Department of Economics- FEA/USP

Floating-rate bonds and monetary policy effectiveness: insights from a DSGE model

PAULO DE CARVALHO LINS
MARCIO ISSAO NAKANE

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Paulo de Carvalho Lins (paulo.lins@fgv.br)

Marcio Issao Nakane (minakane@usp.br)

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JEL Codes: E52; E63; H63.

Floating-rate bonds and monetary policy effectiveness: insights from a DSGE model

Paulo de Carvalho Lins[1] and Márcio Issao Nakane*[2]

[1]Instituto Brasileiro de Economia - IBRE/FGV
[2]Universidade de São Paulo - IPE/USP

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Abstract

In Brazil, there exists a government bond whose return is directly indexed to short-term interest rate set by the Central Bank. Some economists suggest that its existence decreases the effectiveness of monetary policy, mainly by clogging the wealth transmission channel. We introduce a floating-rate bond as a new financial asset in a canonical DSGE model and analyze its effects on the model dynamics. The new bond does not seem to change the dynamics of any variable, even in the presence of rule-of-thumb agents. We interpret these results as evidence against the argument that floating-rate bonds lead to a weaker monetary policy.

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1 Introduction

In the eighties, Brazil experienced a hyperinflation crisis. Attempting to control the inflationary problem, policymakers tried several stabilization plans. In the *Cruzado* plan of 1986, price control mechanisms were used. The plan succeeded in taming inflation in its first three months, and agents formed their expectations in this stabilized environment. Banks, in special, leveraged their loan portfolios expecting a low and stable inflation. However, when some price pressure began to arise, policymakers realized they would have to set prices free and this would cause big losses in the financial sector. As a solution, a new government bond whose return was set daily by the basic interest rate was created, a floating-rate bond. This new bond, named LFTs (*Letras Financeiras do Tesouro*), ended the mismatch between assets and liabilities in the banks' balance sheets[1]. However, even after the stabilization of the Brazilian economy with the *Real* plan in 1994, the LFT bonds continue existing and, they currently (2016) represent almost 30% of the Brazilian debt stock.

The existence of the LFT bonds are central in the Brazilian economic debate. Policymakers, politicians, and economists expected that, with the price stabilization and with the adoption of the inflation target, Brazilian interest rate would converge to a level similar to the one found in other emerging economies. However, Brazil continues to exhibit one of the highest real interest rates in the world. Some economists blame the LFT bonds for this problem. They believe that the existence of this floating-rate bond weakens monetary policy effectiveness, mainly by shutting off the wealth transmission channel[2]. We contribute to the literature by analyzing this issue using a DSGE model. As far as we are aware, we are the first ones to take this approach to analyze the impact of a floating-rate bond in the effectiveness of the monetary policy.

Our model is based on a canonical New Keynesian model developed by Galí (2015). We expand Gali's model by first adding a long-term debt structure, and later by adding rule-of-thumb agents. The first modification follows closely Krause & Moyen (2016), who model long-term debts using a recursive structure. However, they consider only traditional long and one-period bonds. We modify their rule that sets the average long-term debt interest rate to allow for a contemporaneous effect of the monetary policy interest rate. We show explicitly that our new rule is the result of their structure when a floating-rate bond is added. To justify our second modification, we observe that, as pointed by Barro (1999), the Ricardian equivalence is present in models with lump-sum taxes, certainty about the future,

[1]For more complete analyses of the period when LFT bonds were created, see Arida (2006) and Lara Resende (2006).

[2]For other explanations for the Brazilian high real interest rates, see Favero & Giavazzi (2002), Arida *et al.* (2005), and Segura-Ubiergo (2012).

perfect financial markets, and representative agents with rational expectations. In other words, it is redundant to add a new financial asset in such environment. For that reason, we introduce distortionary taxes in the labor market and Rule-of-Thumb agents in our model.

We believe that the transmission mechanisms working in any model are better understood when the latter is kept as clean as possible. For that reason, we decided not to include more complex (yet realistic) features in the model, such as consumption habits, or risk premia. Also, this is the reason why we chose a closed economy model instead of a small open one. The introduction of such elements would make our economy to resemble more the Brazilian economy. However, it would complicate our analysis of the effect of floating-rate bonds in the monetary transmission, because other factors would also affect this transmission.

In our simulations, we find the expected effects when a monetary shock occurs. However, the behavior of some variables is different from the one expected when a fiscal shock occurs. In particular, in relation to our paper's main inquiry, we find no effect of floating-rate bonds in any variable dynamics. Even in the model with rule-of-thumb agents, no effect of the existence of your new financial asset was found. We interpret this result as evidence against the LFTs' hypothesis.

The available theoretical literature has analyzed this issue using traditional models, such as IS/LM. In those models, consumption is a function of the stock of public debt, and the monetary policy only affects the market values of standard bonds; the value of LFTs do not change with monetary policy. The mechanism by which the effect of the bond is found in these models is clear. However, in our model, agents are forward-looking and do not see changes in the bonds' price as an increase of wealth. Our results highlight the importance of the hypothesis of consumption responsiveness to changes in wealth for the LFT bonds to have real effects on the economy.

The article is organized as follows. Section 2 surveys the related literature. Section 3 develops the model. Section 4 presents the calibration for the relevant model parameters. Section 5 shows the simulation results of a monetary shock. Section 6 extends the basic model to consider rule-of-tumb agents. Section 7 concludes the paper.

2 Related Literature

The first to formulate the LFTs' hypothesis was Pastore (1996). He constructs his argument using a IS/LM model with two monetary regimes: one in which the public deficit is financed only with traditional long-term nominal bonds, and another in which the deficit is financed only with perpetuities whose remuneration is directly indexed to the short-term interest rate set by the Central Bank. In the first regime, changes in the interest rate impact the wealth stock, but have no effect on the interest payment flow; there is a wealth effect, but no income effect. On the other hand, in the second regime, changes in the interest rate impact the flow of interest rate payments, but have no effect in the wealth stock; there is an income effect, but no wealth effect.

As pointed by Carneiro (2006), the supply of LFTs is justified by preferences and conveniences of the Brazilian National Treasury. Mainly, the LFTs minimize debt rollover costs. On the other hand, the demand is justified because Brazilian financial regulation sets average maturity liabilities that banks need to hold. Floating-rate bonds are a way of meeting the regulations without taking too much risk. Although the LFT bonds are often cited as a major problem of the Brazilian economy, there are some contrary views. For example, Loyo (2006) argues that the wealth effect seems not quantitatively important for monetary transmission. Also, he points out that the Ricardian equivalence seems to be true for public debt. We contribute to the literature by analyzing this issue using a DSGE model[3].

In our model, we model long-term debt as Krause & Moyen (2016). Another way of modeling it is following Woodford (2001), who proposes exponential decay payment streams on consols. Dias & Andrade (2016) and Lins (2016) use this kind of structure to investigate how debt maturity impacts the economic dynamics. However, we think that the Woodford structure is not flexible enough to allow floating-rate bonds. Two other papers, Alfaro & Kanczuk (2010) and Divino & Silva Junior (2013), use general equilibrium models to study public debt composition in Brazil. However, instead of analyzing the choice between floating-rate and nominal bonds, they only consider the choice between inflation-indexed bonds and nominal bonds. As far as we are aware, we are the first ones to analyze the impact of the existence of LFT bonds using a DSGE model.

3 Model

Our model is based on the canonical model of Galí (2015), in which we introduce a recursive public debt. Our modeling strategy was based on Krause & Moyen (2016), but we added another bond by which our representative agent can postpone consumption: a floating-rate bond. So there are three bonds in the economy: a floating-rate bond, a long-term nominal bond, and a standard one-period bond. The monetary authority follows a Taylor rule by which it sets the short-term nominal interest rate responding to changes in the inflation rate gap. Firms are monopolistic competitors *a la* Calvo (1983). Our representative agent maximizes

[3]In 2006, a book celebrating the 20 years of existence of the LFT bonds was edited by Bacha & Oliveira Filho (2006). One can find there several critical essays and a comprehensive literature review.

lifetime utility from consumption and labor input. Government dynamics are determined by a fiscal rule and by its budget constraint.

3.1 Recursive Public Debt

In the model of Krause & Moyen (2016), public debt has a recursive maturity structure. In each period, nominal bonds pay the interest determined when the bond was issued and they mature with a given probability α. The movement equation for the nominal bond's value, $B_t^{Nominal}$, can be written as:

$$B_t^{Nominal} = (1-\alpha)B_{t-1}^{Nominal} + B_t^{Nominal,n} \tag{1}$$

where $B_t^{Nomianl,n}$ denotes the value of the newly issued bonds, while $(1-\alpha)B_{t-1}^{Nominal}$ is the value of the bonds not maturing in period t. Since only a fraction α of the bonds matures in each period, bonds of all ages will exist in any given period.

The interest rate of a bond issued in period t is given by $i_t^{L,n}$, and the average interest rate of the current stock of long-term bonds is given by i_t^L. Krause & Moyen (2016) define the average long-term interest rate as a weighted average:

$$i_t^L = \frac{B_t^{Nominal,n}}{B_t^{Nominal}} i_t^{L,n} + (1-\alpha)\frac{B_{t-1}^{Nominal,n}}{B_t^{Nominal}} i_{t-1}^{L,n} + (1-\alpha)^2 \frac{B_{t-2}^{Nominal,n}}{B_t^{Nominal}} i_{t-2}^{L,n} + \dots$$

The weights on the interest rates of previously issued bonds, $i_{t-i}^{L,n}$, on the average long-term interest rate, i_t^L, depend on the value of the fraction of those bonds that are still left on the value of the current total stock of long-term debt. The value of those fractions is represented by $\frac{(1-\alpha)^i B_{t-i}^{Nominal,n}}{B_t^{Nominal}}$. This means that the more recently the bond was issued, the higher is the weight of its interest rate when issued on the average long-term interest rate.

We can combine the two previous equations to track together the average interest rate and the value of the long-term debt stock in a recursive form:

$$B_t^{Nominal} i_t^L = (1-\alpha)B_{t-1}^{Nominal} i_{t-1}^L + B_t^{Nominal,n} i_t^{Nominal,n} \tag{2}$$

As pointed out by Krause & Moyen (2016), the parameter $\frac{1}{\alpha}$ determines the average maturity of the long-term debt current stock. Similarly to what they did, we calibrate this parameter to match the actual average maturity of the Brazilian debt.

3.2 Model with Floating-rate Debt

The recursive public debt structure is modified to allow for two types of long-term public debt. In our model, the total stock of long-term public debt , B_t^L, is composed by floating-rate debt, $B_t^{F.R.}$, and by long-term nominal debt, $B_t^{Nominal}$:

$$B_t^L = B_t^{F.R.} + B_t^{Nominal} \tag{3}$$

$$B_t^{L,n} = B_t^{F.r.,n} + B_t^{Nominal,n} \tag{4}$$

In addition, in line with equation (1), we define the law of motions for our two kinds of bonds. Each bond has the same maturing probability α:

$$B_t^{F.R.} = (1-\alpha)B_{t-1}^{F.R.} + B_t^{F.R.,n} \tag{5}$$

$$B_t^{Nominal} = (1-\alpha)B_{t-1}^{Nominal} + B_t^{Nominal,n} \tag{6}$$

Using all these new equations, we obtain a law of motion for the total long-term public debt:

$$B_t^L = (1-\alpha)B_{t-1}^L + B_t^{L,n} \tag{7}$$

Those new bonds modify the equation which determines the average long-term interest rate, equation (2). We continue defining this interest rate as a weighted average of the stock of long-term public debt value, but now we take into account each type of long-term debt, the floating-rate and the nominal one:

$$i_t^L = \frac{B_t^{F.R.,n}}{B_t^L} i_t + \frac{B_t^{Nominal,n}}{B_t^L} i_t^{L,n} + (1-\alpha)[\frac{B_{t-1}^{F.R.,n}}{B_t^L} i_t + \frac{B_{t-1}^{Nominal,n}}{B_t^L} i_{t-1}^{L,n}] + \\ (1-\alpha)^2[\frac{B_{t-2}^{F.R.,n}}{B_t^L} i_t + \frac{B_{t-2}^{Nominal,n}}{B_t^L} i_{t-2}^{L,n}] + \dots$$

Because now the stock of public debt is compounded by two different bonds, we have to specify the interest rate of each kind. For floating-rate bonds, $B_{t-i}^{F.R.,n}$, the interest rate is equal to the short-term rate set by the monetary authority, i_t. For nominal bonds, $B_{t-i}^{Nominal,n}$, the interest rate is equal to the original issued rate, $i_{t-i}^{L,n}$. The weights of the interest rate of a bond previously issued depend on the fraction of this bond in the total stock of public debt. This equation can also be written in a recursive form:

$$i_t^L = \frac{B_t^{F.R.}}{B_t^L} i_t + \frac{B_t^{Nominal}}{B_t^L} i_t^{L,n} + (1-\alpha)\frac{B_{t-1}^{F.R.}}{B_t^L}(i_{t-1}^L - i_{t-1}) + \\ (1-\alpha)\frac{B_{t-1}^{Nominal}}{B_t^L}(i_{t-1}^L - i_t^{L,n}) \tag{8}$$

The long-term interest rate is equal to the sum of the floating-rate bond's and nominal bond's return, being both weighted by their relative participation in the total public stock, plus the differential of long-term interest rate and the short-term rate, and plus the differential of the last-period long-term interest rate and the current-period issuing long-term interest rate. Both differentials are weighted by their relative participation in the total value of the debt stock[4]. Krause and Moyen's definition of the average interest rate, equation (2), is not recoverable from the new equation (8).

3.3 Household

The household is characterized by a representative agent whose problem is to maximize the present value of his utility

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right),$$

in which C_t is an index of consumption and N_t is total amount of working labor hours, subject to the budget constraint

$$C_t + \frac{B_t}{P_t} + \frac{B_t^{L,n}}{P_t} = (1+i_{t-1})\frac{B_{t-1}}{P_t} + (\alpha + i_{t-1}^L)\frac{B_{t-1}^L}{P_t} + (1-\tau_t)\frac{W_t}{P_t}N_t + Z_t,$$

to the total debt movement equation (7)

$$B_t^L = (1-\alpha)B_{t-1}^L + B_t^{L,n},$$

to the average long-term interest rate (8)

$$i_t^L = \frac{B_t^{F.R.}}{B_t^L} i_t + \frac{B_t^{Nominal}}{B_t^L} i_t^{L,n} + (1-\alpha)\frac{B_{t-1}^{F.R.}}{B_t^L}(i_{t-1}^L - i_{t-1}) + (1-\alpha)\frac{B_{t-1}^{Nominal}}{B_t^L}(i_{t-1}^L - i_t^{L,n}),$$

to the equations determining the debt stock composition, (3) and (4)

$$B_t^L = B_t^{F.R.} + B_t^{Nominal}$$
$$B_t^{L,n} = B_t^{F.R.,n} + B_t^{Nominal,n},$$

and to the bond movement equations, (5) and (6)

$$B_t^{F.R.} = (1-\alpha)B_{t-1}^{F.R.} + B_t^{F.R.,n}$$
$$B_t^{Nominal} = (1-\alpha)B_{t-1}^{Nominal} + B_t^{Nominal,n}$$

In the budget constraint, P_t is the price index, i_t is the nominal short-term interest rate, B_t is a standard one-period bond, W_t is the nominal wage, Z_t is a

[4]When the stock of public debt is exclusively composed by nominal bonds, i.e. $B^{F.R.} = 0$, equation (8) is simplified and it is equal, as one would expect, to equation (2).

transfer from the firms, and τ_t is a distortionary tax. The variables $B_t^{L,n}$, B_t^L, $i_t^{L,n}$, and i_t^L are, respectively, the value of recently issued long-term bonds, the value of the total stock of long-term debt, the issuing long-term interest rate, and the average long-term interest rate.

The consumption index, C_t, is defined as a basket of goods of a continuum of differentiated products, $C_t(i) \in [0,1]$. The aggregation is done by the function $C_t \equiv (\int_0^1 C_t(i)^{1-1/\epsilon} di)^{\epsilon/\epsilon - 1}$, where ϵ is the constant elasticity of substitution between the products $C_t(i)$. The price index aggregator is given by $P_t \equiv (\int_0^1 P_t(i)^{1-\epsilon} di)^{1/1 - \epsilon}$.

The easiest way to solve the household problem is through combining the composition equations, (3) and (4), and the floating-rate and nominal bonds' motion equations, (5) and (6), with all the other restrictions. The consumer problem is now to maximize:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right)$$

subject to the modified budget constraint

$$C_t + \frac{B_t}{P_t} + \frac{B_t^{F.R.}}{P_t} - (1-\alpha)\frac{B_{t-1}^{F.R.}}{P_t} + = (1+i_{t-1})\frac{B_{t-1}}{P_t}\frac{B_t^{Nominal}}{P_t} - (1-\alpha)\frac{B_{t-1}^{Nominal}}{P_t}$$
$$+ (\alpha + i_{t-1}^L)\frac{B_{t-1}^{Nominal}}{P_t} + (\alpha + i_{t-1}^L)\frac{B_{t-1}^{F.R.}}{P_t} + (1-\tau_t)\frac{W_t}{P_t}N_t + Z_t$$

and to the average long-term interest rate equation

$$B_t^{F.R.} i_t^L + B_t^{Nominal} i_t^L = B_t^{F.R.} i_t + B_t^{Nominal} i_t^{L,n} + (1-\alpha)B_{t-1}^{F.R.}(i_{t-1}^L - i_{t-1})$$
$$+ (1-\alpha)B_{t-1}^{Nominal}(i_{t-1}^L - i_t^{L,n})$$

Krause & Moyen (2016) argue that consumers take the issuing long-term interest rate as given because it is decided by the market players. However, the average long-term interest rate, i_t^L, depends on the composition of newly issued relative to the total bonds that the households choose to hold. So, the average interest rate must be taken into account when solving the household's optimization problem. The representative household maximizes its intertemporal utility with respect to C_t, B_t, $B_t^{F.R.}$, $B_t^{Nominal}$, i_t^L, and N_t.

The first interesting result of our model is evident when one looks to the optimal conditions resulting from the consumer optimization problem[5]. The Euler equation of the floating-rate bond is equal to the Euler equation of the standard one-period bond. This means that the floating-rate bond behaves as a bond with the shortest maturity. However, the nominal bond in our model behaves as the equivalent bond

[5]The calculations are presented in the appendix B.

of Krause & Moyen (2016). The optimal equations are:

$$1 = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} (1 + i_t) \right] \tag{9}$$

$$1 = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \left(1 + i_t^{L,n} - \mu_{t+1}(1 - \alpha)\Delta i_{t+1}^{L,n} \right) \right] \tag{10}$$

$$\mu_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} (1 + (1 - \alpha)\mu_{t+1}) \right] \tag{11}$$

Also, we found the optimal equation for the labor supply:

$$N_t^{\varphi} C_t^{\sigma} = (1 - \tau_t) \frac{W_t}{P_t} \tag{12}$$

The Euler equation (9) relates the intertemporal discount factor, $(\frac{\lambda_t}{\lambda_{t+1}})^{-\sigma} \frac{P_t}{P_{t+1}}$, with the short-term interest rate. In an analogous way, the Euler equation (10) relates the intertemporal discount factor with the issuing long-term interest rate. The main difference between the two equations is that the latter is corrected by changes in the expectations, $\Delta i_{t+1}^{L,n}$, while the former is not. This correction factor is weighted by the Lagrange multiplier μ_t, which follows the process given by equation (11). The marginal utility of wealth is equal to the marginal utility of consumption,

$$\lambda_t = c_t^{-\sigma} \tag{13}$$

Intuitively, the term $\mu_{t+1}\Delta i_{t+1}^{L,n}$ in equation (10) captures the capital loss in period $t + 1$ caused by the increase in the issuing long-term interest rate. This effect is the wealth channel of the monetary policy transmission. It reduces the incentive to invest in long-term bonds in the current period. Consequently, the average long-term interest rate has to be higher than the short-term interest rate for the consumer to be indifferent between both bonds. It is worth noting that the steady-state values of all interest rate are equal.

3.4 Firms

In the production sector, we follow closely Galí (2015) and Krause & Moyen (2016). There is price rigidity *a la* Calvo. The aggregate price dynamic follows:

$$1 = [\theta \pi_t^{\epsilon - 1} + (1 - \theta)(\frac{P_t^*}{P_t})^{1-\epsilon}] \tag{14}$$

in which θ is the probability of firm i adjusting its price, π_t is the inflation rate, and $\frac{P_t^*}{P_t}$ is the ratio between recently adjusted prices and the price index in the economy.

The firms' optimal conditions are:

$$\frac{P_t^*}{P_t} = \mu \frac{Z_{2,t}}{Z_{1,t}} \tag{15}$$

in which

$$Z_{1,t} = C_t^{1-\sigma} + \beta\theta E_t \pi_{t+1}^{\epsilon-1} Z_{1,t+1} \tag{16}$$

$$Z_{2,t} = C_t^{1-\sigma} mc_t + \beta\theta E_t \pi_{t+1}^{\epsilon} Z_{2,t+1} \tag{17}$$

and in which μ is the mark-up ratio. $Z_{1,t}$ and $Z_{2,t}$ are two auxiliary variables created to facilitate writing the problem in a recursive form, and mc_t is the firm marginal cost, which is given by the equation:

$$mc_t = \frac{W_t}{P_t} \tag{18}$$

in which W_t/P_t is the real wage.

3.5 Fiscal and Monetary Authorities

We will use a fiscal rule exactly equal to the one used by Krause & Moyen (2016):

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + \phi_b(\hat{b_t^L} + \hat{b_t}) + s_t \tag{19}$$

in which $\hat{\tau}_t$ is the deviation of the lump-sum tax from its steady-state value, $\hat{\tau}_t \equiv \tau_t - \bar{\tau}_t$, $\hat{b_t^L}$ is the deviation of the long-term public debt from its steady-state value, $\hat{b_t^L} \equiv b_t^L - \bar{b_t^L}$, $\hat{b_t}$ is the deviation of the standard one-period debt from its steady-state value, $\hat{b_t} \equiv b_t - \bar{b_t}$, and s_t is a fiscal shock with standard deviation equals to σ_s.

The government budget constraint is:

$$\tau_t \frac{W_t}{P_t} N_t + \frac{B_t}{P_t} + \frac{B_t^{L,n}}{P_t} = g + (1 + i_{t-1})\frac{B_{t-1}}{P_t} + (\alpha + i_{t-1}^L)\frac{B_{t-1}^L}{P_t} \tag{20}$$

in which g is a constant representing public spending.

The Monetary Authority follows a monetary rule given by:

$$i_t = (1 - \rho_i)\bar{i} + \rho_i i_{t-1} + \phi_\pi(\pi_t - 1) + v_t \tag{21}$$

in which i_t is the short-term interest rate which the monetary authority controls and by which the one-period bond and the floating-rate bond are remunerated, $\bar{i}$ is the steady-state value of the interest rate, π_t is the inflation rate, and v_t is a monetary policy shock with standard deviation equals to σ_v.

3.6 Equilibrium

Aggregate demand is equal to family consumption plus government consumption:

$$Y_t = C_t + g \tag{22}$$

The market clearing condition of the labor market is:

$$\Delta_{p,t} Y_t = N_t \tag{23}$$

where $\Delta_{p,t}$ is the price dispersion coefficient, whose law of motion is given by the equation:

$$\Delta_{p,t} = \theta \pi_t^{\epsilon} \Delta_{p,t-1} + (1-\theta)[\frac{P_t^*}{P_t}]^{-\epsilon} \tag{24}$$

The system formed by equations (3), (4), (8), (5), (6) and (7) are linearly dependent, and we need more hypothesis to solve the model. No new movement equation was added to the model – because they are redundant –, and the value of one of the bonds was fixed in its steady-state value. So, we assume that ${}^{B_t^i}/_{P_t} = {}^{\bar{B^i}}/_{P}, \forall i \in [Nominal, F.R.]$.

The model equilibrium is determined by the stationary processes for i_t, i_t^L, $i_t^{L,n}$, C_t, λ_t, π_t, ${}^{W_t}/_{P_t}$, N_t, ${}^{P_t^*}/_{P_t}$, $Z_{1,t}$, $Z_{2,t}$, mc_t, τ_t, ${}^{B_t}/_{P_t}$, ${}^{B_t^L}/_{P_t}$, ${}^{B_t^{L,n}}/_{P_t}$, Y_t, $\Delta_{p,t}$, μ_t, ${}^{B_t^{F.R.}}/_{P_t}$, ${}^{B_t^{Nominal}}/_{P_t}$, ${}^{B_t^{F.R.,n}}/_{P_t}$ and ${}^{B_t^{Nominal,n}}/_{P_t}$ which satisfy relations (3), (4), (7), (8), (9), (10), (11), (12), (13), (14), (15), (16), (17), (18), (19), (20), (21), (22), (23) and (24) and by the stochastic processes for v_t and s_t. To find the equilibrium, we need three more restrictions. We will use ${}^{B_t}/_{P_t} = 0$, ${}^{B_t^i}/_{P_t} = {}^{\bar{B^i}}/_{P}, \forall i \in [Nominal, F.R.]$ and ${}^{B_t^{i,n}}/_{P_t} = {}^{B^{\bar{i},n}}/_{P} \quad \forall i \in [Nominal, F.R.]$. Since our last restriction allows for two parameterizations, $i = Nominal$ or $i = F.R.$, we will simulate both.

We believe that the first restriction ${}^{B_t}/_{P_t} = 0$ is important because it allows us to focus on the long-term debt transmission mechanism. Following the same idea, fixing one kind of the long-term debt, the nominal or the floating-rate bond, allows us to better understand how the long-term debt transmission mechanism works in our model. We expect the model dynamics to be different in each calibration, or, in other words, to be different when the model adjusts exclusively through the floating-rate bonds from when it adjusts through the nominal bonds.

4 Calibration

We calibrate our model using commonly used parameters in the Brazilian general equilibrium literature. De Castro *et al.* (2015) introduce the DSGE model used by the Brazilian Central Bank (Bacen) in economic policy analyses and in forecasting exercises. Using their estimated parameters, our intertemporal discount factor, β, is equal to 0.989, our intertemporal substitution elasticity, σ, is equal to 1.3, our Frisch's labor supply elasticity, φ, is equal to 1, our tax's response to debt deviation from its steady-state level, ϕ_b, is equal to 0.02, our Taylor coefficient, ϕ_π, is equal to 2.43, our fiscal rule autoregressive coefficient, ρ_T, is equal to 0.8, and the monetary rule autoregressive coefficient, ρ_R, is equal to 0.79. Besides these parameters, De Castro *et al.* (2015) estimate the standard deviation of the monetary and fiscal shocks, σ_v and σ_s, being the first equals to 1.73 and the second equals to 0.32.

Table 1: Calibration for the first model

Symbol	Values	Description
Ratios		
S_g	22.20%	Government-GDP ratio
S_b	36.81%	Debt-GDP Ratio
$S_{F.R.}$	24.01%	Fraction of pos-indexed debt
Preferences and technology parameters		
α	0.055	Debt probability of mature
β	0.989	Time discount factor
σ	1.3	Intertemporal elasticity of substitution
φ	1	Frisch's labor supply elasticity
Firms parameters		
θ	0.67	Proportion of firms adjusting prices
ϵ	6	Mark-up of 20%
Fiscal and Monetary Parameters		
ϕ_b	0.02	Tax response to debt deviation
ϕ_π	2.43	Taylor coefficient
ρ_T	0.80	Autoregressive coefficient of the fiscal rule
ρ_i	0.79	Autoregressive coefficient of the monetary rule
Exogenous shocks coefficients		
σ_v	0.32	Fiscal shock standard deviation (v_t)
σ_s	1.73	Monetary shock standard deviation (s_t)

De Castro *et al.* (2015) estimate the elasticity of substitution among different intermediate goods for several markets. Because in this work we have only one final good, our substitution coefficient, ϵ, and the duration of the firm's price contract, θ, were taken from Krause & Moyen (2016). The first is equal to 6 – implying a price markup of 20% – and the second is equal 2/3 – implying an average duration of the price contract being equal to three quarters.

The parameter α, besides controlling the debt maturity probability, it also controls the average maturity of the public debt, which is equal to $^1/_\alpha$. The Brazilian public-debt had an average maturity of 4.57 years in December 2015, so the value of α was set to be equal to 0.055. To calibrate the amount of floating-rate bonds, we used the ratio of the public debt whose return was given by the short-term interest rate – *taxa* SELIC – and it was equal to 24.01%. In addition, we find the steady-state values by setting the government-GDP ratio, S_g, to be equal to 22.20%, and the public debt-GDP ratio, S_b, to be equal to 36.81%. We used the values found in the Brazilian National Accounts for both ratios. All parameters used are presented in Table 1.

5 Simulation

In this section, we analyze how changes in the debt structure impact the dynamic adjustment of key variables. Our principal instruments of analysis are impulse response functions. In each figure, we plotted two lines: one in which all adjustment is done by the floating-rate bonds, and another in which all adjustment is done by nominal bonds[6]. To easily identify our two models, we called one as Nominal-Adjusting – blue dash line –; and the other as F.R.-Adjusting – red line. It is worth remembering that the floating-rate bond was modeled to mimic how LFT bonds work in Brazil. We believe that the existence of differences (or lack of existence) between the two models is an evidence in favor (or against) the LFTs' hypothesis of Pastore (1996) and others. In all simulations, the variables are presented in deviations from its steady-state. We solved our model in Dynare.

Figures 1 and 2 compare the impulse response functions of the main variables of the model to a one-standard deviation shock in the short-term interest rate. We interpret this shock as a monetary policy one. Consumption, inflation, real wage, and output show the expected behavior, all falling after the shock. Inflation returns to its steady-state level after 4 quarters; consumption returns after 10 quarters, real wage returns after 4 quarters, and output returns after 10 quarters. The real value of the long-term-debt stock increases, so the tax rate also increases. The tax response has a hump-shaped format, mainly due to the functional form of the fiscal rule – equation (19). The short-term interest rate slowly returns to its steady-state value, governed by the monetary rule (21). The long-term interest rate and the average interest rate also increase after the monetary shock, however both increase less than the short-term rate.

[6]To better understand why in our model only one kind of long-term debt is allowed to adjust, see the last paragraph of section 3.6.

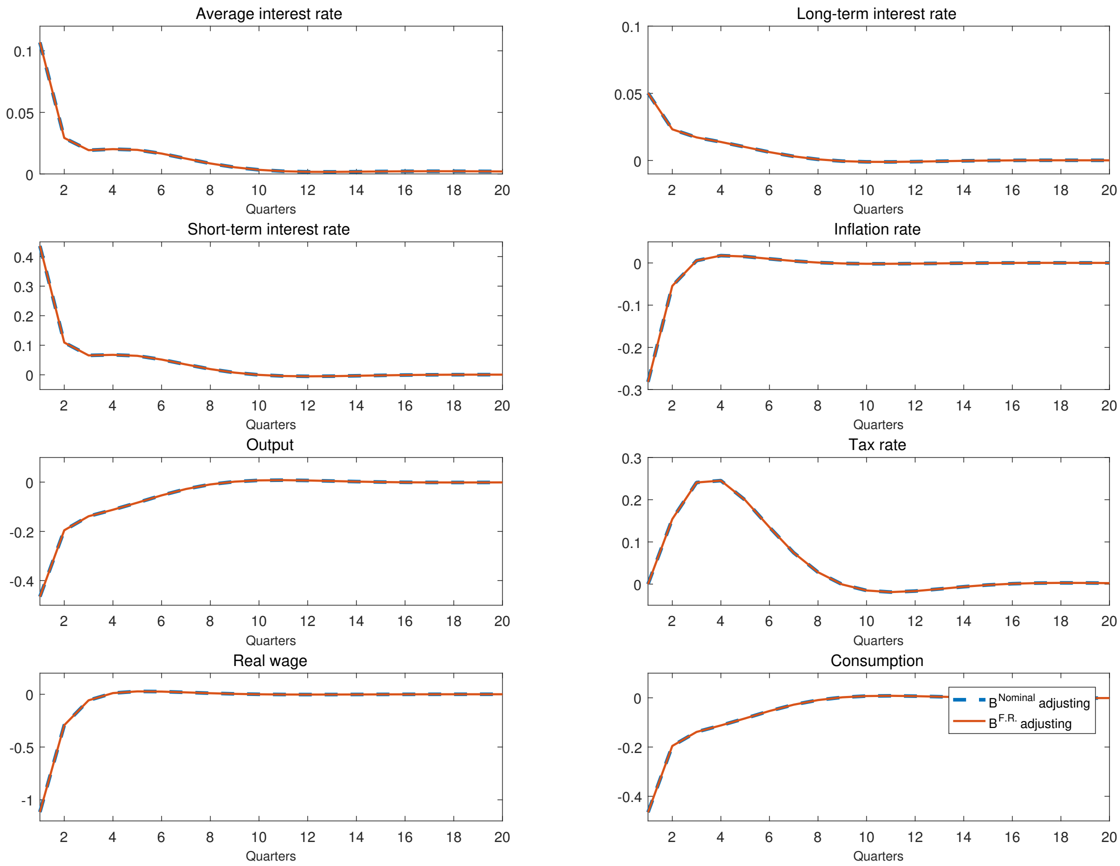

Figure 1: Monetary Shock

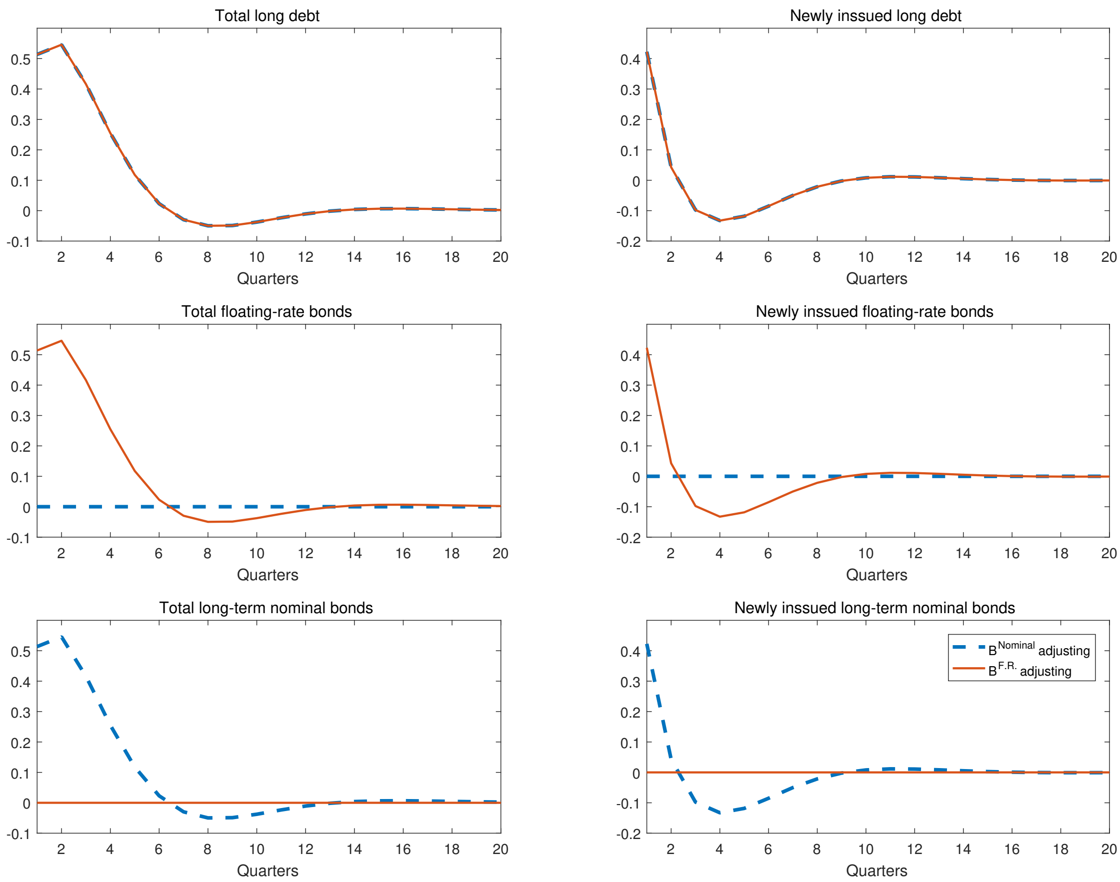

Figure 2: Monetary Shock

Comparison of figures 1 and 2 makes clear that the dynamics of the two models are the same. The observation of the bottom graphs of figure 2 are especially revealing. The behavior of the floating-rate bond when the nominal bond is held constant is equal to the behavior of the nominal bond when the floating-rate bond is held constant. This is also valid for the newly issued bonds. We interpret these results as evidence against the argument that floating-rate bonds lead to a weaker monetary policy. We believe that previous studies, as Pastore (1996), document effects of the presence of LFTs in real variables because they relied on static models in which consumption was a function of the current stock of public debt. However, in our model, agents are forward-looking and do not see changes in the bonds' price as an increase in wealth. Our results highlight how important the hypothesis that consumption responds to changes in wealth is for the LFT bonds to have real effects on the economy.

6 Rule-of-thumb Agents

As pointed out by Barro (1999), the Ricardian equivalence is present in models with lump-sum taxes, certainty about the future, perfect financial markets, and representative agents with rational expectations. We have already introduced distortionary taxes in the labor market in our model. In this section, we introduce a new type of household: rule-of-thumb agents. We follow closely the modeling strategy of Galí *et al.* (2007). We want to study if the results of the previous section on the irrelevance of floating-rate bonds do survive when rule-of-thumb agents are introduced.

We continue assuming a continuum of infinitely-lived households, indexed by $i \in [0, 1]$. However, a faction γ of these households do not maximize the present value of their utility, because they don't have access to capital markets. Brazil is a developing country where there are many credit market imperfections, so using this type of agents is a realistic hypothesis and a desired one. The consumption and the labor supplied by the rule-of-thumb agents are represented by C_t^r and N_t^r. Following the same notation, the consumption and labor supplied by the optimizing agents are represented by C_t^o e N_t^o.

The utility function of the rule-of-thumb agent is:

$$\frac{C_t^{r\,1-\sigma}}{1-\sigma} - \frac{N_t^{r\,1+\varphi}}{1+\varphi}$$

and they are subject to their budget restriction. Because these agents do not have access to the financial market, they will consume their labor income net of taxes[7]:

$$C_t^r = (1-\tau_t)\frac{W_t}{P_t}N_t^r$$

Because the labor market is competitive, rule-of-thumb agents have the same labor supply equation:

$$N_t^{r\varphi} C_t^{r\sigma} = (1-\tau_t)\frac{W_t}{P_t}$$

Combining the two previous equations, we have:

$$(C_t^r)^{1-\sigma} = (N_t^r)^{\varphi+1}$$

6.1 Aggregation

The aggregation is given by the weighted average of the corresponding variables for each consumer type:

$$C_t = \gamma C_t^o + (1-\gamma)C_t^r$$

[7]We assume that the government taxes all agents by the same income fraction, τ_t.

and

$$N_t = \gamma N_t^o + (1-\gamma) N_t^r$$

where C_t^o is the optimizing households' consumption and C_t^r is the rule-of-thumb households' consumption. The same notation is used for the quantity of hours supplied.

Similarly, the aggregate stocks of each bond are given by:

$$B_t = (1-\gamma)B_t^o$$
$$B_t^L = (1-\gamma)B_t^{L,o} \quad \text{and} \quad B_t^{L,n} = (1-\gamma)B_t^{L,n,o}$$
$$B_t^{F.R.} = (1-\gamma)B_t^{F.R.,o} \quad \text{and} \quad B_t^{F.R.,n} = (1-\gamma)B_t^{F.R.,n,o}$$
$$B_t^{Nominal} = (1-\gamma)B_t^{Nominal,o} \quad \text{and} \quad B_t^{Nominal,n} = (1-\gamma)B_t^{Nominal,n,o}$$

6.2 Calibration

The parameter γ can be thought as the ratio of the population that doesn't have access to the credit market. Following De Castro *et al.* (2015), we calibrated the value of γ as 0.40, the fraction of Brazilian population that receives less than 2.5 minimum wage in the National Household Sample Survey (PNAD). As Galí *et al.* (2007), we have to reduce the calibrated value of the labor supply elasticity to find an equilibrium. We follow the authors and set φ as 0.2. Equilibrium still found until when φ is equal to 0.7.

6.3 Simulation

As in the previous simulation section, our main instrument of analysis are impulse response functions. Figures 3, 4, and 5 compare the impulse response functions of the main variables of the models for an one-standard deviation shock in the short-term interest rate. To easily identify our two models, we called one as Nominal-Adjusting – blue dash line –, and the other as F.R.-Adjusting – red line.

As in the model with optimizing agents only, the key variables display the expected effect after a shock of one standard deviation in the short-term interest rate. Consumption, inflation, real wage, and output all fall after the shock. The real value of the long-term-debt stock increases, so the tax rate also increases, but the latter exhibit a hump-shaped response. The short-term interest rate slowly returns to its steady-state value. The long-term interest rate and the average interest rate also increase after the monetary shock, however both increase less than the short-term rate. Also, looking to the financial assets, the behavior of the floating-rate bond when the nominal bond is held constant is equal to the behavior of the nominal bond when the floating-rate bond is held constant. When analyzing figures 3 and 4, it is clear how the dynamics of the two models are the same.

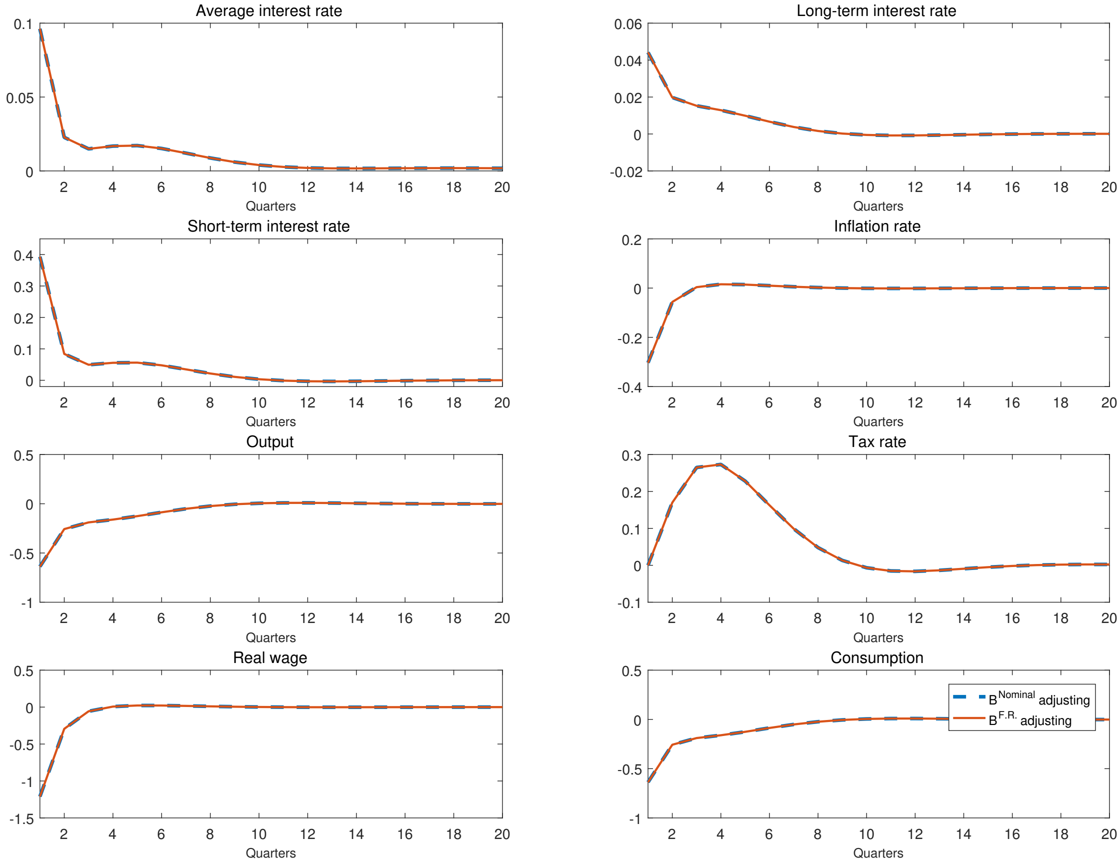

Figure 3: Monetary Shock with Rule-of-Thumb Agents

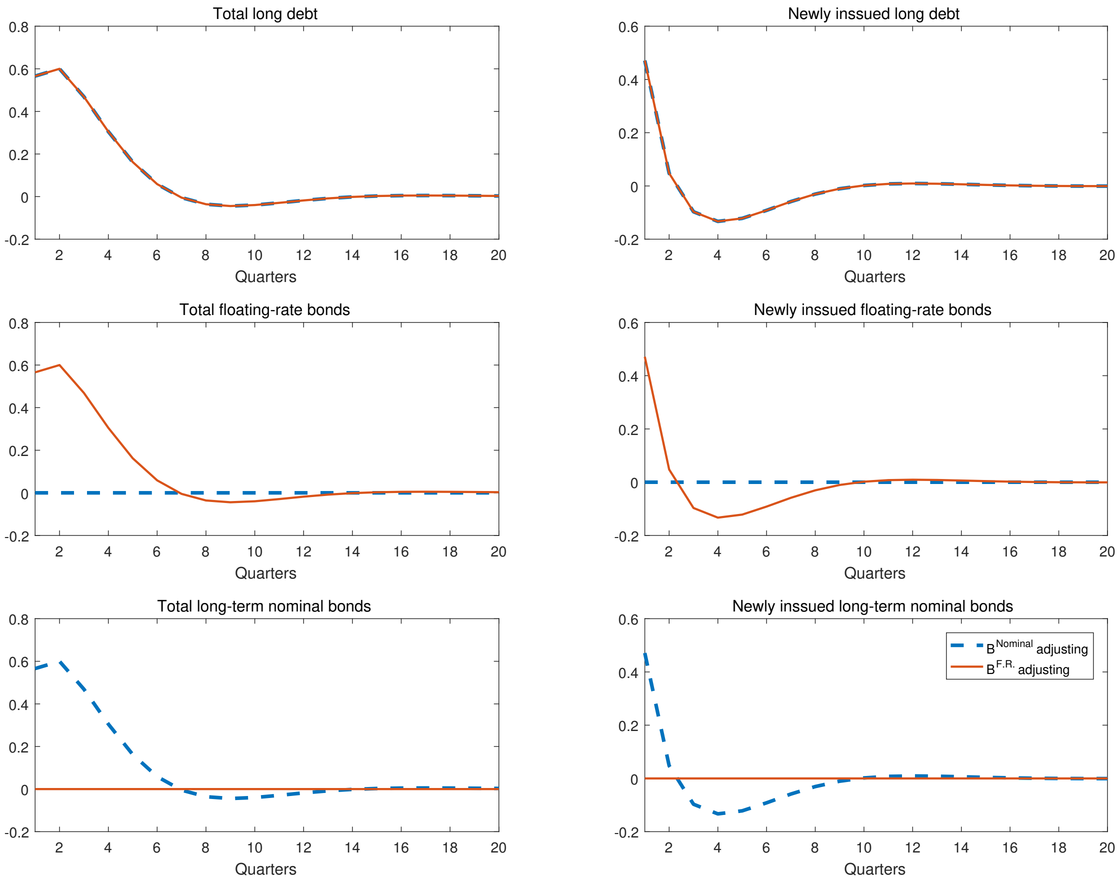

Figure 4: Monetary Shock with Rule-of-Thumb Agents

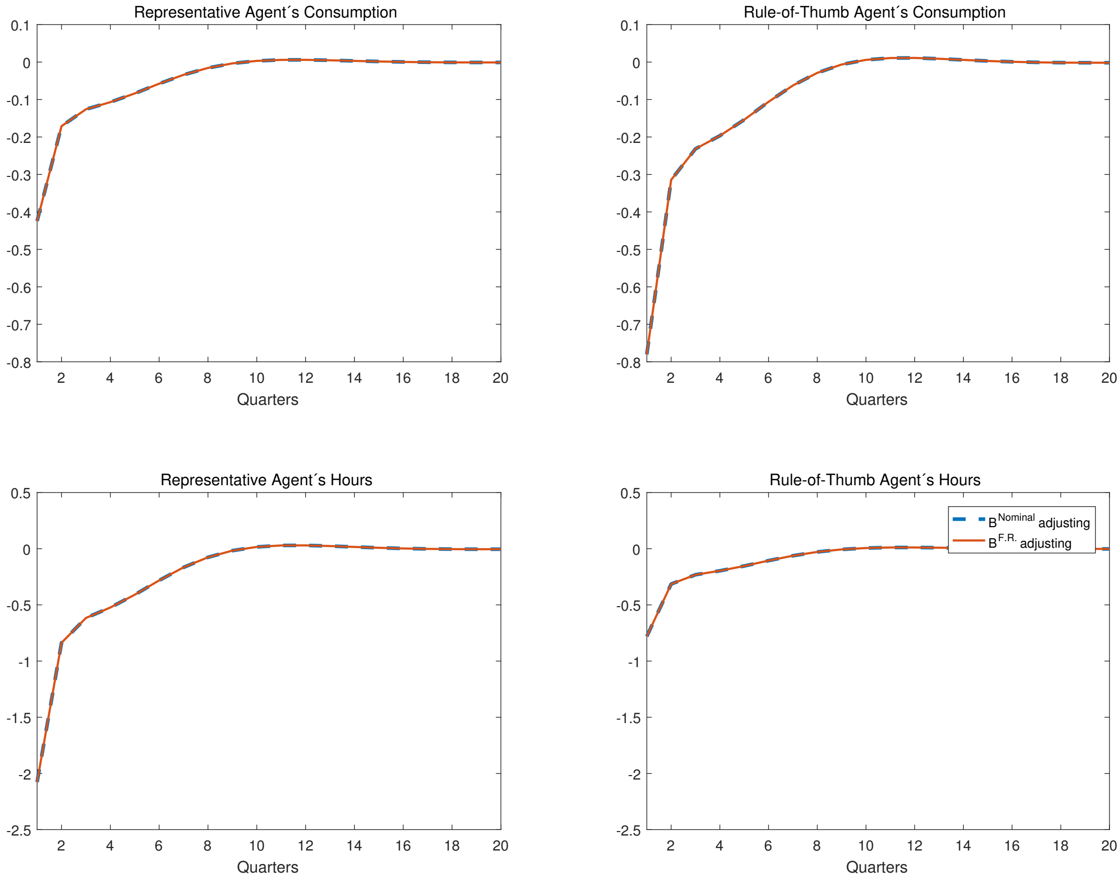

Figure 5: Monetary Shock with Rule-of-Thumb Agents

A possible explanation for these results is that, in our model, optimizing agents and rule-of-thumb agents respond to the monetary policy shock in the same way. This is clear by observing figure 5. We interpret these results as strong evidence against the argument that floating-rate bonds lead to a weaker monetary policy. Even in a model with agents that do not smooth consumption, we did not find any effect of introducing a floating-rate bond.

7 Conclusion

The LFT bonds were created during the *Cruzado* plan to help the Brazilian financial sector cope with the hyperinflation crisis. However, they continue existing until today and some economists believe this floating-rate bond weakens monetary policy effectiveness, mainly by shutting off the wealth channel of the monetary transmission. We contribute to the literature by analyzing this issue using a DSGE model. We use the recursive debt structure proposed by Krause & Moyen (2016) to introduce a new public bond whose return is similar to the LTFs's one. The rest of our model is similar to standard models in the literature.

We simulate two models, one in which the nominal bond is held constant and all adjustments are done through the floating-rate bond, and one in which the floating-rate bond is held constant and all adjustments are done through the nominal bond. In our results, it is clear how the dynamics of the two models are the same. We interpret these results as evidence against the argument that floating-rate bonds lead to a weaker monetary policy. We believe that previous studies, as Pastore (1996), found effects of the existence of LFTs on real variables because they relied on static models in which consumption was a function of the stock of public debt. However, in our model, agents are forward-looking and do not see changes in the bonds' value as an increase of wealth. Our results highlight the importance of the hypothesis of consumption responsiveness to changes in wealth for the LFT bonds to have real effects on the economy. Even when we allow for the presence of rule-of-thumb agents we still do not find any evidence for a special role for floating-rate bonds in the monetary policy transmission mechanism.

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A Average Long-term Interest Rate

The definition of average long-term interest rate is

$$i_t^L = \frac{B_t^{F.R.,n}}{B_t^L} i_t + \frac{B_t^{Nominal,n}}{B_t^L} i_t^{L,n} + (1-\alpha)[\frac{B_{t-1}^{F.R.,n}}{B_t^L} i_t + \frac{B_{t-1}^{Nominal,n}}{B_t^L} i_{t-1}^{L,n}] + (1-\alpha)^2[\frac{B_{t-2}^{F.R.,n}}{B_t^L} i_t + \frac{B_{t-2}^{Nominal,n}}{B_t^L} i_{t-2}^{L,n}] + \dots$$

We can write this equation in a recursive form:

$$B_t^L i_t^L = B_t^{F.R.,n} i_t + B_t^{Nominal,n} i_t^{L,n} + (1-\alpha)[B_{t-1}^{F.R.,n} i_t + B_{t-1}^{Nominal,n} i_{t-1}^{L,n}] + (1-\alpha)^2[B_{t-2}^{F.R.,n} i_t + B_{t-2}^{Nominal,n} i_{t-2}^{L,n}] + \dots$$

$$B_t^L i_t^L - (1-\alpha) B_{t-1}^L i_{t-1}^L = B_t^{F.R.,n} i_t + B_t^{Nominal,n} i_t^{L,n} + (1-\alpha)[B_{t-1}^{F.R.,n} i_t + B_{t-1}^{Nominal,n} i_{t-1}^{L,n} - B_{t-1}^{F.R.,n} i_{t-1} - B_{t-1}^{Nominal,n} i_{t-2}^{L,n}] + (1-\alpha)^2[B_{t-2}^{F.R.,n} i_t + B_{t-2}^{Nominal,n} i_{t-2}^{L,n} - B_{t-2}^{F.R.,n} i_{t-1} + B_{t-2}^{Nominal,n} i_{t-2}^{L,n}] + \dots$$

$$B_t^L i_t^L - (1-\alpha) B_{t-1}^L i_{t-1}^L = B_t^{F.R.,n} i_t + B_t^{Nominal,n} i_t^{L,n} + (1-\alpha)[B_{t-1}^{F.R.,n} i_t - B_{t-1}^{F.R.,n} i_{t-1}] + (1-\alpha)^2[B_{t-2}^{F.R.,n} i_t - B_{t-2}^{F.R.,n} i_{t-1}] + \dots$$

$$B_t^L i_t^L - (1-\alpha) B_{t-1}^L i_{t-1}^L = B_t^{F.R.,n} i_t + B_t^{Nominal,n} i_t^{L,n} + [i_t - i_{t-1}](1-\alpha) \underbrace{[B_{t-1}^{F.R.,n} + (1-\alpha) B_{t-2}^{F.R.,n} (1-\alpha)^2 B_{t-3}^{F.R.,n} + \dots]}_{\text{by the law of motion } = B_{t-1}^{F.R.}}$$

$$B_t^L i_t^L - (1-\alpha) B_{t-1}^L i_{t-1}^L = B_t^{F.R.,n} i_t + B_t^{Nominal,n} i_t^{Nominal,n} + (1-\alpha) B_{t-1}^{F.R.} [i_t - i_{t-1}]$$

$$B_t^L i_t^L - (1-\alpha) B_{t-1}^L i_{t-1}^L = B_t^{Nominal,n} i_t^{L,n} + B_t^{F.R.} i_t - (1-\alpha) B_{t-1}^{F.R.} i_{t-1}$$

$$B_t^L i_t^L - (1-\alpha)(B_{t-1}^{Nominal} + B_{t-1}^{F.R.}) i_{t-1}^L = [B_t^{Nominal} - (1-\alpha) B_{t-1}^{Nominal}] i_t^{L,n} + B_t^{F.R.} i_t - (1-\alpha) B_{t-1}^{F.R.} i_{t-1}$$

Simplifying this last expression, we obtain our equation (8):

$$B_t^L i_t^L = B_t^{F.R.} i_t + B_t^{Nominal} i_t^{L,n} + (1-\alpha) B_{t-1}^{F.R.} (i_{t-1}^L - i_{t-1}) + (1-\alpha) B_{t-1}^{Nominal} (i_{t-1}^L - i_t^{L,n})$$

B Household Problem

We solve the household problem by maximizing the present value of utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t (\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi})$$

subject to the budget constraint and to the average long-term interest rate:

$$C_t + \frac{B_t}{P_t} + \frac{B_t^{F.R.}}{P_t} - (1-\alpha)\frac{B_{t-1}^{F.R.}}{P_t} + = (1+i_{t-1})\frac{B_{t-1}}{P_t}\frac{B_t^{Nominal}}{P_t} - (1-\alpha)\frac{B_{t-1}^{Nominal}}{P_t} + (\alpha + i_{t-1}^L)\frac{B_{t-1}^{Nominal}}{P_t} + (\alpha + i_{t-1}^L)\frac{B_{t-1}^{F.R.}}{P_t} + (1-\tau_t)\frac{W_t}{P_t}N_t + Z_t$$

$$B_t^{F.R.} i_t^L + B_t^{Nominal} i_t^L = B_t^{F.R.} i_t + B_t^{Nominal} i_t^{L,n} + (1-\alpha)B_{t-1}^{F.R.}(i_{t-1}^L - i_{t-1}) + (1-\alpha)B_{t-1}^{Nominal}(i_{t-1}^L - i_t^{L,n})$$

with respect to C_t, B_t, $B_t^{F.R.}$, $B_t^{Nominal}$, i_t^L, and N_t. Deriving for C_t and for N_t, we obtain:

$$C_t^{-\sigma} - \lambda_t = 0$$
$$-N_t^{\phi} + \lambda_t(1-\tau_t)\frac{W_t}{P_t} = 0$$

which, when combined, yield the labor supply equation

$$\frac{N_t^{\phi}}{C_t^{-\sigma}} = (1-\tau_t)\frac{W_t}{P_t}$$

Deriving for B_t, we obtain:

$$-\frac{\lambda_t}{P_t} + \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}[1+i_t] = 0$$

which can be simplified to obtain the standard one-period bond's Euler equation:

$$1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}[1+i_t]$$

We now have to derive for $B_t^{F.R.}$, $B_t^{Nominal}$, and i_t^L. One should remember that, in the model, the average long-term interest rate, i_t^L, depends on the composition of newly issued bonds relative to the total bonds that the households choose to hold. So, the average interest rate must be taken into account when solving the household's optimization problem. The three derivatives are:

$$-\frac{\lambda_t}{P_t} + \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}(1+i_t^L) - \mu_t(i_t^L - i_t) + \beta E_t(1-\alpha)\mu_{t+1}(i_t^L - i_t) = 0$$
$$-\frac{\lambda_t}{P_t} + \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}(1+i_t^L) - \mu_t(i_t^L - i_t^{L,n}) + \beta(1-\alpha)E_t\mu_{t+1}(i_t^L - i_{t+1}^{L,n}) = 0$$
$$\beta E_t \frac{\lambda_{t+1}}{P_{t+1}}[B_t^{F.R.} + B_t^{Nominal}] - \mu_t[B_t^{F.R.} + B_t^{Nominal}] + \beta(1-\alpha)E_t\mu_{t+1}[B_t^{F.R.} + B_t^{Nominal}] = 0$$

The first step to obtain the Euler equations that we used in the paper is to simplify the derivative for i_t^L. Dividing it for $[B_t^{F.R.} + B_t^{Nominal}]$, we obtain an expression for $\beta E_t \frac{\lambda_{t+1}}{P_{t+1}}$:

$$\beta E_t \frac{\lambda_{t+1}}{P_{t+1}} - \mu_t + \beta(1-\alpha)E_t\mu_{t+1} = 0$$
$$\beta E_t \frac{\lambda_{t+1}}{P_{t+1}} = \mu_t - \beta(1-\alpha)E_t\mu_{t+1}$$

Now it is possible to simplify the derivative for $B_t^{F.R.}$: first isolating $(i_t^L - i_t)$; and them using our expression for $\beta E_t \frac{\lambda_{t+1}}{P_{t+1}}$:

$$-\frac{\lambda_t}{P_t} + \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}(1+i_t^L) - \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}(i_t^L - i_t) = 0$$

or, rearranging the terms, gives our equation (9):

$$1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}(1+i_t)$$

We can also simplify the derivative for $B_t^{Nominal}$: first isolating i_t^L; then using our expression for $\beta E_t \frac{\lambda_{t+1}}{P_{t+1}}$:

$$
\begin{aligned}
&-\frac{\lambda_t}{P_t} + \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}(1 + i_t^L) - E_t(\mu_t - \beta(1-\alpha)\mu_{t+1})i_t^L + \mu_t i_t^{L,n} \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad - \beta E_t (1-\alpha)\mu_{t+1} i_{t+1}^{L,n} \\
&-\frac{\lambda_t}{P_t} + \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}(1 + i_t^L) - \beta E_t \frac{\lambda_{t+1}}{P_{t+1}} i_t^L + E_t[\beta \frac{\lambda_{t+1}}{P_{t+1}} + \beta(1-\alpha)\mu_{t+1}] i_t^{L,n} \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad - \beta(1-\alpha) E_t \mu_{t+1} i_{t+1}^{L,n} \\
&-\frac{\lambda_t}{P_t} = \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}(1 - i_{t+1}^{L,n}) - \beta E_t (1-\alpha)\mu_{t+1} (\underbrace{i_{t+1}^{L,n} - i_t^{L,n}}_{\Delta i_{t+1}^{L,n}}) \\
&-\frac{\lambda_t}{P_t} = \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}(1 - i_{t+1}^{L,n}) - \beta E_t (1-\alpha)\mu_{t+1} (\Delta i_{t+1}^{L,n})
\end{aligned}
$$

To close our model, we need to use the same normalization used by Krause & Moyen (2016) in the Lagrange multipliers. We set

$$\tilde{\mu}_t = \mu_t \frac{\lambda_t}{P_t}$$

Now, we can rewrite the Euler equation for $B_t^{Nominal}$ as our equation (10):

$$1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} [1 + i_t^{L,n} - \tilde{\mu}_{t+1}(1-\alpha)\Delta i_{t+1}^{L,n}]$$

We can also use this normalization to find an Euler equation for $\tilde{\mu}_t$ – our equation (11) – simplified derivative for i_t^L:

$$
\begin{aligned}
\mu_t &= \beta E_t \frac{\lambda_{t+1}}{P_{t+1}} + \beta(1-\alpha) E_t \mu_{t+1} \\
\tilde{\mu}_t &= \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} (1 + (1-\alpha)\tilde{\mu}_{t+1})
\end{aligned}
$$

C Fiscal shocks in the floating-rate debt model

In this section, we analyze the dynamic adjustment of selected variables using impulse response functions. As we can see in figure 6 and 7, there are no differences between the model in which the agents only can save through floating-rate bonds and the one in which they only can save through nominal bonds.

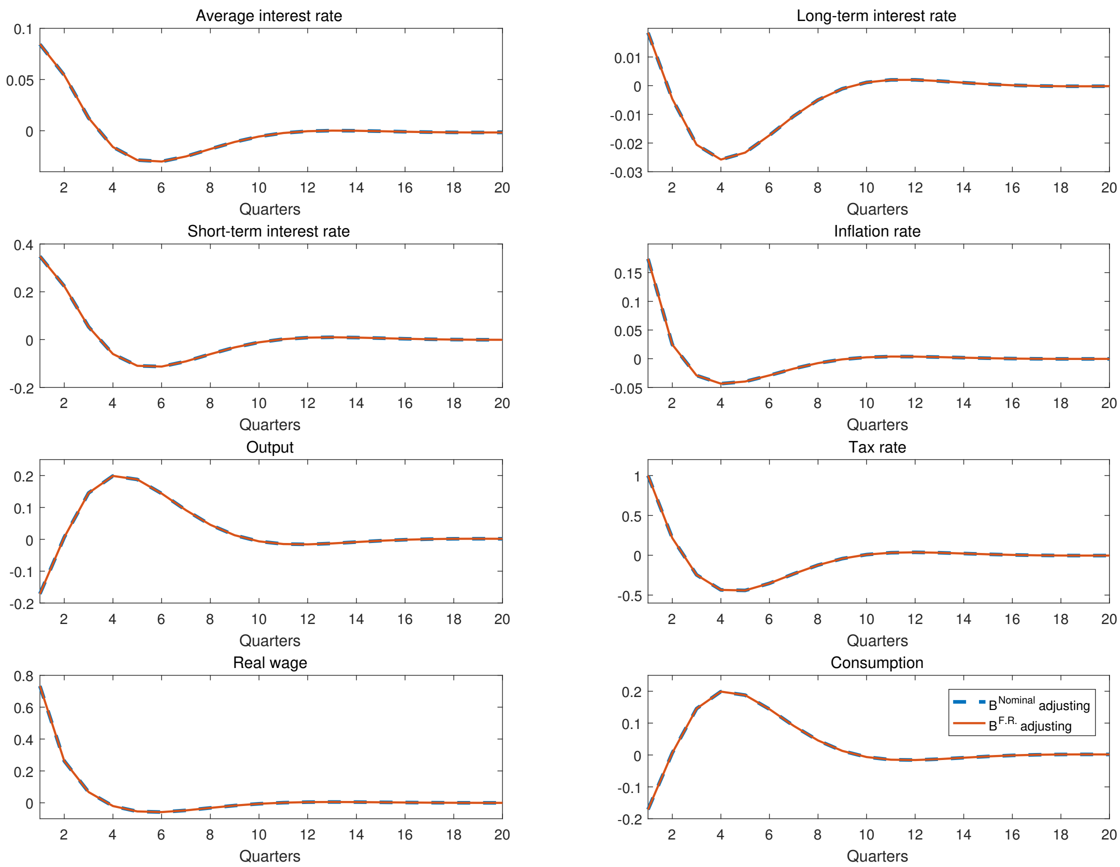

Figure 6: Fiscal Shock

After the fiscal shock, in both situations, the higher taxes lead to higher inflation rates, higher short-term interest rate, and higher real wage. Also, the fiscal shock leads to lower consumption and lower output. These variables exhibited the expected dynamics. The higher average interest rate increased after the increase in the short-term interest rate. However, the issuing long-term interest rate almost does not change, increasing a little after the fiscal shock, and then decreasing. These patterns can be easily identified looking to figure 6.

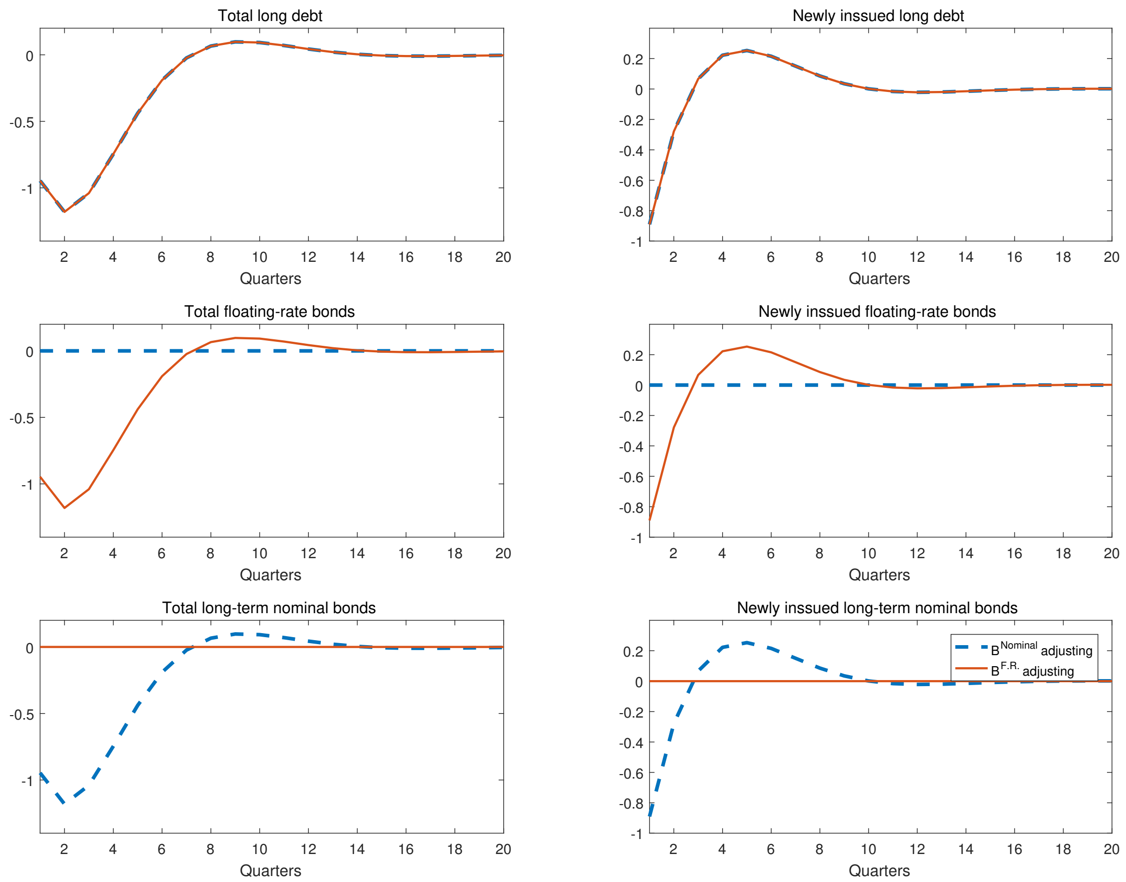

Figure 7: Fiscal Shock

Figure 7 makes clear that the behavior of the floating-rate bond when the nominal bond is held constant is equal to the behavior of the nominal bond when the floating-rate bond is held constant. Also, it is possible to see that the value of the stock of long-term debt decreases after the fiscal shock. Because of the parametrization of our fiscal rule – equation 20 –, the increase in taxes is higher than the decrease in the debt stock.

In figures 8, 9, and 10, we analyze the behavior of the variables in the model with rule-of-thumb agents after the fiscal shock. The dynamics of all variables are exactly equal to the dynamics of the model by only optimizing agents. We see, in figure 8, the same pattern, higher average interest rate, higher short-term interest rate, higher inflation, and higher real wage. In the opposite direction, we see lower output and lower consumption. In figure 9, we see that the value of the stock of long-term debt decrease after the fiscal shock. In figure 10, we see how the optimizing agents and the rule-of-thumb agents respond in exactly the same way to the fiscal policy shock. However, the representative agent's consumption is less volatile than the rule-of-thumb agent's consumption. The opposite is valid for the number of hours worked: the representative agent's hours are more volatile than the rule-of-thumb agent's hours.

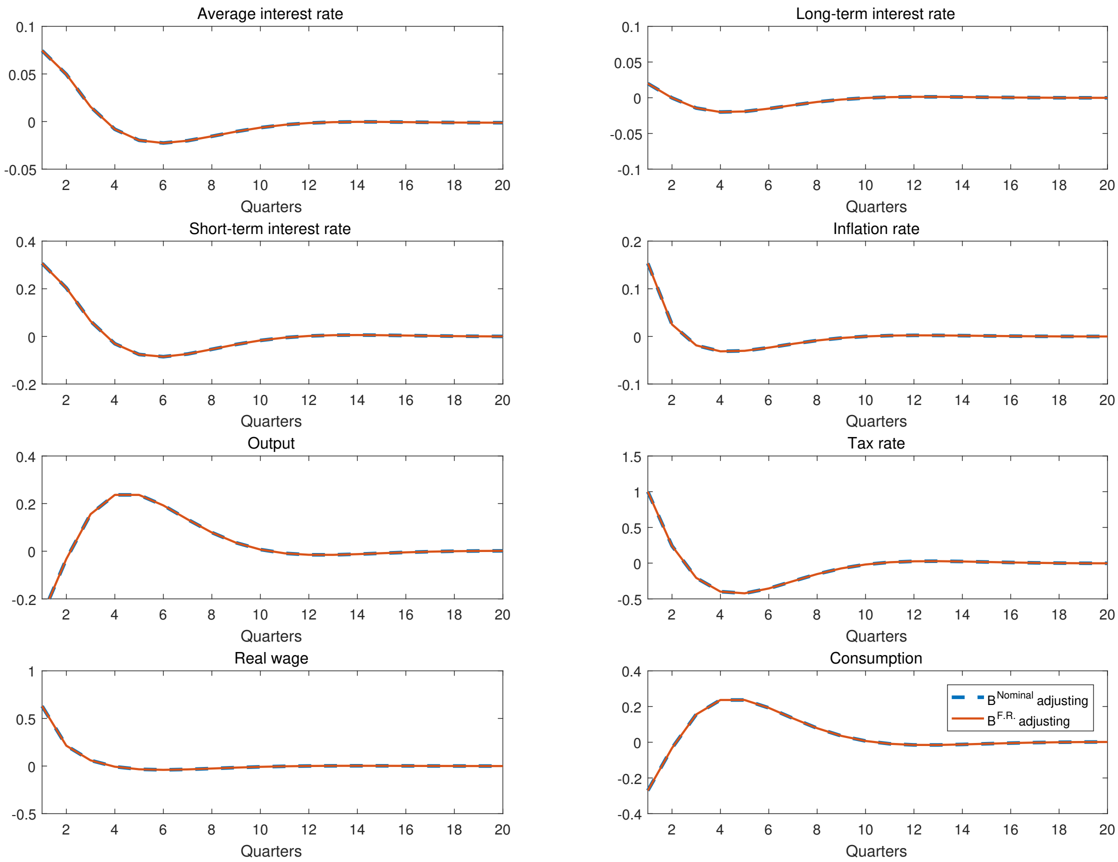

Figure 8: Fiscal Shock with Rule-of-Thumb Agents

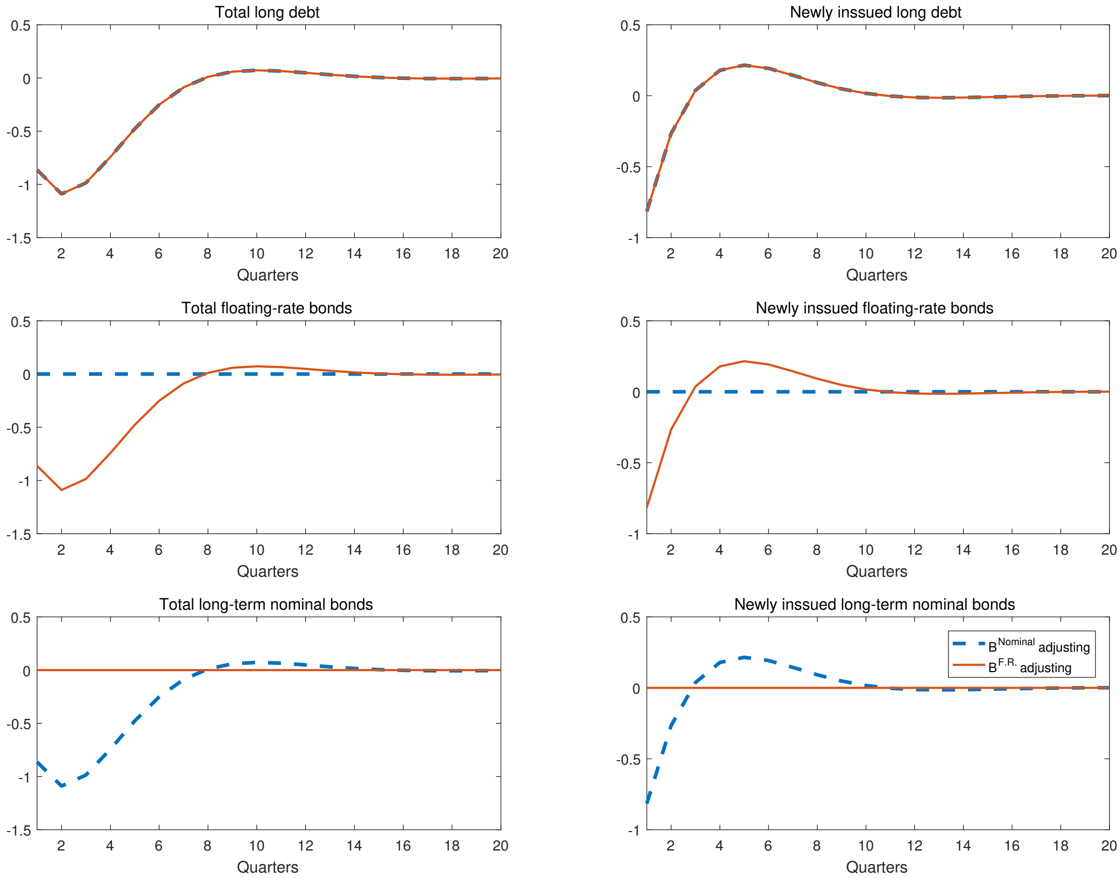

Figure 9: Fiscal Shock with Rule-of-Thumb Agents

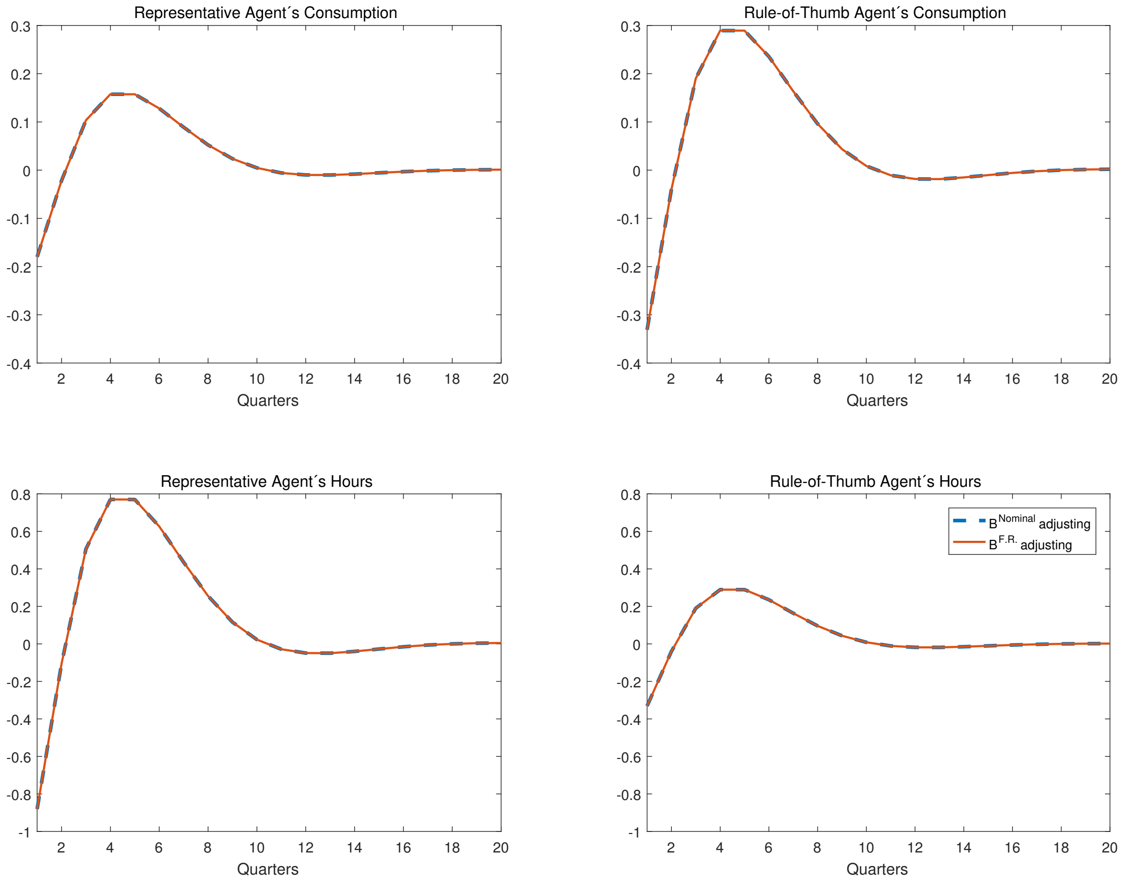

Figure 10: Fiscal Shock with Rule-of-Thumb Agents