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Consumer confidence/expectation indexes are frequently used by the media and the market in order to forecast the behavior of the economy. Agents' expectations are believed to explain output and employment fluctuations, either moderate or drastic as the ".com" and the American subprime crisis. In Brazil, more attention has been drawn to this topic due to the recent economic crisis. The estimation of a VAR with Brazilian data for consumption, output and expectations suggests that innovations to the expectation indexes do have impact on aggregate consumption and GDP in the medium/long-run, as well as the indexes themselves. Inspired by this evidence, a DSGE model is used in order to assess how much of these impacts are due to anticipation of future economic fundamentals and how much are due to animal spirits. The results indicate that animal spirits and index-specific noise are responsible for a non-negligible amount of fluctuations up to 2 quarters, whereas news of future economic conditions prevail on lower frequencies

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March 21, 2019

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Consumer confidence/expectation indexes are frequently used by the media and the market in order to forecast the behavior of the economy. Agents' expectations are believed to explain output and employment fluctuations, either moderate or drastic as the ".com" and the American subprime crisis. In Brazil, more attention has been drawn to this topic due to the recent economic crisis. The estimation of a VAR with Brazilian data for consumption, output and expectations suggests that innovations to the expectation indexes do have impact on aggregate consumption and GDP in the medium/long-run, as well as the indexes themselves. Inspired by this evidence, a DSGE model is used in order to assess how much of these impacts are due to anticipation of future economic fundamentals and how much are due to animal spirits. The results indicate that animal spirits and index-specific noise are responsible for a non-negligible amount of fluctuations up to 2 quarters, whereas news of future economic conditions prevail on lower frequencies.

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# 1 Introduction

The role of expectations of economic agents is of paramount importance to describe the dynamic behavior of an economy. Two classic examples on the subject are the works of Pigou (1927) and Keynes (1936), who popularized the term *animal spirits* to designate some behavior that would represent more spontaneous decision-making than a rational one. Pigou (1927) conjectures that difficulties in forecasting the evolution of an economy can lead to waves of optimism without economic foundation, resulting in subsequent disappointments, followed by economic downturns and recessions. The arising of bubbles can be interpreted using this framework and episodes like the crisis of Japanese assets in the 1980s, the ".com" crisis of the 2000s and the subprime crisis in 2008 appear to serve as good real world examples.

Indexes of confidence and expectations are survey-based indicators that aggregate perceptions on current and future economic conditions by the population. The intuition for the existence of such indicators is that they consolidate, in a single and easy to interpret number, the general sentiment level of economic agents. Carroll et al. (1994) showed that, using U.S. data, consumer confidence indexes were able to help to predict future levels of consumption even after controlling for other variables such as income. On the other hand, the magnitude of this predictive capacity was small. This study was replicated for several other economies around the world, with some mixed results. The overall empirical evidence suggests that while confidence indexes have useful predictive content in most cases, their relevance after controlling for other variables is a more fragile result.

On the theoretical side, the advent of the news literature in the context of dynamic stochastic general equilibrium models (DSGE) in the mid-2000s has made it possible to explore the mechanism transmissions between confidence indexes and the real side of the economy, by allowing agents to anticipate future economic conditions, whether correct or not. The profound period of economic downturn in the recent Brazilian economy lived up the debate on the link between economic recovery and confidence. Despite the general view amongst pundits that there ought to be a link between confidence/expectations and economic recovery (or lack of it), the specialized literature is rather scarce. The main purpose of this paper is to contribute to this literature by providing answers to the following: (i) do consumer expectation indexes really have any relation to the real side of the economy in the Brazilian context? and (ii) if so, how much of this effect is related to future economic fundamentals and how much is due to short-term disturbances, that is, to animal spirits?

We follow the methodology of Barsky and Sims (2012), and we first show that innovations in consumer expectation indicators have real effects on consumption and on output; such effects slowly increase over time and are highly persistent, providing a positive answer to the first question. Subsequently, in the light of a DSGE model with a signal extraction problem, in which agents do not perfectly observe some state variable (in this case, technology growth) and seek to infer it from what they can observe, we show that a relevant fraction of the movements in the expectation index for frequencies higher than or equal to one semester is due to animal spirits and to exogenous shocks to the indexes. However, over time, their dynamics becomes mainly determined by the anticipation that agents make of technological conditions. This answers the second question.

It should be stressed that our model assumes that confidence and expectations do not cause changes on the levels of consumption and GDP by themselves. What actually happens is that a reasonable degree of information about perceptions of future technological conditions - which affect consumption and output - are contained in the indices, making them good leading indicators of what will occur some periods ahead, but not being a direct source of causality.

The main contribution of the present study is therefore to expand the knowledge of news shocks in the context of an emerging market economy that has recently been subject to a deep economic downturn.

The paper is organized as follows: section 2 reviews the relevant literature and presents the empirical evidence relating consumer confidence indexes and consumption and GDP for Brazil. Section 3 follows with the discussion of the news literature shocks, the anticipation of future economic conditions by the agents. The idea is to relate the theory of this section with the data of the previous one. Section 4 describes the structure of the DSGE used to study the behavior of the Brazilian economy. This model is then estimated for Brazil and the main results are presented and discussed. Finally, section 5 concludes.

# 2 Indexes of Consumer Confidence and Expectations

Whenever an economy enters a recession or experiences a period of long-lasting economic boom, it is common to hear or read about a depressed or euphoric state of consumer and business "sentiment", particularly on the media. However, sentiment is a very broad and complex concept, given how many variables should affect an agent's perception of economic conditions. Indexes of consumer confidence and expectations are specifically designed to target this issue, summarizing in a single number the perceived current and/or future states of an economy. Typically, surveys contain questions related to consumer perceptions of several topics such as personal income, employment, inflation and the general state of the economy[2]. An index that focuses on *current* conditions will hereafter be denominated as a *confidence* index, whereas an index that focuses on the *future* conditions of an economy will be denominated as an *expectations* index.

Despite sentiment being regarded as an economically important variable for a while (e.g. Keynes (1936) animal spirits), academic studies about it have only flourished after the 90's, perhaps because of the US recession of 90-91 and its subsequent *jobless recovery*, which Blanchard (1993) attributed mainly to a depressed state of consumer confidence. In addition, it was around that time that (macro)econometric techniques such as vector autoregressions and real business cycle models emerged, making it easier to empirically evaluate measures of the so-called "sentiment" and its economic implications. Seminal papers may be attributed to Fuhrer (1993) and Carroll et al. (1994). In his article, Fuhrer (1993) lists "popular" theories of the role of consumer sentiment, notably that (i) sentiment independently causes economic fluctuations, that (ii) sentiment is a useful forecaster of economic fluctuations and that (iii) sentiment indexes provide information about consumers' expectations even when controlled by other variables. In short, the first theory states that a shock to consumer sentiment may *cause* a recession or a boom, whereas the second theory is only concerned with predictability, not with causation. Lastly, the third theory states that consumer sentiment indexes are not merely aggregations of information already contained in other economic variables such as GDP and consumption, which are commonly observed only with a significant lag.

## 2.1 Worldwide Evidence

First of all, it is important to stress three key aspects of academic investigations around the world: firstly, the literature has carried out studies using either confidence or expectation indexes, sometimes using both and comparing results. Secondly, most of these studies are concerned only with *predictability* of future consumption and are hence focused on reduced-form methods. A structural approach is suggested by Bram and Ludvigson (1998) and made feasible only in the late 2000's with the advent of the *news shocks* literature. Last but not least, the empirical evidence is *not* homogeneous among countries. Whereas some researchers find evidence of a strong anticipation component in consumer sentiment indexes, others find less robust results.

In order to empirically evaluate the relevance of consumer sentiment measures for the

[2]Although one may question the relevance of an individual's assessment of economic variables, aggregation of these perceptions may have relevant implications for the business cycle. See Cochrane (1994).

US, Carroll et al. (1994) estimate the following regression:

$$\Delta ln(C_t) = \alpha_0 + \sum_{i=1}^{4} \beta_i S_{t-i} + \gamma Z_{t-1} + \epsilon_t \tag{1}$$

Where $\Delta ln(C)$ stands for the log difference of household consumption, $Z$ is a set of control variables (namely, four lags of labor income growth and four lags of the dependant variable) and $S$ represents the Index of Consumer Sentiment (ICS)[3] from the University of Michigan. The idea is to evaluate the incremental $\bar{R}^2$ when using the ICS to assess how much forecasting power does the index itself have. The authors conclude that the inclusion of lagged sentiment has only a marginal increment of 3% on the predictability of future consumption, which despite being a small number economically, has interesting implications such as providing evidence against the permanent income hypothesis as in Hall (1978), which would not allow for other than contemporaneous relationships between consumption and sentiment.

Berg and Bergstrom (1996) analyze the Swedish index of consumer expectations, using an approach very similar to that of Carroll et al. (1994). Their results show that the index of consumer expectations regarding personal financial conditions 12 months ahead has a much larger predictive power in Sweden than the ICS does in the US, even when controlled by other economic variables. On the other hand, when Goh (2003) uses New Zealand data, he finds that adding stock prices and interest rates to the control variables set greatly reduces the predictive power of local confidence indexes. This, as a matter of fact, would amount to a favourable evidence for Fuhrer's second theory, but not the third. Easaw and Heravi (2004) focus on UK and enrich the analysis by using out-of-sample forecasting tests for confidence and expectation indexes. Both appear to improve predictability for durable goods, but only the headline index - which includes both current and future perceptions of the economy - is relevant when forecasting aggregate consumption. Finally, Kwan and Cotsomitis (2006) reveal that in Canada questions related to expectations have the highest incremental $\bar{R}^2$ for aggregate consumption.

## 2.2 What does Brazilian data show us?

The work of Bentes (2006) appears to be one of the first to address the relevance of consumer sentiment in Brazil, analyzing the theories of Fuhrer (1993) through the use of the Fecomercio-SP ICC confidence index, a measure of local consumption and a few control variables within a VAR system. The ICC index does relatively well in explaining future

[3] Using the terminology of this paper, the ICS may be classified as a confidence index since it has questions related to the current state of the economy.

variations of total consumption, providing favourable evidence to Fuhrer's second main theory. However, when a credit measure is added to the system, the incremental explicative power of the ICC vanishes, weakening the support for Fuhrer's third theory. As it seems, the confidence index contains important information about future economic fundamentals, but evidence hints that this piece of information may also be contained in other economic variables.

More recently, Graminho (2015) and Simões et al. (2016) also shed light on the relationship between brazilian confidence/expectations indexes and economic activity. Graminho (2015) uses both an (i) expectations index, (ii) current conditions index and a (iii) headline consumer confidence index (ICC-FGV) - which is a mixture of the other two - calculated by the Getúlio Vargas Foundation (FGV). Furthermore, she also carries a similar analysis using the ICI-FGV business confidence index. Even when controlled by other meaningful economic variables, all consumer-related indexes are relevant in explaining future movements of consumption, and all business-related indexes are relevant in explaining future industrial production. Despite not being a formal test of Fuhrer's theories, her evidence confirms the support for the second popular theory but contrasts with Bentes (2006) in terms of the third. Finally, Simões et al. (2016) reveal through vector error correction models that, albeit not having a significant effect on impact, shocks to the National Consumer Expectations Index (INEC) have *permanent and slowly-building* effects on aggregate consumption and GDP. We stress that a strikingly similar result is found by Barsky and Sims (2012) for the US economy and we see this as an interesting opportunity to investigate whether their findings are robust to an emerging economy such as Brazil. If they are, then we believe there may be a space for theory to develop further.

Following the steps of Barsky and Sims (2012) and in line with Simões et al. (2016), we estimate a trivariate VAR composed of (i) Fecomercio-SP IEC expectations index, (ii) real aggregate consumption and (iii) real GDP. The IEC index is calculated in a similar fashion to the Michigan University expectations index, which selects only questions related to future perceptions from the headline index (in the US case, the ICS). Questions asked comprise personal financial conditions 12 months ahead and national economic conditions both 12 months and 5 years ahead. Respondents answer whether they are "positive", "neutral" or "negative" about each topic and question-specific indexes are calculated as 100 + (%positive answers - %negative answers), being therefore bounded below and above by 0 and 200, respectively. The headline index is then merely a combination of each question-specific index.

[3]Notably, the controls are, for the consumer-related analysis: (i) real labor income, (ii) unemployment, (iii) industrial production, (iv) SELIC variation and (v) the IBOV stock market index. For the business-related analysis, she uses: (i) SELIC variation and (ii) exchange rate.

Real consumption and GDP series are extracted from the Quarterly National Accounts calculated by the Instituto Brasileiro de Geografia (IBGE) and are subsequently seasonally adjusted.

The system is estimated in log-levels[4] with a constant term and four lags. Orthogonalized[5] impulse-response functions (henceforth IRFs) and forecast error variance decompositions are reported in Figure 1. As a robustness check, we estimate the same VAR but with the expectations index as the last variable in the system. As shown in Appendix A, our results remain unchanged even in this extreme setup.

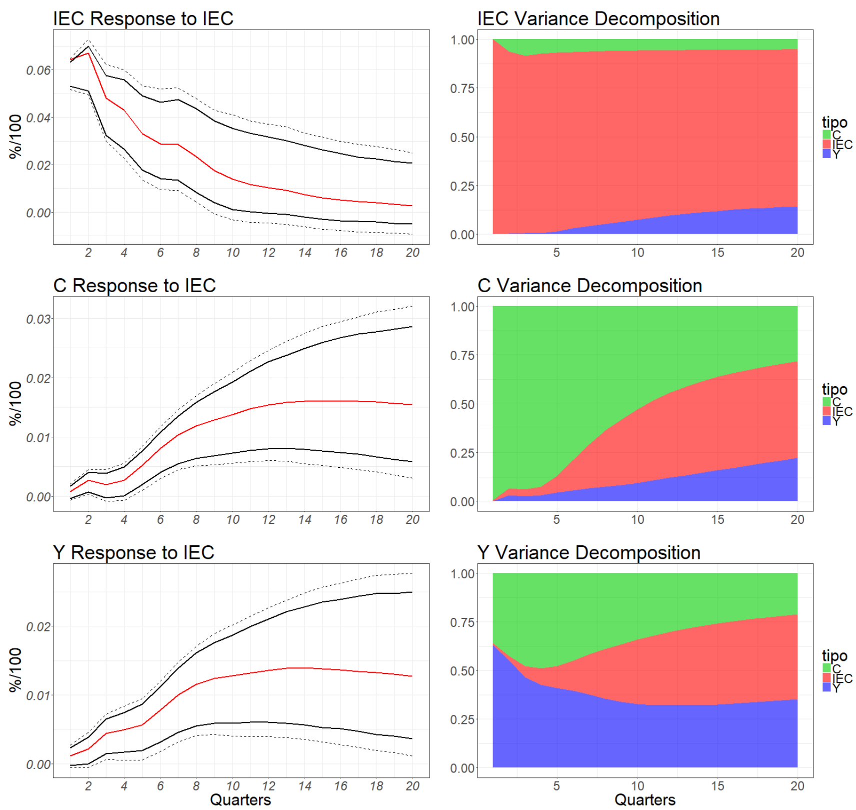


Figure 1: Impulse-response functions from a one standard deviation innovation in the IEC variable and forecast error variance decomposition from the trivariate VAR with the IEC variable ordered first. Black lines are the confidence bands calculated via the *bootstrap-after-bootstrap* method of Kilian (1998), which deals with small-sample bias. Intervals represent the 17th-83rd and 10th-90th percentiles. Our sample period is 1996Q1-2016Q4.

[4]We take this conservative approach of not imposing a specific cointegration relationship among variables since Sims, Stock, and Watson (1990) showed that a levels VAR delivers consistent estimates of impulse-response functions and variance decompositions, even when unit-root processes are present. Imposing cointegration would only improve efficiency, but a misspecified relationship could significantly bias estimates.

[5]Note that the reported orthogonalized IRFs are identical to the generalized impulse-response functions that would arise when using the expectation index as the first variable in the VAR. Since only innovations in the expectation index are crucial to this paper, using IRFs or GIRFs do not change the results.

Two aspects of the IRFs shall be highlighted here. First, the expectations index appears to be highly persistent, casting doubt on the existence of a unit root. Indeed, standard ADF and Ng-Perron tests do not reject the presence of a unit root, and a quick inspection of the time series itself suggest that this can be due to different means along time. Using dummy variables for post-Lula election and the euphoric period after the 2008 financial crisis (which initially had a relatively low impact on the brazilian economy) do render the series stationary. However, they also do not change our qualitative results of Figure 1 for consumption and income. Four reasons lead us not to carry on with the dummy variables for the rest of this paper: (i) it is not necessary to make the IEC stationary for our purposes of IRF analysis, (ii) including them in an augmented VAR with real interest rates and inflation - necessary for our estimation of monetary parameters in a structural model in Section 4 - yield an unstable VAR, (iii) longer time series around the world such as the University of Michigan Expectations Index and Japan's Cabinet Office Expectations Index do not seem to support the existence of unit roots in these types of indices, rather suggesting they move very slowly around a neutral state of euphoria and (iv) this kind of reasoning does not hold for another important expectation index in Brazil, the National Consumer Expectations Index (INEC), which yield very similar IRFs and variance decompostions to those reported in Figure 1. For the sake of simplicity, we do not report results[6] using the INEC index.

Secondly, we note that innovations in consumer expectations have *permanent and slowly-building* effects on consumption and output, just as reported by Simões et al. (2016) for the brazilian economy (using the INEC index) and Barsky and Sims (2012) for the US. Furthermore, variance decompositions show that the IEC is driven mainly by its own dynamics rather than consumption or output-related movements. On the other hand, the IEC is responsible for a relevant share of consumption and output forecast error variance. These results *per se* and their proximity to the ones found by Barsky and Sims (2012) for the US suggest that a similar transmission mechanism operates in the two countries, providing a motivation for further investigation of expectations indexes and real economic activity, along the lines of a structural approach as suggested by Bram and Ludvigson (1998).

One candidate for explaining such permanent moves in real variables is technology, which is commonly used in macroeconomics as a source of trend growth. The data, as well as the nature of an expectations index itself, suggest that there may be a relevant anticipation of technological conditions by individuals in the brazilian economy. The *news shocks* literature

[6] Albeit being an index with respondents only in the city of São Paulo, we choose the IEC in this paper for a variety of reasons. It has a longer sample, its questions focus exclusively on future conditions (whereas the INEC has two questions about current conditions, corresponding to 1/3 of the index) and not only do these questions assess the 12-month ahead perceptions of the whole economy - not just São Paulo -, but also the 5-years ahead period.

focuses specifically on including these kind of anticipation by economic - and rational - agents in macroeconomic (mainly DSGE) models.

# 3 News Shocks

Precisely because of the inclusion of anticipation of future structural shocks in macroeconomic models, this strand of the literature is called *news shocks*. The name, as one could imagine, suggests that such anticipation may be a product of news seen on the media, government statements or private information agents acquire in general. It is important to note that there is no conflict between rational agents and making mistakes about future fundamentals: news received may be false or noisy, leading agents to efficiently use their information sets and still predict wrong trajectories of economic variables, in a manner close to what Pigou (1927) described back in the 20's. Real-world examples appear to at least support this kind of reasoning. Take, for example, the crises of Japanese assets in the 80's, the ".com" bubble and even the subprime crisis of 2008. Despite being complex crises, all of them had individuals' forecast errors as a significant fuel.

## 3.1 Important Results and Methodological Approaches

An important catalyst for the literature was the empirical work of Cochrane (1994), which emphasizes that shocks to consumption appear to explain most (50%-70%) of US business cycles fluctuations. Since this is an endogenous variable and it is somewhat difficult to imagine what could be an exogenous shock to consumption, Cochrane (1994) states that they may in fact be news about future economic fundamentals. The seminal papers of Beaudry and Portier (2004) and Beaudry and Portier (2006) take a critical step in combining evidence and theory. In Beaudry and Portier (2006), the authors use short and long-run restrictions (separately) in a VECM framework with stock prices and TFP in order to investigate whether short-run movements in stock prices had long-run effects on TFP. They find robust evidence that agents anticipate future TFP growth, as innovations in stock prices (contemporaneously orthogonal to TFP) and a shock that brings permanent effects on TFP are almost perfectly correlated. Beaudry and Portier (2004), on the other hand, explore the theoretical side of news and add to a RBC model the possibility of anticipation of future technology shocks: individuals receive at time $t$ a signal about technology innovation at time $t+j$, with probability $q$ that the signal is right and $1-q$ that it is false. Two main conclusions arise: first, they show that economic fluctuations such as booms and busts may occur *even when fundamentals have not yet changed*, due to the possibility of news shocks. In addition, they demonstrate that (positive) news shocks in a RBC framework are not able to induce the so-called "Pigou cycles", in which favourable future technological conditions trigger a rise in output, investment, consumption and hours worked. This occurs mainly because good news about the future increases consumption through the wealth effect, reducing the incentive to work and hence having a negative impact on investment. This can - and probably will - cause a recession before news are finally realized at time $t + j$, running counter to what economic intuition would suggest. As discussed in Jaimovich and Rebelo (2009), even though models with real and nominal frictions make it easier for Pigou cycles to arise, they do not guarantee this result. Some features that facilitate this kind of dynamics are investment adjustment costs, variable capital utilization and a weak short-run wealth effect on labor supply. All of them should help a model to generate co-movement between consumption, investment and hours worked.

News shocks may also be relevant to analyze monetary policy. Christiano et al. (2010) discuss how news about future technology affect monetary policy in an inflation-targeting regime, possibly in a perverse way. By using a DSGE model they compare the responses of an inflation-targeting Central Bank and the Ramsey-efficient[7] response to a news shock, which does not follow a Taylor-type rule. Since the authors are not concerned with the quality of the news received by individuals, they assume that news may be either right or wrong (that is, $q = 1$ or $q = 0$). We note that a similar approach is used by Schmitt-Grohé and Uribe (2012), Kobayashi and Nutahara (2010) and Milani and Treadwell (2012)[8]. Christiano et al. (2010) reveal that the real interest rate rises more under a Ramsey regime than under an inflation-targeting regime. This arises because positive news shocks are deflationary, triggering a response from the Central Bank, which sees an opportunity to lower interest rates. The Ramsey-response, however, is to raise interest rates, because the increased consumption stemming from the news shock is inefficient since no technological improvement has yet occurred. The lesson from the article is that, as news may be false, Central Banks could in fact be fueling an economic boom when they should not. A (partial) solution to this, since it is very difficult to assess whether a boom is driven by fundamentals or agents' potential false expectations, is to include nominal credit growth in a Taylor-type rule[9]. Christiano et al. (2010) show that proceeding in this way makes the inflation-targeting response closer to the Ramsey one.

[7] Efficient in the sense of maximizing some objective function, for example social utility.

[8] As an illustrative example, an exogenous shock $z_t$ in Schmitt-Grohé and Uribe (2012) follows the process $ln(\frac{z_t}{z}) = \rho_z ln(\frac{z_{t-1}}{z}) + \epsilon_{z,t}$, where $\epsilon_{z,t} = \epsilon^0_{z,t} + \epsilon^4_{z,t-4} + \epsilon^8_{z,t-8}$ e $\epsilon^j_{z,t} \sim N(0, \sigma^j_z), \quad j = 0, 4, 8$. In short, this means that agents receive a non-noisy signal at time $t - 8$, but may revise them four quarters ahead. $\epsilon^0_{z,t}$ represents a standard surprise shock.

[9] The inclusion of nominal credit growth is based on historical data. The authors note that in almost every stock market boom there were declines of price levels and rapid expansion of credit.

### 3.2 Signal Extraction Problem

More recently, specifically after the work of Lorenzoni (2009), the literature focused again on the quality of signals received by individuals and the impact of noise on the business cycles. The basic idea is that agents receive each period a noisy signal $s_t$ about the future state of a non-observable variable $x_t$, so that:

$$s_t = x_{t+1} + v_t, \quad v_t \sim N(0, \sigma^2) \tag{2}$$

Where the term $v_t$ represents noise, which itself may be a source of economic fluctuations. Barsky and Sims (2012), for instance, refer to $v_t$ as *animal spirits* since it generates fluctuations without being related to any fundamentals. In the model of Lorenzoni (2009), agents receive a noisy signal about the permanent component of technology, but observe perfectly the transitory term. A significant contribution of his work is to demonstrate that noise shocks may, just as news shocks do, influence the business cycle. However, they have different effects on an economy: noise shocks represent in fact a demand shock, leading to an overheated economy and therefore inflation rises. The "real news" part, in contrast, is related to future fundamentals and technological improvement, representing a deflationary scenario.

In order to evaluate how significant are noise and news shocks in the US economy, Blanchard, L'Huillier, and Lorenzoni (2013) estimate a medium-sized DSGE model embedded with a signal extraction problem very similar to the one found in Lorenzoni (2009). They find that noise shocks are responsible for almost half of the consumption variance on impact, and around 20% in the case of output. These effects vanish over time, however.

Barsky and Sims (2012) take one step further and not only try to assess how news and noise shocks affect the economy, but also how these theoretical variables relate to a consumer expectations index. For this reason, we see their work as the cornerstone of our investigation in this paper and, since we apply their model to the Brazilian economy, we postpone the details to the next section and the appendices. In a nutshell, what they do is to assume that the index of consumer expectations does not, *per se*, have any impact on real economic activity, which would resemble a rejection of Fuhrer's first theory. However, they model innovations in consumer expectations as a linear combination of structural shocks such as news and noise regarding future technology. Thus, an expectations index may reflect future fundamentals of an economy even without being itself a source of fluctuations. Hence, there is a support for Fuhrer's second theory and also, to a certain extent, his third. They conclude that noise shocks - or *animal spirits shocks*, as they call them - have only moderate impact on the short-run and virtually none on the long-run. Nonetheless, news shocks have moderate effects on impact but become increasingly important as time goes by[10].

## 4 Model, Estimation and Results

Inspired by the evidence seen on Figure 1, we believe there is an interesting story behind the data. A natural way of carrying out a structural analysis would be the use of structural VARs (SVARs). Unfortunately, as noted by Sims (2012), Leeper, Walker, and Yang (2013) and Blanchard, L'Huillier, and Lorenzoni (2013), VAR systems in the presence of foresight are very likely to suffer from a non-invertibility problem as described by Fernández-Villaverde et al. (2007), meaning we would almost certainly not be able to recover the history of economic shocks and hence inference would be compromised. As shown in Leeper et al. (2013), DSGE models with foresight yield a VARMA representation of the solution (as opposed to a VAR form). This happens, as Sims (2012) notes, because of missing information: news are state variables unobservable to the econometrician, meaning that observable variables such as GDP, inflation and so on do not span the full state space of the DSGE model. Blanchard, L'Huillier, and Lorenzoni (2013) demonstrate that in the presence of noise in a signal extraction problem the issue of non-invertibility persists, even though in a slightly different manner. The bottom line is, then, that it is a difficult task to estimate a SVAR in order to assess the impact of news shocks. Schmitt-Grohé and Uribe (2012), Blanchard, L'Huillier, and Lorenzoni (2013) and Leeper, Walker, and Yang (2013) show, however, that DSGE models may be used to overcome the issue, at the expense of being necessary to explicitly define how the information set of the agent is formed.

We proceed using the New Keynesian model of Barsky and Sims (2012) in order to assess the interactions between the IEC and the real side of the Brazilian economy. The model is composed of (i) households, (ii) final goods firm, (iii) capital goods firms, (iv) intermediate goods firms and a (v) government. Households are homogeneous and hence its actions may be summarized into a single representative agent. He consumes final goods and offers his labor services to intermediate goods firms, which he owns. Also, he leases capital goods to these firms, which operate under monopolistic competition, producing differentiated goods that serve as input to the final good production, operating under perfect competition. After production, capital is returned to the households, which then lease it to capital goods firms that use both capital and investment goods to produce capital for the next period. These relations are summarized in Figure 2, already using the notation that will follow.

[10] Animal spirits explain about 5% of consumption forecast error variance after one quarter and almost zero after two years. News shocks explain 11% of consumption forecast error variance after one quarter, 36% after two years and nearly half after five years.

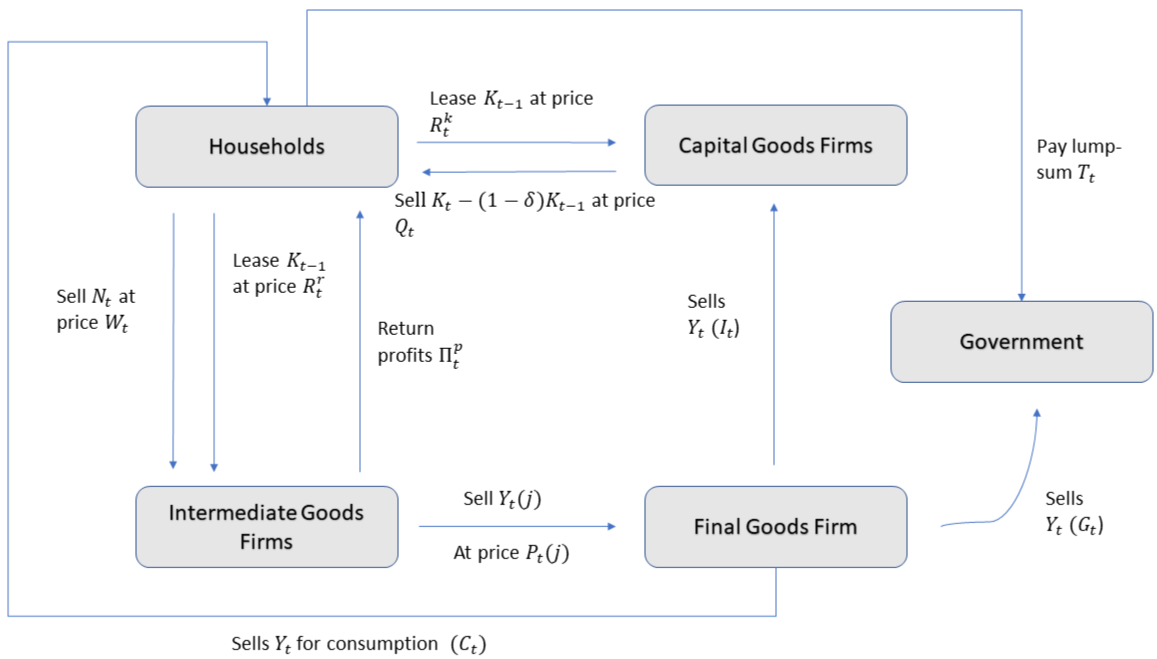


Figure 2: Internal logic of the model.

The model also contains both real and nominal frictions. Habit formation in consumption is included in order to produce smoother responses of consumption to structural shocks, capital adjustment costs allow hump-shaped responses of investment via a variable Tobin's Q, and price rigidity a la Calvo (1983) gives a role to monetary policy. A further important aspect of the model is that it is not stationary, due to an assumed random walk with drift process for technology. This is necessary in order to reproduce the permanent IRFs seen on Figure 1. The model is presented here in levels for the sake of simplicity, but details on how to render it stationary are given in Appendix B.

**Households** maximize their expected utility flow by choosing real consumption $C$, hours worked $N$ and savings either in the form of bonds $B$ or capital $K$, subject to their budget constraint:

$$\max_{C_t,N_t,B_t,K_t} \quad \sum_{t=0}^{\infty} \beta^t E_0 \left[ ln(C_t - \kappa C_{t-1}) - \frac{N_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] \quad s.t.$$

$$C_t + B_t = W_t N_t - T_t + R_{t-1} B_{t-1} + \Pi_t^p + R_t^r K_{t-1} - Q_t[K_t - (1-\delta)K_{t-1}]$$

Where $W$ stands for real wage per hour worked, $T$ is the lump-sum tax paid to the government, $R$ is the gross real interest rate, $\Pi^p$ are real profits received from intermediate good firms, $R^r$ is the gross rental rate of capital and $Q$ is the relative price of capital. As for the parameters, $\kappa$ translates the degree of habit formation in consumption, $\eta$ is the Frisch elasticity of labor supply and $\beta$ discounts future utilities. The first-order conditions are:

$$\Lambda_t = \frac{1}{C_t - \kappa C_{t-1}} - E_t \left[ \frac{\beta\kappa}{C_{t+1} - \kappa C_t} \right] \tag{3}$$

$$N_t^{\frac{1}{\eta}} = \Lambda_t W_t \tag{4}$$

$$\Lambda_t = \beta E_t [\Lambda_{t+1} R_t] \tag{5}$$

$$\Lambda_t = \beta E_t \left[ \frac{R^r_{t+1} + (1-\delta) Q_{t+1}}{Q_t} \Lambda_{t+1} \right] \tag{6}$$

Where $\Lambda$ is the Lagrange multiplier of the budget constraint, interpretable as the marginal utility of consumption.

**Final good firm.** The final good ($Y$) producer operates under perfect competition and uses intermediate goods $Y(j)$ as inputs. The production function is given by the CES aggregator:

$$Y_t = \left[ \int_0^1 Y_t(j)^{\frac{\xi-1}{\xi}} dj \right]^{\frac{\xi}{\xi-1}}, \ \ \xi > 1$$

Where the parameter $\xi$ governs the substitution degree among intermediate goods. Cost-minimization yields the following demand curve for intermediate goods, with $P_t$ as the aggregate price index:

$$Y_t(j) = Y_t \left[ \frac{P_t(j)}{P_t} \right]^{-\xi}, \qquad P_t = \left[ \int_0^1 P_t(j)^{1-\xi} dj \right]^{\frac{1}{1-\xi}}$$

**Intermediate goods firms.** There is a continuum of intermediate goods firms indexed by $j \in [0, 1]$ operating under monopolistic competition. They use capital goods and labor as inputs in order to produce the firm-specific intermediate good $Y(j)$, according to the production function:

$$Y_t(j) = A_t K_t(j)^{\alpha} N_t(j)^{1-\alpha}$$

Note that $A$ represents the current state of technology in the sense that given $N$ and $K$, a higher value of $A$ means one can produce a higher level of output. In addition, note that this technology is shared by all firms, hence it is not indexed by $j$. We find it important to view $A$ as a very broad concept, reflecting not only sophisticated machinery or a highly-skilled labor force, but also business conditions as a whole, for example strong institutions and a market-friendly environment (well defined and enforced property rights). Cost-minimization choosing $N_t(j)$ and $K_t(j)$ may be formalized as:

$$\min_{N_t(j),K_t(j)} \quad W_t P_t N_t(j) + R_t^r P_t K_t(j) \ s.t. \ A_t K_t^{\alpha}(j) N_t^{1-\alpha}(j) \geq Y_t \left(\frac{P_t(j)}{P_t}\right)^{-\xi}$$

Yielding the following first-order conditions, where $MC$ stands for marginal cost:

$$W_t = MC_t(j)(1-\alpha)A_t K_t(j)^{\alpha} N_t(j)^{-\alpha} \tag{7}$$

$$R_t^r = MC_t(j)\alpha A_t K_t(j)^{\alpha-1} N_t(j)^{1-\alpha} \tag{8}$$

Now turning to the price-setting decision, let $TC$ represent total real costs faced by the firm, which will be a function of their desired output $Y(j)$, $\theta$ the probability that the firm will not be able to reset its prices in the current period and $\beta^k \frac{\Lambda_{t+k}}{\Lambda_t}, k = 0, 1, 2...$ the stochastic discount factor. Then, profit maximization may be formulated as:

$$\max_{P_t^*(j)} \ E_t \sum_{k=0}^{\infty} (\theta\beta)^k \left\{ \frac{\Lambda_{t+k}}{\Lambda_t} \left[ Y_{t+k} \left( \frac{P_t^*(j)}{P_{t+k}} \right)^{1-\xi} - TC_{t+k} \left( Y_{t+k} \left( \frac{P_t^*(j)}{P_{t+k}} \right)^{-\xi} \right) \right] \right\}$$

Let $(1+\tilde{\mu}) = \frac{\xi}{\xi-1}$. In equilibrium, this will be the gross mark-up. Then, the first order condition is:

$$E_t \sum_{k=0}^{\infty} (\theta\beta)^k \left\{ \frac{\Lambda_{t+k}}{\Lambda_t} Y_{t+k}(j) \left[ \frac{P_t^*(j) - (1+\tilde{\mu}) MC_{t+k} P_{t+k}}{P_{t+k}} \right] \right\} = 0 \tag{9}$$

**Capital goods producers.** Just as intermediate goods firms, capital producers are indexed by $\nu \in [0, 1]$. Their production funtion is given by:

$$Y_t^k(\nu) = \phi\left(\frac{I_t(\nu)}{K_t(\nu)}\right) K_t(\nu)$$

Where $I$ denotes investment and $\phi(.)$ concave and increasing implies that producers face capital adjustment costs. With $R^k$ denoting the gross real rental rate of capital for these producers, profit-maximizations may be written as:

$$\max_{I_t(\nu),K_t(\nu)} \quad Q_t \phi\left(\frac{I_t(\nu)}{K_t(\nu)}\right) K_t(\nu) - I_t(\nu) - R_t^k K_t(\nu)$$

First-order conditions are:

$$Q_t \phi' \left( \frac{I_t(v)}{K_t(v)} \right) = 1 \tag{10}$$

$$Q_t \left[ \phi \left( \frac{I_t(v)}{K_t(v)} \right) - \phi' \left( \frac{I_t(v)}{K_t(v)} \right) \frac{I_t(v)}{K_t(v)} \right] = R_t^k \tag{11}$$

One may notice from the equations above that, in equilibrium, all firms choose the same investment/capital ratio.

**Government.** The government finances its debt and expenses via lump-sum taxes and new debt issuance:

$$G_t + R_{t-1} B_{t-1} = T_t + B_t$$

Assume further that the share of government expenses relative to GDP follows an AR(1) in log:

$$g_t^{sh} = (1 - \rho_g) g^{sh*} + \rho_g g_{t-1}^{sh} + \epsilon_t^{gsh}, \qquad g_t^{sh} = ln \left( \frac{G_t}{Y_t} \right) \tag{12}$$

Monetary policy is carried on by an authority such as a Central Bank, following a Taylor-type rule of the form:

$$R_t^n = (R_{t-1}^n)^{\rho_r} (\Pi_t)^{(1-\rho_r)\phi_\pi} \left( \frac{Y_t}{Y_{t-1}} \frac{1}{\Gamma^{\frac{1}{1-\alpha}}} \right)^{(1-\rho_r)\phi_y} e^{\epsilon_t^{rn}} \tag{13}$$

Where $\Pi_t$ stands for gross inflation rate given by $\Pi_t = \frac{P_t}{P_{t-1}}$. This kind of monetary rule allows Central Banks not to rely on unobservable variables such as the output gap. Rather, it focuses on inflation and output deviations from trend growth, given by the $\Gamma$ term[11]. In addition, a measure of smoothness in introduced via $\rho_{rn}$. The parameters $\phi_\pi$ and $\phi_y$ quantify how much does the monetary authority care about inflation and/or output deviations from target.

**Exogenous Processes and Information Flows.** The logarithm of technology $a$ follows a random walk with drift, where the drift $ga$ represents its growth rate, following itself a stationary AR(1) process. This formulation allows for stochastic growth in the model, necessary to replicate the responses in the data. As individuals perfectly observe the level of technology but only a noisy measure of its growth rate, we have a signal-extraction problem

[11] See Appendix B for details on $\Gamma$.

(imperfect information) framework:

$$a_t = a_{t-1} + ga_{t-1} + \epsilon_t^a \tag{14}$$

$$ga_t = (1-\rho_{ga})ga^* + \rho_{ga} ga_{t-1} + \epsilon_t^{ga} \tag{15}$$

$$s_t = ga_t + \epsilon_t^s \tag{16}$$

Where $\rho_{ga}$ governs the persistence in the growth rate of technology. All shocks follow zero mean normal distributions with standard deviations denoted as $\sigma_{\epsilon_a}, \sigma_{\epsilon_s}$ and $\sigma_{\epsilon_{ga}}$. The shock $\epsilon_t^s$ represents noise within the signal and may be viewed as animal spirits as in Barsky and Sims (2012). This is due to the fact that it introduces a disturbance in the model that is not related to any fundamentals of the economy, having only a temporary and reversible effect. The $\epsilon_t^{ga}$ shock, on the contrary, may be viewed as a news shock, since it only affects technology at time $t+1$ and beyond, but is (imperfectly) known at time $t$. This formulation is also used by Barsky, Basu, and Lee (2015).

Given the linear and gaussian formulation of the problem, the optimal filter for agents to form their estimate of the current state of the economy is the Kalman Filter. As discussed in Hürtgen (2014), this relaxes the perfect information assumption and introduces a passive learning mechanism.

Finally, the expectations index (IEC) follows a stationary AR(1) in log. The crucial aspect here, as noted previously, is that *the index itself does not have any causal impact on real variables*. Rather, innovations $u$ in the consumer expectations index will be a linear combination of the perceived technological shocks - influenced by noise - and a “pure” noise shock $\epsilon^{IEC}$ which reflects non-modelled movements from the IEC. These perceptions may enable the index to have an anticipation power, depending on how much weight estimated parameters give them:

$$IEC_t = (1-\rho_{IEC})IEC^* + \rho_{IEC} IEC_{t-1} + u_t \tag{17}$$

$$\begin{aligned} u_t = {} & \zeta_1(a_t - a_{t-1} - ga_{t-1|t-1}) + \\ & + \zeta_2(ga_{t|t} - \rho_a ga_{t-1|t-1} - (1-\rho_{ga})ga^*) + \zeta_3 \epsilon_t^{IEC} \end{aligned} \tag{18}$$

The term $ga_{t-1|t-1}$ above translates the estimate at time $t-1$ formed about the unobserved variable $ga_{t-1}$.

Details on aggregation, model linearization and rendering variables stationary are given in Appendix B.

## 4.1 Estimation strategy

Model parameters are estimated via indirect inference, a flexible technique introduced by Gourieroux et al. (1993) and also denomited as Extended Simulated Method of Moments by Smith (1993). Instead of choosing parameters to match all dimensions of the data, indirect inference focuses on some key aspects (moments) of the data that the researcher is interested in. A few applications that employ this strategy using VAR impulse-responses are Rotemberg and Woodford (1997), Christiano et al. (2005) and Barsky and Sims (2012). Our objective here is to choose model parameters so a VAR estimated on simulated series match brazilian data-driven VAR IRFs as closely as possible. Since the structural model presented above has monetary-related parameters, it is necessary to augment our VAR from Section 2 in order to better identify them. We do so as in Barsky and Sims (2012) by adding observed series of inflation as measured by the IPCA index and the ex-ante real interest rate based on the SELIC nominal interest rate and FOCUS market inflation expectations[12]. Resulting IRFs shown in Figure 3 reveal that the responses of IEC, consumption and output remain roughly the same as seen on the motivational VAR of Section 2. Inflation rates appear not have a significant response for most of the time, judging by the confidence bands. When the mean response is concerned, however, we conclude that an innovation in the expectations index is slightly deflationary up to 10 quarters. As for the real interest rates, we find a different result from Barsky and Sims (2012): although we cannot reject that a positive innovation in the IEC may briefly raise the real interest rate, on average its response is negative for a long time.

In addition to the five IRFs showed in Figure 3, we match two additional moments of the data: the autocorrelation and the standard deviation of the growth rate of output per hour worked ($\Delta(Y/N)$). The aim is to better estimate technology-related parameters. An absence of autocorrelation suggests, for example, that surprise technology shocks are more relevant than growth rate shocks, which, by the AR(1) form assumed to the $ga$ process, should exhibit a higher degree of persistence. Brazilian data for hours worked are extracted from the Pesquisa Mensal do Emprego (PME), which spans the period 2002M3-2016M2, almost our entire sample period for the 5 VAR variables. Although the PME survey has the drawback of covering only a few metropolitan areas, the national survey (PNAD Contínua) has too few data points for us to carry on with estimation. We proceed by using the PME

[12]IPCA series represent the annualized %QoQ inflation rate, whereas the real interest rate is constructed using three-months ahead inflation expectations and SELIC target rates. Since the latter is available on a daily basis, we calculate the geometric mean for the whole quarter. After calculating the real ex-ante rate as $1 + r_t = \frac{1+r_t^n}{1+E[\pi_{t+1}]}$ we seasonally adjust them and take logs of gross rates in order to make real-world data and model data compatible. Furthermore, since $ln(1 + x) \approx x$, we may still interpret the IRFs as either percentage or percent points variations, at least for small values of $x$.

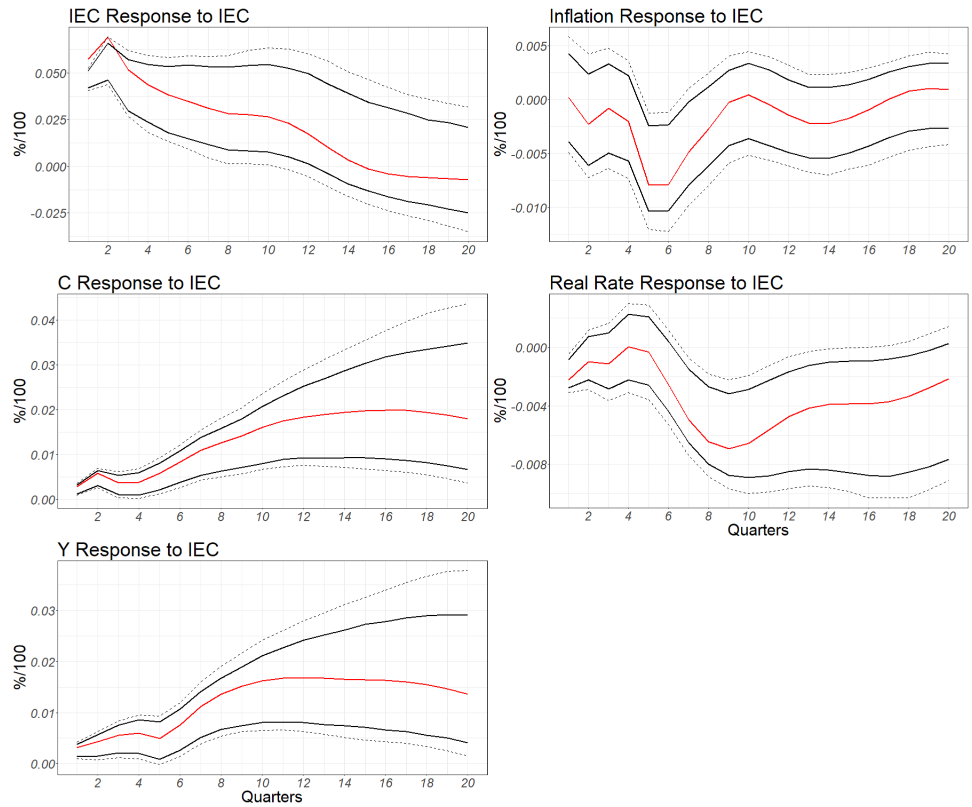


Figure 3: Impulse-response functions from a one standard deviation innovation in the IEC. Black lines are the confidence bands calculated via the *bootstrap-after-bootstrap* method of Kilian (1998). Intervals represent the 17th-83rd and 10th-90th percentiles. Our sample period is 2000Q1-2016Q4.

survey transforming data from monthly to quarterly by computing the quarterly average. In summary, we use $std(\Delta(Y/N)) = 1.28\%$ and $autocorr(\Delta(Y/N)) = 0.42$.

We partition the vector of deep paramenters into two sets: a calibrated and an estimated set. Let us start with the calibrated ones. The parameters $\alpha$, $\delta$, $\eta$ e $\tilde{\mu}$, which represent, respectively, the capital share of income, the net quarterly depreciation rate, the Frisch labor supply elasticity and the net mark-up rate, follow the values found on the SAMBA (Castro et al. (2015)) model for the Brazilian economy. Hence, $\alpha = 0.448$, $\delta = 0.015$, $\eta = 1$ and $\tilde{\mu} = 10\%$. The share of government consumption relative to GDP is calibrated according to data stemming from the IBGE Quarterly National Accounts and set to 19%. The parameter $\Gamma$ is calibrated to 1.0031 reflecting the average quarterly growth rate of GDP found in the data[13] during our VAR sample period. As for $\rho_g$ and $\sigma_{\epsilon_{gsh}}$, we estimate an AR(1) process for the log of $G/Y$ just as in the model and use results as calibrated parameters. Therefore, we

[13]In fact, the non-stochastic gross growth rate of GDP as predicted by the model is equivalent to $\Gamma^{\frac{1}{1-\alpha}}$. We do the appropriate calculations to find $\Gamma$.

set $\rho_g = 0.745$ and $\sigma_{\epsilon_{gsh}} = 2.79\%$. Finally, we use $\beta = 0.98$ based on the average real interest rate during our sample period and normalize the pure noise shock standard deviation to $\sigma_{\epsilon_{IEC}} = 1\%$.

Consequently, we are left with the vector of estimated parameters denoted as $\Theta = [\kappa, \phi_\pi, \phi_y, \theta, \gamma, \rho_{rn}, \rho_{ga}, \rho_{IE}, \zeta_1, \zeta_2, \zeta_3, \sigma_{\epsilon_s}, \sigma_{\epsilon_{ga}}, \sigma_{\epsilon_a}, \sigma_{\epsilon_{rn}}]$. Our goal is to find the optimal vector $\Theta^*$ such that it minimzes the distance between model-implied and "empirical" (in the sense of real-world) data:

$$\min_{\Theta} \quad F(\Theta) = (M - M(\Theta))' S^{-1} (M - M(\Theta)) \tag{19}$$

Where $M$ represents data-based moments and $M(\Theta)$ are moments derived from the model. $S$ is a diagonal matrix composed of empirical variances for each moment, thus estimation cares less about moments with high variances and more with those that are more precisely measured. For each combination of $\Theta$, we conduct $H = 1000$ simulations of the model with $h = 268$ observations, with a burn-in of the first 200 in order to keep simulated time series the same size as those in the empirical VAR. For each one of the $H$ simulations we estimate the same VAR system on the simulated series, computing their average afterwards as suggested by Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016).

As a final technicality on the estimation strategy, we note that we limited the parameter space of our optimization algorithm in order to restrict the search to a non-negative region as well as exclude the possibility of additional unit roots[14] or extremely unreasonable values. Table 1 summarizes their possible values:

| **Parameters** | $\kappa$ | $\theta$ | $\phi_\pi$ | $\phi_y$ | $\gamma$ | $\rho_{rn}$ | $\rho_{ga}$ | $\rho_{IEC}$ |
|---|---|---|---|---|---|---|---|---|
| Lower | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Upper | 0.999 | 0.999 | 6 | 2 | 80 | 0.999 | 0.999 | 0.999 |
| **Parameters** | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ | $\sigma_{es}$ | $\sigma_{ea}$ | $\sigma_{ega}$ | $\sigma_{ern}$ | |
| Lower | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| Upperr | 50 | 50 | 50 | 10 | 10 | 10 | 20 | |

Table 1: Parameter space

[14]We give them, however, the benefit of doubt by allowing extreme borderline values.

## 4.2 Results and Discussion

Parameter estimates are found in Table 2. The value of $\kappa$ indicates that brazilian consumers have a very significant degree of habit formation in consumption. This result was in a certain way expected due to the high persistence of both consumption and output persistence seen on the empirical VAR impulse-response functions to a innovation in IEC. Since they were modelled as a linear combination of perceived technology shocks, this means that when a positive assessment of future economic conditions is made, some mechanism acts in order to make agents smooth consumption and increase it very slowly. A likely candidate would be habit formation, and the estimation confirms this hypothesis.

The values of $\phi_\pi$ and $\phi_y$ show that the Brazilian Central Bank indeed follows the Taylor Principle and reacts strongly to inflation deviations from the target. On the other hand, it becomes quite clear that unlike the US Federal Reserve the Central Bank does not follow a dual mandate and output deviations are not much of a concern[15]. There is a high degree of price rigidity as captured by $\theta$, but by no means this is an absurd parameter. As an example, Blanchard, L'Huillier, and Lorenzoni (2013) find a very similar estimate (0.877) for the US economy when using a signal-extraction problem. In our case, we see the high value of $\theta$ as crucial to reproduce the slow movement of inflation to an innovation in the IEC as seen in Figure 3. Were the value of $\theta$ much lower, price would adjust rapidly and the inflation IRF would have a stronger movement on impact, as well as almost none on the medium run. There is also some relevant degree of capital adjustment costs as reflected in $\gamma$. While the literature lacks consensus on which values would be reasonable for either an emerging or a developed economy, we note that a few papers have chosen parameter values similar to ours[16]. Nevertheless, as will be shown at the end of this section, values of $\gamma$ do not have any striking effect on our relevant conclusions.

Results also clarify that technology growth is rather persistent and, as expected, we would not reject the presence of a unit root on the expectations index. As it is not clear whether this arises in fact because adjustment dynamics is very slow (as suggested by longer time series such as the US and Japanese ones), we maintain a skeptical stance on the presence of an unit root process. The monetary policy smoothing parameter is relatively low, advocating for a more reactive rather than inertial monetary policy.

As for the $\zeta_1, \zeta_2$ and $\zeta_3$ trio, we see their relative magnitudes as hinting that news shocks may have a very significant role on the dynamics of the expectations index. Indeed, a high value of $\zeta_2$ is not sufficient to conclude this, since the relative size of the news shock

[15]Which is consistent with the Brazilian Central Bank mission of pursuing *price* stability.

[16]Carvalho and Nechio (2017) calibrate a multi-sector model for the US economy using $\gamma = 40$, whereas Souza-Sobrinho (2011) use values between 60 and 78.

| **Parameter** | Model Estimate | (d.p.) |
|---|---|---|
| $\kappa$ | 0.943 | (0.002) |
| $\phi_\pi$ | 1.505 | (0.049) |
| $\phi_y$ | 0.000 | (0.065) |
| $\theta$ | 0.890 | (0.005) |
| $\gamma$ | 30.47 | (2.760) |
| $\rho_{rn}$ | 0.148 | (0.029) |
| $\rho_{ga}$ | 0.813 | (0.008) |
| $\rho_{IE}$ | 0.999 | (0.001) |
| $\zeta_1$ | 0.508 | (0.645) |
| $\zeta_2$ | 12.01 | (0.797) |
| $\zeta_3$ | 1.410 | (0.229) |
| $100\sigma_{\epsilon_s}$ | 1.074 | (0.091) |
| $100\sigma_{\epsilon_a}$ | 0.366 | (0.023) |
| $100\sigma_{\epsilon_{ga}}$ | 0.602 | (0.013) |
| $100\sigma_{\epsilon_{rn}}$ | 19.99 | (1.094) |

Table 2: Parameter estimates. Standard deviations inside parentheses.

as given by their standard deviation also plays a role. This number is given by $\sigma_{\epsilon_{ga}}$ and amounts to roughly 0.6%, almost two times larger than that of surprise shocks. $\sigma_{\epsilon_s}$ also provides evidence that animal spirits/noise shocks are relevant for brazilian business cycle fluctuations. Figure 4 shows the empirical IRFs along with model estimates when estimating a VAR on its simulated series. We note that almost all point estimates of the model are inside the empirical VAR confidence bands, indicating that the model is indeed capable of explaining the relevant dimensions of the data. While there is a slight lack of amplification when it comes to consumption and output responses, we highlight that the qualitative aspect of permanent responses is well reproduced. Also, given the high value of $\theta$, the dynamics of inflation are more muted than those of the data. Still, the smooth deflationary movement is captured by the model.

When model confidence bands are introduced we see that albeit there are no significant changes to the expectations index, consumption, output and inflation, there is a significant uncertainty regarding real interest rate dynamics, so not many conclusions may be drawn for this specific variable. This result is mainly the product of our estimated high value of $\sigma_{\epsilon_{rn}}$. However, as it was the case with capital adjustment costs $\gamma$, the magnitude of monetary policy shocks will not be a crucial aspect of the model when evaluating the relevance of news, animal spirits and noise shocks.

Up until this point we analyzed IRFs from innovations in variables of the VAR system.

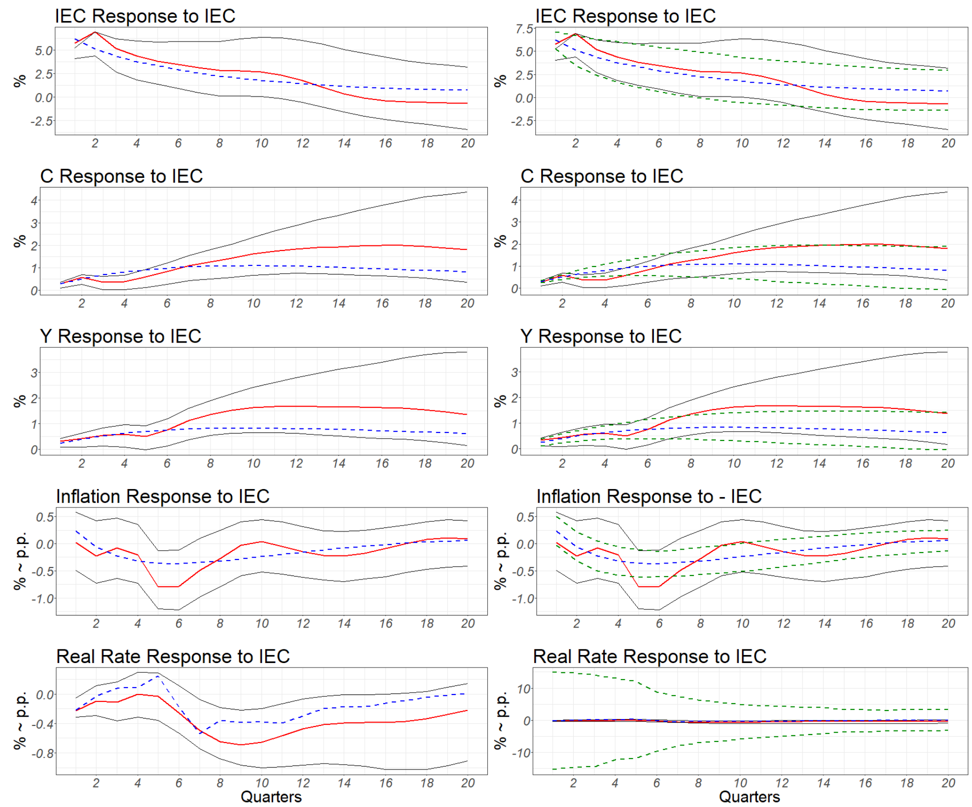


Figure 4: Empirical and model-implied (blue lines) IRFs given an innovation in IEC. Left-hand side figures show the pointwise estimates, whereas right-hand side also show the model 10% and 90% confidence bands (green lines).

Now we turn to the theoretical impulse-response functions that would arise from *structural shocks* such as news and noise. Figure 5 reports the theoretical IRFs that arise after a one standard deviation news shock, whereas Figure 6 refers to a noise (or animal spirits) shock.

A news shock triggers, by construction, an immediate response of the signal received by the agents and the technology growth itself, $ga$. Because of the AR(1) process assumed to $ga$, both of them gradually revert to their initial states according to the parameter $\rho_{ga}$. Much more interesting is to analyze the behavior of the Kalman filtered assessment of $ga$, given by $ga_{t|t}$: individuals are initially skeptic about the true nature of the shock, but as technology advances over time they adjust their beliefs and learn that the shock was indeed related to future fundamentals. Current and expected future consumption are fostered by the wealth effect perceived by the agents, affecting current marginal utility of consumption $\Lambda$, which, given optimal parameter estimates, falls. Consequently, the incentive to work decreases and, given that the capital stock is predetermined and technology does not change at time $t$, real wages must go up in order to prevent output from falling.[17]. This puts an

[17]For further details, see Equations 23, 24, 29, 33, 36, and 37 on Appendix B.

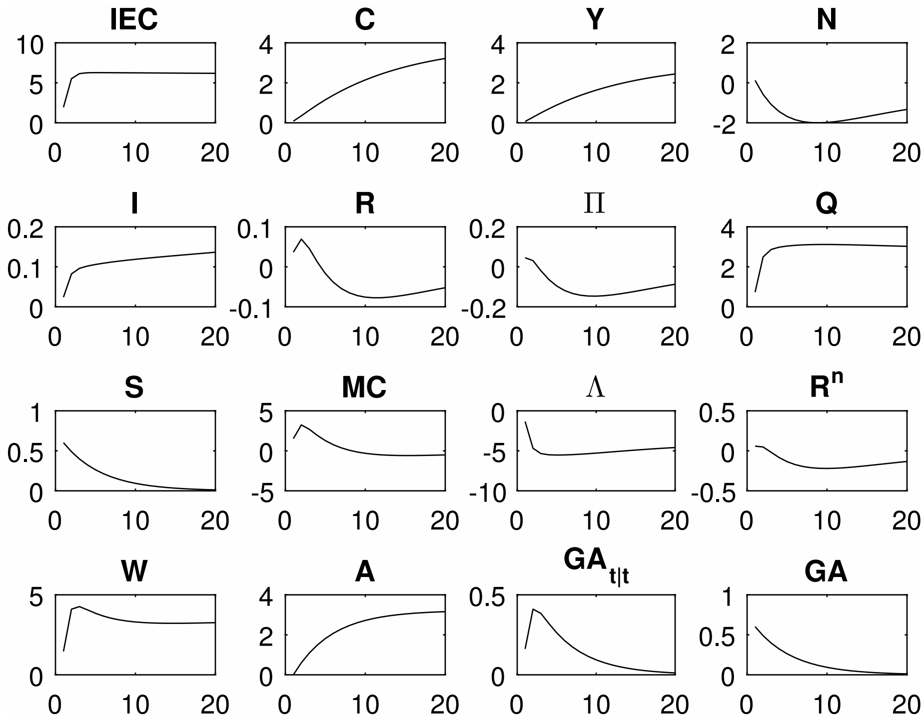


Figure 5: Theoretical IRFs after a one standard deviation news shock.

upward pressure on marginal costs and thus on inflation via the Phillips curve. However, this effect is short-lived and occurs mainly because agents take some time to separate noise from fundamentals. After a while inflation and marginal costs reverse and remain on the negative territory. There is even room for the monetary authority to respond by lowering interest rates and further promoting economic expansion.

A noise/animal spirits shock, on the other hand, has no true relation to fundamentals. Note that there is indeed no movement on technology, but the signal that agents received is affected. Therefore, as seen on the dynamics of $ga_{t|t}$, they take a while to fully understand that they anticipated a non-existent technology improvement. As such, there is initially a period of overheated demand (via the wealth effect) but no improvement on the supply side of the economy. Hence marginal costs and inflation go up and the central bank is forced to raise interest rates. The economy reverts to its original state, even though consumption and output do it slowly due to the high degree of habits in consumption.

So as to gauge how relevant each of the structural shocks are in explaining business cycles fluctuations in Brazil, we present the forecast error variance decomposition in Figure 7. We notice that news shocks have little importance in the very short run (one quarter), but become increasingly pronounced as time goes by and transitory shocks loose their effects. Nevertheless, just after two quarters they are responsible for a major share of economic fluctuations. Noise/animal spirits shocks are by no means negligible at least up to two quarters.

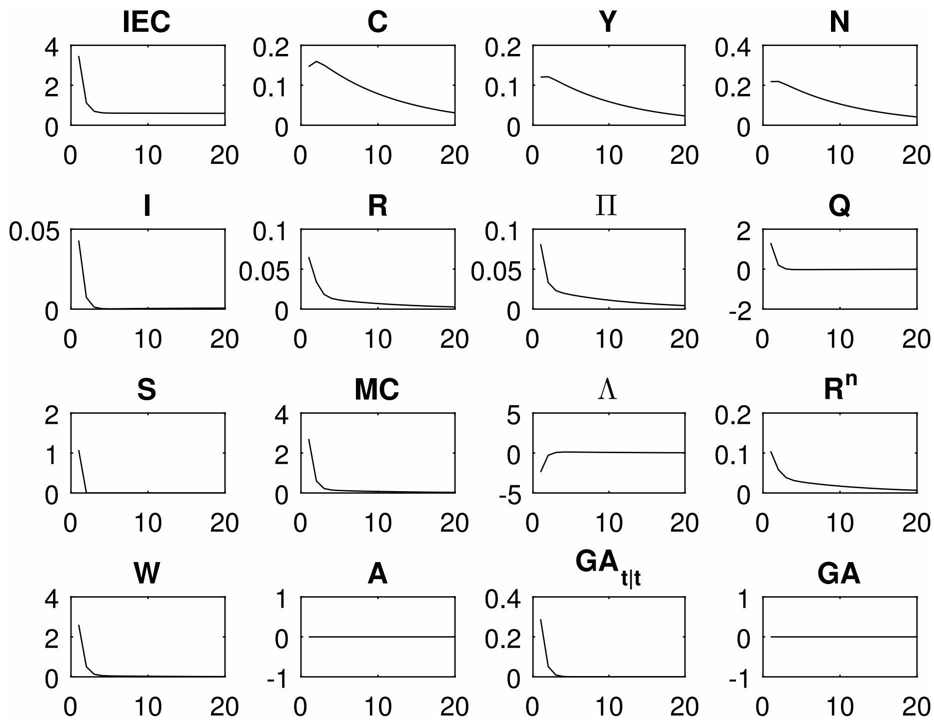


Figure 6: Theoretical IRFs after a one standard deviation noise/animal spirits shock.

More than half of the expectations index and almost half of consumption forecast error variances in the very short run occur due to agents' difficulty in extracting the fundamentals from noise. As they do learn over time the (approximate) true nature of shocks, noise/animal spirits shocks too fade away after a few quarters.

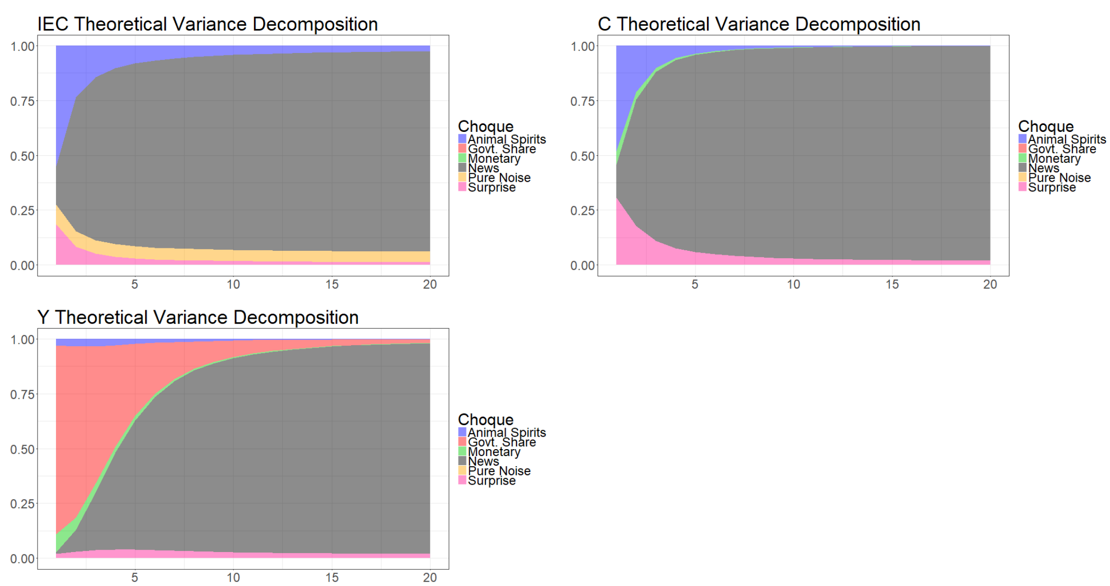


Figure 7: Model forecast error variance decompositions.

Note that, although news shocks account by construction for a sizable share of consumption and output variance error forecasts (after all, they represent technology growth rate shocks), this does *not* hold for the expectations index. Had we estimated $\zeta_1 \approx \zeta_2 \approx 0$, then news shocks would virtually have no relation to the IEC. Our non-zero estimates of these parameters provide evidence that brazilian consumer expectations indexes[18] do significantly reflect agents' anticipation of future technology conditions, even though high-frequency movements (1-2 quarters) are polluted by noise/animal spirits. Therefore, we are able to answer this paper's two main questions: (1) should we care about brazilian consumer expectations indexes? If so, (2) how much information do they carry about future fundamentals and how significantly do animal spirits confound consumers' perceptions? We agree with the conclusions drawn in Barsky and Sims (2012) and Blanchard, L'Huillier, and Lorenzoni (2013) that news shocks account for a relevant share of output fluctuations in Brazil. We also agree with Barsky and Sims (2012) on the fact that consumer expectations indexes have useful anticipation components, but disagree on the relevance of noise/animal spirits shocks. We believe our results are more in line with those of Blanchard, L'Huillier, and Lorenzoni (2013), who find that short-run fluctuations are largely affected by noise.

As a final analysis of the forecast error variance decomposition, we assess how sensitive our results regarding the news-related share would be to extreme key parameter variations. By forcing their values in a *ceteris paribus* analysis, we conclude from Figure 8 that our results are rather unaffected by capital adjustment costs $\gamma$, the size of monetary shock $\sigma_{\epsilon_{rn}}$ and the degree of price rigidity $\theta$. In contrast, when either the degree of habit formation or the relative size of surprise and growth rate technology shocks are far from what we found, conclusions drawn would indeed be different[19].

# 5 Conclusion

The main objective of this paper was to evaluate the role of indexes of consumer expectations in the business cycles of the Brazilian economy. Several articles indicate that they help - even if on a small scale - to predict future levels of consumption and output. However, few studies attempt to unravel the mechanisms behind such results. We combine macroeconometric instruments with the literature on news, and use a new Keynesian model to explore the links between expectations and fluctuations.

We first estimate a VAR to verify that a consumer expectation index for Brazil has

[18]The results showed here also hold when using the - not reported - INEC index.

[19]Notice that since $\gamma$, $\sigma_{\epsilon_{rn}}$, $\kappa$ and $\theta$ do not directly affect the IEC expectations index, they are all represented by the same line labeled as "Original".

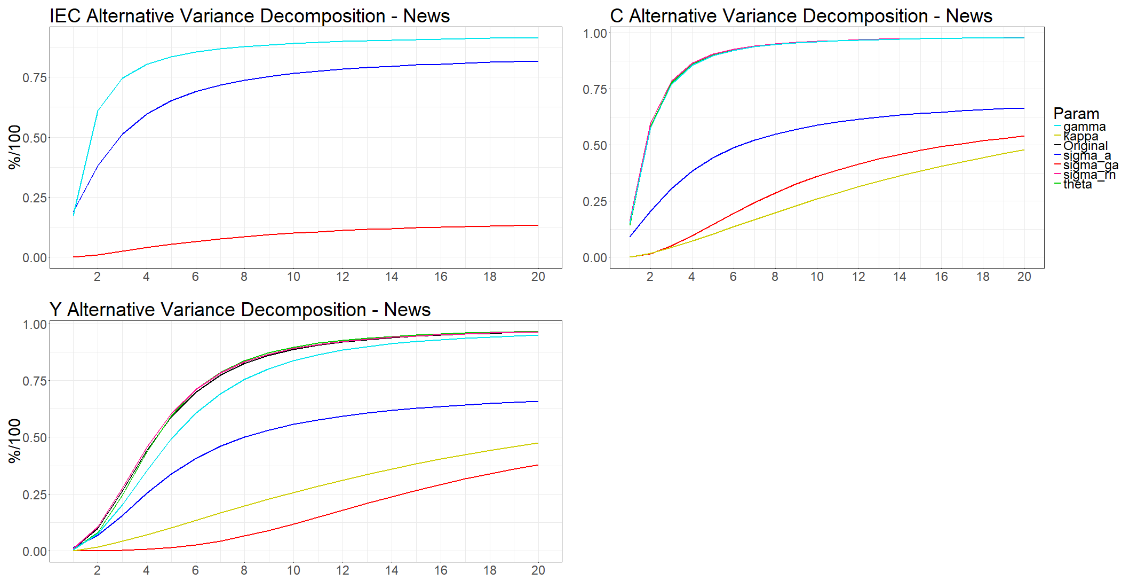


Figure 8: News share of the variance decomposition using alternative parameter values. We use $\sigma_{\epsilon_a} = 2$, $\kappa = 0.1$, $\theta = 0.5$, $\sigma_{\epsilon_{ga}} = 0.1$, $\sigma_{\epsilon_{rn}} = 0.3$ and $\gamma = 5$.

positive long-term effects on the dynamics of of consumption and real GDP, with the magnitude of such effects increasing slowly over time, indicating that they appear to be either permanent or at least very persistent.

We then use Barsky and Sims (2012) modeling to quantify the relevance of the anticipation of the fundamentals and the animal spirits to explain the fluctuations in the expectations index and in other economic variables. This modeling excludes the possibility that the expectation indices are directly related to economic fluctuations, but they may well be a source of relevant information on future economic fundamentals.

Overall, the results indicate that news account for a large part of the variation in the expectations index and in consumption in the medium-long term (after a semester), but animal spirits and noise inherent in the index contaminate the short-term movements in magnitudes well above those found by Barsky and Sims (2012) for the USA, resembling the results of Blanchard, L'Huillier and Lorenzoni (2013).

Another result of our paper that contrasts with Barsky and Sims (2012) is the real interest rate response in the empirical VAR to innovations in the expectation index. While the authors find a positive response for the U.S. real interest rate, we found a negative medium-long term real interest rate response for Brazil after a positive innovation in the expectation index. However, the large confidence bands for this variable when estimating the VAR with the simulated series indicate that the model has little to say about the direct impact of innovations to expectations on the real interest rate.

The estimation of the model also shows that news is not only relevant for the fluctuation of the expectation index, but also for consumption and economic activity (after one year). This result suggests that tracking the more permanent movements of the index of expectations can provide valuable information about future economic fundamentals. The permanent responses found from the VAR suggest that this information is related to technological conditions.

It is tempting to inquire about the possibility of policy recommendations in scenarios of subdued confidence and / or expectations. Larry Summers, for example, argues that policies focused on resumption of confidence would be the cheapest method for stimulating economic activity: "John Maynard Keynes [...] rightly emphasized the need for policy approaches that both promoted business confidence - the cheapest form of stimulus - and increased labor compensation. "(Summers (2016)). However, our results suggest that there are no easy shortcuts involved on the subject, at least when it comes to consumer confidence: any possible action will have to affect the part related to the fundamentals in the aggregate "signal" perceived by the agents. Without this there will be no medium-long-term movements in the index of confidence / expectations. Confidence/expectations indexes move in response to what is expected of the economy and not the other way round.

# References


R. B. Barsky and E. R. Sims. Information, animal spirits, and the meaning of innovations in consumer confidence. *The American Economic Review*, 102(4):1343–1377, 2012.

R. B. Barsky, S. Basu, and K. Lee. Whither news shocks? *NBER Macroeconomics Annual*, 29(1):225–264, 2015.

P. Beaudry and F. Portier. An exploration into pigou's theory of cycles. *Journal of Monetary Economics*, 51(6):1183–1216, 2004.

P. Beaudry and F. Portier. Stock prices, news, and economic fluctuations. *The American Economic Review*, 96(4):1293–1307, 2006.

F. G. M. Bentes. O poder preditivo do índice de confiança do consumidor no brasil: Uma análise através de vetores autorregressivos. *Faculdade de Economia e Finanças IBMEC, Rio de Janeiro*, 2006.

L. Berg and R. Bergstrom. Consumer confidence and consumption in sweden. *Department of Economics, Uppsala University Working Paper 1996:7*, 1996.

O. Blanchard. Consumption and the recession of 1990-1991. *The American Economic Review*, 83(2):270–274, 1993.

O. J. Blanchard, J.-P. L'Huillier, and G. Lorenzoni. News, noise, and fluctuations: An empirical exploration. *The American Economic Review*, 103(7):3045–3070, 2013.

J. Bram and S. C. Ludvigson. Does consumer confidence forecast household expenditure? a sentiment index horse race. *Federal Reserve Bank of New York, Economic Policy Review*, 4(2):59–78, 1998.

G. A. Calvo. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398, 1983.

C. D. Carroll, J. C. Fuhrer, and D. W. Wilcox. Does consumer sentiment forecast household spending? if so, why? *The American Economic Review*, 84:1397–1408, 1994.

C. Carvalho and F. Nechio. Approximating multisector new keynesian models — extended version. *Federal Reserve Bank of San Francisco, Working Paper Series 2017-12*, 2017.

M. R. Castro, S. N. Gouvea, A. Minella, R. Santos, and N. F. Souza-Sobrinho. Samba: Stochastic analytical model with a bayesian approach. *Brazilian Review of Econometrics*, 35(2):103–170, 2015.

L. J. Christiano, M. Eichenbaum, and C. L. Evans. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113(1):1–45, 2005.

L. J. Christiano, C. L. Ilut, R. Motto, and M. Rostagno. Monetary policy and stock market booms. *Proceedings - Economic Policy Symposium - Jackson Hole, Federal Reserve Bank of Kansas City*, pages 85–145, 2010.

J. H. Cochrane. Shocks. *Carnegie-Rochester Conference Series on Public Policy*, 41:295–364, 1994.

J. Durbin and S. J. Koopman. *Time Series Analysis by State Space Methods*, volume 38. OUP Oxford, Oxford, 2012.

J. Z. Easaw and S. M. Heravi. Evaluating consumer sentiments as predictors of uk household consumption behavior: Are they accurate and useful? *International Journal of Forecasting*, 20(4):671–681, 2004.

J. Fernández-Villaverde, J. F. Rubio-Ramírez, T. J. Sargent, and M. H. Watson. Abcs (and ds) of understanding vars. *The American Economic Review*, 97(3):1021–1026, 2007.

J. Fernández-Villaverde, J. F. Rubio-Ramírez, and F. Schorfheide. Solution and estimation methods for dsge models. *Handbook of Macroeconomics*, 2:527–724, 2016.

J. C. Fuhrer. What role does consumer sentiment play in the us macroeconomy? *New England Economic Review*, pages 32–44, 1993.

K. L. Goh. Does consumer confidence forecast consumption expenditure in new zealand? *New Zealand Treasury, Working Paper 03/22*, 2003.

C. Gourieroux, A. Monfort, and E. Renault. Indirect inference. *Journal of Applied Econometrics*, 8(S1), 1993.

F. M. Graminho. Sentimento e macroeconomia: uma análise dos índices de confiança no brasil. *Trabalhos para Discussão, 408*, 2015.

R. E. Hall. Stochastic implications of the life cycle-permanent income hypothesis: theory and evidence. *Journal of Political Economy*, 86(6):971–987, 1978.

P. Hürtgen. Consumer misperceptions, uncertain fundamentals, and the business cycle. *Journal of Economic Dynamics and Control*, 40:279–292, 2014.

N. Jaimovich and S. Rebelo. Can news about the future drive the business cycle? *The American Economic Review*, 99(4):1097–1118, 2009.

J. M. Keynes. *The General Theory of Money, Interest and Employment.* Macmillan, London, 1936.

L. Kilian. Small-sample confidence intervals for impulse response functions. *The Review of Economics and Statistics*, 80(2):218–230, 1998.

K. Kobayashi and K. Nutahara. Nominal rigidities, news-driven business cycles, and monetary policy. *The BE Journal of Macroeconomics*, 10(1):24, 2010.

A. C. C. Kwan and J. A. Cotsomitis. The usefulness of consumer confidence in forecasting household spending in canada: A national and regional analysis. *Economic Inquiry*, 44 (1):185–197, 2006.

E. M. Leeper, T. B. Walker, and S.-C. S. Yang. Fiscal foresight and information flows. *Econometrica*, 81(3):1115–1145, 2013.

G. Lorenzoni. A theory of demand shocks. *The American Economic Review*, 99(5):2050–2084, 2009.

F. Milani and J. Treadwell. The effects of monetary policy "news" and "surprises". *Journal of Money, Credit and Banking*, 44(8):1667–1692, 2012.

A. C. Pigou. *Industrial Fluctuations*. Macmillan, London, 1927.

J. J. Rotemberg and M. Woodford. An optimization-based econometric framework for the evaluation of monetary policy. *NBER Macroeconomics Annual*, 12:297–346, 1997.

S. Schmitt-Grohé and M. Uribe. What's news in business cycles. *Econometrica*, 80(6): 2733–2764, 2012.

J. E. M. Simões, M. J. T. Diniz, M. B. Diniz, and J. N. de Oliveira Junior. Índice de expectativas do consumidor no brasil: uma análise do poder preditivo no período entre 2001 e 2014. *Planejamento e Políticas Públicas*, 2016.

C. A. Sims, J. H. Stock, and M. W. Watson. Inference in linear time series models with some unit roots. *Econometrica: Journal of the Econometric Society*, pages 113–144, 1990.

E. R. Sims. News, non-invertibility, and structural vars. In *DSGE Models in Macroeconomics: Estimation, Evaluation, and New Developments*, pages 81–135. Emerald Group Publishing Limited, 2012.

A. A. Smith. Estimating nonlinear time-series models using simulated vector autoregressions. *Journal of Applied Econometrics*, 8(S1), 1993.

N. F. Souza-Sobrinho. The role of interest rates in the brazilian business cycles. *Revista Brasileira de Economia*, 65(3):315–336, 2011.

L. Summers. The age of secular stagnation, Fevereiro 2016. URL `http://larrysummers.com/2016/02/17/the-age-of-secular-stagnation`. Acessado em: 16/09/2017.

# Appendices

## A Alternative Cholesky Ordering: Robustness of Results

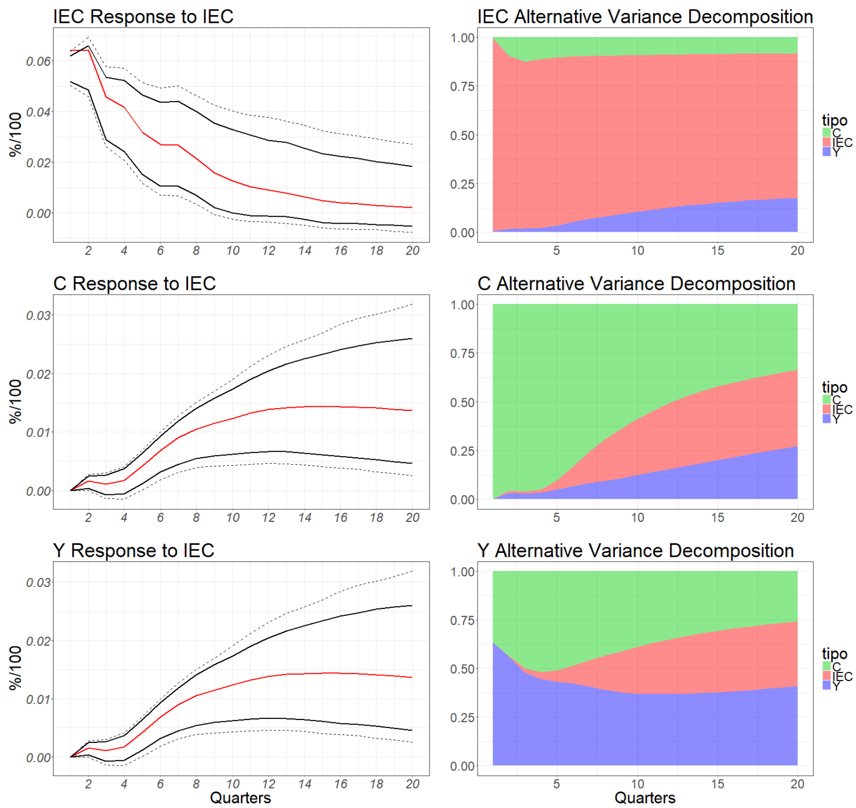


Figure 9: Impulse-response functions from a one standard deviation innovation in the IEC variable and forecast error variance decomposition from the trivariate VAR with the IEC variable ordered *last*. Black lines are the confidence bands calculated via the *bootstrap-after-bootstrap* method of Kilian (1998), which deals with small-sample bias. Intervals represent the 17th-83rd and 10th-90th percentiles. Our sample period is 1996Q1-2016Q4.

# B Model details

## B.1 Aggregation

Aggregate labor is given by $N_t = \int_0^1 N_t(j)dj$. The capital stock, on the other hand, is predetermined for the economy as a whole but not at the firm level. Hence: $K_{t-1} = \int_0^1 K_t(j)dj$. Intermediate goods supply and demand yield:

$$\int_0^1 A_t K_t^\alpha(j) N_t^{1-\alpha}(j)dj = \int_0^1 Y_t \left(\frac{P_t(j)}{P_t}\right)^{1-\xi} dj$$

Since the capital/labor ratio is equal across firms:

$$A_t \left(\frac{K_{t-1}}{N_t}\right)^\alpha \int_0^1 N_t(j)dj = Y_t \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{1-\xi} dj$$

Near the zero inflation deterministic growth rate (yet to be shown), one can write:

$$Y_t = A_t K_{t-1}^\alpha N_t^{1-\alpha} \tag{20}$$

Aggregating nominal firm profits:

$$P_t \Pi_t^p = \int_0^1 \left[P_t(j)Y_t(j) - W_t P_t N_t(j) - R_t^r P_t K_t(j)\right] dj$$

Using the expression for the demand of intermediate goods:

$$P_t \Pi_t^p = Y_t \int_0^1 P_t(j) \left(\frac{P_t(j)}{P_t}\right)^{-\xi} dj - W_t P_t N_t - R_t^r P_t K_{t-1}$$

By using the definition of aggregate price index we may rewrite profits as:

$$\Pi_t^p = Y_t - W_t N_t - R_t^r K_{t-1}$$

Using the above equation and the government budget constraint on the consumer's budget constraint, we get:

$$C_t = -G_t + Y_t - Q_t[K_t - (1-\delta)K_{t-1}]$$

From the first-order conditions of capital producers one may verify that around the non-stochastic growth rate $R_t^k \approx 0$[20]. Therefore one may also verify that $I_t = Q_t \phi\left(\frac{I_t}{K_{t-1}}\right) K_{t-1}$. The law of motion for capital dictates that next-period capital is the sum of non depreciated capital goods and new capital produced, so that:

$$K_t = \phi\left(\frac{I_t}{K_{t-1}}\right) K_{t-1} + (1-\delta)K_{t-1} \tag{21}$$

Hence, at least around the non-stochastic growth rate, the aggregate resource constraint is given by:

$$Y_t = C_t + G_t + I_t \tag{22}$$

## B.2 Stationary Model

In order to proceed with model linearization, we must have a steady-state around which to approximate. However, our model has a trend growth rate and therefore no such steady-state exists. A solution to this is to detrend variables and then carry out approximations around the non-stochastic growth rate of the model. Hence, variables fluctuations around zero should be interpreted in fact as deviations from this deterministic growth trajectory.

Using lowercase letters to denote the logarithm of a variable, one may write the aggregate production function as:

$$y_t = a_t + \alpha k_{t-1} + (1-\alpha)n_t$$

First-differencing returns:

$$\Delta y_t = \Delta a_t + \alpha \Delta k_{t-1} + (1-\alpha)\Delta n_t$$

The model has no population growth, so $\Delta n_t = 0$. Stationary real interest rate, marginal costs and relative price of capital are stationary imply a constant capital/output ratio in equilibrium. Thus capital and output must grow at the same rate, leading to:

$$\Delta y_t = \frac{1}{1-\alpha}\Delta a_t$$

Defining $\Gamma = exp\left\{ga^*\right\}$, variables are rendered stationary via the following transformations:

$$\hat{X}_t \equiv \frac{X_t}{\Gamma^{\frac{t}{1-\alpha}}}, X_t = [Y_t, C_t, W_t, I_t, G_t], \quad \hat{K}_{t-1} \equiv \frac{K_{t-1}}{\Gamma^{\frac{t-1}{1-\alpha}}}, \quad \hat{\Lambda}_t \equiv \Lambda_t \Gamma^{\frac{t}{1-\alpha}}, \quad \hat{A}_t = \frac{A_t}{\Gamma^t}$$

[20]Anticipating a little, note that the ratio $\frac{I}{K}$ is stationary, despite the fact that individually $I_t$ and $K_t$ have a stochastic trend. Assuming a functional form for $\phi(.)$ such that $\phi\left(\frac{I}{K}\right) = \frac{I}{K} = \frac{\hat{I}}{\hat{K}}\Gamma^{\frac{1}{1-\alpha}}$ and $\phi'\left(\frac{I}{K}\right) = 1$ and combining this with the two first-order conditions mentioned before, we get the result.

We now turn to the log-linearization of the model, starting with the consumer Euler equation. Denoting log deviations from the deterministic growth rate as $\tilde{\hat{X}}_t = ln(\hat{X}_t) - ln(\hat{X})$:

$$\hat{\Lambda}_t = E_t \left[ \frac{\Gamma^{\frac{1}{1-\alpha}}}{\Gamma^{\frac{1}{1-\alpha}} \hat{C}_t - \kappa \hat{C}_{t-1}} - \frac{\beta\kappa}{\Gamma^{\frac{1}{1-\alpha}} \hat{C}_{t+1} - \kappa \hat{C}_t} \right]$$

Throwing everything inside the expectation term:

$$1 = E_t \left\{ exp \left[ ln \left( \frac{1}{\hat{\Lambda}_t} \left( \frac{\Gamma^{\frac{1}{1-\alpha}}}{\Gamma^{\frac{1}{1-\alpha}} \hat{C}_t - \kappa \hat{C}_{t-1}} - \frac{\beta\kappa}{\Gamma^{\frac{1}{1-\alpha}} \hat{C}_{t+1} - \kappa \hat{C}_t} \right) \right) \right] \right\}$$

This expression amounts to $1 = \frac{1}{\hat{\Lambda}} \left[ \frac{\Gamma^{\frac{1}{1-\alpha}} - \beta\kappa}{(\Gamma^{\frac{1}{1-\alpha}} - \kappa)\hat{C}} \right]$ around the non-stochastic growth rate. Log-linearization yieds:

$$\tilde{\hat{\Lambda}}_t = \left\{ \frac{\kappa \Gamma^{\frac{1}{1-\alpha}} \tilde{\hat{C}}_{t-1} - (\Gamma^{\frac{2}{1-\alpha}} + \beta\kappa^2) \tilde{\hat{C}}_t + \beta\kappa \Gamma^{\frac{1}{1-\alpha}} E_t[\tilde{\hat{C}}_{t+1}]}{(\Gamma^{\frac{1}{1-\alpha}} - \kappa)(\Gamma^{\frac{1}{1-\alpha}} - \beta\kappa)} \right\} \tag{23}$$

Rewriting Equation 4, we have:

$$N_t^{\frac{1}{\eta}} = \hat{\Lambda}_t \hat{W}_t$$

Log-linearizing:

$$\frac{1}{\eta} \tilde{N}_t = \tilde{\hat{\Lambda}}_t + \tilde{\hat{W}}_t \tag{24}$$

From Equation 5:

$$1 = \beta E_t \left[ \frac{\hat{\Lambda}_{t+1}}{\hat{\Lambda}_t} \frac{R_t}{\Gamma^{\frac{1}{1-\alpha}}} \right]$$

It follows that:

$$\tilde{\hat{\Lambda}}_t = E_t \left[ \tilde{\hat{\Lambda}}_{t+1} \right] + \tilde{R}_t \tag{25}$$

Labor demand as given by Equation 7 may be rewritten as:

$$\hat{W}_t = MC_t (1-\alpha) \hat{A}_t \hat{K}_{t-1}^{\alpha} N_t^{-\alpha} \Gamma^{\frac{-\alpha}{1-\alpha}}$$

Hence:

$$\tilde{\hat{W}}_t = \tilde{MC}_t + \tilde{\hat{A}}_t + \alpha \tilde{\hat{K}}_{t-1} - \alpha \tilde{N}_t \tag{26}$$

Note from Equations 6 and 8 that:

$$E_t \left[ R_t \frac{\Lambda_{t+1}}{\Lambda_t} \right] = E_t \left[ \frac{R_{t+1}^r + (1-\delta) Q_{t+1}}{Q_t} \frac{\Lambda_{t+1}}{\Lambda_t} \right]$$

Near the non-stochastic growth rate:

$$R = R^r + (1 - \delta)$$

Adopting $v = \frac{1-\delta}{R^r+(1-\delta)}$:

$$\tilde{R}_t = (1 - v)E_t[\tilde{MC}_{t+1} + \tilde{\hat{Y}}_{t+1} - \tilde{\hat{K}}_t] + vE_t[\tilde{Q}_{t+1}] - \tilde{Q}_t \tag{27}$$

As discussed with respect to Equation 10, the investment/capital ratio is stationary, despite its components being individually non-stationary. Taylor expansion gives:

$$Q_t\phi^{'}\left(\frac{I_t}{K_{t-1}}\right) \approx Q\phi^{'}\left(\frac{I}{K}\right) + \phi^{'}\left(\frac{I}{K}\right)(Q_t - Q) + Q\phi^{''}\left(\frac{I}{K}\right)\left(\frac{I_t}{K_{t-1}} - \frac{I}{K}\right)$$

Denoting the elasticity of the capital adjustment cost function as $\gamma = -\frac{\phi^{''}(\frac{I}{K})\frac{I}{K}}{\phi^{'}(\frac{I}{K})}$ we arrive at:

$$\tilde{Q}_t = \gamma\widetilde{\left(\frac{I_t}{K_{t-1}}\right)}$$

Note that:

$$\widetilde{\left(\frac{I_t}{K_{t-1}}\right)} = \widetilde{\left(\frac{\hat{I}_t}{\hat{K}_{t-1}}\Gamma^{\frac{1}{1-\alpha}}\right)}$$

Since both $\hat{I}$ and $\hat{K}$ are stationary variables, one may verify that:

$$\widetilde{\left(\frac{\hat{I}_t}{\hat{K}_{t-1}}\Gamma^{\frac{1}{1-\alpha}}\right)} = ln\left(\frac{\hat{I}_t}{\hat{K}_{t-1}}\right) - ln\left(\frac{\hat{I}}{\hat{K}}\right)$$

Finally, we have:

$$\tilde{Q}_t = \gamma(\tilde{\hat{I}}_t - \tilde{\hat{K}}_{t-1}) \tag{28}$$

Rewriting the aggregate prouction function:

$$\hat{Y}_t = \hat{A}_t\hat{K}_{t-1}^{\alpha}N_t^{1-\alpha}\Gamma^{\frac{-\alpha}{1-\alpha}}$$

That is:

$$\tilde{\hat{Y}}_t = \tilde{\hat{A}}_t + \alpha\tilde{\hat{K}}_{t-1} + (1 - \alpha)\tilde{N}_t \tag{29}$$

The resource constraint becomes:

$$\hat{Y}_t = \hat{C}_t + \hat{I}_t + \hat{G}_t$$

Leading us to:

$$\tilde{\hat{Y}}_t = \frac{\hat{C}}{\hat{Y}}\tilde{\hat{C}}_t + \frac{\hat{I}}{\hat{Y}}\tilde{\hat{I}}_t + \frac{\hat{G}}{\hat{Y}}\tilde{\hat{G}}_t \tag{30}$$

Turning to the law of motion for capital:

$$\frac{K_t}{K_{t-1}} = \phi\left(\frac{I_t}{K_{t-1}}\right) + (1-\delta)$$

Or:

$$\frac{\hat{K}_t}{\hat{K}_{t-1}}\Gamma^{\frac{1}{1-\alpha}} = \phi\left(\frac{\hat{I}_t}{\hat{K}_{t-1}}\Gamma^{\frac{1}{1-\alpha}}\right) + (1-\delta)$$

Using a Taylor expansion and the fact that $\hat{I} = \delta_2\hat{K}$, where $\delta_2 \equiv 1 - \frac{1-\delta}{\Gamma^{\frac{1}{1-\alpha}}}$:

$$\tilde{\hat{K}}_t = \delta_2\tilde{\hat{I}}_t + (1-\delta_2)\tilde{\hat{K}}_{t-1} \tag{31}$$

Log-linearizing the Taylor rule:

$$\tilde{R}^n_t = \rho_{rn}\tilde{R}^n_{t-1} + (1-\rho_{rn})\left(\phi_\pi\tilde{\Pi}_t + \phi_y\left(\tilde{\hat{Y}}_t - \tilde{\hat{Y}}_{t-1}\right)\right) + \epsilon^{rn}_t \tag{32}$$

From Equation 9 and the definitions of aggregate price index as well as inflation, it is possible to derive the New Keynesian Phillips Curve:

$$\tilde{\Pi}_t = \varphi\tilde{MC}_t + \beta E_t[\tilde{\Pi}_{t+1}], \quad \varphi = \frac{(1-\theta)(1-\theta\beta)}{\theta} \tag{33}$$

Moving on to the exogenous processes, rewrite Equation 14 as:

$$a_t - tga^* = a_{t-1} - (t-1)ga^* + ga_{t-1} - ga^* + \epsilon^a_t$$

Then[21]:

$$\tilde{\hat{A}}_t = \tilde{\hat{A}}_{t-1} + \tilde{ga}_{t-1} + \epsilon^a_t \tag{34}$$

Accordingly:

$$\tilde{ga}_t = \rho_{ga}\tilde{ga}_{t-1} + \epsilon^{ga}_t \tag{35}$$

$$\tilde{g}^{sh}_t = \rho_g\tilde{g}^{sh}_{t-1} + \epsilon^{gsh}_t\tilde{s}_t = \tilde{ga}_t + \epsilon^s_t \tag{36}$$

The Kalman Filter is constructed keeping in mind that agents perfectly observe each

[21]We decided to write $\tilde{ga}$ instead of $\tilde{\Gamma}$ for convenience, since this variable was only presented in logs throughout the text. The same reasoning applies to $\tilde{g}^{sh}$ and $\tilde{s}$.

period the *level* of technology but not its growth rate, which they only observe a partially informative signal. Nonetheless, as rational agents, individuals know the true value of $\Gamma$ and are capable of observing (or at least computing) $\tilde{\hat{A}}_t$ and $\tilde{s}_t$. Formally, we have the following state-space specification:

$$\underbrace{\begin{pmatrix} \tilde{\hat{A}}_t \\ \tilde{s}_t \end{pmatrix}}_{O_t} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{Z} \underbrace{\begin{pmatrix} \tilde{\hat{A}}_t \\ \tilde{ga}_t \end{pmatrix}}_{\alpha_t} + \begin{pmatrix} 0 \\ \epsilon^s_t \end{pmatrix}, \quad \Sigma_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sigma^2_{\sigma_{\epsilon_s}} \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} \tilde{\hat{A}}_{t+1} \\ \tilde{ga}_{t+1} \end{pmatrix}}_{\alpha_{t+1}} = \underbrace{\begin{pmatrix} 1 & 1 \\ 0 & \rho_{ga} \end{pmatrix}}_{T} \begin{pmatrix} \tilde{\hat{A}}_t \\ \tilde{ga}_t \end{pmatrix} + \begin{pmatrix} \epsilon^a_{t+1} \\ \epsilon^{ga}_{t+1} \end{pmatrix}, \ \Sigma_2 = \begin{pmatrix} \sigma^2_{\epsilon_a} & 0 \\ 0 & \sigma^2_{\epsilon_{ga}} \end{pmatrix}$$

We follow Durbin and Koopman (2012) to filter unobserved states. Denote $P_t = Var(\alpha_t)$ and $\alpha_{t|t} = E[\alpha_t|Y_t]$. If matrices $Z, T, \Sigma_1$ and $\Sigma_2$ are constant over time - which they are - then $P = TPT' - TPZ'F^{-1}ZPT' + \Sigma_2$, where $F = ZPZ' + \Sigma_1$. The expression is numerically solveable provided that $F$ is invertible. Then, agents form their projections for the current state of the economy according to:

$$\alpha_{t|t} = \underbrace{(I - PZ'F^{-1}Z)T}_{\Upsilon} \alpha_{t-1|t-1} + \underbrace{PZ'F^{-1}}_{\Psi} O_t$$

Notice that $\alpha_{t|t}$ is a $2 \times 1$ vector. The first equation refers to $\tilde{\hat{A}}_{t|t}$, so the coefficient of $\tilde{s}_t$ will actually be zero because technology level is perfectly observed. From the second equation we have the desired relationship between $\tilde{ga}_{t|t}$ and $\tilde{s}_t$:

$$\tilde{ga}_{t|t} = \Upsilon_{2,1}\tilde{\hat{A}}_{t-1} + \Upsilon_{2,2}\tilde{ga}_{t-1|t-1} + \Psi_{2,1}\tilde{\hat{A}}_t + \Psi_{2,2}\tilde{s}_t \tag{37}$$

Where the term $\Upsilon_{i,j}$ represents the row $i$ and column $j$ element of matrix $\Upsilon$.

Last but not least important, the log-linearized dynamics of the expectations index will be given by:

$$I\tilde{E}C_t = \rho_e I\tilde{E}C_{t-1} + \zeta_1(\tilde{\hat{A}}_t - \tilde{\hat{A}}_{t-1}) + \zeta_2\tilde{ga}_{t|t} - (\zeta_1 + \rho_{ga}\zeta_2)\tilde{ga}_{t-1|t-1} + \zeta_3\epsilon^{IEC}_t \tag{38}$$