Department of Economics - FEA/USP

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**GILBERTO TADEU LIMA**

**MARK SETTERFIELD**

**JAYLSON JAIR DA SILVEIRA**

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Gilberto Tadeu Lima (giltadeu@usp.br)

Mark Setterfield (mark.setterfield@trincoll.edu)

Jaylson Jair da Silveira (jaylson@cse.ufsc.br)

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Drawing on an extensive empirical literature that suggests persistent and time-varying heterogeneity in inflation expectations, this paper embeds two inflation forecasting heuristics – one based on the current rate of inflation, the second anchored to the official inflation target – in a simple macrodynamic model. Decision makers switch between these forecasting heuristics based on satisficing evolutionary dynamics. We show that convergence towards an equilibrium consistent with the level of output and rate of inflation targeted by policy makers is achieved regardless of whether or not the satisficing evolutionary dynamics that guide the choices agents make between inflation forecasting strategies are subject to noise. We also show that full credulity – a situation where all agents eventually use the forecasting heuristic based on the target rate of inflation – is neither a necessary condition for realization of the inflation target, nor an inevitable consequence of the economy's achievement of this target. These results demonstrate that uncertainty in decision making resulting in norm-based inflation expectations that are both heterogeneous and time-varying need not thwart the successful conduct of macroeconomic policy.

**Keywords:** Inflation target, macroeconomic stability, heterogeneous expected inflation, satisficing evolutionary dynamics.

**JEL Codes:** C73, E12, E52.

**Inflation Targeting and Macroeconomic Stability with Heterogeneous Inflation Expectations***

Gilberto Tadeu Lima
Department of Economics, University of São Paulo, Brazil
giltadeu@usp.br

Mark Setterfield
Department of Economics, Trinity College, CT USA
mark.setterfield@trincoll.edu

Jaylson Jair da Silveira
Department of Economics and International Relations
Federal University of Santa Catarina, Brazil
jaylson@cse.ufsc.br

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## 1. Introduction

There is considerable empirical evidence from both survey data and laboratory experiments that inflation expectations are persistently heterogeneous and formed through boundedly rational, norm-based mechanisms that suggest decision makers confront uncertainty rather than just calculable risk when forming expectations. In fact, the full rationality assumption associated with inflation expectations formation in the vast majority of macroeconomic models is invariably proved empirically invalid.

Motivated by this evidence and in the spirit of the contributions of Paul Davidson and Herbert Simon, we extend a simple macroeconomic model developed in Lima and Setterfield (2008) to explore the implications for macroeconomic stability and the efficacy of inflation and output targeting of heterogeneous, norm-based inflation expectations which vary over time in accordance with two alternative *satisficing evolutionary dynamics* (with and without mutation). In line with the empirical evidence on inflation expectations, the rate of change of expected inflation is modeled as a weighted average of the rate of change of *credulous* agents' expected inflation (based on the inflation target set by policy makers) and the rate of change of *incredulous* agents' expected inflation (reflecting the belief that current inflation will prevail). It is shown that *full credulity* (all agents eventually become credulous) is an evolutionarily satisficing equilibrium configuration in the absence of perturbation (noise), analogous to mutation in natural environments. The equilibrium configuration is also characterized by both policy targets (viz. inflation and output) being achieved through an appropriate policy mix. Hence an equilibrium solution consistent with the achievement of the inflation and output targets does not emerge because all agents are fully credulous. Instead, all agents eventually become fully credulous because the inflation and output targets are reached. Another novel result is that *both* targets will be reached even if monetary policy is the *only* policy instrument in use, despite the apparent violation of the Tinbergen (1952) principle.

As the general case, the model features mutation as an exogenous disturbance in the satisficing evolutionary mechanism, leading some agents to choose an inflation forecasting strategy at random. This disturbance component is intended to capture the effect, for instance, of exogenous institutional factors, such as changes of administration in the monetary authority. Two other rationales for the existence of mutation are that an agent exits the economy with some (fixed) probability and is replaced with a new agent knowing nothing about the decision-making environment, or that each agent simply "experiments" occasionally with exogenously fixed probability. A question that arises is whether the occurrence of such noise thwarts convergence towards an equilibrium consistent with the two policy targets, and the answer is no. Yet the equilibrium distribution of inflation forecasting strategies *does* depend on whether the satisficing evolutionary dynamics are perturbed: in the absence (presence) of mutant agents, the equilibrium configuration, which is a local attractor, is given by the two policy targets and the extinction of the incredulous inflation forecasting strategy (survival of both inflation forecasting strategies). Therefore, full credulity is neither a necessary condition for the achievement of both policy targets, nor an inevitable consequence of the achievement of these targets. Ultimately, our results demonstrate that uncertainty in decision making resulting in norm-based inflation expectations formation need not thwart successful macroeconomic policy intervention. This is true even when norm-based inflation expectations are both heterogeneous and, as a result of the propensity of decision makers to switch between forecasting heuristics based on satisficing criteria, time-varying.

The remainder of the paper is organized as follows. The next section presents the motivating empirical evidence and briefly discusses the related literature, while Section 3 lays out the structure of the basic model on which our analysis is based. Section 4 then examines the consequences for macroeconomic stability and the efficacy of inflation and output targeting of heterogeneous inflation expectations that decision makers switch between in accordance with satisficing evolutionary dynamics. Section 5 performs the same examination for an alternative specification of the satisficing evolutionary dynamics, and section 6 concludes.

## 2. Motivating evidence and related literature

According to Dequech (2004), different visions of reality undergird different concepts of uncertainty in economics. One view, associated with Davidson (1996, pp.479-80), is that social reality is transmutable in novel (and therefore innately unpredictable) ways. A second view, associated with Simon (1959, p.273; 1978, pp.8-9), is that reality is complicated relative to the information processing and decision making capacities of the individual. These visions correspond (respectively) to Davidson's distinction between "ontological" and "epistemological" uncertainty, respectively. But as Dequech (2004, pp.368-9) argues, both visions have an epistemological component: in each case (albeit for different reasons), there are aspects of reality that are unknown to the decision maker so that in each case, rationality is bounded in the sense that decision makers are partially ignorant of their external environment. This common epistemological component suggests that, denied the ability to optimize based on knowledge of the “true model” characterizing reality, decision makers must instead “muddle through” using norms/conventions and satisficing criteria as bases for their expectations and decision making. These insights inform our modeling of expectations in this paper. Specifically, we postulate that, facing some form of uncertainty (as opposed to calculable stochastic risk), decision makers adopt forecasting strategies based on simple heuristics, switching between these “rules of thumb” based on satisficing criteria.

The already vast empirical literature on inflation expectations formation offers support for the themes introduced above. This literature can be divided into two groups depending on the type of evidence used in the investigation: studies based on survey data; and laboratory experiments with human subjects. A recurrent finding is the strong empirical support for time-varying heterogeneity in the formation of inflation expectations characterized by pervasive bounded rationality and the absence of anything approximating rational expectations (see, e.g., Duffy, 2008, and Hommes, 2011, 2013, for detailed overviews of experiments dealing with the formation of expectations about macroeconomic variables). The main reasons for this heterogeneity that have been proposed in the literature are that agents rely on different models, have access to different information sets, or have different cognitive capacities for processing information.

Let us start our brief (but representative) review with the experimental literature on inflation expectations formation. Adam (2007) finds that subjects’ inflation expectations are not captured by the predictor implied by the rational expectations equilibrium, while predictors based on lagged inflation capture inflation expectations quite well. Assenza et al. (2011) ask subjects to forecast inflation under different scenarios that vary according to the underlying assumptions made about output gap expectations. In all treatments, the most popular significant regressor is last period’s value of the inflation rate, followed in most treatments by either the most recent own prediction or the second last available (the period prior to the previous one) value of the forecasting variable. Overall, the authors find that individuals tend to base their predictions on past observations, following simple forecasting heuristics, with individual learning taking the form of switching from one heuristic to another.

Meanwhile, Pfajfar and Zakelj (2011) provide substantial evidence in support of heterogeneity in the forecasting process both across subjects and time. They find that subjects form expectations in accordance with different theoretical models: in fact, on average, in each period 4.5 different models are used in groups of 9 subjects. Although the most popular rule is trend extrapolation, a significant share of the population uses adaptive expectations, adaptive learning or sticky information type models. While adaptive learning assumes that subjects act as econometricians when forecasting, i.e. re-estimating their model each time new data becomes available (see Evans and Honkapohja, 2001), sticky information models assumes that in each period only a random fraction of subjects updates their knowledge of the state of the economy (see Mankiw and Reis, 2002). The authors also find that rather than adhering to one model, subjects switch between alternative models (on average, every 4 periods). Inflation expectations heterogeneity in experimental data is also documented in Roos and Luhan (2013), who find that adaptive and static (or naïve) inflation expectations are frequently observed.

Empirical analyses based on survey data have also cast serious doubt on the validity of the assumption that inflation expectations are continuously homogeneously rational. In Carroll (2003), the typical US household is estimated to update expectations infrequently, and does so to the most recently reported past statistics rather than to the rational forward-looking forecast. Mankiw, Reis and Wolfers (2004) use different survey data for the US to document extensive heterogeneity in inflation expectations. They also obtain only modest confirmation (at best) of the perfect rationality assumption, as there is considerable (and persistent) under- or over-prediction. Meanwhile, Branch (2004) estimates a simple switching model with heterogeneous expectations using US survey data and provides evidence for dynamic switching that depends on the relative mean squared errors of the predictors.

Capistrán and Timmermann (2009) find that heterogeneity of inflation expectations among US professional forecasters varies over time depending on the level and the variance of current inflation. Blanchflower and MacCoille (2009), using survey data for the UK, find strong support for heterogeneous and backward-looking inflation expectations. In fact, individuals' perceptions of current inflation are found to be a highly significant determinant of their inflation expectations, a finding which lends support to a salient feature of the model set forth in the subsequent sections of this paper. Weber (2010), using survey data for five core European economies, finds that there is very little evidence that the inflation expectations of households and professional forecasters are rational.

While a substantial number of studies focus on measures of central tendency (such as the mean or the median forecast), Pfajfar and Santoro (2010) measure the degree of heterogeneity in private agents' inflation forecasts by exploring time series of percentiles from the empirical distribution of US survey data. Interestingly, they identify three regions of the distribution that correspond to different mechanisms of expectation formation: a static (or naïve) or highly autoregressive region on the left hand side of the median, a nearly rational or unbiased region around the median and a fraction of forecasts on the right hand side of the median formed in accordance with adaptive behavior (expectations are revised according to the last observed forecast error) and sticky information (fewer than 10 percent of the forecasts reflect regular information updating).

Meanwhile, Diron and Mojon (2005) provide evidence that the forecast error incurred when assuming that future inflation will be equal to the inflation target announced by the central bank is typically at least as small as (and often smaller than) the forecast errors of model-based and published inflation forecasts. Using data for seven inflation targeting countries, they compare the forecasting performance of benchmark (model-based and published) forecasts of inflation to the performance of forecasts which are set equal to the inflation target. They find that forecasting inflation consistent with the inflation target implies a smaller forecasting error than either a random walk or an autoregressive model of inflation for both 4- and 8-quarter horizon forecasts. For some countries, forecasting inflation consistent with the target also beats professional forecasts. Along similar (albeit theoretical) lines, Kapadia (2005) assumes that firms may simply expect future inflation to equal the inflation target instead of forming rational expectations. However, the proportion of firms with inflation-target expectations is exogenously given rather than endogenously varying, as in the model developed below. Moreover, inflation-target expectations are only formed by near-rational decision makers who cannot afford to form rational expectations. In the environment of fundamental uncertainty with which the model of the present paper deals, decision makers are unable to form rational expectations, and revise expected inflation according to either current inflation or the inflation target. We also allow the distribution of strategies for forming inflation expectations to vary endogenously in accordance with satisficing dynamics.

The recent literature also contains other mainstream macroeconomic models featuring heterogeneous inflation expectations from which predictions are sometimes tested empirically. Branch (2004) develops a model where agents form their forecasts of inflation by selecting a predictor function from a set of costly alternatives whereby they may rationally choose a method other than the most accurate. Agents are seen as rationally heterogeneous in the sense that each predictor choice is optimal for them; strictly speaking, agents' expectations are seen as boundedly rational and consistent with optimizing behavior. The model is then used to test whether US survey data exhibit these rationally heterogeneous expectations, revealing that there is dynamic switching that depends on the relative mean squared errors of the predictors. Agents are identified with the following forecast methods: a vector autoregressive forecast (which is seen as a boundedly rational predictor that is in the spirit of rational expectations); adaptive expectations; and static expectations. The author finds that, on average, agents use the vector autoregressive method more often than the other methods. However, when a costly rational predictor is included instead of the vector autoregressive forecast, the vast majority of agents behave adaptively.

Branch (2007) uses US survey data on inflation expectations to compare two models of sticky information (in which the information set is updated only infrequently) against the rationally heterogeneous expectations model developed in Branch (2004). The author finds evidence of sticky information: on average, the largest proportion of agents update their information sets every 3–6 months, while a smaller proportion of agents update their expectations every period and few agents update their expectations after periods of 9 months or more. Another finding is that the distribution of agents across predictors is time varying.

Meanwhile, Brazier et al. (2008) develop a model in which agents use two heuristics to forecast inflation: one is based on one-period lagged inflation, the other on an inflation target announced by the central bank (which is the steady-state value of inflation). Agents switch between these heuristics based on an imperfect assessment of how each has performed in the past. Agents observe such performance with some noise, but the better the true past performance of a heuristic, the greater chance there is that an agent uses it to make the next period's forecast. The authors find that, on average, the majority of agents use the inflation-target heuristic, even though there are times when everyone does, and times when no one does. While Brazier et al. (2008) embed those two forecasting heuristics in a mainstream monetary overlapping-generations model and heuristic switching is described by a discrete choice model, in this paper a different pair of forecasting heuristics is embedded in a non-mainstream model and decision makers switch between heuristics based on satisficing evolutionary dynamics with and without noise.

The model of Capistrán and Timmermann (2009) dealing with the dynamics of the disagreement in inflation expectations of US professional forecasters relies on three features: asymmetric costs of over- and under-predicting inflation, heterogeneity in agents' loss functions, and agents' tendency to over predict inflation. Three implications of the model for which the authors claim empirical support are that: (i) inflation forecasts are generally biased; (ii) forecast errors are positively serially correlated; and (iii) cross-sectional dispersion rises with the level and variance of the inflation rate.

De Grauwe (2010) develops a macro model in which agents have cognitive limitations and use simple but biased heuristics to forecast future inflation. The author follows Brazier et al. (2008) in allowing for two inflation forecasting rules. One heuristic is based on the announced inflation target, while the other heuristic uses last period's inflation to forecast next period's inflation. The market forecast is a weighted average of these two forecasts, with these weights being subject to predictor selection dynamics based on discrete choice theory. While De Grauwe (2010) formulates an extended three-equation model generating endogenous and self-fulfilling waves of optimism and pessimism, the model of this paper explores whether there is convergence towards an equilibrium consistent with the level of output and rate of inflation targeted by policy makers when private decision makers switch between inflation forecasting heuristics based on satisficing evolutionary dynamics. Branch and McGough (2009) develop a model in which agents are (exogenously) split between rational and adaptive expectations and monetary policy follows a standard (Taylor-type) interest rate rule. As a result, the dynamic properties of the model depend crucially on the distribution of agents across forecasting models; in particular, its dynamic properties differ from those implied by the rational expectations model. In the model developed below, expectational heterogeneity involves two kinds of boundedly rational behavior (static and inflation-target expectations) and is endogenously varying rather than exogenously fixed.

Branch and McGough (2010), meanwhile, introduce dynamic predictor selection into a macro model with heterogeneous inflation expectations and examine its implications for monetary policy. They extend Branch and McGough (2009) by following Brock and Hommes (1997), assuming that the degree of heterogeneity varies over time in response to past forecast errors (net of a fixed cost), thereby coupling predictor choice with the dynamics of inflation and output. Agents choose between using a costly rational predictor and using a costless adaptive forecasting model. They find that for sufficiently low costs, the model's steady state is stable. For higher costs, however, the steady state may destabilize and the dynamic system may bifurcate. In the model set forth in this paper, the choice of foresight strategy is also coupled with the dynamics of inflation and output, but expectational heterogeneity involves two kinds of boundedly rational behavior (static and inflation-target expectations) and varies over time endogenously according to two different kinds of satisficing evolutionary dynamics (with and without mutation).

## 3. The basic model

The basic macroeconomic model on which the analysis in this paper is based can be stated as follows:

(1) $$y = y_0 - \delta r ,$$

(2) $$p = \beta + \varphi p^e + \alpha y + \theta Z ,$$

(3) $$\dot{r} = \lambda (y - y^T) ,$$

(4) $$\dot{Z} = -\mu (p - p^T) ,$$

(5) $$\dot{p}^e = -(1-k)(p - p^T) ,$$

where $y$ denotes the level of real output, $y_0$ represents non-interest sensitive components of aggregate spending, $r$ is the real interest rate, $p$ and $p^e$ are the actual and expected rates of inflation, respectively, $Z$ captures the willingness and ability of workers to bid up the rate of growth of nominal wages independently of the level of economic activity, $y^T$ and $p^T$ denote the policy authorities' target levels of real output and rate of inflation, respectively, and lower case Greek letters denote positive parameters. As usual, a dot over a variable denotes its rate of change (i.e., $\dot{x} = dx/dt$). Meanwhile, $k \in [0,1] \subset \mathbb{R}$ denotes the fraction of incredulous agents who form expectations in accordance with observed inflation, and $1-k$ denotes the fraction of credulous agents whose expectations are anchored to the inflation target. Both $k$ and (by extension) $1-k$ vary endogenously over time in a manner that is described below.

Equation (1) is simply an IS curve, equation (2) is an expectations-augmented Phillips curve and equations (3) and (4) are policy reaction functions. Equation (3) describes the conduct of monetary policy which, in keeping with the Post Keynesian theory of endogenous money, takes the form of an interest rate operating procedure.[1] The instrument for achieving changes in $Z$ in equation (4), meanwhile, must ultimately be some form of incomes policy. Incomes policies are defined as formal and/or informal institutions that frame and mediate aggregate wage and price setting behavior in such a way as to reduce conflict over income shares and better reconcile conflicting income claims (Setterfield, 2007).[2] Incomes policies so-defined can be either cooperative or coercive, depending on whether the objective is to reconcile competing income claims in a mutually satisfactory manner, or simply force one party (either firms or workers) to accept the distributional claims of the other. Hence the precise policy pursued in order to, for example, reduce $Z$ might involve centralizing wage bargaining, creating a tax-based or market anti-inflation plan (see, for example, Colander, 1986), or changing labor law to reduce the job-security of workers.

In keeping with the Post Keynesian structure of our model, we make no assumption that the coefficients in equations (1)—(4) nor even the precise form of these structural equations are time-invariant. We do, however, abstract from these dynamics in what follows. In equation (2), the assumption that $\varphi < 1$ is consistent with the notion that workers lack the bargaining power to fully index expected inflation into nominal wage growth, and no reference is made to a supply-determined equilibrium or "natural"

[1] The control over longer rates that results from manipulation of the short rate may, of course, be imperfect, while there also exists the zero lower bound constraint on the nominal interest rate. We abstract from both of these issues in what follows.

[2] It might be objected that the manipulation of an incomes policy is likely to prove difficult in the short-run. We would argue, however, that whilst the initial *construction* of an incomes policy is likely to be time consuming, its subsequent *manipulation* need not be. For example, in the classic tax-based incomes policy proposed by Wallich and Weintraub (1971), the ultimate instrument of policy is a tax rate that should be amenable to change (at least within limits) even in the short run.

level of output.[3] All these features of our basic model are, in turn, consistent with five essential tenets of Post Keynesian macroeconomics. The potential for parametric instability is congruent with the notion that decision-making is conducted under conditions of fundamental uncertainty (which, in turn, validates the assumption of satisficing behavior). Meanwhile, incomplete indexation and the absence of a unique, supply-determined real sector equilibrium coincide with the idea that the wage bargain is conducted in nominal terms (with real wages being determined only after the wage bargain is complete), the non-neutrality of money, and the principle of effective demand (the central role of aggregate demand in determining the equilibrium values of real variables). Finally, our model reflects the importance of cost-push sources of inflation (in particular, wage inflation associated with conflict over the distribution of income). The correspondence between our model and this last tenet of Post Keynesian macroeconomics is reflected in the inclusion of the variable $Z$ in the Phillips curve, which is exclusively associated with the capacity of conflicting claims over nominal income to create inflation. Hence suppose there exists an incomes policy or "social bargain" (Cornwall and Cornwall, 2001) that creates a conventional and mutually acceptable distribution of income between capital and labor. This will reduce the willingness of workers to use the bargaining power vested in them by the current level of economic activity to bid up wages in pursuit of a larger income share.[4] The result in equation (2) will be a lower value of $Z$ and hence, *ceteris paribus*, a lower rate of inflation.

The sub-model in equations (1)—(4) constitutes the basic model developed in Lima and Setterfield (2008). Sound Post Keynesian behavioral foundations for this model – and in particular, equations (1) and (2) – are provided in Lima and Setterfield

[3] Note that these features are mutually consistent. Under the mainstream "full indexation" assumption ($\varphi = 1$) of the accelerationist hypothesis, according to which workers bargain for wages in real terms, and imposing the equilibrium condition $p^e = p$, the solution to equation (2) becomes:

$$y_n = \frac{-(\beta + \theta Z)}{\alpha},$$

where, with $\alpha$, $\theta > 0$, $\beta < 0$ and $|\beta| > \theta Z$ together imply that $y_n > 0$. In this formulation, $y_n$ is the *unique* level of output consistent with *any* rate of inflation, interpreted as the "natural" level of output (a counterpart of the more familiar natural rate of unemployment).

[4] Note that firms may increase prices independently of changes in costs in pursuit of distributional goals, so there is no intent here to suggest that workers are uniquely responsible for inflation. Instead, we are merely abstracting from the behavior of firms for the sake of simplicity.

(2014).[5] Under the further assumptions that $p^e = kp_{-1} + (1-k)p^T$ , $k = 0$ , and $\dot{p}^e = \dot{p}^T = 0$, this sub-model is then used to analyze the possibilities for successful inflation targeting and macroeconomic stability. In Lima and Setterfield (2008), $k$ is thought of as decreasing in the credibility of the policy authorities' commitment to achieve $p^T$, so the assumption that $p^e = p^T$ implies that commitment is fully credible. In this paper, while $k \in [0,1] \subset \mathbb{R}$ can once again be thought of as measuring the credibility of the policy authorities' commitment to achieve $p^T$, it varies endogenously over time as a result of satisficing evolutionary dynamics (with and without noise) in the spirit of the contributions of Herbert Simon. As classically elaborated by Simon (1955, 1956), satisficing is a theory of choice centered on the process through which available alternatives are examined and evaluated. By conceiving of choice as intending to meet an acceptability threshold rather than to select the best of all alternatives, satisficing theory contrasts with optimization theory. As Simon suggests, this contrast is analogous to 'looking for the sharpest needle in the haystack' (i.e., optimizing) versus 'looking for a needle sharp enough to sew with' (i.e., satisficing) (Simon, 1987, p. 244).

In line with the empirical evidence on heterogeneous inflation expectations reported in the preceding section, equation (5) is derived as follows. The rate of change of expected inflation is a weighted average of the rate of change of expected inflation by incredulous agents ( $\dot{p}_i^e$) and the rate of change of expected inflation by credulous ( $\dot{p}_c^e$) agents: $\dot{p}^e = k\,\dot{p}_i^e + (1-k)\dot{p}_c^e$. As credulous agents expect the convergence of current inflation to the policy convention, $p^T$, while incredulous agents expect inflation to remain unchanged, it follows that $\dot{p}_c^e = p^T - p$ and $\dot{p}_i^e = 0$ . Substituting these expressions for $\dot{p}_c^e$ and $\dot{p}_i^e$ into the equation for $\dot{p}^e$ stated above yields equation (5). As elaborated at the beginning of the preceding section, note that in the environment of fundamental uncertainty with which we are dealing, *no* economic agent is *able* to form rational expectations. As validated by the extensive empirical evidence reported earlier,

[5] Lima and Setterfield (2014) show that equation (1) approximates the relationship between real output and the real interest rate in a standard Kaleckian growth model in which firms' net profits (and hence their investment decisions) are sensitive to their debt-servicing obligations. Equation (2), meanwhile, approximates the relationship between inflation and the level of real activity arising from conflicting-claims wage and price dynamics when firms practice target-return pricing that is sensitive to their debt-servicing obligations, but there is no active cost channel of monetary transmission.

agents instead form expectations using one of two heuristics, but can (and do) switch between these heuristics over time.

Let us now describe the satisficing evolutionary dynamics which yields the law of motion of the degree of credibility in the policy authorities' commitment to achieve $p^T$ – i.e., the proportion of credulous agents, 1 – *k*. An agent *j* takes the gap between current inflation and the inflation target, $p - p^T$, and compares it (ignoring its sign) with the gap he considers acceptable, $p^j - p^T$. If the observed gap is smaller than or equal to (in module) the acceptable gap, agent $j$ does not consider changing his strategy for forming inflation expectations. Otherwise agent $j$ becomes a strategy reviser. Clearly, the fact that policy authorities typically target a range rather than a single rate of inflation, thus implicitly expecting (and tolerating) some variation of actual inflation within this range, provides further behavioral motivation for the way we model agents as accepting some tolerable departure of inflation from their forecasts. The gap that is acceptable to an agent depends, inter alia, on idiosyncratic features. We assume that acceptable gaps are randomly and independently determined across agents and over time. More precisely, we assume that the square of the tolerable gap, $\left(p^j - p^T\right)^2$, is a random variable with cumulative distribution function $F:\mathbb{R}_+ \to [0,1] \subset \mathbb{R}$ which is continuously differentiable. Therefore, the probability of randomly choosing an agent $j$ who considers the current observed deviation $p - p^T$ as unacceptable is given by:

(6) $$\Pr\left((p^j - p^T)^2 < (p - p^T)^2\right) = F\left((p - p^T)^2\right).$$

Therefore, the probability that a randomly drawn agent $j$ will consider that the currently observed gap is acceptable is simply:

(7) $$\Pr\left((p^j - p^T)^2 \geq (p - p^T)^2\right) = 1 - F\left((p - p^T)^2\right).$$

The measure of credulous agents who become incredulous is then given by:

(8) $$(1-k)F\left((p - p^T)^2\right).$$

Analogously, the measure of incredulous agents who becomes credulous is given by:

(9) $$k\left[1 - F\left((p - p^T)^2\right)\right].$$

Hence subtracting (9) from (8) yields the following satisficing evolutionary dynamics:

$$\dot{k} = (1-k)F\left((p-p^T)^2\right) - k\left[1-F\left((p-p^T)^2\right)\right]. \tag{10}$$

Next, we consider the possibility that the satisficing evolutionary dynamics in (10) operate in the presence of a noise term, analogous to mutation in natural environments. In a biological setting, mutation is interpreted literally as consisting of random changes in genetic codes. In economic settings, as pointed out by Samuelson (1997, ch. 7), mutation refers to a situation in which a player refrains from comparing payoffs and changes strategy at random. Hence the present extension features mutation as exogenous noise in the satisficing evolutionary mechanism leading some agents to choose an inflation foresight strategy at random. This disturbance component is intended to capture the effect, for instance, of exogenous institutional factors such as changes of administration in the monetary authority or other changes in the policy-making framework (which nonetheless do not involve an abandonment of the inflation targeting regime). Or, as in Kandori, Mailath and Rob (1993), two other rationales for random choice are that an agent exits the economy with some (fixed) probability and is replaced with a new agent who knows nothing about (or is inexperienced in) the decision-making process, or that each agent simply "experiments" occasionally with exogenously fixed probability.

Drawing on Gale, Binmore and Samuelson (1995), mutation can be incorporated into the satisficing evolutionary mechanism in (10) as follows. Let $\varepsilon \in (0,1) \subset \mathbb{R}$ be the measure of mutant agents that choose an inflation foresight strategy in a given revision period independently of the respective payoffs. Therefore, there are $\varepsilon(1-k)$ credulous agents and $\varepsilon k$ incredulous agents behaving as mutants. We assume that mutant agents choose either one or the other of the two inflation foresight strategies with equal probability, so that there are $\varepsilon(1-k)/2$ credulous mutant agents and $\varepsilon k/2$ incredulous mutant agents changing foresight strategy. The net flow of mutant agents becoming incredulous agents in a given revision period, which can be either positive or negative, is then the following:

$$\varepsilon\left(1-k\right)\frac{1}{2} - \varepsilon k\frac{1}{2} = \varepsilon\left(\frac{1}{2}-k\right). \tag{11}$$

Following Gale, Binmore and Samuelson (1995), this noise can be added to the evolutionary mechanism (14) to yield the following *noisy satisficing evolutionary dynamics*:

(12) $$\dot{k} = (1-\varepsilon)\left\{(1-k)F\left((p-p^T)^2\right) - k\left[1-F\left((p-p^T)^2\right)\right]\right\} + \varepsilon\left(\frac{1}{2}-k\right).$$

## 4. The behavior of the basic model

By combining equations (1)—(5) and (12), we can analyze the implications of inflation and output targeting with evolving heterogeneous inflation expectations. First, note that from (1):

(13) $$\dot{y} = -\delta\dot{r},$$

which, using (3), can be written as:

(14) $$\dot{y} = -\delta\lambda(y-y^T).$$

Similarly, (2) yields:

(15) $$\dot{p} = \varphi\dot{p}^e + \alpha\dot{y} + \theta\dot{Z}.$$

Combining this expression with (4), (5) and (14), we arrive at:

(16) $$\dot{p} = -[\varphi(1-k) + \theta\mu](p-p^T) - \alpha\delta\lambda(y-y^T).$$

Equations (12), (14) and (16) constitute an autonomous three-dimensional system of differential equations in which the rates of change of $k$, $y$ and $p$ depend on the levels of $k$, $y$ and $p$ and accompanying parameters. It follows from (14) that $\dot{y} = 0$ if, and only if:

(17) $$y = y^T.$$

Given (17) and (16), $\dot{p} = 0$ obtains if, and only if:

(18) $$p = p^T.$$

Given that $F(0) = 0$, substituting (18) into (12) and setting $\dot{k} = 0$ yields:

(19) $$(1-\varepsilon)\left\{(1-k)F(0) - k\left[1-F(0)\right]\right\} + \varepsilon\left(\frac{1}{2}-k\right) = -(1-\varepsilon)k + \varepsilon\left(\frac{1}{2}-k\right) = 0,$$

the solution to which is given by:

(20) $$k = \frac{\varepsilon}{2}.$$

Therefore, the (unique) equilibrium configuration of the dynamic system represented by (14), (16) and (12) is given by $\left(y^T, p^T, \varepsilon / 2\right)$. Or, in the absence of mutation ($\varepsilon = 0$), the (likewise unique) equilibrium solution is given by $\left(y^T, p^T, 0\right)$. Note that while the equilibrium values of *y* and *p* are identical in both cases, the equilibrium distribution of inflation foresight strategies depends on whether or not the satisficing evolutionary dynamics is perturbed: in the presence of perturbation represented by mutant agents, the equilibrium solution is given by $\left(y^T, p^T, 0 < \varepsilon / 2 < 1/2\right)$, whereas in the absence of mutant agents the equilibrium solution is given by $\left(y^T, p^T, 0\right)$. Only when the satisficing evolutionary dynamics are not subject to mutation does full credulity ($k = 0$) obtain. It is worth recalling that the empirical evidence reported in the preceding section shows that heterogeneous inflation expectations are an empirically persistent phenomenon even when current inflation is nearly at or converging to the official target – a possibility that is clearly allowed for by our model. Note also that since $\varepsilon$ represents the measure of mutant agents that choose an inflation foresight strategy independently of payoffs, the mixed strategy equilibrium solution with $k^* = \varepsilon / 2$ will be further away from the pure strategy equilibrium solution with $k^* = 0$ (i.e., extinction of the incredulous strategy) the larger the presence of mutant agents. Nonetheless, the presence of mutants implies that, in equilibrium, the incredulous strategy is never played by the majority of agents, as $\lim_{\varepsilon \to 1}(\varepsilon / 2) = 1/2$.

Let us now conduct the corresponding stability analysis. The Jacobian matrix evaluated around the equilibrium (where $0 \le \varepsilon / 2 < 1/2$) is given by:

(21) $$J\left(y^T, p^T, \varepsilon/2\right) = \left[\begin{array}{c|c|c} -\delta\lambda & 0 & 0 \\ \hline -\alpha\delta\lambda & -\left(\varphi\left(1-\varepsilon/2\right)+\theta\mu\right) & 0 \\ \hline 0 & 0 & -1 \end{array}\right].$$

Let $\xi$ be an eigenvalue of the Jacobian matrix (21). We can then set the following characteristic equation of the linearization around the equilibrium:

(22) $$|J-\xi I| = \left| \begin{array}{c|c|c} -\delta\lambda-\xi & 0 & 0 \\ \hline -\alpha\delta\lambda & -\left(\varphi\left(1-\varepsilon/2\right)+\theta\mu\right)-\xi & 0 \\ \hline 0 & 0 & -1-\xi \end{array} \right| = 0.$$

This characteristic equation can be re-written as follows:

(22-a) $$-(\delta\lambda+\xi)\left[\left(\varphi\left(1-\varepsilon/2\right)+\theta\mu\right)+\xi\right](1+\xi)=0,$$

whose solutions are the eigenvalues of the Jacobian matrix (21), which are given by:

(23) $$\xi_1=-\delta\lambda<0,\ \xi_2=-[\varphi(1-\varepsilon/2)+\theta\mu]<0 \text{ and } \xi_3=-1<0.$$

Given that all these eigenvalues have strictly negative real parts no matter the value of $\varepsilon$, the equilibrium configurations given by $\left(y^T, p^T, \varepsilon/2\right)$, with mutation, and $\left(y^T, p^T, 0\right)$, without mutation, are both local attractors.

This result establishes what Setterfield (2006) defines as the *full* compatibility of inflation targeting with the underlying structure of the economy: not only are the policy authorities able to both set and achieve an inflation target (establishing the *partial* compatibility of inflation targeting with the economy), they are able to do so without real costs and hence without thwarting the achievement of any output target set independently of $p^T$ (establishing *full* compatibility). Indeed, the policy authorities can change their inflation target and still meet this target without affecting the real economy (and hence their ability to achieve any freely chosen, as far as inflation is concerned, output target). By the same token, policy makers can also set and pursue an output target without any fear of it having inflationary consequences.

Unlike Lima and Setterfield (2008), where full credulity ( $k=0$ ) prevails throughout by assumption, full credulity is here shown to be an evolutionarily satisficing equilibrium solution when the satisficing evolutionary dynamics are not subject to noise. Even though full credulity is not assumed to prevail to begin with, the equilibrium configuration is also (as in Lima and Setterfield, 2008) characterized by both policy targets (viz. inflation and output) being achieved. In other words, an equilibrium solution consistent with the achievement of the inflation target does not emerge because all agents are fully credulous, but rather all agents eventually become fully credulous (in the absence of noise) because the inflation target is reached.

Moreover, full credulity is neither a necessary condition for both policy targets to be achieved nor an inevitable consequence of the achievement of these targets, since in general (allowing for noise in the satisficing evolutionary dynamics) the equilibrium configuration of the system is given by $\left(y^T, p^T, 0 < \varepsilon / 2 < 1/2\right)$.

Finally, note that the existence, uniqueness and local asymptotic stability of both equilibrium configurations (with and without mutation) are all preserved even if the incomes policy in equation (4) is shut down ( $\mu = 0$ ). Therefore, another novel result of our model is that both the inflation and output targets will now be reached even if monetary policy is the only policy instrument in use. The intuition is that the Tinbergen (1952) principle (that there needs to be as many policy instruments as policy goals) is still satisfied, as there are still two adjusting variables (viz. the interest rate, $r$ , and now the degree of credibility of the policy authorities, measured by $k$ ) ensuring the achievement of two targets (inflation and output). In fact, full credulity (or full credibility of the monetary authority) can be interpreted as another implicit policy target, the achievement of which is a by-product of achieving the inflation target (at least when $\varepsilon = 0$ ).

**5. The behavior of the model under an alternative evolutionary dynamics**

Following Vega-Redondo (1996, p. 91), let us now suppose that satisficing behavior is only a trigger that transforms the agent into a potential strategy reviser. The reviser will switch to the other inflation foresight strategy with probability given by the fraction of agents who have previously adopted the alternative strategy. This is an imitation effect which can be associated with the idea of conventional behavior in the present context of decision making under uncertainty. Under this premise, the inflow to the population of incredulous agents is given by:

(24) $$(1-k)F\left((p-p^T)^2\right)k\,.$$

Meanwhile, the efflux from the population of incredulous agents is given by:

(25) $$k\left[1-F\left((p-p^T)^2\right)\right](1-k)\,.$$

Combining equations (24) and (25), the evolutionary dynamics with mutation are therefore given by:

(26) $$\dot{k} = (1-\varepsilon)k(1-k)\left[2F\left((p-p^T)^2\right)-1\right]+\varepsilon\left(\frac{1}{2}-k\right).$$

Equations (14), (16) and (26) constitute another autonomous three-dimensional system of differential equations in which the rates of change of $y$, $p$ and $k$ depend on the levels of $y$, $p$ and $k$ and accompanying parameters. As a result, the equilibrium configuration is again characterized by $y = y^T$ and $p = p^T$. Given that $F(0) = 0$, (26) implies that $\dot{k} = 0$ obtains if, and only if, the following condition is satisfied:

(27) $$(1-\varepsilon)k^2 - k + \frac{\varepsilon}{2} = 0.$$

As shown in the Appendix, however, there is one, and only one, $k^* \in (0,1) \subset \mathbb{R}$ such that (27) is satisfied, which is given by:

(28) $$k^* = \frac{1-\sqrt{1-2(1-\varepsilon)\varepsilon}}{2(1-\varepsilon)}.$$

It is instructive to compute the first derivative of (28) with respect to the measure of mutant agents:

(29) $$\frac{\partial k^*}{\partial \varepsilon} = \frac{\sqrt{1-2(1-\varepsilon)\varepsilon}\ -\varepsilon}{2(1-\varepsilon)^2\sqrt{1-2(1-\varepsilon)\varepsilon}}.$$

We can show that $\partial k^*/\partial \varepsilon > 0$ for all $\varepsilon \in [0,1) \subset \mathbb{R}$.[6] Therefore, as in the specification of the satisficing dynamics adopted in the preceding section, the higher the measure of mutant agents, the higher the proportion of credulous agents in the population in equilibrium (recall (20)). Moreover, since (28) is a continuous function for all $\varepsilon \in [0,1) \subset \mathbb{R}$ and $\lim_{\varepsilon \to 1^-} k^* = 1/2$, it follows that the survival of both inflation foresight strategies in equilibrium occurs with a predominance of credulous agents, given that $k^* \in (0,1/2) \subset \mathbb{R}$ for all $\varepsilon \in (0,1) \subset \mathbb{R}$.

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[6] The denominator in (29) is clearly strictly positive for all $\varepsilon \in [0,1) \subset \mathbb{R}$. Therefore, the derivative in (29) is strictly positive if, and only if, $\sqrt{1-2(1-\varepsilon)\varepsilon}\ -\varepsilon > 0$. This inequality can be manipulated algebraically to yield $g(\varepsilon) \equiv \varepsilon^2 - 2\varepsilon + 1 > 0$. Given that $g(0) = 1$, $g(1) = 0$ and $g'(\varepsilon) = -2(1-\varepsilon) < 0$ for all $\varepsilon \in [0,1) \subset \mathbb{R}$, it follows that $g(\varepsilon) > 0$ is satisfied for all $\varepsilon \in [0,1) \subset \mathbb{R}$. This then demonstrates that $\sqrt{1-2(1-\varepsilon)\varepsilon}\ -\varepsilon > 0$ holds for all $\varepsilon \in [0,1) \subset \mathbb{R}$.

Let us then conduct the corresponding stability analysis. The Jacobian matrix evaluated around the equilibrium given by $(y^T, p^T, k^*)$ is the following:

$$(30) \qquad J\left(y^T, p^T, k^*\right) = \left[\begin{array}{c|c|c} -\delta\lambda & 0 & 0 \\ \hline -\alpha\delta\lambda & -\left(\varphi\left(1-\varepsilon/2\right)+\theta\mu\right) & 0 \\ \hline 0 & 0 & 2(1-\varepsilon)k^* - 1 \end{array}\right].$$

Let $\xi$ be an eigenvalue of the Jacobian matrix (30). We can then set the following characteristic equation of the linearization around the equilibrium:

$$(31) \qquad \left|J - \xi I\right| = \left|\begin{array}{c|c|c} -\delta\lambda - \xi & 0 & 0 \\ \hline -\alpha\delta\lambda & -\left(\varphi\left(1-\varepsilon/2\right)+\theta\mu\right) - \xi & 0 \\ \hline 0 & 0 & 2(1-\varepsilon)k^* - 1 - \xi \end{array}\right| = 0.$$

This characteristic equation can be re-written as follows:

$$(31\text{-a}) \qquad (\delta\lambda + \xi)\left[\left(\varphi\left(1-\varepsilon/2\right)+\theta\mu\right) + \xi\right][2(1-\varepsilon)k^* - 1 - \xi] = 0,$$

whose solutions are the eigenvalues of the Jacobian matrix (30), which are given by:

$$(32) \qquad \xi_1 = -\delta\lambda < 0,\ \xi_2 = -[\varphi(1-\varepsilon/2)+\theta\mu] < 0 \text{ and } \xi_3 = 2(1-\varepsilon)k^* - 1 < 0.$$

Therefore, $\xi_1$ and (as can be easily checked) $\xi_2$ are strictly negative. Meanwhile, using (28) and (A2) in the appendix, it follows that $\xi_3 = -\sqrt{1-2(1-\varepsilon)\varepsilon} < 0$.

Let us now analyze the system given by equations (14), (16) and (26) under the assumption that mutation is absent (i.e., $\varepsilon = 0$). As a result, the equilibrium solution is again characterized by $y = y^T$ and $p = p^T$. Given that $F(0) = 0$, in the absence of mutant agents (26) implies that $\dot{k} = 0$ obtains if, and only if, either $k = 0$ or $k = 1$. Therefore, as $\varepsilon$ reaches (from above) the critical value of zero, the system bifurcates from a unique equilibrium point to a multiple equilibrium configuration.

The Jacobian matrix evaluated around the equilibrium given by $(y^T, p^T, 0)$ is the following:

$$(33) \qquad J\left(y^T, p^T, 0\right) = \left[\begin{array}{c|c|c} -\delta\lambda & 0 & 0 \\ \hline -\alpha\delta\lambda & -\left(\varphi+\theta\mu\right) & 0 \\ \hline 0 & 0 & -1 \end{array}\right].$$

Let $\xi$ be an eigenvalue of the Jacobian matrix (33). We can then set the following characteristic equation of the linearization around the equilibrium:

(34) $$|J-\xi I| = \begin{vmatrix} -\delta\lambda-\xi & 0 & 0 \\ -\alpha\delta\lambda & -(\varphi+\theta\mu)-\xi & 0 \\ 0 & 0 & -1-\xi \end{vmatrix} = 0.$$

This characteristic equation can be re-written as follows:

(34-a) $$(\delta\lambda+\xi)\left[(\varphi+\theta\mu)+\xi\right](-1-\xi)=0,$$

whose solutions are the eigenvalues of the Jacobian matrix (33), which are given by:

(35) $$\xi_1 = -\delta\lambda < 0,\ \xi_2 = -(\varphi+\theta\mu) < 0 \text{ and } \xi_3 = -1 < 0.$$

Given that all these eigenvalues have strictly negative real parts, the equilibrium configuration given by $(y^T, p^T, 0)$ is a local attractor.

Meanwhile, the Jacobian matrix evaluated around the equilibrium given by $(y^T, p^T, 1)$ is as follows:

(36) $$J\left(y^T, p^T, 1\right) = \begin{bmatrix} -\delta\lambda & 0 & 0 \\ -\alpha\delta\lambda & -(\varphi+\theta\mu) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Let $\xi$ be an eigenvalue of the Jacobian matrix (36). We can then set the following characteristic equation of the linearization around the equilibrium:

(37) $$|J-\xi I| = \begin{vmatrix} -\delta\lambda-\xi & 0 & 0 \\ -\alpha\delta\lambda & -(\varphi+\theta\mu)-\xi & 0 \\ 0 & 0 & 1-\xi \end{vmatrix} = 0.$$

This characteristic equation can be re-written as follows:

(37-a) $$(\delta\lambda+\xi)\left[(\varphi+\theta\mu)+\xi\right](1-\xi)=0,$$

whose solutions are the eigenvalues of the Jacobian matrix (36), which are given by:

(38) $$\xi_1 = -\delta\lambda < 0,\ \xi_2 = -[\varphi+\theta\mu] < 0 \text{ and } \xi_3 = 1 > 0.$$

Given that one of these eigenvalues has a strictly positive real part, the equilibrium configuration given by $\left(y^T, p^T, 1\right)$ is saddle-point unstable. Therefore, albeit only by chance, the economy may converge to an equilibrium configuration in which both policy targets are achieved and full incredulity ($k = 1$) is the only surviving strategy. In fact, when the evolutionary dynamics in (26) operates in the absence of mutation (i.e. $\varepsilon = 0$), even if full incredulity ($k = 1$) prevails from the beginning (as a result, for instance, of an exogenous shock to agents' credulity), the economy remains in the plane given by $\{(y, p, k) \in \mathbb{R}^3_+ : k = 1\}$ and therefore both policy targets are achieved in equilibrium despite the persistence of full incredulity along the transition dynamics.[7] The reason why this result is not counter-intuitive is that to assume that full incredulity prevails from the beginning is tantamount to assuming that the distribution of inflation foresight strategies is already in equilibrium.

Moreover, as in the preceding specification of the satisficing evolutionary dynamics the existence, uniqueness and local asymptotic stability properties of both equilibrium configurations (with and without mutation) are preserved even if the incomes policy is shut down ($\mu = 0$). As before, the intuition is that the Tinbergen (1952) principle is still satisfied, as there are two adjusting variables (viz. the interest rate, $r$, and the degree of credibility of the policy authorities, measured by $k$), ensuring the achievement of two targets (inflation and output).

## 6. Conclusions

Drawing on Post Keynesian behavioral foundations and an extensive empirical literature that suggests persistent heterogeneity and time variation in inflation expectations that depart from the rational expectations ideal, this paper embeds two inflation forecasting heuristics – one based on the current rate of inflation, the second anchored to the policy authorities' inflation target – in a simple macrodynamic model. Decision makers are then allowed to switch between these heuristics in accordance with satisficing evolutionary dynamics that may be subject to noise. The resulting model allows us to study whether or not a macroeconomic equilibrium consistent with the

[7] Recall that when an equilibrium solution of a dynamic system is saddle-point unstable, the stable arm corresponds to the eigenvector(s) associated with the negative eigenvalue(s), so that the dimension of the stable arm is given by the number of negative eigenvalues. Since two of the eigenvalues of the Jacobian matrix (36) have strictly negative real parts, the corresponding stable arm (or manifold) is therefore a plane going through the equilibrium solution given by $\left(y^T, p^T, 1\right)$.

realization of policy makers' targets can be achieved, and whether or not this involves the extinction of either of the inflation forecasting strategies with which decision makers begin.

Our results show that, in general, convergence towards an equilibrium consistent with the level of output and rate of inflation targeted by policy makers is achieved regardless of whether or not the satisficing evolutionary dynamics that guide the choices agents make between inflation forecasting strategies are subject to noise. This is true even when satisficing behavior acts only as a trigger that transforms individual agents into potential strategy revisers. The equilibrium proportion of credulous agents (who form inflation expectations anchored to the inflation target) does, however, vary with the amount of noise (i.e., the level of exogenous mutation) in the model's satisficing evolutionary dynamics. Taken together, these results demonstrate that full credulity – a situation where all agents eventually adopt the forecasting heuristic based on the target rate of inflation – is neither a necessary condition for realization of the inflation target, nor an inevitable consequence of the economy's achievement of this target. Our results also show that in general, the endogenous adjustment of inflation expectations in accordance with our satisficing evolutionary dynamics relaxes the constraint imposed on policy makers by the Tinbergen (1952) principle, allowing policy makers to pursue two targets (output and inflation) using only one instrument (monetary policy). The violation of the Tinbergen principle is only apparent, though, as the degree of heterogeneity in inflation foresight strategies becomes another adjusting variable. Interestingly, therefore, the existence of time-varying heterogeneity in inflation expectations formation by private agents may actually facilitate rather than hinder policy-making by the monetary authorities.

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## Appendix

Let us show that there is one, and only one, mixed-strategy equilibrium under the alternative satisficing evolutionary dynamics with mutation analyzed in Section 5.

Recall that $F(0) = 0$ implies that $\dot{k} = 0$ obtains if, and only if, condition (27) is satisfied. Therefore, we have to show that there is one, and only one, $k^* \in (0,1) \subset \mathbb{R}$ such that $f(k^*) \equiv (1-\varepsilon)(k^*)^2 - k^* + \frac{\varepsilon}{2} = 0$.

It follows from (27) that:

(A1) $$k = \frac{1 \pm \sqrt{1-2(1-\varepsilon)\varepsilon}}{2(1-\varepsilon)}.$$

Now define $y = 2(1-\varepsilon)\varepsilon$. Then $\lim_{\varepsilon \to 0} y = \lim_{\varepsilon \to 1} y = 0$, while:

$$\frac{dy}{d\varepsilon} = 2 - 4\varepsilon > 0 \text{ for all } 0 < \varepsilon < \frac{1}{2},$$

$$\frac{dy}{d\varepsilon} = 2 - 4\varepsilon = 0 \text{ for } \varepsilon = \frac{1}{2},$$

and

$$\frac{dy}{d\varepsilon} = 2 - 4\varepsilon < 0 \text{ for all } \frac{1}{2} < \varepsilon < 1.$$

It follows that when $\varepsilon = 1/2$, the maximum value of $y$ is $y = 1/2$. Hence we have:

(A2) $$1 > 1 - 2(1-\varepsilon)\varepsilon \geq \frac{1}{2} \Rightarrow 1 > \sqrt{1-2(1-\varepsilon)\varepsilon} \geq \sqrt{1/2} \Rightarrow 1 \pm \sqrt{1-2(1-\varepsilon)\varepsilon} >> 0.$$

Using (A2), we can show that there is one, and only one, $k^* \in (0,1) \subset \mathbb{R}$ such that $f(k^*) = 0$. It suffices to show that:

(A3) $$k_1^* \equiv \frac{1 - \sqrt{1-2(1-\varepsilon)\varepsilon}}{2(1-\varepsilon)} < 1 \text{ and } k_2^* \equiv \frac{1 + \sqrt{1-2(1-\varepsilon)\varepsilon}}{2(1-\varepsilon)} > 1.$$

Let us prove that $k_1^* < 1$. Since $0 < \varepsilon < 1$, it follows that:

(A4) $$\begin{aligned}&2(1-\varepsilon)\varepsilon < 4(1-\varepsilon)\varepsilon \Rightarrow 1-2(1-\varepsilon)\varepsilon > 1-4(1-\varepsilon)\varepsilon \Rightarrow 1-2(1-\varepsilon)\varepsilon > (2\varepsilon-1)^2 \Rightarrow\\ &\Rightarrow 1-\sqrt{1-2(1-\varepsilon)\varepsilon} < 2(1-\varepsilon) \Rightarrow \frac{1-\sqrt{1-2(1-\varepsilon)\varepsilon}}{2(1-\varepsilon)} < 1,\end{aligned}$$

which completes the proof that $k_1^* < 1$.

Let us now show that $k_2^* > 1$. Let us suppose rather that $k_2^* \leq 1$.

If $k_2^* = 1$, it follows that $f(1) \equiv (1-\varepsilon)1^2 - 1 + \frac{\varepsilon}{2} = -\frac{\varepsilon}{2} < 0$. Therefore, $k_2^* = 1$ is not an equilibrium solution. If $k_2^* < 1$, the following inequality must be satisfied:

$$k_2^* \equiv \frac{1+\sqrt{1-2(1-\varepsilon)\varepsilon}}{2(1-\varepsilon)} < 1,$$

from which it follows that:

(A5) $$1+\sqrt{1-2(1-\varepsilon)\varepsilon} < 2(1-\varepsilon) \Rightarrow \sqrt{1-2(1-\varepsilon)\varepsilon} < 1-2\varepsilon.$$

If $\varepsilon \geq \frac{1}{2}$, (A2) implies that (A5) is not satisfied. Meanwhile, we can re-write (A5) as follows:

(A6) $$1-2(1-\varepsilon)\varepsilon < (1-2\varepsilon)^2 \Rightarrow 1-2(1-\varepsilon)\varepsilon < 1-4\varepsilon+4\varepsilon^2 \Rightarrow 2(1-\varepsilon)\varepsilon > 4(1-\varepsilon)\varepsilon.$$

Since, as stated earlier, $2(1-\varepsilon)\varepsilon < 4(1-\varepsilon)\varepsilon \ \forall\ 0 < \varepsilon < 1$, the last inequality in (A6) is not satisfied for $0 < \varepsilon < \frac{1}{2}$. This completes the proof that $k_2^* > 1$.

Therefore, it follows that the unique equilibrium is given by $k^* = k_1^* \equiv \dfrac{1-\sqrt{1-2(1-\varepsilon)\varepsilon}}{2(1-\varepsilon)}$.