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Determinants of Agricultural Fires: An Aggregative Games Approach

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WORKING PAPER SERIES Nº 2023-12

DEPARTMENT OF ECONOMICS, FEA-USP
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Keywords: Aggregative games, land use, deforestation.

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September 15, 2023

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1 Introduction

Countries like Brazil, which have large areas of Amazon rainforest and a diversity of biomes, and at the same time have the agriculture as one of their main economic activities, face the dilemma of intensifying the productivity of this activity versus preserving the environment. According to data from the Center for Advanced Studies in Applied Economics (CEPEA-ESALQ/USP) in partnership with the Confederation of Agriculture and Livestock in Brazil (CNA), in 2019 agribusiness generated goods and services equivalent to 21.4% of the GDP, with a growth rate of 3.3% in this aggregate in the first quarter of 2020.

At the same time, deforestation accompanied by use of fire to clean and improve soil fertility is a practice that unfortunately accompanies the growth of the sector. According to the National Institute for Space Research (INPE), from January to June 2020, 23.143 fires were detected in the country and in 2019 more than 10.000 square kilometers of deforestation.

The damage caused by these practices is twofold. On the one hand, fires and deforestation harm the ecosystem, air and water quality, as well as the quality of life of people living in the region. On the other hand, it reduces investments from institutions that are increasingly concerned about protecting the environment. In 2020, seven of the largest European investors published a statement communicating their divestment in the Brazilian economy if measures to protect the environment and reduce the progress of destruction of the Amazon rainforest were not to be adopted (REUTERS (2020))). Therefore, it is important to analyze the individual positive impacts of burning and deforestation activities with the collective damage that the aggregate decision of these activities produces.

To this end, in this work we propose an aggregative game model that embodies these two effects in the individual payoff. On the one hand, fires and deforestation increase the cultivable area of a landowner; however, in the aggregate the productivity of the land will decrease due to environmental effects. Furthermore, there is a cost for the deforestation that an economic unit has, which can be interpreted as the fine that the establishment will receive in case of be detected. Both effects will generate an optimal individual decision and the equilibrium will be a profile of individual decisions from which none of the farmers has the incentive to deviate. In this way we will have the aggregate decision which will depend on the number of establishments involved, the fines applied and structural variables such as the reduction in the land productivity due to deforestation. After obtaining the theoretical results of the model, we perform an empirical analysis of these determinants of the aggregate decision to burn and deforest. This is not the first time that game theoretical frameworks are used to model the externality that deforestation produces in individual agricultural activities (Rodrigues et al. (2009), Martín-Herrán et al. (2006)); however, to the best of our knowledge, this is the first attempt of using the aggregative game modeling to that problem in the proposed form.

In the literature we have extensive documentation and studies that show the relationship between deforestation and economic growth, as well as the policies used to reduce the practice of burning and deforestation for agricultural purposes. In Mendes and Pôrto Júnior (2012) there is a positive and significant relationship between economic growth and deforestation. Cuaresma and Heger (2019) show that low-income countries in general have higher development-deforestation elasticities. With a methodology based on machine learning and a broad database, Andrée et al. (2019) confirm the U-inverted shape in the relationship between per capita income and indicators of environmental degradation. Although growth is positively related to deforestation, Santiago and do Couto (2020) show that the ephemeral improvement in development is in contrast with an increase in income concentration.

Regarding the policies that are used to avoid or reduce deforestation caused by agricultural activities, we have the following. Cammelli and Angelsen (2019) found that the policy of command and control to reduce the farmer's fire use is more effective than the payment for environmental services. West and Fearnside (2021) assessed the effectiveness of the Action Plan for the Prevention and Control of Deforestation in the Legal Amazon, which is a conservation reform launched in Brazil in 2004. The article highlights some activities that were responsible for significant reduction of deforestation in the region. Martín-Herrán et al. (2006) analyzed the effect of foreign transfer form developed to developing countries to reduce deforestation. Fonseca-Morello et al. (2017) argued that the difficulty in controlling farmer's fires in Brazil is that they are necessary for the agricultural activities of smallholders. When studying the impact of two strategies to reduce deforestation in the Peruvian Amazon rainforest through private concessions - sustainable commitments and fines - Anderson et al. (2019) concluded the inefficiency of both and argue that it can be a result of insufficient monitoring for sustainability commitments, and insufficiently punitive fines or low enforcement levels. In Jung and S. (2018) it is found that the partnership between private firms and non-governmental organizations committed to forest protection significantly reduces deforestation in the activity area in Brazil.

This paper is divided into five sections. After this introductory section, we have the Section 2, where the aggregative game representing the individual decisions of land fires to prepare the soil and the effect of the aggregate decision on the farmers' payoff is stated. In Section 3 we present the main theoretical results of the model, specially the effects of the variations of cost fires and number of farmers on the equilibrium aggregate deforestation. Using data from the National Institute for Space Research (INPE), the Brazilian Institute of Environment and Renewable Natural Resources (IBAMA) and the Brazilian Institute of Geography and Statistics (IBGE), we perform some empirical exercises in Section 4 to test the impact of changes in some exogenous parameters on the endogenous aggregate deforestation decision. In Section 5 we resume some conclusions of our work and the proofs of the results presented in Section 3 are given in the Appendix.

2 Framework

There are $N \geq 2$ farmers located in a land with total dimension T. The farmer $n \in \{1, 2, \cdots, N\}$ owns part of that land with size t_n, thus $\sum_{n=1}^{N} t_n = T$. To prepare his land for the agricultural activity, the farmer n may burn a part $\alpha_n \in [0, 1]$ of it, and in this case, $\alpha_n t_n$ represents the size of his land where this strategy is applied.

The aggregate strategy $D = \sum_{n=1}^{N} \alpha_n t_n$ generates a negative externality on the agricultural production of each farmer through a multiplicative shock defined by the function $A : [0, T] \to \mathbb{R}$, so the production of the farmer n who owns a land of size t_n and decides to burn a fraction α_n to prepare it is:

$$q_n = A(D)\,(\alpha_n t_n)^\rho = [A(D) t_n^\rho]\,\alpha_n^\rho, \tag{1}$$

where $\rho \in (0, 1]$ is the production elasticity of the land.

To obtain our main results we are going to use the following assumption for the production externality shock A :

Hypothesis 1. *The function $A : [0, T] \to \mathbb{R}$ is a C^2, $A'(D) < 0$ and $A''(D) \leq 0$ for all $D \in (0, 1)$.*

The Hypothesis 1 above expresses the negative externality that the increase in aggregate burning produces on the agricultural activity (A is a decreasing function). Moreover, the falling in land productivity due to the burning is at an increasing rate, namely, the greater the size of the land that is burnt, the more significant the negative impact of increasing that size becomes (A is a strictly concave function).

As an example (for numerical simulations) we may consider the following function:

$$A(D) = A_0 \left[1 - \left(\frac{D}{T}\right)^r\right]^{1/r} \tag{2}$$

In the functional form (2) A_0 represents the maximum productivity of land and $r > 1$ is a structural parameter. In Figure 1 we show the shape of that function. It is worth noting that the elasticity of the land productivity $\varepsilon = (D/A)(dA/dD) < 0$ is a strictly decreasing function of D, $\varepsilon(0) = 0$ and $\varepsilon(D) \to -\infty$, as $D \to T$.

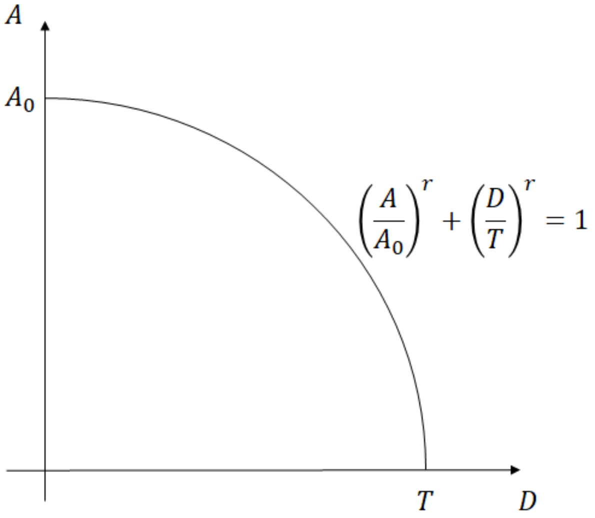

Figure 1: Productivity of land $A(D)$ depending on the total agricultural land burning D.

To end the specification of the farmers payoff, we will assume that the production real cost is linear in the total land prepared for agriculture, namely,

$$c_n(\alpha_n) = k_n t_n \alpha_n. \tag{3}$$

where k_n is the unitary cost of land fire. That parameter can be interpreted as fines applied to illegal land burning; thus we can perform a comparative static analysis of the equilibrium with respect to this parameter.

Therefore, with the specifications (1), (2) and (3), we define the payoff function for farmer n, depending on his individual strategy of land burning α_n and the other farmer's strategies $\alpha_{-n} = (\alpha_1, \cdots, \alpha_{n-1}, \alpha_{n+1}, \cdots, \alpha_N)$ by,

$$u_n(\alpha_n, \alpha_{-n}) = \left[A(D)t_n^{\rho}\right] \alpha_n^{\rho} - (k_n t_n)\alpha_n \tag{4}$$

We can consider $S_n = [0,1]$ as the individual's strategy set for each n, since $\alpha_n \in [0,1]$ for all n. Then, we have the aggregative game of agricultural fires $\Gamma = \{u_n, S_n\}_{n=1}^{N}$. For that game, a profile $(\alpha_1^*, \cdots, \alpha_N^*) \in [0,1]^N$ is an equilibrium if, for any profile $(\alpha_1, \cdots, \alpha_N) \in [0,1]^N$, we have $u_n(\alpha_n^*, \alpha_{-n}^*) \geq u_n(\alpha_n, \alpha_{-n}^*)$.

3 Main theoretical results

In this section we will provide some theoretical results that come from our framework presented in Section 2. In addition to proving the existence and uniqueness of interior equilibrium, we will perform some static comparative analysis of the equilibrium. We will examine how it changes when the fine charged varies, when there is heterogeneity in land size and in fines applied to farmers, and when the number of farmers in the game increases.

Proposition 1. *With the Hypothesis 1 the aggregative game* $\Gamma = \{u_n, S_n\}_{n=1}^{N}$ *has a Nash equilibrium.*

The existence of equilibrium given in Proposition 1, as usual, does not provide information whether it is unique or not. Also, it does not specify whether someone is actually causing agricultural fires or, even more concerning, whether all the land is being burned. In the empirical analysis and especially on the sensitivity analysis of the equilibrium with respect to the parameters of the model, it will be useful to know if the equilibrium is interior and at least locally unique. The next proposition states that.

We will say that the Nash equilibrium $(\alpha_1^*, \cdots, \alpha_N^*) \in [0,1]^N$ is interior if for all n we have $\alpha_n^* \in (0,1)$.

Proposition 2. *With the Hypothesis 1, if there is an interior Nash equilibrium for the aggregative game* $\Gamma = \{u_n, S_n\}_{n=1}^N$ *then it is unique.*

In what follows, we will analyze the response of the aggregate burning D to changes in the fundamentals of the model. The analysis will be done in three cases: when the technology is linear ($\rho = 1$), when the returns are strictly decreasing ($\rho < 1$) and the cost of burns is homogeneous among farmers, and finally, when the returns are strictly decreasing ($\rho < 1$) and the cost of burns is heterogeneous.

Proposition 3. *Suppose that the Hypothesis 1 holds, and the equilibrium is interior. If the technology is linear (*$\rho = 1$*), then the size of the burned land depends on the total unitary cost of burning. Furthermore, if that total unitary cost increases, the size of burned land decreases.*

The first order condition for characterizing the interior equilibrium (which is also sufficient, since the second derivative of $u_n(\alpha_n, \alpha_{-n})$ with respect to α_n is strictly negative) is:

$$A(D) + t_n A'(D)\alpha_n = k_n,$$

then, Proposition 3 allows us to conclude two interesting results: First, farmers with a greater cost for land burning will burn a lower amount of their properties, i.e., if $k_n > k_m$ then $\alpha_n t_n < \alpha_m t_m$. Secondly, a redefinition of fine charges for burning that keeps the total unitary cost $K = \sum_{n=1}^N k_n$ constant, will result in a reduction in the share of the land burned by the farmer whose cost increased and an rise in the corresponding share for the farmer whose costs decreased.

Now, let us discuss the case where the technology is not linear, but the cost of burning is homogeneous.

Proposition 4. *Suppose that the Hypothesis 1 holds and the equilibrium is interior. If the technology has strictly decreasing returns to scale ($0 < \rho < 1$) and the unitary cost of burning is homogeneous among the farmers, then the size of burned land of each farmer is the same; namely, if $k_n = k$ for all n, then $\alpha_n t_n = \alpha_m t_m$ for all $n, m = 1, \cdots, N$. In particular, larger farms have lower shares of their land burned.*

This is an important result on the design of policies against excessive burning used for agricultural activities. Using a flat rate of fine charges for agricultural burnt will penalize small farmers favoring the largest. On the other hand, if we observe that farmers are burning areas with different sizes, then either they are facing different costs for that practice, or the technology is linear.

In the case we are analyzing, it is also possible to obtain an important result if the goal is to keep a fixed size of land burned. This is stated in the following corollary.

Corollary 1. *If in addition to the hypotheses of the Proposition 4 we have that $A'(0) = 0$ and $A'(T) = -\infty$ then the total size of land burned D is a strictly decreasing function of k and a strictly increasing function of N. In particular, there exists and increasing relationship between the unitary cost of burning k and the number of farmers N that keeps the total size of the land burned constant.*

We may illustrate the result of Corollary 1. Consider the functional specification (2) with the following parameters: $A_0 = 1$, $T = 2$ and $r = 2$, and the production elasticity parameter $\rho = 0.6$. If we fix the size of the land burned $D = 1$, the Figure 2 shows the relationship between the cost of burning k and the number of farmers to maintain that land burned size. It is worth noting that the increasing shape of k with respect to N is at decreasing rates.

To finalize the static comparative analysis, we will discuss the last case: strictly decreasing returns and heterogeneous costs of burning. To keep the exposition easy, we are going to suppose two farmers facing (in general different) unitary cost for burning and we will see the responses of their equilibrium strategies of burning to changes in those costs.

Proposition 5. *Consider a game Γ with only two farmers. Suppose that the Hypothesis 1 holds and the equilibrium is interior. If the technology has strictly decreasing returns to*

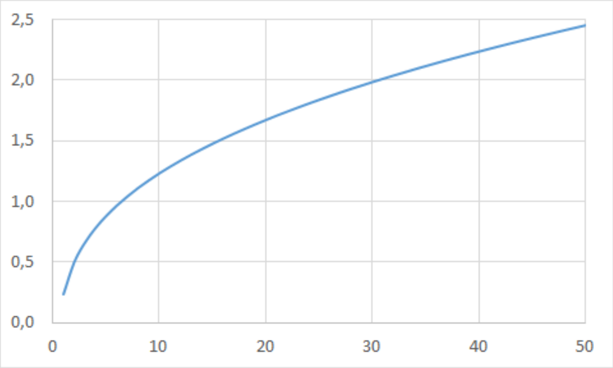

Figure 2: The relationship between the cost of burning k and the number of farmers N to preserve the size of the land burned.

scale $(0 < \rho < 1)$ *then an increase in the unitary cost* k_n *of farmer* n *produces a decrease in his share of land burned* α_n *and an increase in the share of land burned by the other* α_m.

The perverse effect that this general case concludes should be taken into account in the policy design against burning. The externality proposed in the fundamentals of the model (more aggregate land burning leads to a loss in the productivity of the individual lands) is transferred to an externality in the policy design to combat against agricultural burn, since the increase in the fine charged to a farmer (or group of farmers) will be an incentive to increase the burning by others.

4 Testing the determinants of agricultural burning

In the previous section we used an aggregative game theoretical model to show some parameters that may influence the decision of land burning with agricultural purposes. The technology, fine charges and the number of farmers are the most prominent in the proposed model.

In this part of the work we are going to analyze empirically the determinants of the deforestation in the Brazil's Legal Amazon or ”Amazônia Legal”. Since significant part of the deforestation is promoted via land fires which cause individual benefits, although in the aggregate brings prejudicial effects to the productivity of the land, our theoretical model sheds light on those determinants of agricultural activity.

Brazil's Legal Amazon is a socio-geographic division in Brazil composed by nine states in the Amazon basin. In the north region we have the states: Acre, Amapá, Amazonas, Pará, Rondônia, Roraima and Tocantins. In the central-west and western regions, Mato Grosso and Maranhão respectively. The importance of that region conglomerate is based on the fact that it overlaps three biomes (Amazon biome, Cerrado biome and Pantanal biome), containing large tropical vegetation and extensive shares of rainforest. In that region, most of the agricultural activity uses land fires to prepare the soil for crop and large numbers of fire spots have been detected in the last years. When the aggregate land fire is excessive or remains uncontrolled, serious damages to the environment are produced, diminishing rains, prolonging periods of drought and finally, diminishing the productivity of the land itself. For all those reasons, we chose that region to perform our empirical analysis.

The data series that we will use is provided by three sources: The National Institute for Space Research (INPE), the Brazilian Institute of Environment and Renewable Natural Resources (IBAMA) and the 2017 agricultural census applied by the Brazilian Institute of Geography and Statistics (IBGE), and due to the data availability we run two empirical studies, one using a cross-section data analysis with municipal information and the other a panel data analysis with state information.

In both studies we will proceed in two stages. First, we are going to check the relationship between land fires spots and the deforestation in the considered areas. This is done to verify the link between both activities and its intensity. Once we checked that link, we proceed to analyze the determinants of the deforestation, which are the amount of the fines applied for burnt/deforestation, the number of establishments dedicated to agriculture and other possible explanatory variables.

4.1 Cross section data analysis

At the municipal level, the agricultural census carried out by the IBGE in 2017 informs the number of agricultural establishments. For that motive, we use data from 90 municipalities in the Brazil's Legal Amazon to firstly perform the following regression,

$$\ln(\text{Deforest}) = \beta_0 + \beta_1 \ln(\text{Forest fire outbreaks}) + \varepsilon. \tag{5}$$

After checking that relationship, we execute the following estimation,

$$\ln(Y) = \beta_0 + \beta_1 \ln(X_1) + \beta_2 \ln(X_2) + \beta_3 \ln(X_3) + \varepsilon, \tag{6}$$

where the description of the variables are given in Table 1.

Table 1: Variables description in the cross-section model

Variable	Description	Year	Source
Deforest	Deforested area (Km^2)	2017	INPE
Forest fire outbreaks	Forest fire outbreaks detected by satellite	2017	INPE
Y	Deforest area per Municipality area	2017	IBGE
X_1	Value of the infraction notices per municipal GDP	2017	IBGE
X_2	Total value of collected fines per municipal GDP	2017	IBGE
X_3	Number of agricultural establishments	2017	IBGE

The inclusion of the variable X_1 is to analyze to what extent the value of the fine notices inhibits the environmental violation. The difference between that variable and the X_2 variable is that the second measures the impact of the effective punishment on deforestation, rather than only the communication of such punishment.

The result of the first estimation (equation (5) is in Table 2 and as we can see, there is a strong link between deforestation and land fires. An increase in 1% in the forest fire outbreaks implies an increase of 0.922% in the size of deforest land.

Table 2: Effect of forrest fire outbreaks on deforestation at municipal level - 2017

Explanatory variable	Dependent variable ln (Deforest)
ln (Forest fire outbreaks)	0.922^{***}
	(0.0938)
Constant	-4.643^{***}
	(0.663)
R^2	0.523
R^2 (adjusted)	0.518
F statistic	96.6
Prob	$8.09E-16$
Number of observations	90

Note: Robust standard errors in parentheses. $^{*}p < 0.1,^{**} p < 0.05,^{***} p < 0.01$.

Next, we perform the estimation of (6) and the Table 3 shows the results.

Table 3: Cross sectional regression of Deforestation on some explanatory variables - 2017

	Dependent variable: ln (Deforest)		
Explanatory variable	Model 1	Model 2	Model 3
ln (X_1)	0.0266	-0.0573	0.0199
	(0.0734)	(0.0753)	(0.0745)
ln (X_2)		0.285^{***}	0.257^{***}
		(0.0926)	(0.0878)
ln (X_3)			0.687^{***}
			(0.201)
Constant	-6.657^{***}	-4.786^{***}	-9.151^{***}
	(0.837)	(1.004)	(1.589)
R^2	0.0015	0.0994	0.207
R^2 (adjusted)	-0.0099	0.0787	0.18
F statistic	0.131	4.802	7.5
Prob	0.718	0.0105	0.0002
Number of observations	90	90	90

Note: Robust standard errors in parentheses. ${}^{*}p < 0.1, {}^{**}p < 0.05, {}^{***}p < 0.01$.

We can observe that the best model is the one including all variables (well fitted and significant). It is worth noting that the variable corresponding to the notification of fine charges in case of environmental violations is not significant. This, in addition to the positivity of the coefficient associated to the fines effectively charged, means that the low significance of fine notices joint to a precarious environmental supervision are incentives to increase deforestation. This is a perverse conclusion that this empirical analysis reveals. This type of conclusion was also obtained by Anderson et al. (2019) when analyzed whether fines correspond with higher deforestation rates. Regarding to the number of establishments, we find that 1% of increase in the number of farmers has an impact of augmenting 0.687% the relative deforested area in the municipality.

4.2 Panel data analysis

The lack of information about the number of agriculture establishments for long periods of time made us to adopt a *proxy* for that variable. On doing that, we are able to analyze, at the states level, a panel data of the same variables included in the Subsection 4.1, substituting the number of establishments by that *proxy*.

Specifically, we use annual data from the same sources above (INPE, IBAMA and

IBGE), for eight of the nine states of the Brazil's Legal Amazon from 2009 to 2018. We had to exclude Acre due to the lack of information about environmental violations infraction notices in 2013. The data is about deforestation, forest fire outbreaks, infraction notices, and cassava production as a *proxy* for the number of farmers, since its cultivation demands land preparation through burning it in advance and its economic importance (Fonseca-Morello et al. (2017)), and the per capita GDP as an explanatory variable.

For the estimation, we follow the same stages given in Subsection 4.1. First, we estimate the following equation in order to identify the relationship between forest fires and deforestation,

$$\ln(\text{Deforest}) = \beta_0 + \beta_1 \ln(\text{Forest fire outbreaks}) + \varepsilon. \tag{7}$$

After checking that relationship, we execute the following estimation,

$$\ln(Y) = \beta_0 + \beta_1 \ln(X_1) + \beta_2 \ln(X_2) + \beta_3 \ln(X_3) + \beta_4 \ln(X_4) + \varepsilon, \tag{8}$$

The description of the variables can be found in Table 4.

Table 4: Variables description in the cross-section model

Variable	Description	Year	Source
Deforest	Deforested area (Km^2)	2009-2018	INPE
Forest fire outbreaks	Forest fire outbreaks detected by satellite	2009-2018	INPE
Y	Deforest area per State area	2009-2018	IBGE
X_1	Value of the infraction notices per State GDP	2009-2018	IBGE
X_2	Total value of collected fines per State GDP	2009-2018	IBGE
X_3	Cassava production in the State (Tons)	2009-2018	IBGE

The results of the econometric analysis are in the following tables. As the estimations in level were not significant we proceeded to make them in their first difference ($\Delta(\ln(Y))$ and $\Delta(\ln(X_i))$), where it was possible to obtain more robust conclusions.

First, in Table 5 we have the results for the equation (7).

Once again, we found a positive response in the deforested area growth rate (increase of 0.671%) with respect to an increase of 1% in the forest fire outbreaks rate. Thus, we have evidence of the use of land fires to prepare the area for agriculture.

Table 5: Effect of forest fire outbreaks on deforestation at state level - 2009-2018

Explanatory variable	Dependent variable Δln (Deforest)
Δln (Forest fire outbreaks)	0.617^{***}
	(0.115)
Constant	0.0061
	(0.0832)
R^2	0.271
R^2 (adjusted)	0.261
F statistic	28.58
Prob	$8.9E-7$
Number of observations	79

Note: Robust standard errors in parentheses. $^{*}p < 0.1,^{**} p < 0.05,^{***} p < 0.01$.

Then, we proceed to the estimation of the equation (8) and the results are reported in Table 6.

The model including all the proposed variables seems to be the statistically more robust. Differently from the cross-section model of Subsection 4.1, here we have a significant positive effect of the infraction notices for environmental violations on deforestation (0.209). That positive effect contrasted with the negative impact of the effective punishment on the same variable (-0.082) shows how the individuals (offenders) take their own chances in committing the environmental damages, betting that the infraction notice will not be executed. This corroborates the conjecture that the environmental supervision may be insufficient and with low credibility on the offender's part. Another explanation is the possibility of corruption by supervisory authorities that do not effectively apply fines once they are notified. This is the result found by Mendes and Pôrto Júnior (2012) in their analysis of 25 municipalities from the states of Pará and Mato Grosso. The inclusion of the cassava production as a *proxy* for the number of farmers had a well succeeded result: an increase of 1% in the production growth rate of that woody shrub yields to an increase of 0.432% in the growth rate of the deforested area. Finally, we have an aggregate negative effect of deforestation on the state GDP: an increase of 0.375% in the rate of deforestation corresponds to a fall of 1% in the GDP growth rate of the state. In this way, deforestation negatively impacts both the economy and the environment, especially in regions with degraded ecosystems. A study by Andrée et al. (2019) analyzed the relationship between economic growth and the

Table 6: Panel data regression of Deforestation on some explanatory variables - 2009-2018

	Dependent variable: $\Delta\ln$ (Deforest)			
Explanatory variable	Model 1	Model 2	Model 3	Model 4
$\Delta\ln$ (X_1)	0.0915	0.101	0.161^{*}	0.209^{**}
	(0.0947)	(0.0952)	(0.0846)	(0.0854)
$\Delta\ln$ (X_2)		-0.0328	-0.0458	-0.0820^{**}
		(0.0335)	(0.0295)	(0.0332)
$\Delta\ln$ (X_3)			0.460^{***}	0.432^{***}
			(0.095)	(0.0936)
$\Delta\ln$ (X_4)				-0.375^{**}
				(0.172)
Constant	-0.000388	-0.00261	0.000346	0.00406
	(0.0764)	(0.0765)	(0.0672)	(0.0656)
R^2	0.012	0.0243	0.257	0.302
R^2 (adjusted)	-0.00085	-0.00133	0.227	0.264
F statistic	0.934	0.948	8.646	7.998
Prob	0.337	0.392	$5.33E-5$	$2.05E-5$
Number of observations	79	79	79	79

Note: Robust standard errors in parentheses. $^{*}p < 0.1,^{**} p < 0.05,^{***} p < 0.01$.

environment using panel data. They found that higher population densities were associated with decreased deforestation, while lower densities correlated with increased deforestation. Thus, transitioning from an agricultural to a service-based economy would be linked to lower deforestation rates.

5 Conclusions

In this work we presented a theoretical and an empirical analysis of a classic problem in environmental economics: The determinants of deforestation caused by agricultural activities. Through an aggregative game framework, we modeled the farmer's decision of using fire to prepare the land (deforestation), which includes the cost of that land fire and the negative externality that the aggregate deforestation decision has on the farmers' land productivity. As parameters, the model included the land elasticity of production, the cost of land fire and the number of farmers dedicated to the agricultural activity. The theoretical model shed light on the impacts of the variations in the cost of land fire and in the number of smallholders on the aggregate decision of deforestation.

With the theoretical results in hand, we proceeded to estimate the size of the impacts of those parameters on the total deforestation observed in the Brazil's Legal Amazon, which is an important socio-geographic region in Brazil. Using data from The National Institute for Space Research (INPE), the Brazilian Institute of Environment and Renewable Natural Resources (IBAMA) and the Brazilian Institute of Geography and Statistics (IBGE) we have performed two analyses, one fitting a cross-section model for municipalities in that region in 2017 and another with a panel data model for the states of the region for the period 2009 to 2018. In both models we have a strong link between the size of the deforested area and the forest fire outbreaks, suggesting the effective use of land fire for agricultural purposes. Both models also pointed out that the increase in the number of farmers enlarge the deforestation. With respect to the cost of land fire, we have different responses in both models; however, when the value of environmental infraction notices is included, it becomes clear that the environmental authorities' supervision may be insufficient or ineffective for that period. Finally, another important result that the empirical panel data analysis brought was the negative impact of the deforestation on GDP of the states in the region, a well-documented result found in the literature of environmental economics.

Appendix

Proof of Proposition 1

The strategy sets $S_n = [0,1]$ are compact and convex and the payoff functions are continuous, it will be sufficient to prove the quasi-concavity of those functions with respect to their corresponding decision variables. The second derivative of u_n with respect to α_n is:

$$u_n''(\alpha_n, \alpha_{-n}) = A''(D)t_n^{\rho+2}\alpha_n^{\rho} + 2\rho A'(D)t_n^{\rho+1}\alpha_n^{\rho-1} + \rho(\rho-1)A(D)t_n^{\rho}\alpha_n^{\rho-2}$$

which is strictly negative since $\rho \in (0,1]$ and by Hypothesis 1 $A''(D) \leq 0$ and $A'(D) < 0$. □

Proof of Proposition 2

The interior Nash equilibrium must satisfy the first order condition:

$$A'(D)t_n^{\rho+1}\alpha_n^{\rho} + \rho A(D)t_n^{\rho}\alpha_n^{\rho-1} - k_n t_n = 0 \Leftrightarrow \rho A(D) + t_n A'(D)\alpha_n = (k_n t_n^{1-\rho})\alpha_n^{1-\rho}.$$

Let us define $f(\alpha_n) = \rho A(D) + t_n A'(D)\alpha_n$ and $g(\alpha_n) = (k_n t_n^{1-\rho})\alpha_n^{1-\rho}$.

Claim 1. *For each $D > 0$, there exists a unique $\tilde{\alpha}_n(D) > 0$ such that $f(\tilde{\alpha}_n(D)) = g(\tilde{\alpha}_n(D))$*

$f(\alpha_n)$ is a strictly decreasing linear function, since $A'(D) < 0$ and $f(0) = \rho A(D) > 0$. The function $g(\alpha_m)$ is strictly increasing and $g(0) = 0$. Therefore, there is only one $\tilde{\alpha}_n(D) > 0$ such that $f(\tilde{\alpha}_n(D)) = g(\tilde{\alpha}_n(D))$. Notice that $\tilde{\alpha}_n(D)$ could be greater than 1; however, since the hypothesis of the proposition asserts that there is an interior equilibrium, we conclude that the corresponding aggregate burning D^* will lead us to an $\tilde{\alpha}_n(D^*) < 1$. In the aggregative games literature, the function $\tilde{\alpha}_n(D)$ is called the replacement function of player n.

Claim 2. *The replacement function $\tilde{\alpha}_n : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly decreasing function.*

To prove this claim, let $\hat{D} > D$. Then, by Hypothesis 1, $A(\hat{D}) < A(D)$ and $A'(\hat{D}) < A'(D)$. This implies that the function $f(\alpha_n)$ moves down and since the function $g(\alpha_n)$ remains unchanged, then $\tilde{\alpha}_n(\hat{D}) < \tilde{\alpha}_n(D)$, as Figure 3 shows.

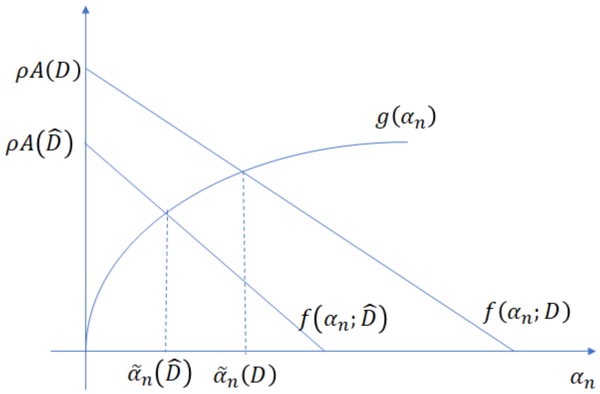

Figure 3: The decreasing behavior of the replacement function $\tilde{\alpha}_n(D)$.

Finally, to prove the Proposition 2, notice that the aggregate burning corresponding to a Nash equilibrium must satisfy:

$$\sum_{n=1}^{N} \tilde{\alpha}_n(D)t_n = D$$

Since the left-hand-side of the equation above is the sum of strictly decreasing functions (by Claim 2) and the right-hand-side is strictly increasing, then such an aggregate is unique and so the corresponding contributions $\alpha_n^* = \tilde{\alpha}_n(D^*)$. $\square$

Proof of Proposition 3

The first order condition for the interior equilibrium is $\rho A(D) + t_n A'(D)\alpha_n = k_n$, then summing up in n:

$$NA(D) + DA'(D) = K = \sum_{n=1}^{N} k_n. \tag{9}$$

Taking the derivative with respect to K in (9) it results:

$$\frac{dD}{dK} = \frac{1}{(N+1)A'(D) + DA''(D)} < 0,$$

therefore, $D(K)$ is a strictly decreasing function. $\square$

Proof of Proposition 4

Using the same $k_n = k$ in the first order condition of the farmer n, results:

$$\rho A(D) + A'(D)(t_n\alpha_n) = k(t_n\alpha_n)^{1-\rho}. \tag{10}$$

Following the same reasoning in the proof of Proposition 2, for each $D > 0$ the left side of the equation above is decreasing in $\alpha_n t_n$ and the right side is a strictly increasing function of $\alpha_n t_n$. Therefore, there exists a unique solution, so $t_n\alpha_n = t_m\alpha_m$ for all n, m. $\square$

Proof of Corollary 1

From Proposition 4 we can write $\alpha_n t_n = D/N$ and substituting this in (10) it results:

$$\rho A(D) + A'(D)\frac{D}{N} = k\left(\frac{D}{N}\right)^{1-\rho}.$$

With a similar reasoning used in the proof of Proposition 2, let $f(D) = \rho A(D) + A'(D)\frac{D}{N}$ and $g(D) = k(D/N)^{1-\rho}$. We have that $f'(D) < 0$ for all $D > 0$, $f(0) = \rho A(0) > 0$ and $f(D) \to -\infty$ as $D \to T$. Analogously, $g'(D) > 0$ for all $D > 0$, $g(0) = 0$. Therefore, there exists a unique $D = D(k, N)$ such that $f(D(, k, N)) = g(D(k, N))$, which is the aggregate land burned in equilibrium (by the uniqueness of interior solutions).

By the implicit function theorem, we have,

$$\frac{dD}{dk} = \frac{(D/N)^{1-\rho}}{(\rho + (1/N))A'(D) + A''(D)(D/N) - (1-\rho)kN^{-}1(D/N)^{-\rho}},$$

and from the Hypothesis 1, it results $\frac{dD}{dk} < 0$. The proof of D being increasing in N is analogous.

Finally, fixing $D = D(k, N)$, we obtain,

$$\frac{dk}{dN} = -\frac{D_N(k, N)}{D_k(k, N)}$$

where D_k and D_N are the derivatives of $D(k, N)$ with respect to each variable. Since $D_k < 0$ and $D_N > 0$, we obtain that $\frac{dk}{dN} > 0$. $\Box$

Proof of Proposition 5

To simplify the exposition, let us define $x_n = \alpha_n t_n$. The first order condition of the farmer n is:

$$k_n = f_n(x_n, x_m) = \rho A(D)x_n^{\rho-1} + A'(D)x_n^{\rho},$$

and an analogous expression for the farmer m, where $D = x_n + x_m$. The derivative and cross derivative of f_n are:

$$\frac{df_n}{dx_n} = A''(D)x_n^{\rho} + 2\rho A'(D)x_n^{\rho-1} + \rho(\rho-1)A(D)x_n^{\rho-2},$$

$$\frac{df_n}{dx_m} = A''(D)x_n^{\rho} + \rho A'(D)x_n^{\rho-1},$$

which are both strictly negative numbers. By the inverse function theorem, we have that,

$$\frac{\partial x_n}{\partial k_n} = \frac{1}{\Delta}\frac{\partial k_m}{\partial x_m},$$

$$\frac{\partial x_n}{\partial k_m} = -\frac{1}{\Delta}\frac{\partial k_n}{\partial x_m},$$

where Δ is the determinant of the matrix $\frac{\partial(f_n,f_m)}{\partial(x_n,x_m)}$, namely:

$$\Delta = \frac{df_n}{dx_n} \times \frac{df_m}{dx_m} - \frac{df_n}{dx_m} \times \frac{df_m}{dx_n}$$

and it is not difficult to see that substituting the expressions above it results $\Delta > 0$, therefore:

$$\frac{\partial x_n}{\partial k_n} < 0 \text{ and } \frac{\partial x_n}{\partial k_m} > 0$$

which allows us to conclude the same signals for α_n and α_m, since $\alpha_n = x_n/t_n$ and t_n is constant. $\square$

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