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THE ROLE PLAYED BY THE WELL-BEHAVED MATCHINGS IN THE COALITION FORMATION PROCESS OF THE STABLE MATCHINGS FOR THE ROOMMATE MARKET

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WORKING PAPER SERIES Nº 2016-06

DEPARTMENT OF ECONOMICS, FEA-USP
WORKING PAPER Nº 2016-06

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JEL Codes: C78; D78.

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by

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07/2005

ABSTRACT

We approach the roommate problem by focusing on *well-behaved* matchings, which are those individually rational matchings whose blocking pairs, if any, are formed with unmatched agents. We show that the set of stable matchings is non-empty if and only if no well-behaved and unstable matching is Pareto optimal among all well-behaved matchings. The economic intuition underlying this condition is that blocking can be done so that the transactions at any well-behaved and unstable matching need not be undone as agents reach the the set of stable matchings. We also give a sufficient condition on the preferences of the agents for the non-emptiness of the set of stable matchings. New properties of economic interest are proved.

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[1] This paper is partially supported by CNPq-Brazil.

INTRODUCTION

The Roommate Problem is one of the three matching problems introduced by Gale and Shapley in their famous paper of 1962. There is a set N with *2n* people who wish to be matched in pairs to be roommate in a college dormitory or partners in paddling a canoe. Each person ranks all the others in accordance with his/her preference for a roommate. This preference is assumed to be strict. Every person is acceptable to any other person. A matching is a bijection from N to N of order two. Therefore it can be identified with a set of pairs of agents so that every agent belongs to exactly one pair. A *stable* matching is a matching such that no two persons who are not roommates both prefer each other to their actual partners. The stability concept is equivalent to the core concept in this model. Gale and Shapley proved, through an example with four people, that the roommate problem may have no stable matching.

Later the roommate model was generalized by allowing that N had any number of people, that the preferences did not need to be strict and that an agent might be unacceptable to other agents (Sotomayor (), …). With this new formulation, the Marriage model could then be considered as a special case of the Roommate model (every man lists as unacceptable all the other men and every woman lists as unacceptable all the other women).

The literature on the roommate model is very small, specially compared with its specific submodel. Only problems related to the existence of core outcomes has been treated.

In this paper we present new results on the structure of the stable matchings for the roommate model, which have some analogue in the two sided matching models, and we also provide new conditions for the core existence.

Our results are concentrated in two parts. The first part concerns the structure of the stable matchings, when these outcomes exist. We show that some properties that are characteristic of the structure of the stable matchings for the Marriage market and the College admissions market, as well as for the continuous one-to-one cases, do not depend on the two-sidedness of the matching as the core existence does. They not only carry over the roommate-model, as well they can be obtained through the use of new techniques, which have in the "well-behaved matchings" their key mathematical tool. Well-behaved matchings are a version of the simple matchings, the concept introduced in Sotomayor (1996). These are certain individually rational matchings, such that none of the matched agents is member of a blocking pair. Well-behaved matchings exist even when stable matchings do not, since the matching where every one is unmatched is well-behaved. Clearly, every stable matching is well-behaved.

The advantage of this approach is that it provides simple and short proofs that only use elementary combinatorial arguments. The proofs of all results use a kind of Decomposition Lemma, very similar to the Decomposition Lemmas for the Marriage model, due to Gale and Sotomayor (1985) and the continuous one-to-one matching model presented in Demange and Gale (1985). Basically we prove that

i) there is a polarization of interests between the players involved in a partnership regarding a well-behaved matching and a stable matching. This property implies the existence of a polarization of interests between the players involved in a partnership regarding two stable matchings.

ii) The matched players at a well-behaved matching are matched among themselves under any stable matching. Consequently, the set of matched players is the same under every stable matching.

iii) The set of unmatched agents under any stable matching is contained in the set of unmatched agents under any well-behaved matching.

iv) The set of unmatched agents who are not part of any blocking coalition of a well-behaved matching remain unmatched at any stable matching. Then, the set of unmatched agents is the same in every stable matching.

In the second part of this paper we address the conditions that guarantee the existence of stable matchings for the roommate model. Our results give an economic intuition about how blockings can be done by non-trading agents, so that the transactions need not be undone as agents approach the core.

Intuitively, given an unstable and well-behaved matching x, it is always possible to obtain a new matching z, that is a Pareto superior of x, by doing the following: Keep the partnerships formed under x, if any, and add some new partnerships. This is always possible because x is unstable. Of course, these new partnerships are formed with blocking pairs of x. The first result asserts that, if the set of stable matchings is non-empty, then matching z can be constructed so that it is stable.

Therefore, when the set of stable matchings is non-empty, no unstable and well-behaved matching can be Pareto optimal among all well-behaved matchings.

In the other direction, suppose that given any unstable and well-behaved matching, there is a Pareto improvement of this matching that is well-behaved and keeps the partnerships done in the original matching. This is to say that the new matching *extends* the original one. It is intuitive that, if we start with any unstable and well-behaved matching, the sequence of well-behaved matchings, where each term extends the previous one, must converge, since it is finite and its terms are distinct, so it does not cycle. Furthermore, the limit of convergence must be a stable matching, for otherwise it would have a well-behave Pareto improvement. Our second theorem confirms such intuition: *if every unstable and well-behaved matching can be extended to a well-behaved maching, the way described above, then the set of stable matchings is non-empty.*

The economic attractiveness of this condition relies on the fact that it reflects the *agents' optimal behavior* (which one would only expect in the core outcomes) along the subsequent stages of a dynamic coalition formation process, in which the well-behaved matchings are the expected resulting outcomes from each stage. Basically, it is assumed that a coalition of players forms a number of partnerships only if the players involved believe that they will not have a better option in the future. If all agents' beliefs are correct, there will not be blocking pairs involving players in the partnerships formed, so the matching at each step is well-behaved. In this procedure, starting with any well-behaved maching, the pairs which form (if any) at a given stage will not dissolve in subsequent stages and are formed with non-trading agents of the previous stage. Therefore, only agents who are not trading at a given stage can be better off at a subsequent stage, by trading among them.

Thus, new trades will occur at each stage until no transaction is able to benefit the agents involved or until that any new interaction requires that some of the agents involved do not behave optimally. In the first case the core has been reached. In the second case, the core is empty. In the example of Gale and Shapley (1962), the matching where every one is unmatched is the only well-behaved matching. Thus, if two players decide to form a partnership, the resulting outcome cannot be well-behaved, so at least one of the two players is not behaving optimally.

The third result of this part gives a sufficient condition on the preferences of the agents for the non-emptiness of the set of stable matchings. It asserts that, if the preferences of the set of players that are part of some blocking pair of a given unstable and well-behaved matching form an even order cycle, such that for any term j_t of the cycle, j_{t+1} is j_t's most preferred agent in the set and j_{t-1} is j_t's second most preferred agent in the set, then it is possible to have an extension of the current matching. Consequently, if this condition holds for every unstable and well-behaved matching, then the set of stable matchings is non-empty.

This paper is organized as follows. In section 2 we describe the model and present the preliminary definitions. Section 3 introduces the well-behaved matchings and proves several of their structural properties. Section 4 is devoted to the existence and non-existence of stable matchings. Section 5 concludes the paper and presents some related works.

2. DESCRIPTION OF THE MODEL AND SOME PRELIMINARIES

There is a finite set of players, $N=\{1,2,\dots,n\}$. Each player is interested in forming at most one partnership with players of N and has complete, transitive and strict preferences over the players in N. Hence, player j's preference can be represented by an ordered list of preferences, $P(j)$, on the set N. Player k is *acceptable* to j if j prefers k to himself/herself. Player j is always acceptable to j. Thus, $P(j)$ might be of the form

$P(j)=k, m, j, \dots, q$

indicating that j prefers k to m, m to himself/herself, and anyone else is *unacceptable* to j.

The model can then be described by (N,P), where $P=\{P(1),\dots,P(n)\}$.

Definition 2.1. *A **matching** x for (N,P) is a one-to-one correspondence from N onto itself of order two (that is, $x^2(j)=j$). We refer to $x(j)$ as the **partner of j at x**.*

The set of matchings for (N,P) will be denoted by X.

If $x(j)=j$ we say that j is **unmatched** at x. Player j prefers matching x to matching y if and only if he/she prefers $x(j)$ to $y(j)$. Therefore, we are assuming that player j cares about who he/she is matched with, but is not otherwise concerned with the partners of other players.

Definition 2.2. *The matching x is **individually rational** if each player is acceptable to his or her partner.*

The key notion is that of stability.

Definition 2.3. *We say that the pair (j,k) **blocks** a matching x if j and k prefer each other to their current partners. A matching x is **stable** if it is individually rational and is not blocked by any pair. If x is not stable we say that it is **unstable**.*

It is a matter of verification that a matching is stable if and only if it is in the core.

Definition 2.4. *Agent j has an **optimal behavior** at matching x if he/she is matched at x and is not part of any blocking pair of x. A matching x is **well-behaved** if it is individually rational and every matched agent has an optimal behavior at x.*

3. STRUCTURE OF THE SET OF STABLE MATCHINGS

In this session we obtain some new and important properties of the stable matchings by using the structure of the well-behaved matchings.

The following result is a powerful lemma that enables us to derive all of our results.

Lemma 3.1 (Decomposition lemma). *Let x be a well-behaved matching and let y be a stable matching. Let $T=\{j\in N;\ x(j)\neq j\}$, $M_x=\{j\in N;\ x(j) >_j y(j)\}$ and $M_y=\{j\in T;\ y(j) >_j x(j)\}$. Then $x(M_x)= y(M_x)=M_y$ and $x(M_y)=y(M_y)=M_x$.*

Proof. All j in M_x are matched under x, since $x(j)>_j y(j) \geq_j j$. Analogously, all j in M_y are matched under y, since $y(j)>_j x(j)\geq_j j$. If j is in M_x then $k=x(j)$ is in M_y, for if not $j=x(k)>_k y(k)$, due to the strictness of the preferences and the fact that $x(k)\neq y(k)$, which contradicts the stability of y. On the other hand, if k is in M_y then $j=y(k)$ is in M_x, for if not $k=y(j) >_j x(j)$, due to the strictness of the preferences and the fact that $x(j)\neq y(j)$, which implies that (j,k) blocks x. However, k is in T, so k is matched under x, which contradicts the fact that x is well-behaved. Therefore, $x(M_x)\subseteq M_y$ and $y(M_y)\subseteq M_x$, so $M_x\subseteq x(M_y)$ and $M_y\subseteq y(M_x)$. It follows that

$$|M_x|=|x(M_x)|\leq|M_y|=|y(M_y)|\leq|M_x| \text{ and } |M_y|\leq|y(M_x)|=|M_x|\leq|x(M_y)|=|M_y| ,$$

which implies $x(M_x)=M_y$, $y(M_y)=M_x$, $y(M_x)=M_y$ and $x(M_y)=M_x$, and the proof is complete. ■

That is, if x is a well-behaved matching and y is a stable matching, then both x and y map the set of people who prefer x to y onto the set of people who prefer y to x and are matched at x.

An immediate consequence of the Decomposition Lemma reflects an opposition of interests between the players involved in a partnership regarding two stable matchings:

Theorem 3.1. *Let x and y be stable matchings. If j prefers x to y then j is matched to some k under x and to some h under y. Furthermore, both k and h prefer y to x.*[2]

The following simple consequence of the Decomposition Lemma implies that the players matched at a well-behaved matching will never be unmatched at a stable

[2] The polarization of interests between the two sides of the Marriage market along the whole core is a restriction of this result to that market (Knuth, 1976).

matching and will be matched among them at any stable matching. That is, the trading agents at a well-behaved matching always make their transactions under a stable matching within the same pool.

Proposition 3.1. *Let* x *be a well-behaved matching and let* y *be a stable matching. Let* $T=\{j\in N;\ x(j)\neq j\}$*. If* $j\in T$ *then* $y(j)\neq j$ *and* $y(j)\in T$*. Consequently, the set of matched agents is the same for every stable matching.*

Proof. Let $j\in T$. The first assertion is immediate if $x(j)=y(j)$. Then, suppose $x(j)\neq y(j)$. Using the notation of Lemma 3.1 we have that $j\in M_x\cup M_y\subseteq T$. If $j\in M_x$ then $y(j)\in M_y$, so $y(j)\neq j$ and $y(j)\in T$. If $j\in M_y$ then $y(j)\in M_x$, so $y(j)\neq j$ and $y(j)\in T$. Hence, in any case the first assertion follows. The second assertion follows from the fact that every stable matching is well-behaved.■

This proposition concurs to the following result, which concerns a set of players who are indifferent between all stable matchings.

Proposition 3.2. *Let* x *be a well-behaved matching and let* y *be a stable matching. Then, the set of unmatched agents under* y *is contained in the set of unmatched agents under* x*.*

Proof. If $y(j)=j$ and $j\in T$ then, by Proposition 3.1, $y(j)\neq j$, contradiction. ■

A simple consequence of Propositions 3.1 and 3.2 is the following:

Theorem 3.2. *Suppose the set of stable matchings is non-empty. Let* x *be a well-behaved matching. If* j *is unmatched at* x *and is not part of a blocking pair then* j *is unmatched at every well-behaved matching. Consequently, if* j *is unmatched at some stable matching, then* j *is unmatched at every stable matching.*

Proof. If x is stable, the result follows from Proposition 3.2. Then suppose x is unstable. Let y be a stable matching. Proposition 3.1 implies that all matched players at

x are matched among themselves at y. This means that, if $y(j)=k$, for some $k\neq j$, k should be unmatched at x, so $\{j,k\}$ would block x, contradiction. Then j is unmatched at y and hence, j is unmatched at every well-behaved matching by Proposition 3.2. The other assertion follows from the fact that every stable matching is well-behaved. ■

4. EXISTENCE OF STABLE MATCHINGS

This session address the conditions that guarantee the non-emptiness of the set of stable matchings. Theorem 4.1 implies that, when the set of stable matchings is non-empty, the matched players at a well-behaved matching keep their partners under some stable matching.

Theorem 4.2 demonstrates that the condition under which the matched players at an unstable and well-behaved matching keep their partners under some well-behaved matching, distinct from the original one, is necessary and sufficient for the non-emptiness of the set of stable matchings. Theorem 4.3 provides a new way of having such condition satisfied for a given unstable and well-behaved matching, by only using the preferences of the agents who are part of some blocking pair of the given outcome. The corollary of Theorem 4.3 shows that if this new condition is satisfied for every unstable and well-behaved matching then the set of stable matchings is non-empty.

We will make use of the following definition.

Definition 4.1. *Let x be a well-behaved matching. Let $T=\{j\in N;\ x(j)\neq j\}$. We say that matching* ***z extends x (or z is an extension of x)*** *if z is a well-behaved matching, $z\neq x$ and $z(j)=x(j)$ for all $j\in T$.*

Notation: Denote $U\equiv\{x\in X\ ;\ x \text{ is unstable and well-behaved}\}$.

Theorem 4.1. *Let $x\in U$. If the set of stable matchings is non-empty, then there exists a stable matching z that extends x.*

Proof. Let y be a stable matching. Let $T=\{j\in N;\ x(j)\neq j\}$. It follows from Proposition 3.1 that all of T are matched among them under y. Then, we can construct the matching z as follows: $z(j)=x(j)$ if $j\in T$; $z(j)=y(j)$ otherwise. It is clear that z extends x ($z\neq x$ due to the fact that $x\neq y$, given that x is unstable). It remains to show that z is stable. That z is individually rational is immediate from the individual rationality of x and y. To see that z does not have any blocking pair, take any pair $\{j,k\}$. The fact that x is well-behaved and y is stable implies that (j,k) does not block z in the cases where $\{j,k\}\subseteq T$ and $\{j,k\}\subseteq N\text{-}T$. Then, without loss of generality, suppose $k\in T$ and $j\in N\text{-}T$. If (j,k) blocks z then $j>_k z(k)=x(k)$ and $k>_j z(j)=y(j)\gtrsim_j j=x(j)$, so (j,k) blocks x. However, k is matched at x, which contradicts the fact that x is well-behaved.

Hence, in any case, $\{j,k\}$ does not block z, so z is stable and the proof is complete. ■

For the statement and proof of Theorem 4.2 we need one more concept.

Definition 4.2. *Matching x is called **well-behaved Pareto optimal matching (PO for short)** if it is well-behaved and there is no well-behaved matching y such that:*

(i) *all players weakly prefers y to x, and*

(ii) *at least one player prefers y to x.*

Therefore, if x is PO and some player prefers a well-behaved matching y to x, then there is some other player who prefers the opposite. The existence of such a matching x is guaranteed by the fact that the set of well-behaved matchings is non-empty and finite and the preferences are transitive.

Clearly, if x is PO then x cannot be extended by any well-behaved matching. Then, if every unstable and well-behaved matching has an extension, x must be stable. This is the main argument for the proof of Theorem 4.2 below.

Theorem 4.2. *The set of stable matchings is non-empty if and only if every $x\in U$ has an extension.*

Proof. If the set of stable matchings is non-empty, the result follows immediately from Theorem 4.1. In the other direction, let x be a PO. We are going to show that x is stable. In fact, if x is unstable then, by hypothesis, x has an extension, which is absurd. Hence x is stable and the proof is complete. ■

Corollary 4.1 follows immediately.

Corollary 4.1. *If the set of stable matchings is non-empty then a well-behaved Pareto efficient matching is stable.*

Proof. Let x be a PO matching. If x is unstable then, by Theorem 4.1, x has an extension, which is absurd. ■

For Theorem 4.3 we need the following definition.

Definition 4.2. *An S-cycle of even order is a sequence $c=(j_1, \ldots, j_{2s})$, where s is some positive enteger such that $j_t=Ch_{t-1}(S)$ for all $t=2,\ldots,2s$ and $j_1=Ch_{2s}(S)$. Furthermore, $j_{2t-1}=Ch_{2t}(S-\{j_{2t+1}\})$ for all $t=1,\ldots,s-1$ and $j_{2s-1}= Ch_{2s}(S-\{j_1\})$. The cycle c is called an **S-cycle of order 2s.***

That is, $Ch_t(S)$ is the most preferred agent for j_t among the players in S and, if $Ch_t(S)=k$ then $Ch_t(S-\{k\})$ is the second most preferred agent for j_t among the players in S. Thus, if say $c=\{j_1, j_2, j_3, j_4\}$ is an S-cycle of order 4 then $P(j_1)=j_2,\ldots$, $P(j_2)=j_3, j_1,\ldots$, $P(j_3)=j_4, \ldots$, $P(j_4)=j_1, j_3,\ldots$

Notation: *Let $x\in U$. Denote $S(x)$ the set of agents who are part of some blocking pair at x.*

Theorem 4.3. *Let $x\in U$. Suppose $S(x)$ has an $S(x)$-cycle of even order. Then x has an extension.*

Prof. Suppose x has an $S(x)$-cycle of even order. Set $c=(j_1,\ldots,j_{2s})$ such cycle. Construct y as follows: $y(j)=x(j)$ for all $j\in N\text{-}S(x)$ and all $j\in S(x)\text{-}c$; $y(j_1)=j_2$, $y(j_3)=j_4$, …, $y(j_{2s-1})=j_{2s}$. We claim that y is an extension of x. In fact, by construction of y we have that y is an extension of x, so we only need to show that y is well-behaved. Then, suppose by way of contradiction that y is not well-behaved. The definition of an $S(x)$-cycle and the fact that x is well-behaved imply that y is individually rational. Then y has some blocking pair (p,q), and one of the members, say p, is matched at y. Given that y and x agree on $N\text{-}S(x)$ and x is well-behaved, we have that $\{p,q\}\subseteq S(x)$. Then $p\in c$ and $y(p)\in c$. We have two cases.

1st case: $p=j_{2t-1}$, for $t\in\{1,\ldots,s\}$. Then, $y(p)=Ch_p(S(x))$, so $y(p)>_p q$. Hence (p,q) does not block y, contradiction.

2nd case: $p=j_{2t}$, for some $t\in\{1,\ldots,s\}$. Then, $y(p)=Ch_p(S(x)\text{-}\{j_{2t+1}\})$ if $t\neq s$ and $y(p)=Ch_p(S(x)\text{-}\{j_1\})$ otherwise. In any case, if $p>_q y(p)$ then $q\notin\{j_k;\ k=2r\text{-}1$ for some $r\in\{1,\ldots,s\}\}$. In particular $q\neq j_{2t-1}$ and $q\neq j_{2t+1}$, so $y(p)>_p Ch_p(S(x)\text{-}\{j_{2t-1}, j_{2t+1}\})\geq_p q$. Hence $y(p)>_p q$, contradiction.

Hence y is an extension of x and the proof is complete. ■

Corollary 4.2. *Let $x\in U$. Suppose that for every $x\in U$, $S(x)$ has an $S(x)$-cycle of even order. Then the set of stable matchings is non-empty.*

Proof. Theorem 4.3 implies that every $x\in U$ has an extension. The result then follows from Theorem 4.2. ■.

5. CONCLUDING REMARKS AND RELATED WORKS.

A version of the concept of well-behaved matching for the Marriage model, as an individually rational matching where the woman (or the man) involved in a blocking pair is always single, was introduced in Sotomayor (1996) and called simple maching. In that paper, a non-constructive proof of the non-emptiness of the set of stable matchings is presented. The proof consists in demonstrating that the Pareto optimal and simple matching for the men must be stable. The same result is obtained by replacing women for men.

Similar concepts were introduced in Sotomayor (1999) for the discrete many-to-many matching market with substitutable and non-strict preferences and in Sotomayor (2000), for the continuous Assignment game of Shapley and Shubik and for the unified two-sided matching model of Eriksson and Karlander (2000). For all these two-sided matching models, a non-constructive and simple proof of the non-emptiness of the set of pairwise-stable outcomes has been obtained. The main argument of these proofs is that the Pareto optimal simple outcome for one of the sides is pairwise-stable.

Recently, Sotomayor (2005) has introduced the concept of simple allocation for the one-sided market (not matching market) of Shapley and Scarf (1974). There, a non-constructive proof of the non-emptiness of the core has been obtained by proving that every Pareto optimal simple allocation is in the core. That paper remarks that such assertion does not apply to the counter example of Gale and Shapley for the roommate problem. This observation has then raised the question if such condition would be necessary for the existence of stable matchings for the roommate problem. The answer to this question is given in the present work: If the set of stable matchings is non-empty then every unstable simple matching has a simple extension. Given that a Pareto optimal simple matching canot be extended to a simple matching then it must be stable.

In developing the theory to deal with simple matchings we obtained a sort of decomposition lemma that enabled us to derive other important results.

Since Gale and Shapley (1962) the problem of existence of stable matchings for the roommate problem has been the subject of several research articles. In Irvin (1985) an algorithm is presented to find a stable matching when the set of stable matchings is non-empty. Tan (1991) has identified a necessary and sufficient condition, stated in terms of preference restriction, for the existence of stable matchings for the roommate problem. Chung (2000) has identified a condition called *no odd rings* that is sufficient, but not necessary, for the existence of stable matchings for this market. According to this author, this condition is quite abstract and may not have an economic interpretation.

Differently, the condition presented here gives an economic intuition about how blockings can be done by non-trading agents, so that the transactions need not be undone when agents reach the core. At each stage, new interactions are done and none of them is undone, so the core is reached after a finite number of interactions. The core is empty if there is a stage in which any new interaction requires that some of the agents involved do not behave optimally.

While the results proved in this paper are not technically difficult, very intuitive, and very much in line with common well known findings in general matching literature, they can be proved from this new setting, not studied before, but potentially very important. These results are also interesting perse, and could be connected with work on network formation, and with some staff on general cooperative games. This paper provides for more understanding of the under-investigated one-sided matching model.

The approach developed here suggests that the concept of simple matching and the theory built here open a new way to study more general games.

REFERENCES.

Abeledo, H. and Isaak, G. (1991), "A characterization of graphs that ensure the existence of stable matchings", *Mathematical Social Sciences* vol. 22, Issue 1, 93-96.

Chung, Kim-Sau (2000) "On the existence of stable roommate matchings", *Games and Economic Behavior* 33, 206-230.

Demange, G. and Gale, D. (1985) "The strategy structure of two-sided matching markets", *Econometrica,* 53, 873-888.

Eriksson, K. and Karlander, J. (2000) "Stable matching in a common generalization of the marriage and assignment models", *Discrete Mathematics* 217, 135-156.

Gale,D. and Shapley, L. (1962) “College admissions and the stability of marriage”, *American Mathematical Monthly* 69, 9-15.

Gale, D. and Sotomayor M. (1985-a) “Some remarks on the stable matching problem” *Discrete Applied Mathematics,* 11, 223-32.

Gale, D. and Sotomayor M. (1985-b) "Ms. Machiavelli and the stable matching problem", *American Mathematical Monthly,* 92, 261-268.

Gusfield, D. (1988) “The structure of the stable roommate problem: efficient representation and enumeration of all stable assignments”, *SIAM Journal on Computing,* 17, 742-69.

Irving, R. W. (1985) “ An efficient algorithm of the stable room-mates problem”, *Journal of Algorithms,* 6, 577-95.

Knuth, D. (1976) “Marriages estables” Montreal: Les Presses de l’Université de Montreal.

McVitie, D. G. and Wilson L. B. (1970) “ *Stable marriage assignments for unequal sets”, BIT,* 10, 295-309.

Roth, A. (1984) "The evolution of the labor market for medical interns and residents; a case study in game theory", *Journal of Political Economy*, 92, 991-1016.

Roth, A. and Sotomayor, M (1990) “Two-sided matching. A study in game-theoretic modeling and analysis” *Eonometric Society Monographs,* 2nd ed., Vol. 18, Cambridge University Press.

Shapley, L. and Scarf, H. (1974) “On cores and indivisibility” *Journal of Mathematical Economics,* 1, 23-8.

Shapley, L. and Shubik, M. (1972) "The assignment game I: The core", *Journal of Game Theory* 1, 111-130.

Sotomayor, M. (2005) “An elementary non-constructive proof of the non-emptiness of the core of the housing-market of Shapley and Scarf” *Mathematical Social Sciences,* to appear.

___________(2000) "Existence of stable outcomes and the lattice property for a unified matching market", *Mathematical social Science* 39 , 119-132.

___________(1999) "Three remarks on the many-to-many stable matching problem", *Mathematical Social Sciences* 38, 55-70

___________(1996) "A non-constructive elementary proof of the existence of stable marriages", *Games and Economic Behavior* 13, 135-137.

Tan, J.J.M. (1991)