Department of Economics - FEA/USP

Human Capital Accumulation and Output Growth in Demand-led Macrodynamics

GILBERTO TADEU LIMA

LAURA BARBOSA DE CARVALHO

GUSTAVO PEREIRA SERRA

WORKING PAPER SERIES Nº 2024-23

DEPARTMENT OF ECONOMICS, FEA-USP
WORKING PAPER Nº 2024-23

Human Capital Accumulation and Output Growth in Demand-led Macrodynamics

Gilberto Tadeu Lima (giltadeu@usp.br)

Laura Barbosa de Carvalho (lcarvalho@usp.br)

Gustavo Pereira Serra (g.serra@unesp.br)

Abstract:

This paper sets forth two macrodynamic models of capacity utilization and output growth with human capital formation having a positive impact on labor productivity. Human capital formation is costly, however, and two financing mechanisms are considered, one in each model. In the first model, public investment in human capital is financed by some part of income tax revenues, configuring a further source of aggregate demand in addition to private consumption, government consumption, and private investment in physical capital. The second model features public investment in human capital formation financed by all income tax revenues supplemented by workers' own investment in human capital financed through debt. Hence public and private investment in human capital configure further sources of aggregate demand formation, together with private consumption and investment in physical capital. Analogously to firms' desired investment in physical capital, workers' desired investment in human capital is independent from saving out of current income. The excess of workers' desired investment in human capital formation over the public investment in human capital is financed through debt made available by an endogenous supply of credit money. Although this second model is not intended to describe specifically debt-financed human capital formation through student loans, it incorporates some features of the U.S. experience with student debt such as a fixed interest rate and income-based repayment.

Keywords: Human capital; household debt; capacity utilization; employment rate; output growth.

JEL Codes: E12, E22, E24, J24.

Human capital accumulation and output growth in demand-led macrodynamics*

Gilberto Tadeu Lima† Laura Barbosa de Carvalho‡

Gustavo Pereira Serra§

August 26, 2024

1 Introduction

Accumulation of human capital (or of knowledge more broadly) features as a driver of economic growth in fundamental models in contemporary mainstream theory. Mankiw et al. (1992) incorporate accumulable human capital, along with physical capital and labor, as another production factor into a model featuring constant returns to scale in aggregate production. The steady-state level but not the growth rate of output per worker varies positively with the level of physical capital per worker and the level of human capital per worker. In Lucas (1988), human capital accumulation, by exhibiting constant returns to the existing stock of human capital, is a source of sustained long-run growth. Romer (1990) shows that the production of new ideas or knowledge, which owing to their non-rivalrous nature gives rise to increasing returns to scale in aggregate production, lies at the heart of sustained long-run growth.

However, mainstream growth models usually assume that the level of economic activity is supply-determined, with full utilization of existing capacity in capital (broadly defined) and labor services being the normal state of affairs. These mainstream models ignore or bypass the role of aggregate demand in output growth and the joint but separate effect of investment in human capital on aggregate supply and demand. Meanwhile, demand-led approaches to output growth have typically neglected human capital formation through education or schooling on the grounds that it is too narrowly focused on the supply-sided. This ignores the potential impacts of human capital accumulation on labor productivity and the bargaining power of workers in wage negotiations, and so ultimately on the functional distribution of income and hence aggregate demand formation and the level of economic activity.

In a pioneering contribution, Dutt (2010) explicitly formalizes the process of skill acquisition in a demand-led model, in which both the number of high-skilled and low-skilled workers

*This paper was prepared for the second edition of the *Handbook of Alternative Theories of Economic Growth* edited by Mark Setterfield, to whom we are quite grateful for useful suggestions.

†Department of Economics, University of São Paulo, Brazil. E-mail: giltadeu@usp.br

‡Department of Economics, University of São Paulo, Brazil. E-mail: lcarvalho@usp.br

§Department of Economics, São Paulo State University (UNESP), Brazil. E-mail: g.serra@unesp.br

and their wages vary over time. Skill acquisition thus impacts the interaction between income distribution and output growth. The output growth and distributional effects of education are also explored in Dutt and Veneziani (2019; 2020) but using a classical model of the sort outlined in chapter 1, it being found that human capital accumulation does not necessarily increase output growth or reduce inequality across workers in the long run. Lima et al. (2021) incorporate accumulation of human capital through universal public education provided by a balanced-budget government into a demand-led model of capacity utilization, income distribution and output growth. The level of education, as measured by the stock of human capital, positively impacts on both workers' productivity in output production and their bargaining power in wage negotiations. Serra (2023) develops a demand-led model where workers borrow to consume and invest in human capital formation. The model is calibrated using data for the U.S. to simulate the short- and long-term macroeconomic effects of policies such as student loan forgiveness. Carvalho et al. (2024) incorporate knowledge capital accumulation financed by workers themselves through indebtedness into a demand-led model of physical and knowledge capital utilization and output growth. Average labor productivity varies positively with the average knowledge capital across the labor force, and increases in labor productivity are fully passed on to the real wage. Setterfield (2023) maps out potential links and cross-fertilization between human capital accumulation as explored in Lima et al. (2021) and Serra (2023) and two other new contributions asserting the importance of the supply-side in demand-led output growth: the social reproduction of labor and its emphasis on the broader notion of human capacities (see, e.g., Braunstein et al. (2011)) and the supply-side link according to which the functional distribution of income impacts on both the actual and (via technical change) potential rates of output growth (see, e.g., Rada et al. (2021)). Employing the model in Braunstein et al. (2011), Setterfield (2023) suggests that feminist macroeconomic theory offers a nascent general framework for synthesizing these three new contributions.[1]

This chapter sets forth two macrodynamic models of capacity utilization and output growth with human capital formation having a positive impact on labor productivity. Human capital formation is costly, however, and two financing mechanisms are considered, one in each model. In the first model, public investment in human capital is financed by some part of income tax revenues, configuring a further source of aggregate demand in addition to private consumption, government consumption, and private investment in physical capital. The second model features public investment in human capital formation financed by all income tax revenues supplemented by workers' own investment in human capital financed through debt. Hence public and private investment in human capital configure further sources of aggregate demand formation, together with private consumption and investment in physical capital. Analogously to firms' desired investment in physical capital, workers' desired investment in human capital is independent from saving out of current income. The excess of workers' desired investment in human capital formation over the public investment in human capital is financed through debt made available by an endogenous supply of credit money. Although this second model is not intended to describe specifically debt-financed human capital formation through student loans, it incorporates some features of the U.S. experience with student debt such as a fixed interest rate and income-based repayment.[2]

In both models, the human capital stock is uniformly distributed in the labor force, so that unemployed labor also means unutilized human capital. Firms operate with excess production capacity not solely in physical capital and labor quantities, as frequently assumed in demand-

[1]For further discussion of feminist themes in alternative theories of economic growth, see chapter 21.

[2]A detailed institutional, empirical and modelling analysis of the U.S. experience with student loans is offered in Serra (2021).

led models, but in human capital as well. Labor is always available to firms at the current real wage. The state of the wage bargaining process is such that any increase in labor productivity arising from human capital formation is fully passed on to the real wage, so that the wage share in income remains constant. This constancy results from a steady state of relative bargaining power of workers and capitalists that prevents either of these social classes from lifting its share in income. Though insufficient aggregate demand causes the human capital stock to be underutilized, employed workers receive a full wage return on their human capital. The wage share, which is computed as the ratio of the real wage to labor productivity, measures the wage return on productivity-enhancing human capital. Given that increases in labor productivity brought about by human capital accumulation are fully passed on to the real wage, employed workers succeed in collecting the full wage return on their stock of human capital, so that the functional distribution of income remains constant.[3]

The remainder of this chapter is structured in the following way. The next section presents a simple one-dimensional demand-led macrodynamic model featuring human capital formation through government spending. Section 3 develops a more complete two-dimensional demand-led macrodynamic model, in which human capital formation through government investment is supplemented by workers' own investment in human capital financed through debt. Concluding remarks on possibilities for future research close the chapter.

2 Human capital formation through government spending

The economy is closed to both international trade and international labor and capital flows, producing in a single vertically integrated sector a single good/service for both consumption and investment. The government runs a balanced budget by spending all its tax revenues (out of wage and profit income) on consumption and investment in human capital accumulation. The latter contributes to labor productivity enhancement by taking the form of expenditures on public education in the broadest sense possible, including regular education provided by government schools and universities, professional education for workers offered by government training and development programs, and also the availability of public library services and other public educational services of different kinds.

Two homogeneous factors of production are used by firms in the production of the single good/service, physical capital and labor. The stock of human capital remains uniformly distributed in the labor force due to the externalities associated with public expenditures on human capital formation. These production inputs are combined through a fixed-coefficient technology:

$$X = [Kv, La(h)], \tag{1}$$

where X is the output level, K is the stock of physical capital, L is the employment level, v is the full-capacity output to physical capital ratio, which is an exogenously fixed technological parameter, $h = H/N$ is the human capital stock to labor force ratio (or average human capital in the labor force), and $a(h) = X/L$ is the output to labor ratio (or labor productivity), which

[3]See Lima et al. (2021) for a demand-led model in which human capital impacts positively on labor productivity and workers' bargaining power. Nominal wage and price inflation are determined within an accelerationist framework of endogenous conflicting claims on real income, with the result that the wage share is endogenous.

varies endogenously with the average human capital in the labor force. The technical coefficient v is normalized to one, and we measure the rate of physical capital capacity utilization, u, by the output to capital ratio, X/K. In the production function in (1), we have that $a(0) = 0$, $a'(h) > 0$ and $a''(h) \leq 0$. The existence of positive human capital externalities and spillovers implies that unemployed workers are as skilled (or human capital endowed) as the employed ones. Thus, the employment rate represented by $e = L/N$, which is determined by aggregate demand, also measures the rate of human capital utilization. Though we consider only the case in which there is insufficient aggregate demand to yield full utilization of the available human capital capacity at the current real wage, there is no human capital depreciation or labor deskilling.[4]

Considering that we are dealing with a single sector economy, the 'production' of human capital does not constitute another productive sector. The single good/service that can be used for both consumption and physical capital accumulation purposes can also be used for human capital accumulation. Conceptually, we conceive of the government as doing no production, owning no capital, and hiring no labor. Instead, the government simply buys a flow of the single good/service from firms at the current price level. These purchased goods/services, which the government makes available to workers free of charge, correspond to the inputs that matter for human capital accumulation. In long-run equilibrium, therefore, output growth can be measured by the growth rate of either kind of capital, given that both physical and human capital grow at the same rate in the long-run equilibrium.[5]

The economy is composed of two social classes, firm-owner capitalists and labor-selling workers, who receive profits and wages, respectively. The functional division of pre-tax aggregate income is given by:

$$X = VL + R, \tag{2}$$

where V is the pre-tax real wage and R is the level of pre-tax aggregate profits. From (1) and (2), the share of labor in pre-tax income, σ, is given by:

$$\sigma = \frac{V}{a(h)}. \tag{3}$$

As intimated earlier, any increase in labor productivity resulting from human capital accumulation is fully passed on to the wage rate, so that the wage share is constant.

Firms produce (and hire labor) according to aggregate demand. Considering that we model only the case featuring excess productive capacity (in labor and overall capital), employment is determined by production:

[4]See (Serra, 2021, Chapter 2) for a demand-led model of capacity utilization and output growth with human capital accumulation by heterogeneously skilled workers. The paper innovates by treating debt-financed human capital accumulation as a strategy for keeping up with the Joneses, which has implications for the dynamics of wage inequality.

[5]A more inclusive model could also feature investment in entrepreneurial human capital financed either by the government or by capitalists themselves. This could positively affect physical capital accumulation or even the full-capacity output to physical capital ratio in (1). Ehrlich et al. (2017) provide evidence that investment in entrepreneurial human capital may contribute positively to long-run output growth.

$$L = \frac{X}{a(h)}. \tag{4}$$

The employment rate is linked to the state of the goods/services market as follows:

$$e = \frac{L}{X}\frac{X}{K}\frac{K}{N} = uk. \tag{5}$$

where k is the ratio of physical capital to labor force in productivity units, $k = K/(Na(h))$. This formal link between u and e is necessary given that the fixed-coefficient nature of the technology implies that an increase in output in the short run will necessarily be accompanied by an increase in employment. Recall that as the stock of human capital is uniformly distributed in the labor force, the employment rate also measures the degree of utilization of the stock of human capital.

Firms' decisions regarding the accumulation of physical capital are made independently from any prior savings. The implied desired growth rate of the stock of physical capital, g_K, assuming no depreciation, is given by:

$$g_K = \frac{I_K}{K} = \alpha + \beta u, \tag{6}$$

where α and β are strictly positive parameters and I_K denotes firms' desired investment in physical capital.

At any point in time, the technological parameters are given, having resulted from previous human and physical capital accumulation. Over time, however, human capital accumulation occurs, which results in labor productivity growing at a proportionate rate $\hat{a}$:

$$\hat{a}(h) = \rho(\hat{h}), \tag{7}$$

where $\hat{h}$ is the growth rate of average human capital. The level of labor productivity has a one-to-one correspondence with average human capital, meaning that $a(h) = h$ and hence $\hat{a} = \hat{h}$, whereas the size of the labor force (or size of the labor force in natural units), N, is constant and normalized to one. It follows that the ratio of physical capital to labor force in productivity units in (5) is ultimately represented by $k = K/H$. Nonetheless, there is always excess supply of labor in both natural and productivity units.

Human capital formation is fully financed by the government using an exogenously fixed fraction $0 < \eta < 1$ of its receipts from taxes levied on wages and profits (taxation is costless for the government and there is no tax evasion), with the government's propensity to save being equal to zero. The assumption of a balanced-budget is restrictive, but it is convenient to sharpen focus on the issue of the impact of human capital formation on the rates of capacity utilization and output growth. Workers receive wage income, which is taxed at an exogenously fixed rate $0 < \tau < 1$ and consume all their disposable income. Capitalists receive profit income, which is the entire surplus over the wage bill, pay taxes at the same rate τ, and have a strictly higher saving propensity than workers. This lack of progressivity in taxation (as wage income per worker is assumed strictly lower than profit income per capitalist) is partially compensated by the fact that a fraction η of income tax revenues is used to finance productivity-enhancing human capital accumulation (recall that any rise in labor productivity is fully passed on to the real wage). Thus, the induced level of aggregate investment in human capital created by the government running a balanced budget is given by:

$$I_H = \eta T = \eta[\tau V L + \tau(X - VL)] = \eta\tau X, \tag{8}$$

where I_H is government investment in human capital formation and T denotes government tax revenues. Therefore, the public investment rate represented by $I_H/X = \eta\tau$ is an exogenously fixed rate which varies positively with the tax rate and the fraction of government tax revenues allocated to investment in human capital formation. However, using (5) and (8) and recalling that $k = K/H$, the actual rate of growth of the stock of human capital, g_H, is endogenously given by:

$$g_H = \frac{I_H}{H} = \frac{I_H}{X}\frac{X}{K}\frac{K}{H} = \eta\tau uk = \eta\tau e. \tag{9}$$

Therefore, the rate of growth of the stock of human capital features an accelerator effect, as the employment rate e also measures the rate of human capital utilization. The intuition for this result is that government investment in human capital formation is financed out of government income tax revenues. For future reference, government investment in human capital formation as a proportion of the physical capital stock is given by:

$$\frac{I_H}{K} = \frac{I_H}{X}\frac{X}{K} = \eta\tau u, \tag{10}$$

which of course also features an accelerator effect, now operating through the rate of physical capital utilization.

2.1 Short-run equilibrium

The short-run is defined as the time period in which the stock of physical capital, K, the stock of human capital, H, and the output-labor ratio, a, can all be taken as given. The supply-demand equilibrium in the goods/services market is:

$$X = C_w + C_p + G_c + I_H + I_K, \tag{11}$$

where C_w is workers' consumption, C_p is capitalists' consumption, and $G_c = (1-\eta)T$ is government's consumption. Human capital formation financed out of government tax revenues is a source of aggregate demand together with investment in physical capital by firms and consumption by workers, capitalists, and the government itself. All these expenditures are on the single good/service which can be used for consumption and investment purposes.

Given that workers do not save and capitalists save an exogenously fixed fraction $0 < s < 1$ of their post-tax profit income, consumption by these classes is respectively represented by:

$$C_w = (1-\tau)VL, \tag{12}$$

and:

$$C_p = (1-s)(1-\tau)(X - VL). \tag{13}$$

Normalizing (11)-(13) by the physical capital stock and using (6) and (10) yields:

$$u = [(1-\tau)\sigma u] + [(1-s)(1-\tau)(1-\sigma)u] + \tau u + \alpha + \beta u. \tag{14}$$

Since aggregate output is determined by aggregate demand, whereas labor is always in excess supply, physical capital utilization adjusts for the goods/services market short-run equilibrium defined in (14) to be achieved. The short-run equilibrium value of physical capital utilization rate is:

$$u^* = \frac{\alpha}{s(1-\tau)(1-\sigma) - \beta}. \tag{15}$$

For stability of the short-run equilibrium value of physical capital utilization, we suppose that $s(1-\tau)(1-\sigma)-\beta > 0$, which is equivalent to a strictly positive denominator in (15). The substance of this Keynesian stability condition is that after-tax saving (out of profits) must react strictly more than investment in physical capital to a change in physical capital utilization, so that any excess demand or supply is eliminated instead of exacerbated by changes in physical capital utilization.

As the government spends all its tax revenue on the single good/service either for consumption or investment in human capital, physical capital utilization varies positively with the tax rate in the short-run equilibrium. The running of a balanced budget by the government means that its propensity to save out of income tax revenues is equal to zero, and as workers do not save but capitalists do, a strictly higher tax rate reduces aggregate demand leakages. In this balanced-budget context, taxation is a mechanism of forced dissaving. The short-run equilibrium value of physical capital utilization varies positively with the parameters of firms' desired growth rate of the stock of physical capital, α and β, and the wage share, σ, and negatively with capitalists' saving propensity, s. Considering that k is given in the short run, all these comparative statics also apply to the short-run equilibrium employment rate, $e^* = u^*k$. Changes in the composition of government expenditures between consumption and investment in human capital as measured by $(\eta, 1-\eta)$, in turn, do not have any effect on capacity utilization in either physical or human capital in the short-run equilibrium.

2.2 Long-run equilibrium

In the long run, the short-run equilibrium values of the endogenous variables are always attained, with the economy moving over time due to changes in the stock of physical capital, K, and the stock of human capital, H, and hence in labor productivity, a (recall that labor productivity is equal to the average human capital in the labor force, $a = h$, and the labor force is normalized to one, so we have $k = K/H$ in (5)). Using (5)-(6) and (9), the long-run dynamics are driven by:

$$\hat{k} = \hat{K} - \hat{H} = \alpha + \beta u - \eta\tau u k. \tag{16}$$

Solving (16) for the long-run equilibrium value of k characterized by $\hat{k} = 0$ gives:

$$k^* = \frac{\alpha + \beta u^*}{\eta\tau u^*} = \frac{s(1-\tau)(1-\sigma)}{\eta\tau}. \tag{17}$$

The unique long-run equilibrium in (17) is stable, as $\partial\hat{k}/\partial k = -\eta\tau u^* < 0$. Using (5), (15) and (17), the employment rate (which also measures human capital utilization) in the long-run equilibrium, e^{**}, is given by:

$$e^{**} = u^* k^* = \frac{\alpha + \beta u^*}{\eta\tau}. \tag{18}$$

Therefore, given u^*, which is the equilibrium physical capital utilization rate in both the short and long run, the long-run equilibrium value of the employment rate in (18) varies positively with the parameters of firms' desired growth rate of the stock of physical capital, α and β, and negatively with the tax rate, τ, and the fraction of government expenditures allocated to investment in human capital, η. As u^* in (15) varies positively with the two former parameters and the wage share, σ, and negatively with capitalists' saving rate, s, while e^{**} varies positively with u^*, the employment rate varies positively with α, β and σ, and negatively with s in both the short- and long-run equilibrium (recall that k is given in the short run). As per (6) the numerator in (18) is the output growth rate in long-run equilibrium, g^*, the latter also varies positively with α, β and σ, and negatively with s. As is the case with u^*, note that g^* also varies positively with the tax rate, τ.

Recall from the preceding sub-section that changes in the composition of government expenditures between consumption and investment in human capital as measured by $(\eta, 1-\eta)$ do not have any impact on the rates of physical capital utilization and employment in the short-run equilibrium. In fact, the same neutrality result applies to the long-run equilibrium values of the rates of physical capital utilization and output growth. Nevertheless, per (18), the long-run equilibrium employment rate varies negatively with the fraction of government tax revenues allocated to investment in human capital formation, η. A rise in the latter, despite leaving the long-run equilibrium physical capacity utilization unchanged, reduces the long-run equilibrium ratio of physical to human capital in (17), therefore raising the availability of human capital relative to physical capital.

Finally, the impact of a change in the tax rate on the long-run equilibrium employment rate is ambiguous. Although in (18) the long-run equilibrium physical capital utilization rate varies positively with the tax rate, the long-run equilibrium physical to human capital ratio varies negatively with the same variable:

$$\frac{\partial k^*}{\partial \tau} = -\frac{s(1-\sigma)}{\eta\tau^2} < 0. \tag{19}$$

The impact of a change in the tax rate on the long-run equilibrium employment rate is therefore given by:

$$\frac{\partial e^{**}}{\partial \tau} = \frac{\partial u^*}{\partial \tau} k^* + u^* \frac{\partial k^*}{\partial \tau}. \tag{20}$$

We can re-write the expression above as follows:

$$\varepsilon_{e^{**},\tau} = \varepsilon_{u^*,\tau} + \varepsilon_{k^*,\tau}, \tag{21}$$

where $\varepsilon_{e^{**},\tau} = (\partial e^{**}/\partial \tau)(\tau/e^{**})$ is the tax rate elasticity of the long-run equilibrium employment rate and the two terms on the right-hand side, which are strictly positive and strictly negative, respectively, are defined analogously. Therefore, a rise in the tax rate will result in a strictly higher (lower) long-run equilibrium employment rate if the long-run equilibrium physical capital utilization rate is strictly more (less) tax rate elastic than the long-run equilibrium ratio of physical to human capital.

The two preceding results, that the long-run equilibrium employment rate (or human capital utilization rate) varies negatively with the fraction of the government tax revenues allocated to investment in human capital and varies ambiguously with the tax rate, imply that greater availability of labor in human capital units does not automatically create its own greater demand. Recall from (9) that the fraction of government tax revenues allocated to investment in human capital and the tax rate are the parameters that measure the magnitude of the accelerator effect in the human capital accumulation rate.[6]

A further noteworthy result concerns the dynamics underlying the achievement of the ratio of physical to human capital in (17) and hence the employment rate in (18) as stable long-run equilibria. In (16), the growth rate of the stock of human capital (and hence of labor productivity) adjusts to the growth rate of the stock of physical capital, doing so through changes in the ratio of physical to human capital and therefore in the employment rate (human capital utilization rate). Or, to phrase it in Harrodian parlance, the output growth rate warranted by the human capital accumulation rate (labor productivity growth rate) adjusts to the output growth rate warranted by the physical capital accumulation rate, so that the growth rates of output, physical capital and human capital in long-run equilibrium are one and the same. Equality of the actual (equilibrium) and Harrodian natural (maximum rate of growth allowed by growth of the labor force in natural and productivity units) rates of growth is achieved in the long run by means of adjustment of the latter to the former (recall that the size of the labor force is normalized to unity). Human capital accumulation thus constitutes a mechanism capable of solving the first Harrod problem in demand-led output growth models.[7]

[6] Such an ambiguous effect of a change in the tax rate is also found in Lima et al. (2021) in a model in which the functional distribution of income is endogenous to the rates of physical and human capital utilization as well as the average human capital in the labor force. The impact of a strictly higher tax rate on the employment rate (or human capital utilization rate) in long-run equilibrium is strictly negative (ambiguous) when output growth is wage-led (profit-led).

[7] The so-called first Harrod problem is that equality of the actual and natural rates of output growth is possible but unlikely. That human capital accumulation is a mechanism that can solve the first Harrod problem was first formally demonstrated in (Serra, 2021, Chapter 3) in a different demand-led model in which investment in human capital formation is financed by workers themselves, using a portion of wage income supplemented by debt accumulation.

3 Human capital formation through government spending and workers' indebtedness

In the model set forth in the preceding section, investment in human capital formation is fully induced, financed by an exogenously fixed fraction $0 < \eta < 1$ of government tax revenues. In this section, overall investment in human capital formation is partially publicly financed by tax revenues and partially financed by workers themselves via indebtedness. The government allocates all its tax revenues to publicly fund human capital formation, so that $\eta = 1$ as *mutatis mutandis* in Lima et al. (2021), while workers' desired investment in human capital is driven by an independent investment function. The additional human capital formation resulting from workers' desired investment being strictly greater than the respective publicly funded investment is financed by workers themselves via indebtedness, as *mutatis mutandis* in Carvalho et al. (2024) and Serra (2023).

The basic model structure specified in (1)-(7) remains valid, except that the parameter α in (6) is normalized to zero. As in the case of the preceding model, taxes are levied on both wages and profits (taxation is costless and there is no tax evasion), with the government running a balanced-budget. Wage income is again taxed at an exogenously fixed rate $0 < \tau < 1$, but now all disposable wage income is partially consumed and partially allocated to debt servicing. Profit income, the entire surplus over the wage bill, is again taxed at the same rate τ, and not all disposable profit income is consumed. Once again, the lack of progressivity in taxation (considering that wage income per worker is arguably strictly lower than the profit income per capitalist) is partially compensated by the fact all income tax revenues are used to fund productivity-enhancing human capital accumulation in an environment where increases in labor productivity are fully passed on to the real wage.

The level of investment in human capital formation carried out by the government running a balanced budget is given by:

$$I_G = T = [\tau VL + \tau(X - VL)] = \tau X, \tag{22}$$

where I_G is government investment in human capital formation and T denotes government tax revenues. Hence the public investment rate represented by $I_G/X = \tau$ is an exogenously fixed rate equal to the tax rate. Meanwhile, workers' desired investment in human capital formation is given by:

$$I_H = \gamma VL, \tag{23}$$

where γ is a strictly positive parameter. Analogous to specifications of the desired investment in physical capital in the Cambridge U.K. tradition, which typically have current or expected total profits as a positive determinant, workers' desired investment in human capital formation in (23) varies positively with the wage bill (or its expected value, which is then conventionally proxied by its current value). Workers' desired investment in human capital is always realized, as its persistent excess of over publicly financed investment in human capital is financed by workers themselves through indebtedness, which is accommodated by an endogenous supply of credit money. Hence the actual rate of growth of the stock of human capital, g_H, is equivalent to the rate of growth of workers' desired stock of human capital, and is given by:

$$g_H = \frac{I_H}{H} = \gamma \frac{VL}{X} \frac{X}{K} \frac{K}{H} = \gamma \sigma u k = \gamma \sigma e, \tag{24}$$

where e is given by (5) and there is no human capital depreciation or labor deskilling. Note that the rate of accumulation in (24) features an accelerator effect, given that the rate of employment, e, also measures the rate of utilization of the human capital stock. Interestingly, the strength of this accelerator effect varies positively with the wage share, which is explained by workers' desired investment in human capital depending on the wage bill in (23). For future reference, workers' desired investment in human capital as a proportion of the physical capital stock is represented by:

$$\frac{I_H}{K} = \gamma \sigma u, \tag{25}$$

which of course also features an accelerator effect, now operating through the rate of physical capital utilization.

Workers rely on new borrowing to close the gap between their desired investment in human capital and the publicly financed investment in human capital, $I_H > I_G$, so that $\gamma\sigma > \tau$. Debt servicing by workers follows an income-driven repayment plan bearing some similarity to repayment plans applicable to federal student loan payments in the U.S., the Income-Based Repayment Plan (IBR). This repayment plan establishes that debtors allocate a (usually small) percentage of their disposable income to servicing their debt (such small percentage is currently up to 20%).[8] As workers devote the exogenously fixed fraction $0 < 1 - \phi < 1$ $(0 < \phi < 1)$ of their disposable income to debt servicing (consumption), their consumption and the change in the stock of debt they hold are respectively given by:

$$C_w = \phi(1 - \tau)VL, \tag{26}$$

and:

$$\dot{D} = iD + (\gamma\sigma - \tau)X - (1 - \phi)(1 - \tau)VL, \tag{27}$$

where D is the stock of debt in real terms held by workers, so that $\dot{D} = dD/dt$ is the change in this stock, and $i > 0$ denotes the interest rate. Note that with saving representing abstention from consumption, $1 - \phi$ denotes workers' propensity to save out of disposable wage income, which is nevertheless converted into their coefficient of debt repayment or propensity to repay debt.

The stock of debt held by workers is given in the short run but varies over time as described in (27), while the interest rate is an exogenously fixed constant. The stock of debt, D, and the real wage, V, vary over time (the latter varying at the same rate as the rate of growth of labor productivity), while the output level, X, and the employment level, L, are adjusting variables in the short run, as described shortly. It follows that the proportion of disposable wage income that workers divert from consumption for the purpose of debt repayment, $(1 - \phi)(1 - \tau)VL$, may be greater than, equal to or lower than the interest payment due, iD, plus the amount of

[8] See https://studentaid.gov/manage-loans/repayment/plans/income-driven.

their desired investment in human capital financed through debt, $(\gamma\sigma - \tau)X > 0$, in any given short run. The amount of debt repayment is the adjusting variable when the proportion of the disposable wage income that workers divert from consumption to debt repayment is not enough to cover the interest payment plus the amount of their desired investment in human capital financed through debt. This means that debt servicing does not compromise the minimum level of consumption by workers in (26) or the realization of their desired investment in human capital in (23). When $(1-\phi)(1-\tau)VL = iD$, there is no repayment of principal and all of the increase in the stock of debt is equal to the amount of workers' desired investment in human capital financed through debt, $(\gamma\sigma - \tau)X > 0$. When $(1-\phi)(1-\tau)VL > iD$, there is some repayment of principal, and whether the stock of debt will increase, remain the same, or fall depends on whether the excess of $(1-\phi)(1-\tau)VL$ over iD is lower than, equal to, or greater than $(\gamma\sigma - \tau)X > 0$, respectively. Finally, when $(1-\phi)(1-\tau)VL < iD$, there is no repayment of principal and the increase in the stock of debt is equal to $(\gamma\sigma - \tau)X > 0$ plus the unpaid interest cost added to the outstanding principal.

Using (27), the growth rate of the stock of debt is given by:

$$\hat{D} = \frac{\dot{D}}{D} = i + \left[(\gamma\sigma - \tau) - (1-\phi)(1-\tau)\sigma\right](u/\delta), \tag{28}$$

where $\delta = D/K$ denotes the ratio of workers' stock of debt to the stock of physical capital, or simply the debt ratio.

Capitalists receive not only profit income (as they own the physical capital stock employed in production), but also interest income from workers' debt servicing, given that they also operate as financial capitalists. Interest income is not taxed by the government and capitalists save an exogenously fixed fraction of their total (profit and interest) disposable income, allocating what is left to consumption:

$$C_p = (1-s)[(1-\tau)(X - VL) + iD], \tag{29}$$

where, as in the previous section, $0 < s < 1$ is the saving rate of capitalists.[9] Given that $1-\phi$ represents workers' propensity to save out of disposable wage income, it follows from the strictly lower saving capacity of workers that $s > 1-\phi$.

The short-run is defined as the time period in which the stock of physical capital, K, the stock of human capital, H, the output-labor ratio, a, and the stock of debt, D, can all be taken as given. The supply-demand equilibrium in the goods/services market is represented by:

$$X = C_w + C_p + I_H + I_K. \tag{30}$$

Normalizing (26) and (29)-(30) by the physical capital stock and using (25) yields:

[9]Following Dutt (2006) and Kapeller and Schütz (2015), only the interest payment contributes to capitalists' total disposable income, as debt amortization does not have an impact on their purchasing power. The rationale is that, to ensure the stock-flow consistency of the model, given that new borrowing by workers does not reduce capitalists' total disposable income (as lending generates deposits proportionally), debt amortization only affects their assets, with no impact on their total disposable income. There is, by assumption, no propensity to consume out of wealth.

$$u = \phi(1-\tau)\sigma u + (1-s)[(1-\tau)(1-\sigma)u + i\delta] + \gamma\sigma u + \beta u, \tag{31}$$

where it is to be recalled that the parameter α in (6) is now normalized to zero.

Aggregate output is determined by aggregate demand, while labor (along with the human capital uniformly embodied in the labor force) is always in excess supply at the current real wage. Therefore, physical capital utilization adjusts for the goods/services market short-run equilibrium defined in (31) to be achieved. The rate of physical capital utilization in the short-run equilibrium is then given by:

$$u^* = \frac{(1-s)i\delta}{(1-\tau)[(1-\phi)\sigma + s(1-\sigma)] - (\gamma\sigma - \tau) - \beta} = \frac{(1-s)i\delta}{\Omega}, \tag{32}$$

where $\Omega \equiv (1-\tau)[(1-\phi)\sigma + s(1-\sigma)] - (\gamma\sigma - \tau) - \beta$. To guarantee that the demand-led output-adjustment stability condition is satisfied, we suppose that demand leakages (workers' disposable wage income leaking as debt repayment plus capitalists' total disposable income leaking as saving) as a proportion of the physical capital stock are strictly more responsive to changes in physical capital utilization than demand injections as a proportion of the physical capital stock (I_H/K in (25) minus I_G/K, using (22), plus g_K in (6) with $\alpha = 0$). This condition is equivalent to a strictly positive denominator in (32), $\Omega > 0$, which ensures that the short-run equilibrium physical capital utilization in (32) is also strictly positive.

The short-run equilibrium value of physical capital utilization in (32) varies positively with the parameters of the functions describing investment in physical and human capital, β and γ, respectively, and negatively with the saving rate of capitalists, s, and the repayment coefficient of workers, $1 - \phi$. The positive impact associated with the interest rate, i, and the debt ratio, δ, is explained by their positive impact on capitalists' consumption (per (29)) and lack of impact on workers' consumption (per (26)). Meanwhile, an increase in the wage share, holding everything else constant (i.e., without considering the multiplier effects), raises workers' consumption and investment in human capital, but also raises the demand leakage associated with debt repayment by workers, and reduces capitalists' consumption. The ultimate impact on the short-run equilibrium value of physical capital utilization of a change in the wage share is formally represented by the following expression:

$$\frac{\partial u^*}{\partial \sigma} = \frac{i\delta(1-s)[(1-\tau)(s+\phi-1)+\gamma]}{\Omega^2} > 0. \tag{33}$$

Note that the second term in parentheses in (33) can be re-arranged as $s - (1-\phi)$, which is strictly positive given that capitalists' propensity to save out of their total disposable income is strictly greater than worker's propensity to save out of their disposable wage income. Hence the rate of physical capital utilization in the short-run equilibrium is wage-led, as it varies positively with the wage share.

A rise in the tax rate, holding everything else constant (i.e., without considering the multiplier effects), lowers capitalists' and workers' consumption, but reduces the demand leakage represented by debt repayment by workers and raises government tax revenues and hence government investment in human capital, reducing the debt-financed excess of workers' desired investment in human capital over the same investment realized by the government. The ultimate impact on the short-run equilibrium value of physical capital utilization of a change in the wage share is formally represented by the following expression:

$$\frac{\partial u^*}{\partial \tau} = \frac{i\delta(1-s)[(1-\phi)\sigma + s(1-\sigma) - 1]}{\Omega^2} < 0. \tag{34}$$

Notice that the term within brackets in (34) can be re-arranged as $-\phi\sigma - (1-\sigma) + s(1-\sigma)$ and then further re-arranged as $-\phi\sigma - (1-s)(1-\sigma) < 0$, which explains the resulting strictly negative sign. Bearing in mind that k is given in the short run, it follows that all the comparative statics above also apply to the short-run equilibrium employment rate, $e^* = u^*k$.

In contrast to the model set out in the preceding section, the physical capital utilization rate in the short-run equilibrium varies negatively with the tax rate. In both models, a strictly higher tax rate results in a combined fall in capitalists' and workers' consumption, but leads to an increase in government investment in human capital. In the model set forth in this section, such a strictly higher tax rate also results in a decrease in the portion of workers' desired investment in human capital realized through debt financing made available by an endogenous supply of credit money. In the model developed in Section 2, considering that the government runs a balanced-budget and hence there is no public saving, all income tax collection is allocated either to government consumption or to government investment in human capital formation. Therefore, a strictly higher tax rate results in an increase in that part of the profit income which would otherwise be saved that is instead injected back into the income-expenditure flow as government spending (recall that in both models workers do not save). The net effect of a strictly higher tax rate is an increase in aggregate demand formation and thereby in the rates of physical and human capital utilization. In the model developed in this section, the same aggregate demand-boosting effect occurs, but together with an additional effect. Given that workers' desired investment in human capital is not completely satisfied by the respective government investment, they borrow to finance the resulting strictly positive gap. In the context of an endogenous supply of credit money, workers' new borrowing represents a creation of purchasing power *ex nihilo*, independent from previous savings or loanable funds, resulting in a further aggregate demand injection. When the tax rate rises and thereby increases government investment in human capital, workers need to borrow strictly less in order to attain their desired investment in human capital, which contributes negatively to aggregate demand formation through such an additional channel. As shown in (34), the ultimate effect of a strictly higher tax rate on aggregate demand formation and hence physical capacity utilization in short-run equilibrium is strictly negative.

3.1 Long-run equilibrium

In the long run the short-run equilibrium values of the endogenous variables are always attained, with the economy moving over time driven by changes in the stock of physical capital, K, the stock of human capital, H, and hence in labor productivity, a, and the stock of debt held by workers, D. Recall again that the stock of human capital is uniformly distributed among the labor force (the measure of which we have normalized to one) and that the productivity of labor is equal to the average stock of human capital, which together implies that the growth rates of the stock of human capital and labor productivity are one and the same. Hence we can follow the behavior of the system over time by exploring the dynamics of the short-run state variables $k = K/H$ and $\delta = D/K$. Using (5), (6) with $\alpha = 0$, and (28), the long-run dynamics are driven by:

$$\hat{k} = \hat{K} - \hat{H} = \beta u - \gamma\sigma u k, \tag{35}$$

and:

$$\hat{\delta} = \hat{D} - \hat{K} = i + [(\gamma\sigma - \tau) - (1-\phi)(1-\tau)\sigma]\,(u/\delta) - \beta u, \tag{36}$$

where u is given by (32). Setting $\hat{k} = 0$ in (35), and considering from (32) and (36) that $\hat{\delta}$ is not a function of k, we can compute the latter's (unique and strictly positive) long-run equilibrium value:

$$k^* = \frac{\beta}{\gamma\sigma}. \tag{37}$$

Intuitively, the long-run equilibrium physical to human capital ratio in (37) varies positively with the parameter measuring the magnitude of the accelerator effect in the desired rate of physical capital accumulation in (6), β, and negatively with the parameters measuring the magnitude of the accelerator effect in the desired rate of human capital accumulation in (24), where they feature multiplicatively as $\gamma\sigma$.

Solving for $\hat{\delta} = 0$ using (36) and (32) yields the long-run equilibrium value of the debt ratio:

$$\delta^* = \frac{(1-s)[(\gamma\sigma - \tau) - (1-\phi)(1-\tau)\sigma] + \Omega}{\beta(1-s)}, \tag{38}$$

which is strictly positive as we suppose that $(\gamma\sigma - \tau) > (1-\phi)(1-\tau)\sigma$.[10] The equilibrium debt ratio in (38) varies positively (negatively) with parameters contributing negatively (positively) to aggregate demand formation.[11] As demonstrated in Appendix B, it varies positively with the saving rate of capitalists, s (per (B.1)), and negatively with the parameter measuring the accelerator effect in the rate of physical capital accumulation, β (per (B.2)), and each of the parameters measuring the accelerator effect in the rate of human capital accumulation, γ and σ (per (B.3) and (B.4)). The long-run equilibrium debt ratio is invariant with respect to the interest rate and varies positively with the fraction of workers' disposable income allocated to debt repayment, $1-\phi$ (and so negatively with the fraction allocated to consumption, ϕ), giving rise to the *paradox of debt repayment* (Carvalho et al., 2024).[12] Formally, we have:

$$\frac{\partial \delta^*}{\partial \phi} = -\frac{s(1-\tau)\sigma}{\beta(1-s)} < 0. \tag{39}$$

[10]In long-run equilibrium with constant physical to human capital and debt to physical capital ratios, the debt to human capital ratio is also constant. Moreover, given that in the long-run equilibrium physical capital utilization (as measured by the output to physical capital ratio) is constant, the debt to output ratio is also constant.

[11]Appendix A shows that the numerator in (38) can be re-written as $s[(1-\tau)(1-\phi\sigma) - (\gamma\sigma - \tau)] - \beta$. In the comparative statics that follow, we will use whichever version is more convenient.

[12]This paradox was first formally demonstrated in a similar demand-led model without taxation and public investment in human capital formation in Carvalho et al. (2024). In addition to showing the same invariance of the debt ratio with respect to the interest rate, the paper demonstrates that this paradox may vanish under alternative debt servicing behaviors, for example when workers save and forfeit savings to increase debt servicing, which may raise aggregate demand formation.

Meanwhile, a strictly higher tax rate, which also represents a strictly higher government investment in human capital as a proportion of output (per (22)), results in an increase in the ratio of workers' debt to physical capital utilization:

$$\frac{\partial \delta^*}{\partial \tau} = \frac{s\phi\sigma}{\beta(1-s)} > 0. \tag{40}$$

As noted earlier in footnote 10, in the long-run equilibrium with constant debt to physical capital ratio and physical capital utilization rate (as measured by the output to physical capital ratio, the debt to output ratio is also constant. It follows that the debt to wage income ratio, $\omega = (D/VL) = (D/K)(X/VL)(K/X) = \delta/\sigma u$, is also constant in the long-run equilibrium:

$$\omega^* = \frac{\delta^*}{\sigma u^*(\delta^*)} = \frac{\Omega}{(1-s)i\sigma}, \tag{41}$$

where u^* in (32) is evaluated at δ^* in (38). In fact, the debt to wage income ratio is arguably a more representative measure of the financial health of workers. Note that the above-mentioned paradox of debt repayment also applies to the long-run equilibrium value of the debt to wage income ratio:

$$\frac{\partial \omega^*}{\partial \phi} = -\frac{(1-\tau)}{(1-s)i} < 0. \tag{42}$$

It is demonstrated in Appendix C that, as in the case of the debt to physical capital ratio, the debt to wage income ratio varies positively with capitalists' saving rate, s (per (C.1)), and negatively with the parameter measuring the accelerator effect in the rate of physical capital accumulation, β (per (C.2)), and one of the parameters measuring the accelerator effect in the rate of human capital accumulation, γ (per (C.3)). However, unlike in the case of the debt to physical capital ratio, the long-run equilibrium debt to wage income ratio is not invariant with respect to the interest rate, instead varying negatively with it. The intuition is that interest payments made by workers add to capitalists' total disposable income and hence consumption, and new borrowing by workers does not reduce capitalists' disposable income. Also one should bear in mind that the interest rate in question is the one associated with the specific purpose of financing human capital formation. It is not an interest rate which could directly impact on potentially interest-sensitive components of aggregate demand, from which we abstract here. Again as in the case of the debt to physical capital ratio, the debt to wage income ratio in the long-run equilibrium varies positively with the tax rate, which also represents the rate of government investment in human capital (as a proportion of output):

$$\frac{\partial \omega^*}{\partial \tau} = \frac{1-(1-\phi)\sigma - s(1-\sigma)}{(1-s)i\sigma} > 0. \tag{43}$$

Note that the numerator in (43) can be re-arranged as $(1-s)(1-\sigma) + \phi\sigma > 0$, which explains the resulting strictly positive sign in (43).

As for the impact of a change in the wage share on the debt to wage income ratio, we have:

$$\frac{\partial \omega^*}{\partial \sigma} = -\frac{[(s-\beta)+(1-s)\tau]}{(1-s)i\sigma^2}, \tag{44}$$

the sign of which is ambiguous. It is empirically plausible to consider that $s > \beta$, so that the impact of a change in the wage share on the debt to wage income ratio is likely negative.[13]

Let us now verify whether the long-run equilibrium (k^*, δ^*) is stable. The elements of the Jacobian matrix of partial derivatives evaluated at the long-run equilibrium represented by (37) and (38) are the following:

$$J_{11} = \frac{\partial \hat{k}}{\partial k} = -\gamma\sigma u^* < 0, \tag{45}$$

$$J_{12} = \frac{\partial \hat{k}}{\partial \delta} = (\beta - \gamma\sigma k^*)(\partial u^*/\partial \delta) = 0, \tag{46}$$

$$J_{21} = \frac{\partial \hat{\delta}}{\partial k} = 0, \tag{47}$$

$$J_{22} = \frac{\partial \hat{\delta}}{\partial \delta} = \Big[(\gamma\sigma-\tau)-(1-\phi)(1-\tau)\sigma\Big]\Big[\frac{(\partial u^*/\partial\delta)\delta - u^*}{\delta^2}\Big] - \beta(\partial u^*/\partial\delta) = -\beta\frac{(1-s)i}{\Omega} < 0. \tag{48}$$

Note that the sign of J_{12} is obtained by using (37) and the sign of J_{22} is obtained by recalling from (32) that the debt-ratio-elasticity of physical capital utilization is unitary, so that $(\partial u^*/\partial\delta)\delta - u^* = 0$. The Jacobian matrix represented by (45)-(48) features a strictly positive determinant, $Det(J) = J_{11}J_{22} + J_{12}J_{21} > 0$, and a strictly negative trace, $Tr(J) = J_{11} + J_{22} < 0$, so that the long-run equilibrium with $\hat{k} = \hat{\delta} = 0$ represented by $(k, \delta) = (k^*, \delta^*)$ is locally asymptotically stable.

We can employ the stable long-run equilibrium values k^* and δ^* in (37) and (38) to evaluate how the long-run equilibrium values of the rates of physical capital utilization, human capital utilization and output growth vary with parametric changes. Here we only present these results, with the corresponding calculations being reserved for Appendix D.

First, the paradox of thrift applies to the long-run equilibrium physical capital utilization rate, as demonstrated in (D.1), and hence to the long-run equilibrium rate of output growth and employment rate (which also measures human capital utilization) (per (D.2) and (D.3)), in the case of the latter noticing that k^* does not depend on the capitalists' saving rate. Although an increase in the saving rate of capitalists raises the debt ratio in (38) and hence capitalists' consumption as a proportion of physical capital (per (29)), the net impact on aggregate demand formation in the long-run quilibrium is ultimately strictly negative.

Second, a strictly lower (higher) propensity to consume (repay debt) out of disposable wage income on the part of workers ϕ $(1-\phi)$ reduces the long-run equilibrium values of physical

[13]For example, with a net growth rate of the physical capital stock in the range of 2 to 4 per cent and a rate of physical capital utilization in the range of 75 to 85 percent, the value of the parameter β in (6) with $\alpha = 0$ would approximately be in the range of 0.02 to 0.05, which is strictly lower than even conservative estimates of the saving rate of capitalists.

capital utilization (per (D.4)). Per (D.5) and (D.6), this result carries over to the long-run equilibrium rates of output growth and employment (in the latter case noting that k^* in (37) does not depend on either propensity). The relevant policy implication here is that more affordable debt servicing associated with human capital formation financed through debt, that requires a strictly lower fraction of disposable wage income be devoted to debt servicing, yields a strictly higher level of macroeconomic activity and a strictly lower debt ratio.

Third, recall that a strictly higher wage share exerts a double positive effect on aggregate demand formation, as it impacts positively on both workers' consumption and the accelerator effect in the rate of human capital accumulation. In the long run, the equilibrium physical capital utilization and output growth rate vary positively with the wage share (per (33) and (D.8)). However, given that workers' desired (and actual) rate of human capital accumulation varies positively with the wage share, the higher the latter, the lower the physical to human capital ratio in (38). Recall from (5) that the long-run equilibrium employment rate is represented by $e^* = u^* k^*$, where u^* in (32) is evaluated at δ^* in (38). It follows that the ratio of human capital utilization to physical capital utilization is inversely proportional to the ratio of physical to human capital in long-run equilibrium. Intuitively, the long-run equilibrium is characterized by the equality between the stock of physical capital multiplied by its rate of utilization and the stock of human capital multiplied by its rate of utilization, which is measured by the rate of employment. Ultimately, with a strictly higher wage share generating both a strictly higher rate of physical capital utilization and a strictly lower ratio of physical to human capital, the resulting impact on the employment rate is ambiguous. As formally demonstrated in (D.9), the wage share elasticity of the long-run equilibrium employment rate is the sum of the wage share elasticity of the long-run equilibrium physical capacity utilization rate (which is strictly positive, per (33)) and the wage share elasticity of the long-run equilibrium physical to human capital ratio (which is strictly negative, per (37)). It then follows that a rise in the wage share will result in a strictly higher (lower) long-run equilibrium employment rate if the long-run equilibrium physical capital utilization rate is strictly more (less) wage share elastic than the long-run equilibrium ratio of physical to human capital.

Fourth, (37) indicates that the ratio of physical to human capital in the long-run equilibrium varies negatively with γ, one of the parameters determining the magnitude of the accelerator effect in the rate of human capital accumulation in (24). Meanwhile, the long-run equilibrium values of the rates of physical capital utilization and output growth (per (D.10) and (D.11)) vary positively with γ. Since the employment rate in the long-run equilibrium is given by $e^* = u^* k^*$, where u^* in (32) is evaluated at δ^* in (38), the impact of a strictly higher γ on the long-run equilibrium employment rate is ultimately ambiguous. As shown in (D.12), the elasticity of the long-run equilibrium employment rate with respect to the parameter γ is the sum of the elasticity of the long-run equilibrium physical capacity utilization rate with respect to the parameter γ (which is strictly positive, per (D.10)) and the elasticity of the long-run equilibrium physical to human capital ratio with respect to the same parameter (which is strictly negative, per (37)). Hence an increase in the parameter γ will result in a strictly higher (lower) long-run equilibrium employment rate if the long-run equilibrium physical capital utilization rate is strictly more (less) elastic with respect to the same parameter than the long-run equilibrium ratio of physical to human capital.

Fifth, (37) also indicates that the ratio of physical to human capital in long-run equilibrium varies positively with β, the parameter measuring the magnitude of the accelerator effect in the rate of physical capital accumulation in (6) with $\alpha = 0$. This places an upward pressure on the rate of employment in long-run equilibrium. Although a strictly higher β has an ambiguous impact on physical capital utilization in long-run equilibrium (per (D.13)), it ultimately leads to strictly higher rates of employment and output growth in long-run equilibrium (per (D.14) and (D.15)). As shown in (D.13), the elasticity of the long-run equilibrium physical capacity utilization rate with respect to the parameter β is the difference between the elasticity of the long-run debt ratio in (38) with respect to the parameter β (which is strictly negative, per (B.2)) and the elasticity of the denominator of the short-run equilibrium physical capacity utilization in (32), which is the strictly positive composite parameter Ω, with respect to the parameter β (an elasticity which can be checked to be strictly negative). Hence an increase in the parameter β will result in a strictly higher (lower) long-run equilibrium rate of physical capital utilization if the composite parameter Ω is strictly more (less) elastic with respect to the same parameter than the debt ratio in (38).

Taken together, the three preceding comparative static results interestingly show that, in long-run equilibrium, a higher parameter associated with the magnitude of the accelerator effect in the accumulation rate of one of the two types of capital raises the utilization rate of the other type but has an ambiguous impact on its own utilization rate.

Finally, as shown in (D.16), the impact of a strictly higher tax rate on the long-run equilibrium physical capacity utilization is ambiguous. Per (D.17) and (D.18), this result carries over to the long-run equilibrium rates of output growth and employment (in the latter case noting that k^* in (37) does not depend on the tax rate). As shown in (D.16), the elasticity of the long-run equilibrium physical capacity utilization rate with respect to the tax rate τ is the difference between the elasticity of the long-run debt ratio in (38) with respect to τ (which is strictly positive, per (40)) and the elasticity of the denominator of the equilibrium physical capacity utilization rate in (32), which is the strictly positive composite parameter Ω, with respect to τ (an elasticity which can be shown to be strictly positive). Thus, a strictly higher tax rate will lead to a strictly higher (lower) long-run equilibrium rate of physical capital utilization if the debt ratio in (38) is strictly more (less) elastic with respect to the same parameter than the composite parameter Ω. In long-run equilibrium, if the rate of physical capital utilization varies positively (negatively) with the tax rate, so do the rates of output growth and employment (per (D.17) and (D.18)).

4 Concluding remarks

The importance of the supply-side in demand-led output growth cannot be neglected, and the role of human capital (or knowledge more broadly) is certainly a case in point. Human capital accumulation contributes to the determination of both the demand-led equilibrium rate of growth and the potential (Harrodian natural) rate of growth. However, a key message of this chapter is that the supply of higher human capital-endowed workers does not automatically create its own demand. The relevant policy implication is clear: expansionary aggregate demand policies are a necessary complement to expansionary aggregate supply policies based on human capital formation.

The models developed in this chapter can be extended in several directions. In addition to the suggestions made in the preceding sections, these include making the functional distribution of income endogenous through a conflicting-claims analysis of wage and price inflation dynamics in the presence of labor productivity enhancing human capital formation. The implications of different tax systems for human capital accumulation through debt, income distribution and output growth can also be explored, including taxation on interest income received by lenders and tax exemption on interest payments made by borrowers. A final issue worth exploring concerns the supply-side effects of human capital formation operating through channels other than labor productivity, particularly those with potential implications for income distribution and output growth.

References

Braunstein, E., Van Staveren, I., and Tavani, D. (2011). Embedding care and unpaid work in macroeconomic modeling: a structuralist approach. *Feminist Economics*, 17(4):5–31.

Carvalho, L. B., Lima, G. T., and Serra, G. P. (2024). Household debt, knowledge capital accumulation, and macrodynamic performance. *Journal of Post Keynesian Economics*, 47(1):84–116.

Dutt, A. K. (2006). Maturity, stagnation and consumer debt: A Steindlian approach. *Metroeconomica*, 57(3):339–364.

Dutt, A. K. (2010). Keynesian growth theory in the 21st century. In Arestis, P. and Sawyer, M. C., editors, *Twenty-First Century Keynesian Economics*, pages 39–80. Palgrave Macmillan, London.

Dutt, A. K. and Veneziani, R. (2019). Education and 'human capitalists' in a classical-Marxian model of growth and distribution. *Cambridge Journal of Economics*, 43(2):481–506.

Dutt, A. K. and Veneziani, R. (2020). A classical model of education, growth, and distribution. *Macroeconomic Dynamics*, 24(5):1186–1221.

Ehrlich, I., Li, D., and Liu, Z. (2017). The role of entrepreneurial human capital as a driver of endogenous economic growth. *Journal of Human Capital*, 11(3):310–351.

Kapeller, J. and Schütz, B. (2015). Conspicuous consumption, inequality and debt: the nature of consumption-driven profit-led regimes. *Metroeconomica*, 66(1):51–70.

Lima, G. T., Carvalho, L., and Serra, G. P. (2021). Human capital accumulation, income distribution, and economic growth: a demand-led analytical framework. *Review of Keynesian Economics*, 9(3):319–336.

Lucas, R. E. (1988). On the mechanics of economic development. *Journal of Monetary Economics*, 22(1):3–42.

Mankiw, N. G., Romer, D., and Weil, D. N. (1992). A contribution to the empirics of economic growth. *The Quarterly Journal of Economics*, 107(2):407–437.

Rada, C., Santetti, M., Schiavone, A., and von Arnim, R. (2021). Post-keynesian vignettes on secular stagnation: From labor suppression to natural growth. *Working Paper Series, Department of Economics, University of Utah*, (2021-05).

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5):71–102.

Serra, G. P. (2021). *Three Essays on Human Capital Formation and Student Loans: A Post-Keynesian Perspective*. PhD thesis, The New School.

Serra, G. P. (2023). Household debt, student loan forgiveness, and human capital investment: a neo-kaleckian approach. *Journal of Post Keynesian Economics*, 46(1):173–206.

Setterfield, M. (2023). Post-keynesian growth theory and the supply side: a feminist approach. *European Journal of Economics and Economic Policies: Intervention*, 20(2):299–316.

Appendix A Numerator of the long-run equilibrium ratio of debt to physical capital in (38)

As mentioned in footnote 10, the numerator in (38) can be alternatively expressed as:

$$\begin{aligned}
&(1-s)[(\gamma\sigma-\tau)-(1-\phi)(1-\tau)\sigma]+\Omega \\
&=(1-s)(1-\tau)[(1-\phi)\sigma+s(1-\sigma)]-(\gamma\sigma-\tau)-\beta+[(\gamma\sigma-\tau)-(1-\phi)(1-\tau)\sigma] \\
&=(1-s)(1-\tau)[(1-\phi)\sigma+s(1-\sigma)]-s(\gamma\sigma-\tau)-\beta-(1-\phi)(1-\tau)\sigma \\
&=(1-\tau)[(1-\phi)\sigma+s(1-\sigma)-(1-\phi)\sigma(1-s)]-s(\gamma\sigma-\tau)-\beta \\
&=(1-\tau)[s(1-\sigma)+s(1-\phi)\sigma]-s(\gamma\sigma-\tau)-\beta \\
&=s(1-\tau)(1-\phi\sigma)-s(\gamma\sigma-\tau)-\beta \\
&=s[(1-\tau)(1-\phi\sigma)-(\gamma\sigma-\tau)]-\beta.
\end{aligned} \tag{A.1}$$

Appendix B Comparative statics of the long-run equilibrium ratio of debt to physical capital in (38)

Using (38) and Appendix A, we can compute the following comparative statics:

$$\frac{\partial\delta^*}{\partial s}=\frac{[(1-\tau)(1-\phi\sigma)-(\gamma\sigma-\tau)]-\beta}{\beta(1-s)^2}>0, \tag{B.1}$$

$$\frac{\partial\delta^*}{\partial\beta}=-\frac{s[(1-\tau)(1-\phi\sigma)-(\gamma\sigma-\tau)]}{\beta^2(1-s)}<0, \tag{B.2}$$

$$\frac{\partial\delta^*}{\partial\gamma}=-\frac{s\sigma}{\beta(1-s)}<0, \tag{B.3}$$

$$\frac{\partial\delta^*}{\partial\sigma}=-\frac{s[(1-\tau)\phi+\gamma]}{\beta(1-s)}<0. \tag{B.4}$$

Appendix C Comparative statics of the long-run equilibrium ratio of debt to wage income in (41)

Using (41), we can compute the following comparative statics:

$$\frac{\partial \omega^*}{\partial s} = \frac{(1-s)(1-\tau)(1-\sigma)+\Omega}{(1-s)^2 i\sigma} > 0, \tag{C.1}$$

$$\frac{\partial \omega^*}{\partial \beta} = -\frac{1}{(1-s)i\sigma} < 0, \tag{C.2}$$

$$\frac{\partial \omega^*}{\partial \gamma} = -\frac{1}{(1-s)i} < 0. \tag{C.3}$$

Appendix D Comparative statics of the macroeconomic variables in the long-run equilibrium

First, using u^* in (32) evaluated at δ^* in (38), g_K in (6) with $\alpha = 0$, e^* in (5) evaluated at the long-run equilibrium (k^*, δ^*) in (37)-(38), and the assumption in Section 3.1 that $(\gamma\sigma - \tau) > (1-\tau)(1-\phi)\sigma$, which ensures that $\delta^* > 0$ in (38), we have:

$$\frac{\partial u^*}{\partial s} = -\frac{i}{\beta\Omega^2}\Big\{\{(\gamma\sigma-\tau)-(1-\tau)(1-\phi)\sigma\}\{[(1-\tau)(1-\phi\sigma)-(\gamma\sigma-\tau)]-\beta\}\Big\} < 0, \tag{D.1}$$

$$\frac{\partial g^*}{\partial s} = \beta\frac{\partial u^*}{\partial s} < 0, \tag{D.2}$$

$$\frac{\partial e^{**}}{\partial s} = \frac{\partial u^*}{\partial s}k^* + \frac{\partial k^*}{\partial s}u^* = \frac{\partial u^*}{\partial s}k^* < 0. \tag{D.3}$$

Second, using u^* in (32) evaluated at δ^* in (38) and the numerator in (38), which is strictly greater than Ω, we have:

$$\frac{\partial u^*}{\partial \phi} = \frac{i(1-\tau)\sigma}{\beta\Omega^2}\Big\{\{s[(1-\tau)(1-\phi\sigma)-(\gamma\sigma-\tau)]-\beta\}-s\Omega\Big\} > 0, \tag{D.4}$$

$$\frac{\partial g^*}{\partial \phi} = \beta\frac{\partial u^*}{\partial \phi} > 0, \tag{D.5}$$

$$\frac{\partial e^{**}}{\partial \phi} = \frac{\partial u^*}{\partial \phi}k^* + \frac{\partial k^*}{\partial \phi}u^* = \frac{\partial u^*}{\partial \phi}k^* > 0. \tag{D.6}$$

Third, let us re-express the comparative statics in (B.4) as follows:

$$\frac{\partial \delta^*}{\partial \sigma} = \frac{\delta^*}{\sigma} - \frac{[s(1-\tau)+s\tau-\beta]}{\beta(1-s)\sigma}.$$

and compute:

$$\frac{\partial\Omega}{\partial\sigma} = \frac{\Omega - [s(1-\tau)+\tau-\beta]}{\sigma}.$$

Using u^* in (32) and the expressions above, we can compute:

$$\frac{\partial u^*}{\partial\sigma} = \frac{(1-s)i}{\Omega}\Big(\frac{\partial\delta^*}{\partial\sigma}\Omega - \delta^*\frac{\partial\Omega}{\partial\sigma}\Big) = \frac{(1-s)i}{\Omega}\Big\{\frac{[(s-\beta)+(1-s)\tau]}{\sigma}\Big[\delta^* - \frac{\Omega}{\beta(1-s)}\Big] + \frac{\tau\Omega}{\beta\sigma}\Big\} > 0, \tag{D.7}$$

where $[(s-\beta)+(1-s)\tau] > 0$, as seen in (44), and $\delta^* > \Omega/[\beta(1-s)]$, as can be verified from (38). In addition, we have:

$$\frac{\partial g^*}{\partial\sigma} = \beta\frac{\partial u^*}{\partial\sigma} > 0, \tag{D.8}$$

$$\frac{\partial e^{**}}{\partial\sigma} = \frac{\partial u^*}{\partial\sigma}k^* + \frac{\partial k^*}{\partial\sigma}u^* \Rightarrow \varepsilon_{e^{**},\sigma} = \varepsilon_{u^*,\sigma} + \varepsilon_{k^*,\sigma}, \tag{D.9}$$

where here and henceforth we use $\varepsilon_{y,z}$ to denote the elasticity of the variable y with respect to the parameter z.

Fourth, as the numerator in (38) is strictly greater than Ω, and hence strictly greater than $s\Omega$, so that $\delta^* > s\Omega/[\beta(1-s)]$, we can use u^* in (32) to compute:

$$\frac{\partial u^*}{\partial\gamma} = \frac{(1-s)i}{\Omega}\Big(\frac{\partial\delta^*}{\partial\gamma}\Omega - \delta^*\frac{\partial\Omega}{\partial\gamma}\Big) = \frac{(1-s)i\sigma}{\Omega}\Big(\delta^* - \frac{s\Omega}{\beta(1-s)}\Big) > 0, \tag{D.10}$$

$$\frac{\partial g^*}{\partial\gamma} = \beta\frac{\partial u^*}{\partial\gamma} > 0, \tag{D.11}$$

$$\frac{\partial e^{**}}{\partial\gamma} = \frac{\partial u^*}{\partial\gamma}k^* + \frac{\partial k^*}{\partial\gamma}u^* \Rightarrow \varepsilon_{e^{**},\gamma} = \varepsilon_{u^*,\gamma} + \varepsilon_{k^*,\gamma}. \tag{D.12}$$

Fifth, we can use u^* in (32) and (B.2) to compute:

$$\frac{\partial u^*}{\partial\beta} = \frac{(1-s)i}{\Omega}\Big(\frac{\partial\delta^*}{\partial\beta}\Omega - \delta^*\frac{\partial\Omega}{\partial\beta}\Big) = \frac{(1-s)i}{\Omega}\Big(\frac{\partial\delta^*}{\partial\beta}\Omega + \delta^*\Big) \Rightarrow \varepsilon_{u^*,\beta} = \varepsilon_{\delta^*,\beta} - \varepsilon_{\Omega,\beta}. \tag{D.13}$$

Meanwhile, note that:

$$g^* = \beta\frac{(1-s)i\delta^*}{\Omega} = \frac{i}{\Omega}\{s[(1-\tau)(1-\phi\sigma) - (\gamma\sigma-\tau)] - \beta\},$$

$$e^{**} = u^*k^* = \frac{\beta u^*}{\gamma\sigma} = \frac{g^*}{\gamma\sigma},$$

so that (using the condition that ensures that (38) is strictly positive):

$$\frac{\partial g^*}{\partial \beta} = \frac{i}{\Omega^2}\Big\{\{s[(1-\tau)(1-\phi\sigma)-(\gamma\sigma-\tau)]-\beta\}-\Omega\Big\} > 0, \tag{D.14}$$

$$\frac{\partial e^{**}}{\partial \beta} = \frac{1}{\gamma\sigma}\frac{\partial g^*}{\partial \beta} > 0. \tag{D.15}$$

Finally, using u^* in (32), δ^* in (38), k^* in (17) and recalling that $e^{**} = u^*k^*$, we can compute:

$$\frac{\partial u^*}{\partial \tau} = \frac{(1-s)i}{\Omega^2}\frac{\partial \delta^*}{\partial \tau}\Omega - \frac{(1-s)i}{\Omega^2}\frac{\partial \Omega}{\partial \tau}\delta^* = u^*\left[\frac{\frac{\partial \delta^*}{\partial \tau}}{\delta^*} - \frac{\frac{\partial \Omega}{\partial \tau}}{\Omega}\right] \Rightarrow \varepsilon_{u^*,\tau} = \varepsilon_{\delta^*,\tau} - \varepsilon_{\Omega,\tau}, \tag{D.16}$$

$$\frac{\partial g^*}{\partial \tau} = \beta\frac{\partial u^*}{\partial \tau}, \tag{D.17}$$

$$\frac{\partial e^{**}}{\partial \tau} = \frac{\partial u^*}{\partial \tau}k^*. \tag{D.18}$$

In long-run equilibrium, if the physical capital utilization rate varies positively (negatively) with the tax rate, so, too, do the rates of output growth and employment.