1}\right) R_{t+1}}, \tag{14}$$

in which the value of the innovation to the risk-neutral firm is equal to the stream of profits thereof discounted by the exogenous interest rate r and the risk of losing its monopoly power, i.e., the rate at which its technology becomes obsolete due to being replaced by a new discovery.

If we plug (11) and (5) (which implies $A_{t+1} = \gamma A_t$, since $\chi_t = 1$, i.e., there was an innovation at t) into (14), we obtain

$$V_{t+1} = \frac{\theta\left(1-\theta\right)\gamma Y_t}{r + \lambda\left(1 - l^{a}_{r,t+1}\right) R_{t+1}} \frac{Y_{t+1}/A_{t+1}}{Y_t/A_t}, \tag{15}$$

an expression which will prove useful in the following.

2.6 Individuals' maximization problem

In the absence of a savings mechanism, consumers' problems can be stated in a static form. At each period $t \in \mathbb{N}$, unskilled individuals choose $(C_{u,t}, l_{u,t}) \in \mathbb{R}_{+} \times [0,1]$ so as to maximize $U\left(C_{u,t}, l_{u,t}\right)$ subject to their budget constraint

$$C_{u,t} \leq \left(1-\tau\right)\left(1-l_{u,t}\right) v_t + B_t. \tag{16}$$

This is almost of the standard Cobb-Douglas consumer type, the only difference being the inclusion of the $l_{u,t} \leq 1$ physical constraint. Due to U being strictly increasing in its first argument, (16) must hold with equality at the solution, so that it is a trivial task to find $C_{u,t}$ once $l_{u,t}$ has been found. The well-known solution for $l_{u,t}$ is

$$l_{u,t} = \frac{\eta}{\eta+1} \frac{(1-\tau)\, v_t + B_t}{(1-\tau)\, v_t}.$$

Since there is no guarantee from the start that the above expression cannot be larger than 1, the actual solution to the unskilled workers' problem takes the form

$$l_{u,t} = \begin{cases} \frac{\eta}{\eta+1}\left(1 + \frac{B_t}{(1-\tau)v_t}\right), & \text{if } v_t > \frac{\eta B_t}{1-\tau} \\ 1, & \text{otherwise} \end{cases}. \tag{17}$$

In the same fashion, manufacturers choose $(C_{m,t}, l_{m,t}) \in \mathbb{R}_+ \times [0,1]$ so as to maximize $U(C_{m,t}, l_{m,t})$ subject to

$$C_{m,t} \leq (1-\tau)(1-l_{m,t})\, w_t + B_t. \tag{18}$$

This is the same problem as that of unskilled workers, only with a different wage rate. Thus,

$$l_{m,t} = \begin{cases} \frac{\eta}{\eta+1}\left(1 + \frac{B_t}{(1-\tau)w_t}\right), & \text{if } w_t > \frac{\eta B_t}{1-\tau} \\ 1, & \text{otherwise} \end{cases}. \tag{19}$$

Unskilled workers and manufacturers' optimal leisure choices are therefore decreasing in their post-tax wage rates $(1-\tau)\, v_t$ and $(1-\tau)\, w_t$, and increasing in the transfers B_t they receive. Also, if an individual receives no transfer (i.e., $B_t = 0$), his/her leisure choice will be invariant to taxation and equal to $\eta/(\eta+1)$. This stems from the income effect of τ exactly offsetting its substitution effect, given the

Cobb-Douglas utility function in (1).

The reason a skilled individual will never choose to work in the final goods sector is that he/she would then face the same maximization problem as the one in the manufacturing sector, only with a lower wage rate. In fact, $v_t \geq w_t$ would imply, through (17) and (19), $l_{u,t} \leq l_{m,t}$, whence $v_t(1-l_{u,t}) \geq w_t(1-l_{m,t})$. In equilibrium, this would read, through (12) and (13), $(1-\theta)Y_t/L \geq \theta^2 Y_t/(H-R_t)$, so that $(1-\theta)/\theta^2 \geq L/(H-R_t) \geq L/H$, in opposition to the constraint assumed on θ.[1]

Researchers, in turn, not knowing whether they will make a discovery or not, choose $(C_{1,t}, C_{0,t}, l_{r,t}) \in \mathbb{R}_+^2 \times [0,1]$ to maximize the expected utility

$$\lambda(1-l_{r,t})U(C_{1,t}, l_{r,t}) + (1-\lambda(1-l_{r,t}))U(C_{0,t}, l_{r,t}) \tag{20}$$

subject to

$$C_{0,t} \leq B_t \tag{21}$$

and

$$C_{1,t} \leq (1-\tau)V_{t+1} + B_t. \tag{22}$$

Due to U being strictly increasing in its first argument and $\lambda < 1$ (so that we know the weight $1-\lambda(1-l_{r,t})$ will be positive), (21) must hold with equality at the solution. And unless the solution entails taking $l_{r,t} = 1$ (which is a possibility, as we will see shortly), (22) must also be active. Substituting these expressions for $C_{1,t}$ and $C_{0,t}$ into (20), we obtain a function of $l_{r,t}$ only. It is concave, since its second-order

[1] We are only interested in equilibria with $Y_t > 0, \forall t \in \mathbb{N}$, so that division by Y_t is permissible.

derivative will be

$$
\begin{aligned}
&-2\lambda\left(\left(\left(1-\tau\right)V_{t+1}+B_t\right)^{\alpha}-B_t^{\alpha}\right)\eta\alpha l_{r,t}^{\eta\alpha-1} \\
&+\left[\lambda\left(1-l_{r,t}\right)\left(\left(1-\tau\right)V_{t+1}+B_t\right)^{\alpha}+\left(1-\lambda\left(1-l_{r,t}\right)\right)B_t^{\alpha}\right]\eta\alpha\left(\eta\alpha-1\right)l_{r,t}^{\eta\alpha-2},
\end{aligned}
$$

a sum of two nonpositive summands (this is where the constraint $\eta \in [0, 1/\alpha]$ comes into play). Therefore, this problem's solution is delivered by its first-order condition, which entails equating the first-order derivative

$$
\begin{aligned}
&-\lambda\left(\left(\left(1-\tau\right)V_{t+1}+B_t\right)^{\alpha}-B_t^{\alpha}\right)l_{r,t}^{\eta\alpha} \\
&+\left[\lambda\left(1-l_{r,t}\right)\left(\left(1-\tau\right)V_{t+1}+B_t\right)^{\alpha}+\left(1-\lambda\left(1-l_{r,t}\right)\right)B_t^{\alpha}\right]\eta\alpha l_{r,t}^{\eta\alpha-1}
\end{aligned}
$$

to zero, or checking its nonnegativity at $l_{r,t} = 1$ (the $l_{r,t} = 0$ possibility is excluded because U satisfies the Inada conditions). Therefore,

$$
l_{r,t} = \begin{cases} \frac{\eta\alpha}{\eta\alpha+1}\left(1+\frac{1}{\lambda}\frac{B_t^{\alpha}}{\left(\left(1-\tau\right)V_{t+1}+B_t\right)^{\alpha}-B_t^{\alpha}}\right), & \text{if } V_{t+1} > \frac{\left(1+\frac{\eta\alpha}{\lambda}\right)^{\frac{1}{\alpha}}-1}{1-\tau}B_t \\ 1, & \text{otherwise} \end{cases} . \qquad (23)
$$

We thus see that the researcher will work more at t the higher the value V_{t+1} of an innovation, and the lower the transfer level B_t, as expected. Again, leisure is invariant to taxation when there is no transfer B_t to be received, and in this case equals $\eta\alpha/\left(\eta\alpha+1\right)$.

3 Equilibrium

An allocation is defined as an equilibrium if firms are maximizing profits, agents are maximizing their utilities, skilled workers do not want to change their occupational choice, markets clear and the government runs a balanced budget. Since equilibria will necessarily be symmetric within each sector of the economy, as seen in subsection 2.6, there is no need for including indices for each firm and individual in the definition that follows, and the superscript a used in subsection 2.5, denoting averages, may be dropped.

Accordingly, O_u should equal $(1 - l_u) L$ in equilibrium, whence (2) becomes

$$Y_t = A_t x_t^\theta ((1 - l_{u,t}) L)^{1-\theta} . \tag{24}$$

Similarly, O_m should equal $(1 - l_m)(H - R)$ in equilibrium, whence (3) becomes

$$x_t = (1 - l_{m,t})(H - R_t) . \tag{25}$$

Definition 1 *$(p, v, w, V, \tau, B, \xi, \chi, A, C_u, C_m, C_0, C_1, l_u, l_m, l_r, R, x, Y) \in \mathbb{R}_{++}^{\mathbb{N}} \times \mathbb{R}_{++}^{\mathbb{N}} \times \mathbb{R}_{++}^{\mathbb{N}} \times \mathbb{R}_{+}^{\mathbb{N}\setminus\{0\}} \times [0,1] \times \mathbb{R}_{+}^{\mathbb{N}} \times \mathbb{N}^{\mathbb{N}} \times \{0,1\}^{\mathbb{N}} \times \mathbb{R}_{++}^{\mathbb{N}} \times \mathbb{R}_{+}^{\mathbb{N}} \times \mathbb{R}_{+}^{\mathbb{N}} \times \mathbb{R}_{+}^{\mathbb{N}} \times \mathbb{R}_{+}^{\mathbb{N}} \times [0,1]^{\mathbb{N}} \times [0,1]^{\mathbb{N}} \times [0,1]^{\mathbb{N}} \times [0,H]^{\mathbb{N}} \times \mathbb{R}_{++}^{\mathbb{N}} \times \mathbb{R}_{++}^{\mathbb{N}}$ is an equilibrium if, for every $t \in \mathbb{N}$, the following expressions hold: (4), (5), (6), (10), (12), (13), (15) (these last three with the a superscript dropped), (17), (19), (23), (24), (25), plus the four restrictions (16), (18), (21) and (22) with equality, the arbitrage conditions*

Arb1 $U(C_{m,t}, l_{m,t}) < \lambda (1 - l_{r,t}) U(C_{1,t}, l_{r,t}) + (1 - \lambda (1 - l_{r,t})) U(C_{0,t}, l_{r,t}) \Rightarrow R_t =$

H,

Arb2 $U\left(C_{m,t}, l_{m,t}\right) > \lambda\left(1 - l_{r,t}\right) U\left(C_{1,t}, l_{r,t}\right) + \left(1 - \lambda\left(1 - l_{r,t}\right)\right) U\left(C_{0,t}, l_{r,t}\right) \Rightarrow R_t = 0$,

and the market clearing condition

MC $LC_{u,t} + \left(H - R_t\right) C_{m,t} + R_t C_{0,t} + \chi_t \left(C_{1,t} - C_{0,t}\right) = Y_t$.

Obviously, a laissez-faire equilibrium would correspond to one with $\tau = 0$.

For simplicity, we take $V \in \mathbb{R}_+^{\mathbb{N}\setminus\{0\}}$. If, at a specific period $t \in \mathbb{N}$, no innovation has occurred (i.e., $\xi_t = 0$), then the value attained by V at $t+1$ (indicating the value of a nonexistent patent) is immaterial. Without loss of generality, V can be built according to the rule "$V_{t+1} = 0$ if $\xi_t = 0$".

Arbitrage condition Arb1 refers to a scenario in which manufacturing ceases to exist, which will be proved to be impossible in equilibrium. Arbitrage condition Arb2 refers to a scenario in which research ceases to exist, which is a possible situation and will also be explored in this work.

The last term in the left-hand side of the market clearing condition accounts for the uncertainty related to the occurrence of innovations. If there is an innovation at t, then χ_t equals one and a measure $R_t - 1$ of researchers will have a consumption level of $C_{0,t} = B_t$, while a unit measure of researchers will be able to consume $C_{1,t} = (1 - \tau) V_{t+1} + B_t$. However, if no innovation occurs at t, then all R_t researchers will only consume $C_{0,t} = B_t$.

As can be seen in the above definition, we deem economically relevant only equilibria with $Y > 0$. This in itself implies, through (12) and (13),

Lemma 2 *In equilibrium, $l_{u,t} < 1$, $l_{m,t} < 1$ and $R_t < H$, for all $t \in \mathbb{N}$.*

Given this lemma, in order to compute $l_{u,t}$ in equilibrium, one may plug (6) and (12) into the first line of (17) to obtain

$$l_{u,t} = \frac{\eta}{\eta+1}\left(1 + \frac{\frac{\tau}{N}}{\frac{(1-\tau)(1-\theta)}{(1-l_{u,t})L}}\right),$$

which can be solved for $l_{u,t}$:

$$l_{u,t} = l_u^* := \frac{\eta}{\eta+1}\left(1 + \frac{\frac{\tau}{N}}{(\eta+1)\frac{(1-\tau)(1-\theta)}{L} + \eta\frac{\tau}{N}}\right). \tag{26}$$

This is indeed lower than 1 $(= (\eta/(\eta+1))(1+1/(\eta+0)))$.

Following the same procedure with (13) and (19) instead of (12) and (17) yields

$$l_{m,t} = \frac{\eta}{\eta+1}\left(1 + \frac{\frac{\tau}{N}}{(\eta+1)\frac{(1-\tau)\theta^2}{H-R_t} + \eta\frac{\tau}{N}}\right), \tag{27}$$

dependent on the equilibrium value of R_t.

By plugging (6) and (15) into the first line of (23), one sees that the potentially interior value of $l_{r,t}$ will also depend on $l_{r,t+1}$:

$$l_{r,t} = \frac{\eta\alpha}{\eta\alpha+1}\left(1 + \frac{1}{\lambda}\frac{\left(\frac{\tau}{N}\right)^\alpha}{\left(\frac{1-\tau\theta(1-\theta)\gamma}{r+\lambda(1-l_{r,t+1})R_{t+1}}\frac{Y_{t+1}/A_{t+1}}{Y_t/A_t} + \frac{\tau}{N}\right)^\alpha - \left(\frac{\tau}{N}\right)^\alpha}\right). \tag{28}$$

Henceforth, we will only be interested in stationary equilibria.

3.1 Stationary equilibrium

Definition 3 *A stationary equilibrium is an equilibrium* $(p, v, w, V, \tau, B, \xi, \chi, A, C_u, C_m, C_0, C_1, l_u, l_m, l_r, R, x, Y)$ *in which the sequences* l_m*,* l_r *and* R *are constant.*

If R^* is the number of researchers in a stationary equilibrium, then the leisure level of manufacturers will be given by (27):

$$l_m^* = \frac{\eta}{\eta+1}\left(1 + \frac{\frac{\tau}{N}}{(\eta+1)\frac{(1-\tau)\theta^2}{H-R^*} + \eta\frac{\tau}{N}}\right). \tag{29}$$

From (25), we see that x must also be constant in a stationary equilibrium. Thus (24) says that Y/A is constant in such an equilibrium, and (28) now yields

$$l_r^* = \frac{\eta\alpha}{\eta\alpha+1}\left(1 + \frac{1}{\lambda}\frac{\left(\frac{\tau}{N}\right)^\alpha}{\left(\frac{(1-\tau)\theta(1-\theta)\gamma}{r+\lambda(1-l_r^*)R^*} + \frac{\tau}{N}\right)^\alpha - \left(\frac{\tau}{N}\right)^\alpha}\right),$$

an implicit expression (the best we can do, due to the α exponent in the denominator) for an interior l_r^*.

Now, since the right-hand side above is a decreasing function of l_r^* for $l_r^* \in [0,1]$, it can touch the 45° line (the graph of the identity function, represented by the l_r^* in the left-hand side above) only once. This crossing will occur at a $l_r^* < 1$ if and only if the right-hand side assumes a value strictly lower than 1 at $l_r^* = 1$, that is, if and only if

$$\frac{\eta\alpha}{\eta\alpha+1}\left(1 + \frac{1}{\lambda}\frac{\left(\frac{\tau}{N}\right)^\alpha}{\left(\frac{(1-\tau)\theta(1-\theta)\gamma}{r} + \frac{\tau}{N}\right)^\alpha - \left(\frac{\tau}{N}\right)^\alpha}\right) < 1.$$

If this condition does not hold, then, by tedious algebraic manipulations we arrive at

$$\frac{\theta\left(1-\theta\right)\gamma}{r} \leq \frac{\left(1+\frac{\eta\alpha}{\lambda}\right)^{\frac{1}{\alpha}}-1}{1-\tau}\frac{\tau}{N}, \tag{30}$$

which, in a stationary equilibrium, corresponds (through (15) and (6)) to the condition given in the second line of (23) for the corner solution $l_r = 1$.

Yet another way of writing (30) is

$$\tau \geq \bar{\tau} := \left(1 + r\frac{\left(1+\frac{\eta\alpha}{\lambda}\right)^{\frac{1}{\alpha}}-1}{\theta\left(1-\theta\right)\gamma N}\right)^{-1}.$$

Thus, we obtain

$$l_r^* = \begin{cases} \frac{\eta\alpha}{\eta\alpha+1}\left(1+\frac{1}{\lambda}\frac{\left(\frac{\tau}{N}\right)^{\alpha}}{\left(\frac{(1-\tau)\theta(1-\theta)\gamma}{r+\lambda\left(1-l_r^*\right)R^*}+\frac{\tau}{N}\right)^{\alpha}-\left(\frac{\tau}{N}\right)^{\alpha}}\right), & \text{if } \tau < \bar{\tau} \\ 1, & \text{if } \tau \geq \bar{\tau} \end{cases}. \tag{31}$$

It must be noted that $\bar{\tau} \in (0,1]$, and $\bar{\tau} < 1$ if and only if $\eta > 0$.

Thus, given a positive income tax rate τ, if the preference-for-leisure parameter η is high enough, or if the rate-of-innovations parameter λ is low enough, researchers do not work at all.

With respect to the risk-tolerance parameter α, it can be seen that $\bar{\tau}$ increases with it.[2] Therefore, ceteris paribus, the "bad" equilibrium in the second line of (31)

[2] If we let $\beta := \eta/\lambda$ to save a bit on notation, the derivative of $(1+\beta\alpha)^{\frac{1}{\alpha}}$ with respect to α is $(1+\beta\alpha)^{\frac{1}{\alpha}}\left(1-1/\left(1+\beta\alpha\right)-\log\left(1+\beta\alpha\right)\right)/\alpha^2$, which has the same sign as $1-1/u-\log u$, where $u := 1+\beta\alpha \geq 1$. This expression is nonpositive, since: (i) at $u=1$ it equals zero, and (ii) it is strictly decreasing in u (its derivative, for $u>1$, is $1/u^2-1/u = \left(1-u\right)/u^2 < 0$). Therefore, $\bar{\tau}$ is

can only emerge for a sufficiently low α (i.e., if individuals are sufficiently risk averse). However, since the limit of $\bar{\tau}$ for $\alpha \rightarrow 0_+$ is $\left(1 + r\left(e^{\eta/\tau} - 1\right) / \left(\theta\left(1-\theta\right)\gamma N\right)\right)^{-1}$, for income tax rates lower than this expression, a stationary equilibrium with $l_r^* = 1$ is not a possibility.

A different problem, but with similar economic implications as we shall shortly see, is that of no one wanting to work in the R&D sector of the economy. This will necessarily happen in a stationary equilibrium if the utility level in this sector is lower than that available in the manufacturing sector, according to arbitrage condition Arb2. In order to be able to carry out this analysis involving comparison of utility levels in different sectors, we write down the stationary consumption levels in each of these sectors (plus the final goods one, for completeness).

By simply plugging (12) and (6) into (16); (13) and (6) into (18); (6) into (21); and (15) and (6) into (22), we obtain

$$C_{u,t} = \left(\frac{(1-\tau)(1-\theta)}{L} + \frac{\tau}{N}\right) Y_t, \tag{32}$$

$$C_{m,t} = \left(\frac{(1-\tau)\theta^2}{H - R^*} + \frac{\tau}{N}\right) Y_t, \tag{33}$$

$$C_{0,t} = \frac{\tau}{N} Y_t, \tag{34}$$

$$C_{1,t} = \left(\frac{(1-\tau)\theta(1-\theta)\gamma}{r + \lambda(1-l_r^*) R^*} + \frac{\tau}{N}\right) Y_t. \tag{35}$$

increasing in α.

3.2 The research effort

Given a stationary equilibrium with the notation of the previous subsections, we call $(1 - l_r^*) R^*$ the *research effort* in the economy. This variable is key in determining growth.

In a stationary equilibrium, for any period $t \in \mathbb{N}$, as argued in the previous subsection, one has $Y_{t+1}/A_{t+1} = Y_t/A_t$. Therefore, the growth rate of output at t equals the growth rate of productivity, which we approximate by $\log(A_{t+1}/A_t)$. Obviously, this rate should not be expected to be constant (unless in the equilibrium in which no innovations ever come about and in the equilibrium in which innovations happen at each period), since A follows a stochastic process, according to (5).

In a stationary equilibrium, what is necessarily constant in time is the *expected* growth rate of productivity. It can be computed by multiplying the Poisson rate at which innovations occur, $\lambda(1 - l_r^*) R^*$, by $\log \gamma$ (since $A_{t+1}/A_t = \gamma$ when $\xi_t > 0$), and adding to that the product of $1 - \lambda(1 - l_r^*) R^*$ by $\log 1 = 0$ (since $A_{t+1}/A_t = 1$ when $\xi_t = 0$):

$$g = \lambda(1 - l_r^*) R^* \ln \gamma.$$

If $\tau \geq \bar{\tau}$, then the research effort (whence also the expected growth rate) will be null, as shown in the previous subsection. But this might be the case even if $\tau < \bar{\tau}$, since R^* may be zero. All the following analysis is done with the $\tau < \bar{\tau}$ assumption in mind.

Let us initially consider the $\tau = 0$ case. In this case, (31) gives $l_r^* = \eta\alpha/(\eta\alpha + 1)$. In order to find R^*, we must compare the utility levels in the manufacturing and R&D

sectors. Plugging (33) and (29) into (1) yields the following utility level divided by Y_t^{α}:

$$\bar{U}_m(R^*) = \left(\frac{\theta^2}{H - R^*} \left(\frac{\eta}{\eta + 1} \right)^{\eta} \right)^{\alpha}. \tag{36}$$

Similarly, plugging (34), (35) and the first line of (31) into (20) and dividing by Y_t^{α} gives

$$\bar{U}_r(R^*) = \frac{\lambda}{\eta\alpha + 1} \left(\frac{\theta(1-\theta)\gamma}{r + \frac{\lambda}{\eta\alpha+1} R^*} \left(\frac{\eta\alpha}{\eta\alpha + 1} \right)^{\eta} \right)^{\alpha}. \tag{37}$$

It may be gathered from arbitrage conditions Arb1, Arb2 and Lemma 2 that either $R^* > 0$ and $\bar{U}_m(R^*) = \bar{U}_r(R^*)$, or $R^* = 0$ and $\bar{U}_m(R^*) \geq \bar{U}_r(R^*)$. Now, while $\bar{U}_m(R^*)$ is strictly increasing in R^*, $\bar{U}_r(R^*)$ is strictly decreasing in that same variable. This only assures uniqueness of a positive number of researchers R^* in stationary equilibrium, not existence. Indeed, it will not exist if, and only if, the graphs of $\bar{U}_m$ and of $\bar{U}_r$ do not cross each other. That is, if and only if $\bar{U}_m(0) \geq \bar{U}_r(0)$ or, equivalently,

$$\frac{\theta r}{(1-\theta)\gamma H} \geq \left(\frac{\lambda}{\eta\alpha + 1} \right)^{\frac{1}{\alpha}} \left(\frac{(\eta+1)\alpha}{\eta\alpha + 1} \right)^{\eta}. \tag{38}$$

When $\eta = 0$ and $\alpha = 1$, this becomes the condition for what Aghion and Howitt (1992) call a no-growth trap, $\theta r / ((1-\theta)\gamma H) \geq \lambda$. For this choice of η and α, *trap* is a very suitable word, since, as we shall see below, even by putting into effect a positive income tax rate τ, effort (and hence growth) would remain zero. But if $\alpha < 1$ and/or if $\eta > 0$, once simple redistributive mechanisms are brought into the picture, we realize there is hope for escaping this trap. That is, there might exist $\tau > 0$ (but lower than $\bar{\tau}$) capable of stimulating skilled workers to become researchers and/or making researchers work harder, and hence bring growth to the economy.

Suppose $\tau > 0$. Following the same procedure as before, we obtain

$$\bar{U}_m(R^*) = \left(\left(\frac{(1-\tau)\theta^2}{H-R^*} + \frac{\tau}{N}\right)\left(\frac{\eta}{\eta+1}\left(1 + \frac{\frac{\tau}{N}}{(\eta+1)\frac{(1-\tau)\theta^2}{H-R^*} + \eta\frac{\tau}{N}}\right)\right)^{\eta}\right)^{\alpha}, \quad (39)$$

an expression that reduces to (36) when $\tau = 0$.

When $\eta = 0$, this becomes $\left((1-\tau)\theta^2/(H-R^*) + \tau/N\right)^{\alpha}$, strictly increasing in R^*. When $\eta > 0$, to ease the analysis of the behavior of this expression with respect to R^*, we first pass it through the $\log \circ (\cdot)^{\frac{1}{\alpha}}$ monotonic transformation, thus arriving at an expression of the form

$$\log\left(\frac{a}{H-R^*} + b\right) + \eta \log\left(1 + \frac{b}{(\eta+1)\frac{a}{H-R^*} + \eta b}\right) + \eta \log \frac{\eta}{\eta+1},$$

where $a := (1-\tau)\theta^2 > 0$ and $b := \tau/N > 0$. Its derivative is

$$\begin{aligned}
& \frac{1}{\frac{a}{H-R^*} + b}\frac{a}{(H-R^*)^2} - \eta\frac{1}{1 + \frac{b}{(\eta+1)\frac{a}{H-R^*} + \eta b}}\frac{b(\eta+1)}{\left((\eta+1)\frac{a}{H-R^*} + \eta b\right)^2}\frac{a}{(H-R^*)^2} \\
= & \frac{a}{(H-R^*)^2}\left(\frac{1}{\frac{a}{H-R^*} + b} - \eta\frac{1}{(\eta+1)\frac{a}{H-R^*} + \eta b + b}\frac{b(\eta+1)}{(\eta+1)\frac{a}{H-R^*} + \eta b}\right) \\
= & \frac{a}{(H-R^*)^2}\left(\frac{1}{\frac{a}{H-R^*} + b} - \eta\frac{1}{\frac{a}{H-R^*} + b}\frac{b}{(\eta+1)\frac{a}{H-R^*} + \eta b}\right) \\
= & \frac{a}{(H-R^*)^2\left(\frac{a}{H-R^*} + b\right)}\left(1 - \eta\frac{b}{(\eta+1)\frac{a}{H-R^*} + \eta b}\right) \\
= & \frac{a}{(H-R^*)^2\left(\frac{a}{H-R^*} + b\right)}\frac{(\eta+1)\frac{a}{H-R^*}}{(\eta+1)\frac{a}{H-R^*} + \eta b} > 0.
\end{aligned}$$

Therefore, once again $\bar{U}_m(R^*)$ is strictly increasing in R^*.

Proceeding as before for the R&D sector, we consider two cases. If $\eta = 0$, then

$$\bar{U}_r(R^*) = \lambda\left(\frac{(1-\tau)\,\theta\,(1-\theta)\,\gamma}{r+\lambda R^*} + \frac{\tau}{N}\right)^{\alpha} + (1-\lambda)\left(\frac{\tau}{N}\right)^{\alpha},$$

strictly decreasing in R^*, whence $R^* = 0$ if and only if $\bar{U}_m(0) \geq \bar{U}_r(0)$, i.e.,

$$\left(\frac{(1-\tau)\,\theta^2}{H} + \frac{\tau}{N}\right)^{\alpha} \geq \lambda\left(\frac{(1-\tau)\,\theta\,(1-\theta)\,\gamma}{r} + \frac{\tau}{N}\right)^{\alpha} + (1-\lambda)\left(\frac{\tau}{N}\right)^{\alpha}. \tag{40}$$

In particular, when $\alpha = 1$, it is straightforward to see that this is equivalent to $\theta r / ((1-\theta)\,\gamma H) \geq \lambda$, the no-growth trap condition in Aghion and Howitt (1992). We thus confirm the aforementioned impossibility of escaping this trap through a simple redistributive scheme when $\eta = 0$ and $\alpha = 1$. But if $\alpha < 1$, then it is possible that, for some positive τ, (38) holds while (40) does not.

If $\eta > 0$, then

$$\bar{U}_r(R^*) = \left(\begin{array}{c} \lambda\,(1-\ell(R^*))\left(\frac{(1-\tau)\theta(1-\theta)\gamma}{r+\lambda(1-\ell(R^*))R^*} + \frac{\tau}{N}\right)^{\alpha} \\ +\,(1-\lambda\,(1-\ell(R^*)))\left(\frac{\tau}{N}\right)^{\alpha} \end{array}\right)\ell(R^*)^{\eta\alpha}, \tag{41}$$

where the function ℓ is defined as follows.

Let[3]

$$\underline{l} := \frac{\eta\alpha}{\eta\alpha+1}\left(1 + \frac{1}{\lambda}\frac{\left(\frac{\tau}{N}\right)^{\alpha}}{\left(\frac{(1-\tau)\theta(1-\theta)\gamma}{r} + \frac{\tau}{N}\right)^{\alpha} - \left(\frac{\tau}{N}\right)^{\alpha}}\right)$$

[3] Since a stationary equilibrium must have $R^* \geq 0$, when $\tau \in (0, \bar{\tau})$ it is necessarily the case that $l_r^* \geq \underline{l}$, as can be seen from the first line of (31).

and $R : [\underline{l}, 1) \to \mathbb{R}_+$ be given by[4]

$$R(l) = \frac{1}{\lambda(1-l)} \left[\frac{1-\tau}{\tau} \frac{\theta(1-\theta)\gamma N}{\left(1 + \frac{\eta\alpha}{\lambda((\eta\alpha+1)l - \eta\alpha)}\right)^{\frac{1}{\alpha}} - 1} - r \right]. \tag{42}$$

This formula is obtained by solving for R^* the equation in the first line of (31), so that, in a stationary equilibrium with $\tau \in (0, \bar{\tau})$, if $l_r^* \in [\underline{l}, 1)$ is the choice of leisure of researchers, then $R^* = R(l_r^*)$. Since $R(l)$ is the product of two expressions that are strictly increasing in l and nonnegative (the one outside of the square brackets because $l < 1$, and the one inside of the square brackets because $l \geq \underline{l}$), R itself is strictly increasing, and can be inverted. Let $\ell : \mathbb{R}_+ \to [\underline{l}, 1)$ be the inverse of R.

Thus $\ell(R^*) = \ell(R(l_r^*)) = l_r^*$, and the denominator $r + \lambda(1 - \ell(R^*)) R^*$ appearing in (41) can be rewritten as

$$\begin{aligned} & r + \lambda(1 - \ell(R^*)) R^* = r + \lambda(1 - l_r^*) R(l_r^*) \\ = \; & \frac{1-\tau}{\tau} \frac{\theta(1-\theta)\gamma N}{\left(1 + \frac{\eta\alpha}{\lambda((\eta\alpha+1)l_r^* - \eta\alpha)}\right)^{\frac{1}{\alpha}} - 1} = \frac{1-\tau}{\tau} \frac{\theta(1-\theta)\gamma N}{\left(1 + \frac{1}{\lambda\left(\frac{\eta\alpha+1}{\eta\alpha}\ell(R^*) - 1\right)}\right)^{\frac{1}{\alpha}} - 1}, \end{aligned}$$

[4] The reason why we had to treat the $\eta = 0$ case separately is in the very form of $R(l)$.

where we have used (42). Substituting this back into (41) gives

$$\begin{aligned}
\bar{U}_r(R^*) &= \left(\begin{array}{c} \lambda\left(1-\ell\left(R^*\right)\right)\left(\left(1+\frac{1}{\lambda\left(\frac{\eta\alpha+1}{\eta\alpha}\ell(R^*)-1\right)}\right)^{\frac{1}{\alpha}}\frac{\tau}{N}\right)^{\alpha} \\ +\left(1-\lambda\left(1-\ell\left(R^*\right)\right)\right)\left(\frac{\tau}{N}\right)^{\alpha} \end{array}\right)\ell\left(R^*\right)^{\eta\alpha} \\
&= \left(\begin{array}{c} \lambda\left(1-\ell\left(R^*\right)\right)\left(1+\frac{1}{\lambda\left(\frac{\eta\alpha+1}{\eta\alpha}\ell(R^*)-1\right)}\right) \\ +1-\lambda\left(1-\ell\left(R^*\right)\right) \end{array}\right)\left(\frac{\tau}{N}\right)^{\alpha}\ell\left(R^*\right)^{\eta\alpha} \\
&= \left(\frac{\lambda\left(1-\ell\left(R^*\right)\right)}{\lambda\left(\frac{\eta\alpha+1}{\eta\alpha}\ell\left(R^*\right)-1\right)}+1\right)\left(\frac{\tau}{N}\right)^{\alpha}\ell\left(R^*\right)^{\eta\alpha} \\
&= \frac{\frac{1}{\eta\alpha}\ell\left(R^*\right)}{\frac{\eta\alpha+1}{\eta\alpha}\ell\left(R^*\right)-1}\left(\frac{\tau}{N}\right)^{\alpha}\ell\left(R^*\right)^{\eta\alpha} = \frac{\ell\left(R^*\right)^{\eta\alpha}}{1-\eta\alpha\left(\frac{1}{\ell(R^*)}-1\right)}\left(\frac{\tau}{N}\right)^{\alpha}.
\end{aligned}$$

Since the expression $l^{\eta\alpha}/\left(1-\eta\alpha\left(1/l-1\right)\right)$ is strictly decreasing in l (its derivative equals $-\eta\alpha\left(\eta\alpha+1\right)\left(1/l-1\right)l^{\eta\alpha-1}/\left(1-\eta\alpha\left(1/l-1\right)\right)^2<0$) and ℓ, being the inverse of a strictly increasing function, is strictly increasing as well, $\bar{U}_r(R^*)$ is strictly decreasing in R^*.

Therefore, as before, $R^*=0$ if and only if $\bar{U}_m(0)\geq\bar{U}_r(0)$. Since $\underline{l}=\ell(0)$, one obtains, after a series of algebraic manipulations,

$$\begin{aligned}
\bar{U}_r(0) &= \frac{\underline{l}^{\eta\alpha}}{1-\eta\alpha\left(\frac{1}{\underline{l}}-1\right)}\left(\frac{\tau}{N}\right)^{\alpha} \\
&= \frac{1}{\eta\alpha+1}\left(\lambda\left(\frac{(1-\tau)\,\theta\,(1-\theta)\,\gamma}{r}+\frac{\tau}{N}\right)^{\alpha}+(1-\lambda)\left(\frac{\tau}{N}\right)^{\alpha}\right) \qquad (43) \\
&\quad\times\left(\frac{\eta\alpha}{\eta\alpha+1}\left(1+\frac{1}{\lambda}\frac{\left(\frac{\tau}{N}\right)^{\alpha}}{\left(\frac{(1-\tau)\theta(1-\theta)\gamma}{r}+\frac{\tau}{N}\right)^{\alpha}-\left(\frac{\tau}{N}\right)^{\alpha}}\right)\right)^{\eta\alpha},
\end{aligned}$$

so that the condition for $R^* = 0$ becomes

$$\left(\left(\frac{(1-\tau)\,\theta^2}{H}+\frac{\tau}{N}\right)\left(\frac{\eta}{\eta+1}\left(1+\frac{\frac{\tau}{N}}{(\eta+1)\,\frac{(1-\tau)\theta^2}{H}+\eta\frac{\tau}{N}}\right)\right)^{\eta}\right)^{\alpha} \geq \bar{U}_r\,(0)\,. \qquad (44)$$

It is a trivial task to check that it so happens that, when $\eta = 0$ is plugged in (44), it becomes equivalent to (40). Also, when $\tau = 0$ is plugged in (44), it becomes equivalent to (38). We have therefore obtained the following

Proposition 4 *If* $(p, v, w, V, \tau, B, \xi, \chi, A, C_u, C_m, C_0, C_1, l_u^*, l_m^*, l_r^*, R^*, x, Y)$ *is a stationary equilibrium of this economy, then the research effort* $(1 - l_r^*)\,R^*$ *and the expected growth rate are zero if, and only if, the condition* $\tau \geq \bar{\tau}$ *or (44) hold.*

4 Comparative statics

4.1 Impacts of redistribution on research effort

García-Peñalosa and Wen (2008) discussed two different effects that raising τ can have on research: the incentive effect, that discourages skilled agents to engage in research due to the tax charged on the value of the innovation, and the insurance effect, which partially protects researchers from failures and thus encourages R&D. They showed that the insurance effect is dominant for all levels of taxation (as long as λ is small), and showed that even a τ near 100% ($\bar{\tau} = 1$ in their model) would generate more research effort (number of researchers, in their model) and expected growth than laissez-faire. We propose the consideration of a third effect: the impact of taxation on researchers' choice of leisure.

Proposition 5 *In a stationary equilibrium of this economy with $\tau < \bar{\tau}$ and in which (44) does not hold, l_r^* is strictly increasing in τ.*

Proof. In the works. ■

We shall refer to the effect expressed in this proposition as the *leisure effect of taxation*: being a normal good, the demand for leisure will rise if transfers are raised. Thus, in this more egalitarian society, researchers will end up doing less research.

This proposition qualifies the aforementioned result of García-Peñalosa and Wen (2008) (their Proposition 1(iii)), in that it shows why raising τ indefinitely (or up to $\bar{\tau}$, in our model) should not necessarily bring higher expected growth than $\tau = 0$.

As for the net effect of taxation on the number of researchers, as we shall see shortly, it is ambiguous. In that way, the same will be true of its effect on research effort.

4.2 Social planner

In order to have a benchmark for welfare comparisons, we briefly consider a social planner who chooses the number of researchers in the economy and the level of consumption and leisure of each agent so as to maximize a utilitarian welfare function given by

$$W = \sum_{t=0}^{\infty} \delta^t W_t,$$

where δ is the social discount factor and

$$W_t = LU\left(C_{u,t}, l_{u,t}\right) + \left(H - R_t\right) U\left(C_{m,t}, l_{m,t}\right) + R_t U\left(C_{r,t}, l_{r,t}\right). \tag{45}$$

Recall that there is no saving mechanism in this economy, whence the resource constraint

$$LC_{u,t} + (H - R_t) C_{m,t} + R_t C_{r,t} = Y_t \tag{46}$$

must hold at every period $t \in \mathbb{N}$. The technology available at t is informed by the production function $Y_t = A_t \left((1 - l_{m,t}) (H - R_t) \right)^{\theta} \left((1 - l_{u,t}) L \right)^{1-\theta}$.

Hence, the planner's problem is essentially static. Define $\bar{C}_i = C_{i,t}/Y_t$ for $i \in \{u, m, r\}$. Then (45) can be rewritten as

$$\begin{aligned} W_t = \quad & \left(A_t \left((1 - l_m) (H - R) \right)^{\theta} \left((1 - l_u) L \right)^{1-\theta} \right)^{\alpha} \\ & \times \left(L \left(\bar{C}_u l_u^{\eta} \right)^{\alpha} + (H - R) \left(\bar{C}_m l_m^{\eta} \right)^{\alpha} + R \left(\bar{C}_r l_r^{\eta} \right)^{\alpha} \right). \end{aligned}$$

Except for the productivity index A_t, all terms above are time invariant. Thus, the present value of the expected welfare can be written as

$$\begin{aligned} W &= \sum_{t=0}^{\infty} \delta^t \sum_{s=0}^{t} \frac{t!}{s!\,(s-t)!} \left(\lambda (1 - l_r) R \right)^s \left(1 - \lambda (1 - l_r) R \right)^{t-s} \left(\gamma^{\alpha} \right)^s W_0 \\ &= W_0 \sum_{t=0}^{\infty} \delta^t \sum_{s=0}^{t} \frac{t!}{s!\,(s-t)!} \left(\gamma^{\alpha} \lambda (1 - l_r) R \right)^s \left(1 - \lambda (1 - l_r) R \right)^{t-s} \\ &= W_0 \sum_{t=0}^{\infty} \delta^t \left(\gamma^{\alpha} \lambda (1 - l_r) R + \left(1 - \lambda (1 - l_r) R \right) \right)^t \\ &= W_0 \sum_{t=0}^{\infty} \delta^t \left(1 + (\gamma^{\alpha} - 1) \lambda (1 - l_r) R \right)^t = \frac{W_0}{1 - \delta \left(1 + (\gamma^{\alpha} - 1) \lambda (1 - l_r) R \right)} \\ &= \frac{\left(A_0 \left((1 - l_m) (H - R) \right)^{\theta} \left((1 - l_u) L \right)^{1-\theta} \right)^{\alpha}}{1 - \delta \left(1 + (\gamma^{\alpha} - 1) \lambda (1 - l_r) R \right)} \\ & \quad \times \left(L \left(\bar{C}_u l_u^{\eta} \right)^{\alpha} + (H - R) \left(\bar{C}_m l_m^{\eta} \right)^{\alpha} + R \left(\bar{C}_r l_r^{\eta} \right)^{\alpha} \right), \end{aligned} \tag{47}$$

where we have employed the Binomial Theorem and the formula for an infinite sum of a geometric progression.

The expression above provides some intuition on the factors that impact welfare. The numerator of the first term is the initial product, while the denominator can be understood as a combination of the social discount factor and the rate of innovation (which determines the expected growth rate, as seen in subsection 3.2). The second term captures risk exposure and redistributive effects. As in García-Peñalosa and Wen (2008), research effort has a negative impact on welfare for reducing initial product, but also has a positive impact for increasing the expected innovation (and hence growth) rate.

4.3 Inequality

In order to measure inequality in our simulations, we use the Gini coefficient. It is given by

$$I = \frac{N}{2Y} \sum_{i=1}^{G} \sum_{j=1}^{G} |Y_i - Y_j| \, n_i n_j, \tag{48}$$

where G is the number of different groups in the population, Y_i is the income of group i and n_i is the fraction of the population that belongs to group i.

When an innovation occurs, there are four income groups: the unskilled individuals with income (32), the manufacturers with income (33), the successful innovators with income (35) and the unsuccessful researchers with income (34). Plugging these expressions in (48) yields

$$I_1 = \frac{1-\tau}{2N} \left(\begin{array}{c} \theta^2 L - (1-\theta)(H - R^*) \\ +\theta(1-\theta)\left(\frac{\gamma(N-1)}{r+\lambda(1-l_r^*)R^*} - (R^*-2)\right) + R^* - 2 \end{array} \right).$$

When there is no innovation, however, there is no successful innovator, whence there are three different income groups and we obtain

$$I_0 = \frac{1-\tau}{N} \left(\theta^2 (L + R^*) - (1-\theta)(H - 2R^*)\right).$$

The probability-weighted Gini coefficient is, then,

$$\lambda(1-l_r^*) R^* I_1 + (1 - \lambda(1-l_r^*) R^*) I_0.$$

4.4 Parameters

We make use of the exact same parameter values as in García-Peñalosa and Wen (2008) to facilitate comparisons, except for the leisure elasticity, which was introduced in this model. We use a low value for leisure elasticity ($\eta = 0.05$) and show that, even this small change in parameters, results change significantly. The social and private discount rate are identical and one period of time corresponds to ten years. Table 1 presents the values of these parameters.[5]

[5] Here, $\sigma := 1 - \alpha$.

θ=0.25	γ=1.8	r=1.5
σ=0.1	η=0.05	δ=0.4
λ=0.0005	H=1500	N=20000

Table 1: Baseline parameters

4.5 Results

Figure 1 presents several stationary-equilibrium variables, for different tax rates. The horizontal axis of all graphs correspond to the tax rate, in percentage points. Growth rate and leisure are given in percentage points as well, while welfare and initial output are defined as deviation rates from their laissez-faire values. Inequality corresponds to the probability-weighted Gini coefficient. Purple curves correspond to García-Peñalosa and Wen's (2008) model (with $\eta = 0$), while the blue curves correspond to the present model (with $\eta = 0.05$).

García-Peñalosa and Wen (2008) argue that the number of researchers (whence, in their model, also the growth rate) is always increasing in τ due to the insurance effect, that encourages skilled individuals to engage in research. This effect still exists in our model with positive η, but it is no longer dominant for all levels of taxation. The graph shows that the expected growth rate, and therefore research effort, increases with taxation up to a certain point, then starts to decrease. At first, the number of researchers increases, stimulating growth. It eventually collapses, however. For a high enough income tax rate (approximately 90.6%), (44) holds, whence $R^* = 0$.

For low tax rates, income inequality increases relative to laissez-faire, but for higher tax rates it decreases. Initial output decreases in taxation for low tax rates due to skilled individuals shifting to the research sector, and for high tax rates because of agents' increasing leisure. Although the same goes for the model with $\eta = 0$, the inelastic labor supply smooths the fall of initial output. The welfare maximizing tax rate is not 99.8% as in García-Peñalosa and Wen (2008), but around 58%. A more detailed analysis of how leisure elasticity affects welfare is presented in the next subsection.

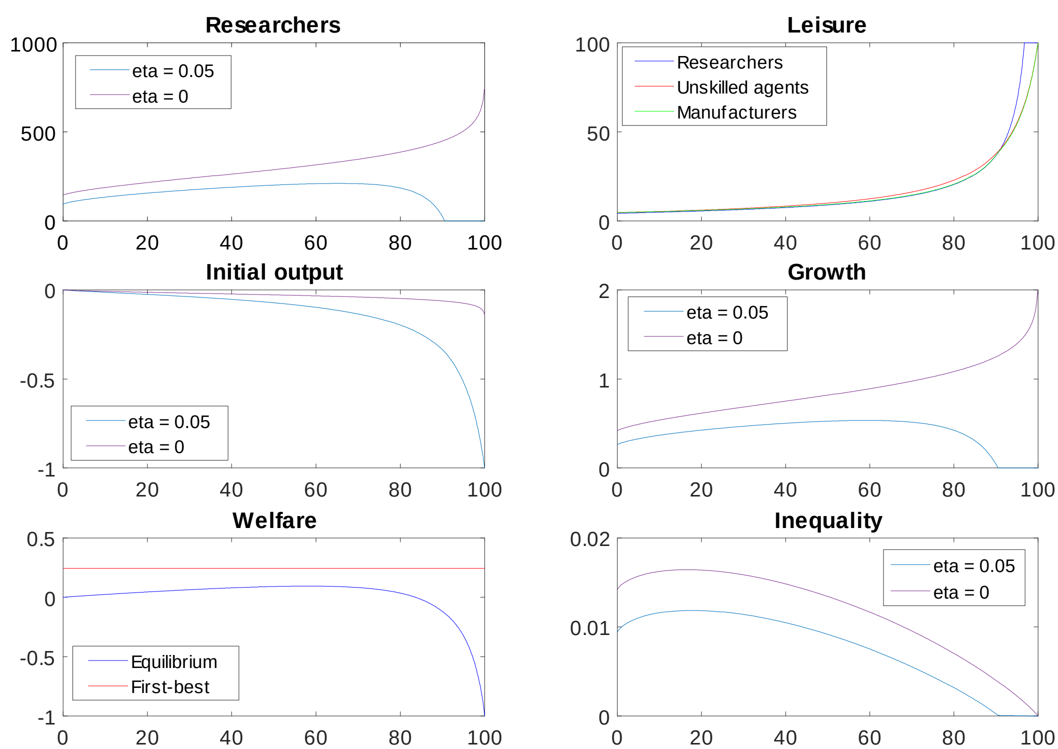

Figure 1: Equilibrium results for different values of the tax rate

4.6 Optimal tax rate and first-best welfare

We define the government's optimal tax rate as the welfare maximizing tax rate under the redistribution policy described in subsection 2.4. The third graph of Figure 1 shows the difference between the optimal tax (the highest point of the blue curve) and the first-best welfare (the red line). García-Peñalosa and Wen (2008) suggest that this difference is very small (first-best welfare is 22.67% greater than in laissez-faire, while their optimal tax rate welfare is 22.63%). However, if we take leisure into consideration, we see that, as η is increased, the greater is the difference between the optimal tax rate welfare and the first-best welfare. Figure 2 shows what happens to this difference as we increase leisure elasticity (η).[6]

[6] As with any welfare level comparison, the reader should take Figure 2 with a pinch of salt. The graph's purpose is simply to suggest how sensitive the aforementioned result of García-Peñalosa and Wen (2008) would be to the parameter η.

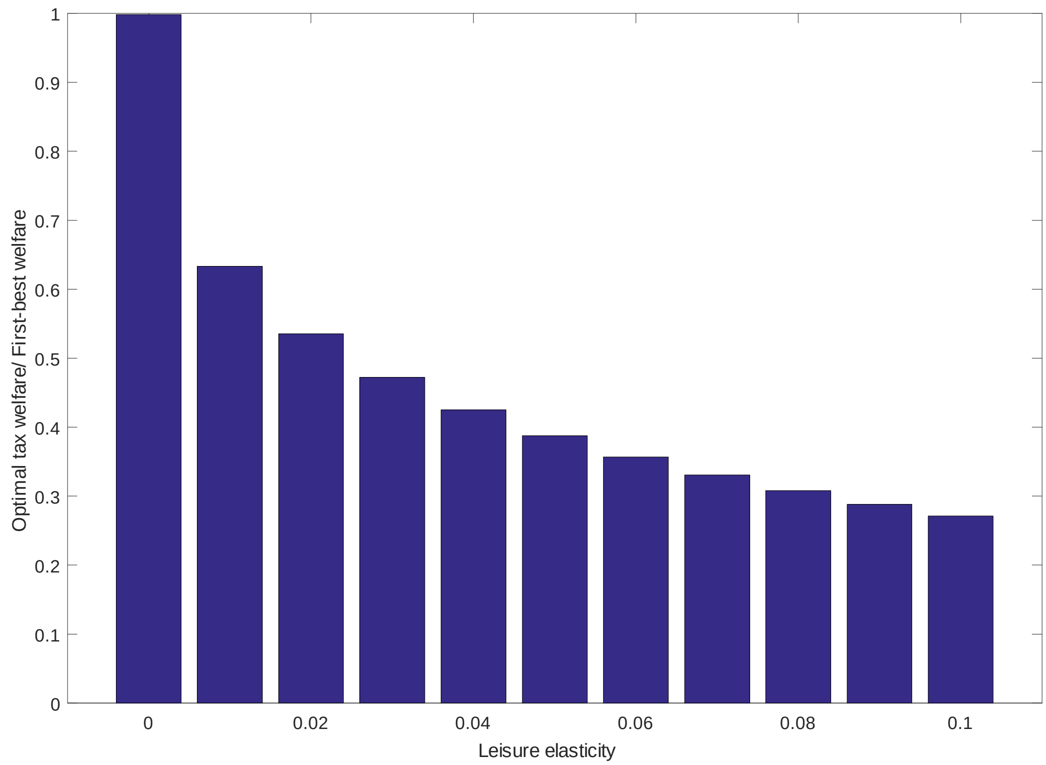

Figure 2: Ratio between optimal tax welfare and first-best welfare for different values of leisure elasticity

4.7 Alternative redistribution policies

If we consider a framework with the possibility of lump-sum taxation and in which the government can transfer different amounts to each agent according to their sector, the results above would still stand. The greater the leisure elasticity, the furthest away from the first best will the economy be. In this new framework, and still with $\eta = 0.05$, optimal redistribution welfare would represent 63% of first-best welfare (while in the original model with a linear income tax and equal redistribution, it reaches 37% of the first-best welfare). That is, although there is a substantial welfare gain in introducing more complex taxation and transfer schemes, it would still not be enough in order to approximate the first best.

5 Concluding remarks

We have studied the impacts of redistributive taxation in a Schumpeterian growth model with risk-averse agents and elastic labor supply. Under an equal redistribution policy, increasing taxation relative to laissez-faire provides insurance to researchers who engage in risky income-generating activities, promoting growth and welfare, but only up to a certain tax rate. For high levels of the tax rate, the negative effects of taxation on research (namely, the incentive effect and the leisure effect, both discouraging skilled agents to engage in research) dominate the insurance effect, and excessive tax rates can generate zero research effort and growth. We have also proven that, as the tax rate converges to 1, the expected growth rate converges to zero, in contrast to García-Peñalosa and Wen's (2008) result that the expected growth rate is greater when τ approaches 100% than it is in laissez-faire equilibrium.

We have found that introducing a leisure elasticity parameter of small magnitude has a great impact on the optimal tax rate (with a 0.05 parameter, optimal tax rate shifts from 99.8% to 58%), and that equal redistribution can only generate close to first-best welfare in a model with inelastic labor supply. We have showed that the greater the leisure elasticity parameter, the harder it is to achieve first-best welfare with redistribution policies (which remains true even in an economy with lump-sum taxation and differentiation in transfers between agents).

Therefore, alternative policies other than income redistribution may be necessary, in order to achieve an equilibrium allocation closer to the first best. For instance, patent protection (for more than one period) and the distribution of bonuses only to successful innovators are possibilities that may be considered.

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