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The Roommate Problem Revisited

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Abstract:

We approach the roommate problem by focusing on simple matchings, which are those individually rational matchings whose blocking pairs, if any, are formed with unmatched agents. We show that the core is non-empty if and only if no simple and unstable matching is Pareto optimal among all simple matchings. The economic intuition underlying this condition is that blocking can be done so that the transactions at any simple and unstable matching need not be undone, as agents reach the core. New properties of economic interest are proved.

Keywords: Core; stable matching

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JEL Codes: C78; D78.

THE ROOMMATE PROBLEM REVISITED

by

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ABSTRACT

We approach the roommate problem by focusing on *simple* matchings, which are those individually rational matchings whose blocking pairs, if any, are formed with unmatched agents. We show that the core is non-empty if and only if no simple and unstable matching is Pareto optimal among all simple matchings. The economic intuition underlying this condition is that blocking can be done so that the transactions at any simple and unstable matching need not be undone, as agents reach the core. New properties of economic interest are proved.

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INTRODUCTION

The Roommate Problem is one of the three matching problems introduced by Gale and Shapley in their famous paper of 1962. There is a set of people who wish to be matched in pairs to be roommate in a college dormitory or partners in paddling a canoe. Each person ranks all the others in accordance with his/her preference for a roommate. This preference is assumed to be strict. The Marriage Problem may be considered as a special case of the Roommate Problem (every man lists as unacceptable all the other men and every woman lists as unacceptable all the other women). A *stable* matching is an individually rational matching such that no two persons who are not roommates both prefer each other to their actual partners. The stability concept is equivalent to the core concept in this model. Gale and Shapley proved, through a simple example, that the roommate problem may have no stable matching.

The literature on the roommate problem is very small, specially compared with its specific submodel. Only the existence problem of core outcomes has been treated.

This paper provides a new way of approaching the roommate problem. We focus on certain individually rational matchings, whose blocking pairs, if any, are formed with unmatched agents. The matchings that satisfy this property are called *simple*. Thus, simple matchings are individually rational matchings in which none of the matched agents is member of a blocking pair. Simple matchings exist even when stable matchings do not, since the matching where every one is unmatched is simple. Clearly, every stable matching is simple.

Our main finding is that the set of agents who are not part of any blocking coalition of a simple matching has the following properties: i) The unmatched players remain unmatched at any stable matching and ii) the matched players are matched among themselves under any stable matching. Moreover, iii) when the core is non-empty, there is a stable matching that contains all partnerships formed under the unstable simple matching. Such a stable matching is a Pareto superior for the given unstable simple matching. Therefore, no unstable simple matching can be Pareto optimal among all simple matchings when the core is non-empty. On the other hand, since a Pareto optimal simple matching always exists, this condition is also sufficient for the non-emptiness of the core. It follows from the fact that the set of simple matchings is non-empty and finite and the preferences are transitive. Finally, iv) there is a polarization of interests between the players involved in a partnership regarding a simple matching and a stable matching. The proofs of all these properties are not straightforward, but they are simple and only use elementary combinatorial arguments.

Property (i) implies that the set of unmatched agents is the same in every stable matching. Property (iv) implies the existence of a polarization of interests between the players involved in a partnership regarding two stable matchings. These are well-known properties of the stable matchings for the Marriage market and the College admissions market and have an analogue in the continuous one-to-one cases.

We think that the investigation of the properties of the simple outcomes for the roommate model provide for more understanding of how properties (i) and (iv), mentioned above, that characterize the stable outcomes of many of the two-sided matching models, are not affected by the number of sizes of the market, as the existence of the core is. This suggests that, indeed, these properties are more general than might have been expected.

This paper is organized as follows. In section 2 we describe the model and present the preliminary definitions. Section 3 introduces the simple matchings and proves several of their properties. Section 4 is devoted to the existence and non-existence of stable matchings. Section 5 concludes the paper and presents some related works.

It is based on the fact that when the core of the roommate market is non-empty, given any unstable simple matching, we can create new partnerships by keeping those already done and still have a simple matching. This fact is a consequence from the properties proved here that relate the unstable simple matchings with the stable matchings. Specifically, given any unstable simple matching i) the unmatched players, who are not part of a blocking coalition, remain unmatched at any stable matching and ii) the matched players are matched among themselves under any stable matching. Moreover, there is a stable matching that contains all partnerships formed under the unstable simple matching. This means that the partnerships formed under an unstable simple matching need not to be dissolved when the core is reached;

It is well-known that the set of unmatched players is the same in every stable matching of the Marriage market. The fact that this property extends to the roommate problem suggests that it does not depend on the two-sidedness of the matching as the core existence does. Indeed, it is more general than might have been expected.

This paper is organized as follows. In section 2 we describe the model and present the preliminary definitions. Section 3 introduces the simple matchings and proves several of their properties. Section 4 is devoted to the existence and non-existence of stable matchings. Section 5 concludes the paper and presents some related works.

The idea seems to be very intuitive. Suppose that given an unstable and simple matching x_1, there is another simple matching x_2 that keeps the partnerships done in the previous matching. This procedure must converge to a stable matching. Conversely, every stable matching can be obtained this way.

Procedures have been provided to reach a stable matching for the Marriage model, as a convergence of some specific sequence of unstable (see, for example, Roth and Vande Vate (1990) and Diamantoudi, Effrosyni, Eiichi Miyagawa, Licun Xue (2004)). Unlike the unstable simple matchings, the blocking pairs of these unstable matchings have at least one player who is not unmatched. However, these procedures are very artificial and there is no intuition to explain the dynamics of the selected blockings to construct the terms of the sequnce. If the sequence is not conveniently formed, it may cycle, even in case the core is non-empty (see Knuth, 1974).

This suggests that the partnerships of a stable matching could be considered to be formed in several steps of a *quasi-dynamic match-formation process*, under the assumption that the players "behave cooperatively": At each step only those pairs form whose members they feel they will have no better opportunities in the future. This implies that there will not be blocking pairs among the partnerships formed, so the matching in each step is simple. Also, the pairs which form at some stage will not dissolve in subsequent stages. Then, if equilibrium exists, this procedure continues until no transaction is able to benefit the agents involved. At this point the core has been reached. The core is empty if there is a stage in which any new interaction requires that some of the agents involved do not behave cooperatively.

In the example of Gale and Shapley (1962), the matching where every one is unmatched is the only simple matching. Thus, if two players decide to form a partnership, the resulting outcome cannot be simple, so at least one of the two players is not behaving cooperatively.

2. DESCRIPTION OF THE MODEL AND SOME PRELIMINARIES

There is a finite set of players, $N=\{1,2,\ldots,n\}$. Each player is interested in forming at most one partnership with players of N and has complete, transitive and strict preferences over the players in N. Hence, player j's preference can be represented by an ordered list of preferences, $P(j)$, on the set N. Player k is *acceptable* to j if j prefers k to himself/herself. Player j is always acceptable to j. Thus, $P(j)$ might be of the form

$P(j)=k, m, j, \ldots, q$

indicating that j prefers k to m, m to himself/herself, and anyone else is *unacceptable* to j.

The model can then be described by (N,P), where $P=\{P(1),\ldots,P(n)\}$.

Definition 1. *A* ***matching*** x *is a one-to-one correspondence from* N *onto itself of order two (that is,* $x^2(j)=j$*). We refer to* $x(j)$ *as the* ***partner of*** j ***at*** x.

If $x(j)=j$ we say that j is **unmatched** at x. Player j prefers matching x to matching y if and only if he/she prefers $x(j)$ to $y(j)$. Therefore, we are assuming that player j cares about who he/she is matched with, but is not otherwise concerned with the partners of other players.

Definition 2. *The matching x is* ***individually rational*** *if each player is acceptable to his or her partner.*

The key notion is that of stability.

Definition 3. *We say that the pair (j,k)* ***blocks*** *a matching x if j and k prefer each other to their current partners. A matching x is* ***stable*** *if it is individually rational and is not blocked by any pair. If x is not stable we say that it is* ***unstable****.*

It is a matter of verification that a matching is stable if and only if it is in the core.

Definition 4. *A matching x is* ***simple*** *if it is individually rational, every player is unmatched at x or, in case $x(j)\neq j$ for some j, then j is not part of any blocking pair of x. Hence, in case a blocking pair (j,k) exists, j and k are unmatched at x.*

Since the matching at which no partnership is formed is simple, **the set of simple matchings is non-empty.** Clearly, every stable matching is simple.

Definition 5. *Let x be an individually rational matching. We say that x admits* ***a stable structure $[S(x), N\text{-}S(x)]$,*** *if $S(x)\subseteq N$ ($S(x)$ might be the empty set) and is such that i) $x(S(x))=S(x)$; ii) $y(S(x))=S(x)$ for every stable matching y; if $j\in S(x)$ then iii) $y(j)\neq j$ if $x(j)\neq j$, and $y(j)=j$, otherwise; iv) j is not part of a blocking pair of x; and v) if $j\notin S(x)$ then j is part of a blocking pair of x. The sets $S(x)$ and $N\text{-}S(x)$ are called* ***stable component of x*** *and* ***unstable component of x, respectivelly.***

Matching x admits ***a strongly stable structure $[S(x),N\text{-}S(x)]$,*** *if* ***$[S(x),N\text{-}S(x)]$*** *is a stable structure of x and there is a stable matching y such that $y(j)=x(j)$ for every*

$j \in S(x)$. *The sets* $S(x)$ *and* $N\text{-}S(x)$ *are called* ***strongly stable component of*** $\boldsymbol{x}$ *and* ***strongly unstable component of*** $\boldsymbol{x}$***, respectivelly.***

Clearly, not every unstable matching has a stable structure. When such structure exists, it is uniquely detemined, by conditions (iv) and (v). Set N is the stable component of every stable matching. We will see in the next section that if x is simple then it admits a stable structure where the set of players who are not part of any blocking pair is its stable component. In addition, this structue is strongly stable, then not only the unmatched players remain unmatched at every stabe matching, but also the matched players keep their partners under some stable matching.

3. PROPERTIES OF THE SIMPLE MATCHINGS.

The players matched at a simple matching will never be unmatched at a stable matching and will be matched among them at any stable matching. The unmatched players who are not part of a blocking pair remain unmatched under any stable matching. Consequently, the unmatched players are the same at every stabe matching. In addition, the matched players keep their partners under some stable matching.

The following result is a powerful lemma that enables us to derive all of our results.

Lemma 1. *Let* x *be a simple matching and let* y *be a stable matching. Let* $T=\{j \in N; x(j)\neq j\}$, $M_x=\{j \in N; x(j) >_j y(j)\}$ *and* $M_y=\{j \in T; y(j) >_j x(j)\}$. *Then* $x(M_x)= y(M_x)=M_y$ *and* $x(M_y)=y(M_y)=M_x$.

Proof. All j in M_x are matched under x, since $x(j)>_j y(j) \geq_j j$. Analogously, all j in M_y are matched under y, since $y(j)>_j x(j)\geq_j j$. If j is in M_x then $k=x(j)$ is in M_y, for if not $j=x(k)>_k y(k)$, due to the strictness of the preferences and the fact that $x(k)\neq y(k)$, which contradicts the stability of y. On the other hand, if k is in M_y then $j=y(k)$ is in M_x, for if not $k=y(j) >_j x(j)$, due to the strictness of the preferences and the fact that $x(j)\neq y(j)$, which implies that (j,k) blocks x. However, k is in T, so k is matched under x, which contradicts the fact that x is simple. Therefore, $x(M_x)\subseteq M_y$ and $y(M_y)\subseteq M_x$, so $M_x\subseteq x(M_y)$ and $M_y\subseteq y(M_x)$. It follows that

$$|M_x|=|x(M_x)|\leq|M_y|=|y(M_y)|\leq|M_x| \text{ and } |M_y|\leq|y(M_x)|=|M_x|\leq|x(M_y)|=|M_y| ,$$

which implies $x(M_x)=M_y$, $y(M_y)=M_x$, $y(M_x)=M_y$ and $x(M_y)=M_x$, and the proof is complete.■

That is, if x is a simple matching and y is a stable matching, then both x and y map the set of people who prefer x to y onto the set of people who prefer y to x and are matched at x.

The following simple consequence of the decomposition lemma implies that trading agents at a simple matching always make their transactions under a stable matching within the same pool. That is:

Proposition 2. *Let x be a simple matching and let y be a stable matching. Let $T=\{j\in N;\ x(j)\neq j\}$. If $j\in T$ then $y(j)\neq j$ and $y(j)\in T$.*

Proof. Let $j\in T$. The result is immediate if $x(j)=y(j)$. Then, suppose $x(j)\neq y(j)$. Using the notation of Lemma 1 we have that $j\in M_x\cup M_y\subseteq T$. If $j\in M_x$ then $y(j)\in M_y$, so $y(j)\neq j$ and $y(j)\in T$. If $j\in M_y$ then $y(j)\in M_x$, so $y(j)\neq j$ and $y(j)\in T$. Hence, in any case the result follows. ■

This proposition concurs to the following result, which concerns a set of players who are indifferent between all stable matchings: *The set of unmatched players under a stable matching is contained in the set of unmatched players of every simple matching. Consequently, the set of unmatched players is the same in every stable matching*[2].

[2] This result, which also holds for the College Admission problem (Gale and Sotomayor , 1985-b and Roth, A., 1984) and has an analogue in the continuous cases (Demange and Gale, 1985), was first proved for the Marriage market by McVitie and Wilson (1970) for the case when all men and women are mutually acceptable. The proof for the general case of the Marriage model is due to Gale and Sotomayor (1985-a).

Formally,

Proposition 3. *Let x be a simple matching and let y be a stable matching. Then, the set of unmatched agents under y is contained in the set of unmatched agents under x. Consequently, the set of unmatched agents is the same at every stable matching.*

Proof. Immediate from Proposition 2.

A simple consequence of Propositions 2 and 3 is the following:

Proposition 4. *Suppose the set of stable matchings is non-empty. Let x be a simple matching. If j is unmatched at x and is not part of a blocking pair then j is unmatched at every simple matching. Consequently, j is unmatched at every stable matching.*

Proof. If x is stable, the result follows from Proposition 4. Then suppose x is unstable. Let y be a stable matching. Proposition 3 implies that all matched players at x are matched among themselves at y. This means that, if $y(j)=k$, for some $k\neq j$, k should be unmatched at x, so $\{j,k\}$ would block x, contradiction. Then j is unmatched at y and hence, j is unmatched at every simple matching by Proposition 3. ■

Given a simple matching x, which is not in the core, it is always possible to obtain a new matching z by doing the following: Keep the partnerships formed under x, if any, and add some new partnerships. Of course, these new partnerships are formed with blocking pairs of x. Theorem 6 asserts that matching z can be constructed so that it is stable.

We will make use of the following definition

Definition 6. *Let x and z be simple matchings. Let $T=\{j\in N;\ x(j)\neq j\}$. We say that* ***z extends x (or z is a simple extension of x)*** *if $z\neq x$ and $z(j)=x(j)$ for all $j\in T$.*

Proposition 5. *Let x be an unstable and simple matching. If the set of stable matchings is non-empty, then there exists a stable matching z that extends x.*

Proof. Let y be a stable matching. Let $T=\{j\in N;\ x(j)\neq j\}$. It follows from Proposition 3 that all of T are matched among them under y. Then, we can construct the matching z as follows: $z(j)=x(j)$ if $j\in T$; $z(j)=y(j)$ otherwise. It is clear that z extends x ($z\neq x$ due to the fact that $x\neq y$, since x is unstable). It remains to show that z is stable. That z is individually rational is immediate from the individual rationality of x and y. To see that z does not have any blocking pair, take any pair $\{j,k\}$. The fact that x is simple and y is stable implies that (j,k) does not block z in the cases where $\{j,k\}\subseteq T$ and $\{j,k\}\subseteq N\text{-}T$. Then, without loss of generality, suppose $k\in T$ and $j\in N\text{-}T$. If (j,k) blocks z then $j>_k z(k)=x(k)$ and $k>_j z(j)=y(j)\gtrsim_j j=x(j)$, so (j,k) blocks x. However, k is matched at x, which contradicts the fact that x is simple.

Hence, in any case, $\{j,k\}$ does not block z, so z is stable and the proof is complete. ■

An immediate corollary of the lemma reflects an opposition of interests between the players involved in a partnership regarding two stable matchings:

Theorem 3. *Let x and y be stable matchings. If j prefers x to y then j is matched to some k under x and to some h under y. Furthermore, both k and h prefer y to x.*[3]

4. EXISTENCE OF STABLE MATCHINGS

Theorem 7 demonstrates that the condition under which every simple and unstable matching has a simple extension is necessary and sufficient for the non-emptiness of the set of stable matchings. We need one more concept.

Definition 5. *Matching x is called* ***Pareto optimal simple matching (PS for short)*** *if it is simple and there is no simple matching y such that:*

(i) *all players weakly prefers y to x, and*

[3] The polarization of interests between the two sides of the Marriage market along the whole core is a restriction of this result to that market (Knuth, 1976).

(ii) *at least one player prefers y to x.*

Therefore, if x is PS and some player prefers a simple matching y to x, then there is some other player who prefers the opposite. The existence of such a matching x is guaranteed by the fact that the set of simple matchings is non-empty and finite and the preferences are transitive.

We can now prove our main result.

Theorem 4. *The set of stable matchings is non-empty if and only if every unstable and simple matching has a simple extension.*

Proof. If the set of stable matchings is non-empty, the result follows immediately from Theorem 6. In the other direction, let x be a Pareto optimal simple matching. We are going to show that x is stable. In fact, suppose by way of contradiction that x is unstable. By hypothesis, there is some simple matching z which extends x. Let S be the set of players who are matched at z but are unmatched at x. We have that $x(j)=z(j)$ for all $j \in N\text{-}S$ (use the construction of z and Theorem 4-a), all of S prefer z to x and $S \neq \phi$, since $x \neq z$.. But this contradicts the fact that x is a PS. Hence x is stable and the proof is complete. ■

5. CONCLUDING REMARKS AND RELATED WORKS.

A version of the concept of simple matching, as an individually rational matching where the woman involved in a blocking pair is always single, was introduced in Sotomayor (1996) for the Marriage market. There, a non-constructive proof of the non-emptiness of the set of stable matchings is presented. The proof consists in demonstrating that the Pareto optimal simple matching for the men must be stable. The same result is obtained by replacing women for men ad vice-versa.

Similar concepts were introduced in Sotomayor (1999) for the discrete many-to-many matching market with substitutable and non-strict preferences and in Sotomayor (2000), for the continuous Assignment game of Shapley and Shubik and for the unified two-sided matching model of Eriksson and Karlander (2000). For all these two-sided matching models, a non-constructive and simple proof of the non-emptiness of the set of pairwise-stable outcomes has been obtained. The argument of these proofs is that the Pareto optimal simple outcome for one of the sides must be pairwise-stable.

Recently, Sotomayor (2005) has introduced the concept of simple allocation for the one-sided market (not matching market) of Shapley and Scarf (1974). There, a non-constructive proof of the non-emptiness of the core has been obtained by proving that every Pareto optimal simple allocation is in the core. That paper remarks that such assertion does not apply to the counter example of Gale and Shapley for the roommate-problem. This observation has then raised the question if such condition would be necessary and sufficient for the existence of stable matchings for the roommate problem. The answer to this question was given in the present work.

In developing the theory to deal with simple matchings we obtained a sort of decomposition lemma that enabled us to derive other important and unexpected results.

Since Gale and Shapley (1962) the problem of existence of stable matchings for the roommate problem has been the subject of several research articles. In Irvin (1985) an algorithm is presented to find a stable matching when the set of stable matchings is non-empty. Tan (1991) has identified a necessary and sufficient condition, stated in terms of preference restriction, for the existence of stable matchings for the roommate problem. Chung (2000) has identified a condition called *no odd rings* that is sufficient, but not necessary, for the existence of stable matchings for this market. According to this author, this condition is quite abstract and may not have an economic interpretation. Differently, the condition presented here gives an economic intuition about how blocking can be done by non-trading agents, so that the transactions need not be undone when agents reach the core.

At each stage, new interactions are done and none of them is undone, so the core is reached after a finite number of interactions. The core is empty if there is a stage in which any new interaction requires that some of the agents involved do not behave cooperatively.

While the results proved in this paper are not technically difficult, very intuitive, and very much in line with common well known findings in general matching literature, they are important, since they show that the results on the structure of the matching model can be extended for this new setting, not studied before, but potentially very important. These results are also interesting perse, and could be connected with work on network formation, and with some staff on general cooperative games. This paper provides for more understanding of the under-investigated one-sided matching model.

The approach developed here suggests that the concept of simple matching and the theory built here open a new way to study more general games.

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That is, a stable matching can be regarded as the resulting outcome from a *quasi-dynamic match-formation process*, in which the partnerships are formed under the assumption that the players "behave cooperatively": *Two agents will only agree about a partnership if they both believe that more favorable terms cannot be obtained elsewhere.*

Our main result asserts that:

The core is non-empty if and only if every simple and unstable matching can be extended to a simple matching.

That is, under the assumption of the non-emptiness of the core, given a simple and unstable matching x, there is a simple matching (actually, there is a stable matching) that keeps all partnerships of x and adds some new partnerships formed with blocking pairs of x. If this condition is satisfied, no simple and unstable matching is Pareto optimal among all simple matchings. Since such a Pareto optimal matching always exist, this condition is also sufficient for the existence of core outcomes.

The economic attractiveness of this condition relies on the fact that it reflects the *agents' cooperative behavior* (which one would only expect in the core outcomes) along the subsequent stages of a dynamic negotiation process, where the resulting outcomes from each stage are simple matchings. The premiss that agents behave cooperatively has the natural meaning: *Two agents will only sign a contract if they believe that more favorable terms cannot be obtained elsewhere.* Thus, once a transaction is done at a given stage, one can expect that it will be maintained along the negotiation process. In addition, only agents who are not trading at a given stage can be better off at a subsequent stage, by trading among them. Then, if equilibrium exists and one believes that every agent behaves cooperatively, new trades can be expected at each stage, until no transaction is able to benefit the agents involved. At this point the core has been reached.

Of course, the core is empty if there is a stage at which any new trade requires that some of the agents involved do not behave cooperatively. In the example of Gale and Shapley (1962), the matching where every one is unmatched is the only simple matching. Thus, if two players decide to form a partnership, the resulting outcome cannot be simple, so at least one of the two players is not behaving cooperatively.