Department of Economics - FEA/USP

Long-run implications of government budget leakage in a Solow-Swan economy: An evolutionary game approach

GILBERTO TADEU LIMA

JAYLSON JAIR DA SILVEIRA

WORKING PAPER SERIES Nº 2024-29

DEPARTMENT OF ECONOMICS, FEA-USP
WORKING PAPER Nº 2024-29

Long-run implications of government budget leakage in a Solow-Swan economy: An evolutionary game approach

Gilberto Tadeu Lima (giltadeu@usp.br)

Jaylson Jair da Silveira (jaylson.silveira@gmail.com)

Abstract:

There is mounting evidence of persistent capture or drainage of government budgetary resources through unlawful means by individuals in the rest of the economy. This paper develops a formal analytical framework in which such persistence arises as a stable evolutionary equilibrium configuration. Also in keeping with the empirical evidence, this evolutionary equilibrium is characterized by behavioral heterogeneity across decision makers who periodically choose whether or not to engage in illegal activities of capture of government resources. A key implication is that the macrodynamics of the capital stock in per capita terms and the per capita income are crucially affected by the frequency of capturing behavior in the economy in a complex way.

Keywords: Economic growth; Solow-Swan model; evolutionary games.

JEL Codes: C73; E13; O46.

Long-run implications of government budget leakage in a Solow-Swan economy: An evolutionary game approach*

Gilberto Tadeu Lima
Department of Economics
University of São Paulo (Brazil)
giltadeu@usp.br

&

Jaylson Jair da Silveira
Department of Economics and International Relations
Federal University of Santa Catarina (Brazil)
Jaylson.silveira@gmail.com

Abstract: There is mounting evidence of persistent capture or drainage of government budgetary resources through unlawful means by individuals in the rest of the economy. This paper develops a formal analytical framework in which such persistence arises as a stable evolutionary equilibrium configuration. Also in keeping with the empirical evidence, this evolutionary equilibrium is characterized by behavioral heterogeneity across decision makers who periodically choose whether or not to engage in illegal activities of capture of government resources. A key implication is that the macrodynamics of the capital stock in per capita terms and the per capita income are crucially affected by the frequency of capturing behavior in the economy in a complex way.

Keywords: Economic growth; Solow-Swan model; evolutionary games.

JEL Codes: C73, E13, 049.

* This study was supported by CNPq (National Council of Scientific and Technological Development – Brazil) [grants 316740/2023-3 (GTL) and 313628/2021-1 (JJS)]. This study was also financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001 and the São Paulo Research Foundation – Brazil (FAPESP) [grant 2019/03148–5].

1. Introduction

A persistent occurrence in developing and developed countries is the capture of some of the government budgetary resources through unlawful means by individuals in the rest of the economy. These illegal means include, for example, different forms of corruption (such as bribery of government officials or bureaucrats and undue influence peddling) and a few specific kinds of rent-seeking behavior (such as the bribing of politicians to gain easier access to public decisions involving the government budget). Yet it is important to differentiate rent seeking from corruption, as not all rent seeking activities are illegal or depend on bribery.

This persistent capture of government budgetary resources has various negative consequences. In fact, several empirical studies show that high levels of corruption are associated with lower investment and entrepreneurship as well as slower development (see, e.g., Weill, 2000; Mo, 2001; Swaleheen and Stansel, 2007; Avnimelech, Zelekha and Sharabi, 2014). There is also evidence that the relationship between corruption and slower economic development is bicausal (Treisman, 2000). Meanwhile, Méon and Weil (2010) contend that corruption might remove bureaucratic obstacles and found some evidence that corruption may act as "grease in the wheels" in countries with sluggish bureaucratic systems.

Against this backdrop, this paper develops an analytical framework in which the capture of some of the government budget through unlawful means may arise and be persistent, in keeping with the existing empirical evidence. The net government revenues (gross government revenues minus the amount of captured revenues not recovered by the government plus the pecuniary penalization imposed on the amount of captured revenues that was recovered by the government) is endogenously time-varying driven by an imitation-augmented satisficing evolutionary dynamic. A key implication is that the macrodynamics of the capital stock in per capita terms and the per capita income in a Solow-Swan economy are crucially affected by the microdynamics of capturing behavior across agents who periodically decide whether or not to engage in illegal activities of capture of government resources.

The remainder of this paper is organized as follows. Section 2 introduces the analytical framework and explores the behavior of the economy when the proportion of agents who engage in illegal activities of capture or drainage of government budgetary resources is predetermined. Section 3 specifies and explores the evolutionary dynamics driving the frequency of capturing behavior in the economy. Section 4 concludes the paper with further remarks.

2. A Solow-Swan model with productive government expenditure under budget leakage

The model economy is closed to international trade and capital movements, producing a single homogeneous good for both investment and consumption purposes. We assume that the government provides free-of-charge a flow of services as an input to private production, as in Barro (1990). The profit-maximizing firms take as given such flow of productive inputs provided by the government and combine them with two other (equally homogeneous) factors of production, capital and labor, as specified by a Cobb-Douglas technology. As the government resources are subject to leakage owing to several illegal diversion and capture mechanisms used by private agents or even government officials, the amount of productive government expenditures, G, may be strictly lower than the tax revenues, T. We assume that there is a fixed and large number H of households in the economy. At a given point in time, there may be a strictly positive proportion $x = H_c/H$ of households that chose to capture government revenues to increase their disposable income (and succeeded in doing so), where H_c is the number of capturer households, which varies over time according to an evolutionary dynamic to be described in subsection 3.1. We assume that the amount of public resources diverted from the government budget is given by:

(1) $$D = \phi(x)T ,$$

where the function $\phi(x) \in [0,1] \subset \mathbb{R}$ denotes the fraction of tax revenues that are diverted from the gross government budget, which satisfies the following conditions: $\phi'(x) > 0$, $\phi(0) = 0$ and $\phi(1) = 1$. In sum, the function $\phi(x)$ in (1) plausibly specifies that the amount of tax revenues diverted from the government budget increases with the fraction of households that chose to capture government revenues to increase their disposable income, x. From now on, for analytical simplicity, we assume that $\phi(x) = x$.

We assume that the government revenues are generated by a flat-rate income tax:

(2) $$T = \tau Y ,$$

where $\tau \in (0,1) \subset \mathbb{R}$ is the income tax rate and Y is the aggregate income level.

The government holds a balanced net budget, employing all collected revenues (including penalties imposed on capturer households who are detected, as detailed in the next section) to cover its expenditures. Therefore, out of the amount deviated from the gross government budget, which using (1)-(2) together with $\phi(x) = x$ is given by $D = x\tau Y$, the government manages to recover an amount represented by $\rho D = \rho x \tau Y$. In sum, the resulting government expenditure is determined as follows:

(3) $$G = T - D + \rho D = [1-(1-\rho)x]\tau Y ,$$

where $\rho \in (0,1) \subset \mathbb{R}$ is the rate of recovery of government revenue per unit of tax revenue deviated by capturer households.

Therefore, when the fraction of households that chose to capture government resources goes down, productive government spending rises. This happens because a higher fraction of households engaging in capturing public resources increases the leakage of tax revenues. When there is full capture of government resources, meaning that $x = 1$, government expenditure is funded solely by recovered revenues, i.e., $G = \rho\tau Y$. When there is no diversion of government resources, meaning that $x = 0$, gross tax revenues are fully employed to finance productive government expenditures, that is, $G = \tau Y$.

Normalizing the expression in (3) by the total labor force, denoted by L, which for simplicity is taken as equal to the population, we can express the net government expenditure in per capita terms (i.e., the gross tax revenues net of budget leakages that are not recovered) as follows:

(4) $$g = [1-(1-\rho)x]\tau y ,$$

where $g \equiv G/L$ and $y \equiv Y/L$ are the levels of productive government expenditure and output in per capita terms, respectively. Hence the government spends all its revenues net of budget leakages that are not recovered on the provision free-of-charge of a flow of goods/services that are used by firms as an input for output production.

In effect, we assume that aggregate output is produced according to a Cobb-Douglas function that satisfies the Inada conditions, that is:

(5) $$Y = K^{\alpha} G^{\beta} L^{1-(\alpha+\beta)},$$

where K is the aggregate stock of capital, while $\alpha \in (0,1) \subset \mathbb{R}$ and $\beta \in (0,1) \subset \mathbb{R}$ are parametric constants, with $\alpha + \beta < 1$. Therefore, the Cobb-Douglas functional form in (5) displays constant returns to scale and specifies that all inputs are necessary for production (i.e., aggregate output is equal to zero when the use of any of the inputs is equal to zero).

The production function in (5) in intensive form is given by:

(6) $$y = k^{\alpha} g^{\beta},$$

where $k \equiv K/L$ is the per capita capital stock.

Substituting the per capita productive government expenditure in (4) into the intensive-form production function in (6), we obtain:

(7) $$y = k^{a} \{[1-(1-\rho)x]\tau\}^{b},$$

where

(8) $$a \equiv \frac{\alpha}{1-\beta} \in (0,1) \subset \mathbb{R} \text{ and } b \equiv \frac{\beta}{1-\beta} \in (0,1) \subset \mathbb{R}.$$

The disposable income of the set of households is composed not only of the net income after taxes, $Y - T = (1-\tau)Y$, as usual, but also of the amount of diverted public revenues that are not recovered by the government, which per (1)-(2), recalling that $\phi(x) = x$, is given by $(1-\rho)D = (1-\rho)x\tau Y$. Therefore, the total disposable income in the hands of the set of households is given by:

(9) $$Y^{d} = (1-\tau)Y + (1-\rho)x\tau Y = \{1-[1-(1-\rho)x]\tau\}Y.$$

Considering that $0 < Y^{d} < Y$, it follows from (9) that the following inequalities have to hold: $0 < 1-[1-(1-\rho)x]\tau < 1$, which are satisfied since $\rho \in (0,1) \subset \mathbb{R}$. In this case, the total disposable income of the set of households varies positively with the proportion of capturer households. More precisely, given that $\frac{\partial Y^{d}}{\partial x} = (1-\rho)\tau Y > 0$, the total disposable income of the set of households is strictly increasing in the proportion of capturer households for all $\rho \in (0,1) \subset \mathbb{R}$. Considering (9), it is worth noting that the total disposable income of the set of households when $x = 0$ is given by the usual expression $(1-\tau)Y$, while it is given $(1-\rho\tau)Y$ when $x = 1$. It then follows that the disposable income differential represented by $(1-\tau)Y - (1-\rho\tau)Y = -(1-\rho)\tau Y$ is strictly negative, as $\rho \in (0,1) \subset \mathbb{R}$.

As in Solow (1956) and Swan (1956), aggregate savings constitute a fraction of the total disposable income in the hands of households, $S = sY^{d}$, where $s \in [0,1] \subset \mathbb{R}$ is the constant saving rate. Using (9), aggregate savings can be written as:

(10) $$S = s\{1-[1-(1-\rho)x]\tau\}Y.$$

Also following Solow (1956) and Swan (1956), we assume that savings are fully and automatically converted into investment demand, so that $I = S$. Besides, we suppose for simplicity that there is no depreciation of the capital stock, so that the rate of change in the aggregate capital stock is equal to the aggregate investment, i.e., $\frac{dK}{dt} \equiv \dot{K} = I$. Thus, we can use (10) to obtain:

(11) $$\dot{K} = s\{1-[1-(1-\rho)x]\tau\}Y .$$

Let us consider that population, which is taken to be equal in size to the labor force, grows at a constant rate $\eta \in \mathbb{R}_{++}$, that is:

(12) $$\frac{\dot{L}}{L} = \eta .$$

As $k \equiv K/L$, it then follows that the growth rate of the capital stock in per capita terms is given by:

(13) $$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} .$$

Substituting (11) and (12) into (13), we obtain:

(14) $$\dot{k} = s\{1-[1-(1-\rho)x]\tau\}y - \eta k .$$

This equation states that the per capita capital stock increases (decreases) if per capita savings are strictly greater (smaller) than the investment necessary to offset population (and hence labor force) growth.

Substituting the function in (7) into (14) we then obtain the Solow-Swan fundamental equation of growth, now parameterized by the proportion of households that chose to follow the strategy of capturing public resources:

(15) $$\dot{k} = sh(x)k^a - nk ,$$

where, using (4) and (9),

(16) $$h(x,\tau) \equiv \{1-[1-(1-\rho)x]\tau\}\{[1-(1-\rho)x]\tau\}^b .$$

As by assumption $\rho \in (0,1) \subset \mathbb{R}$, we know that $h(x) > 0$ for any $x \in [0,1] \subset \mathbb{R}$. Note that $h(x,\tau)$ in the expression above involves the operation of two different effects. The term $\frac{Y^d}{Y} = 1-[1-(1-\rho)x]\tau$ in (16) is related to what can be dubbed *disposable income effect*. When the proportion of households following the capturing strategy and hence the leakage of public revenues increase, households' disposable income and hence saving and investment formation rise. In other words, there is a *disposable income effect*, which can be represented by $\frac{\partial(Y^d/Y)}{\partial x} = (1-\rho)\tau > 0$, given that $\rho \in (0,1) \subset \mathbb{R}$ and $\tau \in (0,1) \subset \mathbb{R}$. However, the same increase in the proportion of households following the capturing strategy reduces productive government spending, as indicated by the expression $\frac{G}{Y} = [1-(1-\rho)x]\tau$ in (16). This *productive government-expenditure effect* is represented by $\frac{\partial(G/Y)}{\partial x} = -(1-\rho)\tau < 0$, as

$\rho \in (0,1) \subset \mathbb{R}$ and $\tau \in (0,1) \subset \mathbb{R}$. Consequently, these two effects operate in different directions when the effectiveness with which the government recovers public revenues that were deviated by capturer households is characterized by $\rho < 1$.

We can rewrite (15) in terms of growth rate as follows:

(17) $$\hat{k} \equiv \frac{\dot{k}}{k} = sh(x,\tau)k^{a-1} - \eta .$$

In the balanced growth path, when the aggregate capital stock grows at the same rate as population, so that $\hat{k} = 0$, the unique economically relevant steady-state level of the capital stock in per capita terms, k^*, is therefore a function of the proportion of households following the capturing strategy, x, that is:

(18) $$k^* = \left[\frac{sh(x,\tau)}{\eta}\right]^{\frac{1}{1-a}} \equiv k^*(x) .$$

Considering that function $h(x,\tau)$ in (16) is bounded in unit interval $[0,1] \subset \mathbb{R}$ and the growth rate in (17), we know that the growth rate of the capital stock in per capita terms goes to infinity when k approaches zero. Meanwhile, as the capital stock in per capita terms goes to infinity, its growth rate approaches $-\eta$. The growth rate of the capital stock in per capita terms is a strictly decreasing and strictly convex function of the capital stock in per capita terms, since from (17) we have:

(19) $$\frac{\partial \hat{k}}{\partial k} = (a-1)sh(x,\tau)k^{a-2} < 0 \text{ and } \frac{\partial^2 \hat{k}}{\partial k^2} = (a-1)(a-2)sh(x,\tau)k^{a-3} > 0 ,$$

for all $k > 0$, recalling that $a \in (0,1) \subset \mathbb{R}$ in (8). These characteristics of the function in (17) implies that $k^*(x)$ is a global attractor for a given $x \in [0,1) \subset \mathbb{R}$.

Let τ^* be the value of tax rate that maximizes the steady-state capital stock in per capita terms in (18). Based on the expression in (18) and recalling that $h(x,\tau) > 0$ for any $x \in [0,1] \subset \mathbb{R}$ and $\tau \in (0,1) \subset \mathbb{R}$, the maximizing tax rate τ^* has then to satisfy the following condition:

(20) $$\frac{\partial k^*}{\partial \tau} = \left(\frac{s}{\eta}\right)^{\frac{1}{1-a}} \frac{[h(x,\tau)]^{\frac{a}{1-a}}}{(1-a)} \frac{\partial h(x,\tau^*)}{\partial \tau} = 0 \Leftrightarrow \frac{\partial h(x,\tau^*)}{\partial \tau} = 0 .$$

Using (16), we have for all $x \in (0,1] \subset \mathbb{R}$ that:

(21)
$$\frac{\partial h(x,\tau)}{\partial \tau} = (1-\rho)x\{[1-(1-\rho)x]\tau\}^{b-1}\left\{[1-(1-\rho)x]\tau(1+b) - b\right\} = (1-\rho)x\{[1-(1-\rho)x]\tau\}^{b-1} b\left(\frac{\tau}{\tau^*} - 1\right) \overset{<}{\underset{>}{=}} 0 \Leftrightarrow \tau \overset{<}{\underset{>}{=}} \tau^* ,$$

where $\tau^* \equiv \dfrac{b}{(1+b)[1-(1-\rho)x]} = \dfrac{\beta}{1-(1-\rho)x}$, given the definition of b in (8). As a result, from the derivatives in (20)-(21), we can infer that the tax rate that maximizes the steady-state capital stock in capita terms for a given level of capture of public resources can be expressed as follows:

(22) $$\tau^*(x) = \min\left\{\frac{\beta}{1-(1-\rho)x}, 1\right\}, \text{ for any } x \in [0,1) \subset \mathbb{R}.$$

Note that when there is no capture of public resources ($x = 0$) the threshold tax rate in (22) is at its minimum value, given by β. When the level of capture of public resources increases, the threshold tax rate in (22) increases monotonically until it reaches its maximum value, given by β/ρ. In other words, the higher the frequency of households capturing public resources, the greater the leakage in the gross public budget and the higher the tax rate associated with the highest steady-state capital stock in per capita terms.

The impact of a change in the frequency of capturer households on the steady-state capital stock in per capita terms also depends on the difference between the actual tax rate and the tax rate τ^*. In fact, given the sign of b in (8), we have from (16) for all $x \in [0,1] \subset \mathbb{R}$ the following result:

(23)
$$\frac{\partial h(x,\tau)}{\partial x} = (1-\rho)\tau\{[1-(1-\rho)x]\tau\}^{b-1}\{[1-(1-\rho)x]\tau(1+b)-b\} = (1-\rho)\tau\{[1-(1-\rho)x]\tau\}^{b-1}b\left(\frac{\tau}{\tau^*}-1\right)\lesseqgtr 0 \Leftrightarrow \tau \lesseqgtr \tau^*,$$

recalling that $\tau^* \equiv \dfrac{b}{(1+b)[1-(1-\rho)x]} = \dfrac{\beta}{1-(1-\rho)x}$ is the threshold tax rate.

Considering (21)-(23), based on (18) we have the following comparative statics:

(24) $$\frac{\partial k^*(x)}{\partial \tau} = \left(\frac{s}{\eta}\right)^{\frac{1}{1-a}} \frac{[h(x,\tau)]^{\frac{a}{1-a}}}{(1-a)} \frac{\partial h(x)}{\partial \tau} \lesseqgtr 0 \quad \Leftrightarrow \quad \tau \lesseqgtr \tau^*$$

and

(25) $$\frac{\partial k^*(x)}{\partial x} = \left(\frac{s}{\eta}\right)^{\frac{1}{1-a}} \frac{[h(x)]^{\frac{a}{1-a}}}{(1-a)} \frac{\partial h(x,\tau)}{\partial x} \lesseqgtr 0 \quad \Leftrightarrow \quad \tau \lesseqgtr \tau^*.$$

3. Coevolution of macrodynamics and government budget leakage

The preceding section has shown that the unique economically relevant steady-state level of the capital stock in per capita terms in (18) is, for a given proportion of capturer households in the economy, a global attractor. Let us then now specify the evolutionary protocol driving the dynamics of the frequency distribution of strategies as regards government resources (extracting and non-extracting) across households, given by $(x, 1-x)$.

These dynamics will be evaluated at the steady-state level of the capital stock in per capita terms in (18), which is parameterized by the frequency distribution of strategies in question. The assumption underlying our analytical approach to explore the evolutionary dynamics of the heterogeneity in strategic behavior across households is that the capital stock in per capita terms adjusts faster than the frequency distribution of behavioral strategies across households. Therefore, we conceive of the steady-state level of the capital stock in per capita terms as a kind of stable temporary equilibrium, in that the latter is parameterized by a variable the adjustment of which occurs only after this temporary steady-state is achieved. In fact, it is plausible to conceive of the behavior of households with respect to whether or not to capture or deviate public resources as having a cultural dimension to it, so that it is a behavioral trait that varies quite slowly. It is then only in a configuration that we dub evolutionary steady-state that both the capital stock in per capita terms and the frequency distribution of behavioral strategies across households become stationary.

3.1 An imitation-augmented satisficing evolutionary dynamic of government budget leakage

The economy is composed of households who may or may not engage in activities of capture or deviation of public resources. There is a large population of households, which is constant over time. More specifically, consider that there are H households, each one consisting of L individuals. Given that H is fixed, each household grows at the same rate $\eta \in \mathbb{R}_{++}$.

Intrahousehold behavior is homogeneous. The representative member of a non-capturer household (herein identified by the subscript n) has a disposable income given by $(1-\tau)y$. As households are concerned about their members' payoffs, the relevant strategy payoff of a non-capturer household, denoted by u_n, is the per capita consumption of its representative member, i.e., $u_n = (1-s)(1-\tau)y$, which can then be re-written using (7) as follows:

(26) $$u_n = (1-s)(1-\tau)k^a\{[1-(1-\rho)x]\tau\}^b \equiv u_n(x,k).$$

In addition to the disposable income enjoyed by all members of the economy, the representative member of a capturer household (identified by the subscript c), deviates for herself a certain fraction of the gross government revenues in (1), which are given by $\frac{1}{(L/H)}\left(\frac{\phi(x)T}{H_c}\right) = \tau y$, recalling that, by assumption, $\phi(x) = x$. Thus, assuming that all capturer households nonetheless comply with their tax obligations, their disposable income before considering the risk of penalization if they are caught can be expressed as follows:

(27) $$(1-\tau)y + \tau y = y.$$

Let $\gamma \in (\tau,1) \subset \mathbb{R}$ be the percentage of its income with which a capturer household is penalized in the case of being detected and hence punished. Thus, net of punishment, the disposable income of a capturer household who is detected is:

(28) $$(1-\gamma)y.$$

Despite the risks associated with capturing resources from the government budget, some households may choose to do so. The probability with which a capturer household is caught and hence punished is given by $\varepsilon \in (0,1) \subset \mathbb{R}$. Thus, the disposable income of the representative member of a capturer household can be expressed as the following expected value:

(29) $$y_c^d = \varepsilon(1-\gamma)y + (1-\varepsilon)y = (1-\varepsilon\gamma)y.$$

Notice that the expected punishment of a capturer household, which is given by $\varepsilon\gamma$, determines the rate of recovery of government revenues per unit of tax revenue deviated by capturer households, i.e., $\rho = \varepsilon\gamma$. We can then specify the expected payoff of the representative member of a capturer household as the portion of her expected disposable income that is allocated to consumption as follows:

(30) $u_c = (1-s)(1-\rho)y$,

which can be re-written using (7) as follows:

(31) $u_c = (1-s)(1-\rho)k^a\{[1-(1-\rho)x]\tau\}^b \equiv u_c(x,k)$.

Therefore, there is homogeneity in the saving propensity across households, so that the saving rate of a given household does not depend on its type (c or n).

The response of the government to leakages in the public budget is determined by a reaction function according to which the rate of recovery of government revenues per unit of tax revenue deviated by capturer households, given by $\rho = \varepsilon\gamma$, varies with the proportion of capturer households:

(32) $\varepsilon\gamma = \rho(x)$,

with $0 < \rho(0) < \tau < \rho(1) < 1$ and $\rho'(x) > 0$ for all $x \in [0,1] \subset \mathbb{R}$. Note that to increase the endogenous rate of recovery specified in (32) the government can rely on an improved effort to monitor and detect unlawful capturing activities, which raises the probability ε with which a capturer household is detected, and/or a raise in the fraction γ of its income with which a capturer household is penalized in the event of being detected.

Drawing on the payoffs in (26) and (31), we assume that the selection process of behavioral strategies across households in the economy is described as an imitation-augmented satisficing evolutionary dynamic. More precisely, we assume that household j has a satisficing payoff that it considers to be the minimum acceptable for her current behavioral strategy, denoted by μ^j. In a given point in time, if the current payoff of household j is such that $u^j = u_l$, with $l = c, n$, is greater than or equal to its satisficing payoff μ^j, it will not consider switching behavioral strategy. Otherwise, when $u^j < \mu^j$, household j becomes a strategy reviser. Let us assume that the satisficing payoff is randomly and independently determined across households, with a cumulative distribution function given by $F : \mathbb{R} \to [0,1] \subset \mathbb{R}$, which is continuously differentiable and strictly increasing. It then follows that the probability with which a household will find its current payoff non-satisficing is given by:

(33) $\text{Prob}(\mu^j > u^j) = 1 - \text{Prob}(\mu^j \le u^j) = 1 - F(u^j)$.

Therefore, if household j is currently behaving as non-capturer, it becomes a strategy reviser with probability given by $1 - F\left(u_n(k,x)\right)$, where $u^j = u_n(k,x)$ is defined in (26). Having become a strategy reviser, a household j that is currently behaving as non-capturer will switch to the capturing strategy with probability given by the proportion of capturing households in the economy, x, which can be interpreted as a popularity or imitation effect. Thus, given that the mass of non-capturer households in a given point in time is represented by $1-x$, the inflow to (outflow from) the segment of capturing (non-capturing) households is given by:

(34) $(1-x)\left[1 - F\left(u_n(x,k)\right)\right]x$.

Analogously, if household j is currently behaving as capturer, it becomes a strategy reviser with probability given by $1 - F\left(u_c(k,x)\right)$, where $u^j = u_c(k,x)$ is defined in (31). Having become a strategy reviser, a household j that is currently behaving as capturer of government revenues will switch to the non-capturing strategy with probability given by the proportion of non-capturing households in the economy, $1-x$, which is another manifestation of a popularity or imitation effect. Consequently, given that the mass of capturer households in a given point in time is represented by x, the inflow to (outflow from) the segment of non-capturing (capturing) households is given by:

(35) $$x\left[1-F\left(u_c(x,k)\right)\right](1-x).$$

Subtracting the outflow in (35) from the inflow in (34) we arrive at the following imitation-augmented satisficing evolutionary dynamic:

(36) $$\dot{x}=x(1-x)\left[F\left(u_c(x,k)\right)-F\left(u_n(x,k)\right)\right].$$

As function F is strictly increasing, a rise in the per capita consumption of non-capturer households will cause the proportion of capturer households that see their per capita consumption as non-satisficing to increase. Analogously, if there is a rise in the per capita consumption of capturer households, the strategy of capturing government revenues becomes relatively more appealing, with the result that the proportion of non-capturer households that see their per capita consumption as non-satisficing increases. In sum, the satisficing evolutionary dynamic in (36) reflects the behavior of a selection mechanism according to which the proportion of capturer households in the economy varies positively with the relative fitness of the strategy of capturing public revenues.

3.2 Evolutionary equilibria along a balanced growth path

Let us explore the possibility that the Solow-Swan economy with productive government expenditures and government budget leakage modelled in this paper achieves an *evolutionary steady-state equilibrium*, i.e., a microeconomic state featuring the constancy of the frequency distribution of behavioral strategies, $(x,1-x)$, across households in the economy. As intimated earlier, while the adjustment of the frequency distribution of behavioral strategies is taking place, the steady-state level of the capital stock in per capita terms in (18) is always attained as a temporary balanced growth path equilibrium.

It follows that the state transition of the economy along the temporary balanced growth path of the capital stock in per capita terms is driven by the following imitation-augmented satisficing evolutionary dynamic:

(37) $$\dot{x}=x(1-x)\left[F\left(u_c(x,k^*(x))\right)-F\left(u_n(x,k^*(x))\right)\right],$$

which was obtained inserting the capital stock per capita along the temporary balanced growth path specified in (18) into the payoffs in (26) and (31) and considering them in the evolutionary dynamic specified in (36). Of course, the space state associated to the evolutionary dynamic in (37) is the unit interval $[0,1]\subset\mathbb{R}$.

Let us show that the evolutionary dynamic in (37) has three equilibria. These are two monomorphic equilibria featuring survival of only one behavioral strategy as regards whether or not to capture public resources in each, and one polymorphic equilibrium, now featuring the survival of both capturer and non-capturer strategies.

Considering (32), and based on the strategy payoffs specified in (26) and (31), we have that $u_c(0,k^*(0))-u_n(0,k^*(0))=(1-s)[k^*(0)]^a\tau^b[\tau-\rho(0)]>0$, where the expression represented by

$k^*(0) = \left[\frac{sh(0,\tau)}{\eta}\right]^{\frac{1}{1-a}} = \left[\frac{s(1-\tau)\tau^b}{\eta}\right]^{\frac{1}{1-a}} > 0$ is the steady-state value of the capital stock in per capita terms when there is no capture of public resources by households ($x = 0$). Thus, considering (37), it is straightforward to verify that $x = 0$ implies $\dot{x} = 0$, so that the economy has a monomorphic evolutionary equilibrium $(0, k^*(0))$ featuring no government budget leakage.

Considering the strategy payoffs in (26) and (31), and given the government reaction function in (32), we then have that $u_c(1,k^*(1)) - u_n(1,k^*(1)) = (1-s)[k^*(1)]^a[\rho(1)]^b[\tau - \rho(1)] < 0$, where $k^*(1) = \left[\frac{sh(1,\tau)}{\eta}\right]^{\frac{1}{1-a}} = 0$ is the steady-state value of the capital stock in per capita terms when all households engage in different illegal activities through which public resources are captured ($x = 1$). Consequently, it follows from the evolutionary dynamic in (37) that at $x = 1$ we then have $\dot{x} = 0$, so that that there is another monomorphic evolutionary equilibrium, given by $(1, k^*(1))$. This evolutionary equilibrium represents an extreme situation, with output production not coming to a halt because the government is able to recovery some of the leakage in public revenues.

In the event that the two strategies yield the same non-null expected payoff, meaning that there is a certain value $x^* \in (0,1) \subset \mathbb{R}$ such that $u_c(x^*, k^*(x^*)) = u_n(x^*, k^*(x^*)) \neq 0$, the evolutionary dynamic also features a polymorphic equilibrium. It should be noted that this evolutionary equilibrium configuration may be characterized by a strictly positive number of households switching behavioral strategies, but the respective outflows and inflows offset each other. As a result, the frequency distribution of behavioral strategies across households remains stationary. Formally, for any $x \in (0,1) \subset \mathbb{R}$ we have $\dot{x} = 0$ if, and only if, the expression within brackets in (37) is equal to zero. As function F is strictly increasing, it follows that the payoff equalization condition represented by $F\left(u_c(k,x)\right) = F\left(u_n(k,x)\right)$ holds if, and only if, $u_c(x,k^*(x)) = u_n(x,k^*(x))$. Taking into account the reaction function in (32), substituting the payoffs in (26) and (31) into the latter condition regarding the equalization of the respective payoffs allows simple algebraic manipulation to yield:

(38) $\rho(x) = \tau$.

In view of the government reaction function in (32), we know that $\rho(0) < \tau < \rho(1)$. As $\rho(x)$ is a continuous function over the unit interval $[0,1] \subset \mathbb{R}$, we can then apply the intermediate value theorem to readily conclude that there is some $x^* \in (0,1) \subset \mathbb{R}$ such that $\rho(x^*) = \tau$. Moreover, as $\rho'(x) > 0$ for all $x \in [0,1] \subset \mathbb{R}$, it follows that this function is strictly increasing in the proportion of capturer households. Thus, given that the function $\rho(x)$ is continuous over the unit interval $[0,1] \subset \mathbb{R}$, it then follows that there is a single polymorphic evolutionary equilibrium represented by $x^* \in (0,1) \subset \mathbb{R}$ implicitly defined by the condition in (38).

We can use (16) to express the capital stock in per capita terms associated with this polymorphic evolutionary equilibrium as:

(39) $$k^*(x^*) = \left[\frac{sh(x^*,\tau)}{\eta}\right]^{\frac{1}{1-a}},$$

which is the steady-state value of the capital stock in per capita terms evaluated at the polymorphic evolutionary equilibrium implicitly defined by the condition in (38). In sum, the evolutionary dynamic of the economy also features a polymorphic equilibrium configuration $(x^*, k^*(x^*))$, which is then characterized by a strictly positive level of capture of gross government revenues by households ($0 < x^* < 1$). However, a certain fraction of these captured public resources will be recovered by the government.

The stability properties of the three evolutionary equilibria can be readily inferred from the sign of the payoff differential given by $\psi(x) \equiv u_c(x, k^*(x)) - u_n(x, k^*(x))$, which can be expressed using (26), (31) and (32) as follows:

(40) $$\psi(x) = (1-s)[k^*(x)]^a\{[1-(1-\rho(x))x]\tau\}^b[\tau - \rho(x)].$$

Since $x(1-x) > 0$ for all $x \in (0,1) \subset \mathbb{R}$ and function F is strictly increasing, we know by (37) that $sign(\dot{x}) = sign(\psi(x))$ for all $x \in (0,1) \subset \mathbb{R}$, where $sign(\cdot)$ stands for the sign function. As $(1-s)[k^*(x)]^a\{[1-(1-\rho(x))x]\tau\}^b > 0$ for all $x \in (0,1) \subset \mathbb{R}$, the differential given by $\tau - \rho(x)$ in the second brackets in (40) determines $sign(\psi(x))$. We know that such differential given by $\tau - \rho(x)$ in (34) is null at $x = x^*$ and, consequently, $\psi(x^*) = 0$. As $\rho(x)$ is strictly increasing in x, we can then conclude that $\psi(x) > 0$ for all $x \in (0, x^*) \subset \mathbb{R}$ and $\psi(x) < 0$ for all $x \in (x^*, 1) \subset \mathbb{R}$. Therefore, $\dot{x} > 0$ for all $x \in (0, x^*) \subset \mathbb{R}$ and $\dot{x} < 0$ for all $x \in (x^*, 1) \subset \mathbb{R}$. It then follows that both monomorphic evolutionary equilibria are unstable (repulsors) and the unique polymorphic evolutionary equilibrium is locally asymptotically stable (an attractor).

It follows from the payoff equalization condition for the existence of a polymorphic evolutionary equilibrium in (38) that:

(41) $$\frac{\partial x^*}{\partial \tau} = \frac{1}{\rho'(x^*)} > 0.$$

Based on the expression above and the derivative in (25), we can compute the impact of a change in the tax rate on the steady-state capital stock in per capita terms evaluated at the unique polymorphic evolutionary equilibrium implicitly defined by the condition in (38):

(42) $$\frac{\partial k^*(x^*)}{\partial \tau} = \frac{\partial k^*(x^*)}{\partial x}\frac{\partial x^*}{\partial \tau} = \frac{1}{\rho'(x^*)}\frac{\partial k^*(x^*)}{\partial x} \begin{smallmatrix}<\\=\\>\end{smallmatrix} 0 \iff \tau \begin{smallmatrix}<\\=\\>\end{smallmatrix} \tau^*.$$

As seen in (25), the impact of a change in the frequency of capturer households on the steady-state capital stock in per capita terms depends on the ratio between the actual tax rate and the threshold tax rate τ^* defined in (21). Therefore, since per (41) an increase (decrease) in the tax rate increases (decreases) the frequency of capturer households, the impact of a change in the tax rate on the steady-state capital stock in per capita terms evaluated at the evolutionary polymorphic equilibrium also depends on the ratio between the actual tax rate and the threshold tax rate τ^*.

Meanwhile, per capita income in the polymorphic evolutionary equilibrium, given by $y^* = [k^*(x^*)]^a\{[1-(1-\rho(x^*))x^*]\tau\}^b$ by using (7), varies with the tax rate as follows:

(43) $\frac{\partial y^*}{\partial \tau} = a[k^*(x^*)]^{a-1}\frac{\partial k^*(x^*)}{\partial \tau}\{[1-(1-\rho(x^*))x^*]\tau\}^b + [k^*(x^*)]^a b\{[1-(1-\rho(x^*))x^*]\tau\}^{b-1}[\rho'(x^*)x^* - (1-\rho(x^*))]\tau$,

which can be re-written as follows:

(44)

$$\frac{\partial y^*}{\partial \tau}\frac{\tau}{y^*} = \frac{\partial y^*}{\partial \tau}\frac{\tau}{y^*} = a\left[\frac{\partial k^*(x^*)}{\partial \tau}\frac{\tau}{k^*(x^*)}\right] + b\left[\frac{\partial (G/Y)^*}{\partial \tau}\frac{\tau}{(G/Y)^*}\right] = aE_{k^*\tau} + bE_{(G/Y)^*\tau},$$

where $E_{y^*\tau} \equiv \frac{\partial y^*}{\partial \tau}\frac{\tau}{y^*}$ is the elasticity of per capita income with respect to the tax rate, $E_{k^*\tau} \equiv \frac{\partial k^*(x^*)}{\partial \tau}\frac{\tau}{k^*(x^*)}$ is the elasticity of the steady-state capital stock in per capita terms with respect to the tax rate, whereas $E_{(G/Y)^*\tau} \equiv \frac{\partial (G/Y)^*}{\partial \tau}\frac{\tau}{(G/Y)^*}$ is the elasticity of the ratio of government productive expenditures to income with respect to the tax rate, all evaluated at the polymorphic evolutionary equilibrium implicitly defined by the condition in (38).

Recalling from (16) that $\frac{G}{Y} = \{1-[1-\rho(x)]x\}\tau$, it follows that:

(45) $$E_{(G/Y)\tau} \equiv \frac{\partial (G/Y)}{\partial x}\frac{x}{\tau} = \left([\rho'(x)x - (1-\rho(x))]\tau\right)\frac{x}{\tau} = \left[E_{\rho x}(x) - \left(\frac{1-\rho(x)}{\rho(x)}\right)\right]x\rho(x),$$

where $E_{\rho,x}(x) \equiv \frac{\partial \rho(x)}{x}\frac{x}{\rho(x)}$ is the elasticity of the rate of recovery of captured government revenues per unit of captured government revenues with respect to frequency of capturer households. Note that the strictly positive ratio given by $\frac{1-\rho(x)}{\rho(x)}$ is related to the effectiveness with which the government recovers public revenues that were captured by households, with a higher such effectiveness corresponding to a lower such ratio. In the polymorphic evolutionary equilibrium configuration, we have $\rho(x^*) = \tau$, so that $\frac{1-\rho(x^*)}{\rho(x^*)} = \frac{1-\tau}{\tau}$ and hence by (45) we have:

(46) $$E_{(G/Y)^*x} = \left[E_{\rho x}(x^*) - \left(\frac{1-\tau}{\tau}\right)\right]x\tau.$$

Let us consider $\tau > \tau^*$, such that $E_{k^*\tau} > 0$ by (24). In this case, if the response of the rate of recovery of captured government revenues to an increase in the frequency of capturer households is sufficiently strong to satisfy $E_{\rho x}(x^*) > \frac{1-\tau}{\tau}$, this intuitively implies that $E_{(G/Y)^*x} > 0$, which reinforces the effect represented by $E_{k^*\tau} > 0$. As a result, per capita income in the polymorphic evolutionary equilibrium increases, given that $E_{y^*\tau} = aE_{k^*\tau} + bE_{(G/Y)^*\tau} > 0$.

Suppose now that $\tau < \tau^*$, such that $E_{k^*\tau} < 0$ by (24). In this case, if the response of the rate of recovery of captured government revenues to an increase in the frequency of capturer households is relatively weak, so that $E_{\rho x}(x^*) < \frac{1-\tau}{\tau}$, this implies that $E_{(G/Y)^* x} < 0$, which reinforces the effect represented by $E_{k^* \tau} < 0$. As a result, per capita income in the polymorphic evolutionary equilibrium unambiguously falls, since $E_{y^* \tau} = aE_{k^* \tau} + bE_{(G/Y)^* \tau} < 0$.

4. Final remarks

The capture or drainage of government budgetary resources through unlawful means by individuals in the rest of the economy is a persistent and widespread reality in most countries. In the evolutionary analytical framework set forth in this paper, the economy evolves over time driven by the interaction between the microdynamics of the frequency of capturing behavior across agents and the macrodynamics of the accumulation of capital and the per capita income.

The evolutionary microdynamics of behavioral strategies (to capture or not to capture government budgetary resources) across the respective decision makers is driven by an imitation-augmented satisficing dynamics. This evolutionary dynamic has three equilibria. These are two monomorphic equilibria featuring survival of only one behavioral strategy capture-wise in each, and one polymorphic equilibrium, now featuring the survival of both behavioral strategies. Both monomorphic evolutionary equilibria are repulsors, while the unique polymorphic evolutionary equilibrium is an attractor. Therefore, in keeping with the empirical evidence, heterogeneity in behavioral strategy capture-wise emerges evolutionarily as a persistent outcome.

In the polymorphic evolutionary equilibrium, the frequency of capturing behavior in the economy varies positively with the tax rate. However, how the steady-state capital stock in per capita terms and the per capita income respond to a change in the tax rate mediated by a change in the frequency of capturing behavior depends on the level of the tax rate. A key role is played in this non-linearity by the effectiveness with which the government manages to recover budgetary resources that were captured, which in turn depends on the probability with each a capture is detected and the size of the resulting pecuniary penalization.

References

AVNIMELECH, G.; ZELEKHA, Y.; SHARABI, E. The effect of corruption on entrepreneurship in developed vs non-developed countries. *International Journal of Entrepreneurial Behavior & Research*, v. 20, n. 3, p. 237-262, 2014.

BARRO, R. J. Government spending in a simple model of endogenous growth. *Journal of Political Economy*, v. 98, n. 5, p. 103-125, 1990.

MÉON, P. G.; WEILL, L. Is corruption an efficient grease? *World Development*, v. 38, n. 3, p. 244-259, 2010.

MO, P. H. Corruption and economic growth. *Journal of Comparative Economics*, Elsevier, v. 29, n. 1, p. 66-79, 2001.

SOLOW, R. M. A contribution to the theory of economic growth. *Quarterly Journal of Economics*, v. 70, n.1, p. 65-94, 1956.

SWALEHEEN, M. u.; STANSEL, D. Economic freedom, corruption, and growth. *The Cato Journal*, v. 27, p. 343-358, 2007.

SWAN, T. W. Economic growth and capital accumulation. *Economic Record*, v. 32, n. 2, p. 334-361, 1956.

TREISMAN, D. The causes of corruption: a cross-national study. *Journal of Public Economics*, v. 76, n. 3, p. 399-457, 2000.

WEI, S. J. Local corruption and global capital flows. *Brookings Papers on Economic Activity*, n. 2, p. 303-346, 2000.