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Abstract:

This paper proposes a methodology to integrate econometric models with Johansen-type computable general equilibrium (CGE) models in instances when it is necessary to generate results consistent with a subset of variables that are endogenous to both models. Results for a subset of the CGE endogenous variables are generated from econometric models, and set as targets to be replicated by the CGE model. The methodology is further extended for robustness testing of the outcomes in cases which the targeted scenarios are random. The proposed methodology is illustrated by simulating the impacts of a monetary shock in Brazil.

Keywords: Model integration, target fitting, sensitivity analysis, CGE models, monetary policy, regional analysis.

JEL Codes: C63, C68, R13, R15.

Target Fitting and Robustness Analysis in CGE Models

Gabriel Garber[1] and Eduardo Haddad[2]

Abstract. This paper proposes a methodology to integrate econometric models with Johansen-type computable general equilibrium (CGE) models in instances when it is necessary to generate results consistent with a subset of variables that are endogenous to both models. Results for a subset of the CGE endogenous variables are generated from econometric models, and set as targets to be replicated by the CGE model. The methodology is further extended for robustness testing of the outcomes in cases which the targeted scenarios are random. The proposed methodology is illustrated by simulating the impacts of a monetary shock in Brazil.

1. Introduction

Large-scale modeling relies heavily on the integration or linkage of different submodels. The usual process of integrating or linking models consists of endogeneizing exogenous components of one of them through components of another or many others. Integration occurs in cases when researchers are able to reconcile models in a unified system in which components are tied through hard links. In the more often instances when models are treated separately, the adopted strategy is to have a series of them linked, as the output of one is used as the input to others through soft links (Boyce, 1988; Hewings et al., 2003).

The use of more straightforward soft links requires only the endogenous variables from one model to be mapped to exogenous variables of other model(s). However, there may be instances in which researchers want to generate results consistent with a given subset of variables that are endogenous to both models. This is usually accomplished by the use of more complex hard links between models. In this paper we look at this issue from a different perspective in the context of CGE models. The problem posed is to carry out simulations that mimic the behavior of a subset of variables of the CGE model. The usual approach to deal with this problem is to re-specify model closures making sure the information one wants to

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replicate is contained in the set of the exogenous variables (Dixon and Rimmer, 2002). However, as closures correspond to different economic environments, the re-specified closure may sometimes lead to a meaningless economic setting.

This paper proposes a diverse strategy. We take a working CGE model and its economically meaningful closure as given. Moreover, results for a subset of the CGE endogenous variables are generated from other models and set as targets to be replicated by the CGE model. The challenge becomes to calibrate exogenous shocks in the CGE model that endogenously generate the same values as the targeted variables.

The shocks in the set of exogenous variables may be very hard to calibrate, as each individual shock usually affects many endogenous variables, and the combined final effects are intertwined. The problem is solved by setting the given scenario as a target for some of the endogenous variables, and taking advantage of standard solution procedures in Johansen-type models. We further extend the methodology to enable to test for robustness of the outcomes in cases in which the targeted scenarios are random.

In the next section, we formalize the proposed methodology in the context of Johansen-type CGE models. The method is illustrated in the following section with an application that looks at the regional impacts of monetary policy in Brazil. This example is particularly compelling as it highlights some of the more interesting features of the methodology. First, it is based on an economic problem that stand-alone standard CGE models are not appropriate to handle. It adds to different applications, which have successfully overcome some of the shortcomings of CGE models, by inserting a core CGE in a broader modeling framework. Second, it shows that one can get more information from initial estimates from reduced-form econometric models whose outcomes are to be further disaggregated and distributed among sectoral and regional variables which are then reconciled with global estimates through the CGE model properties. Finally, the use of robustness tests allows for dealing with uncertainty originated in the scenarios, providing the necessary information for more sound conclusions. The closing section puts the method into perspective.

2. Methodology

2.1. The Johansen Approach

We consider a class of CGE models known as Johansen-type CGE models in that the solutions are obtained by solving the system of *linearized* equations of the model.[3] A typical result shows the percentage change in the set of endogenous variables, after a policy is carried out, compared to their values in the absence of such policy, in a given environment.

In Johansen-type CGE models, the system of equations of the model can be written as:

$$F(V)=0 \tag{1}$$

where V is an equilibrium vector of length n (number of variables), and F is a vector function of length m (number of equations), which is assumed to be differentiable. Regarding the dimensions, n and m, it is assumed that the total number of variables is greater than the total number of equations in the system, i.e., $n > m$. Thus, $(n - m)$ variables must be set exogenously. For the purpose of calibration of the system, it is fundamental to assume that $\exists V = V^{*} s.t.\, F\left(V^{*}\right) = 0$ and the initial solution, V^{*}, is known.

The Johansen approach consists of using a differential or log differential version of (1), which may be represented as:

$$A(V)v=0 \tag{2}$$

where $A(V)$ is a (mxn) matrix containing partial derivatives or elasticities, and v is adequately calculated as changes, log-changes or percentage changes in vector V.

The procedure to obtain approximate estimates of (percentage) changes in endogenous variables is to evaluate $A(.)$ on a known initial equilibrium vector V^{I}, and then solve (2). It is useful to partition matrix A and vector v into two parts each, separating the endogenous and

[3] More details can be found in Dixon *et al*. (1982, 1992), and Dixon and Parmenter (1996).

exogenous variables. The endogenous and exogenous parts of the system are indexed α and β, respectively:

$$A(V^I)v=A_\alpha(V^I)v_\alpha+A_\beta(V^I)v_\beta=0$$

$$v_\alpha= - A_\alpha(V^I)^{-1} A_\beta(V^I)v_\beta$$

$$v_\alpha =B(V^I)v_\beta \tag{3}$$

where A_α is (mxm), v_α is (mx1), A_β is (mx(n-m)), v_β is ((n-m)x1) and $B(V^I)$ is defined as $- A_\alpha(V^I)^{-1} A_\beta(V^I)$.[4]

Now consider the following representation of equation (3):

$$\begin{bmatrix} v_{\alpha 1} \\ v_{\alpha 2} \\ \vdots \\ v_{\alpha m} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1(n-m)} \\ B_{21} & B_{22} & & B_{2(n-m)} \\ \vdots & & \ddots & \vdots \\ B_{m1} & \cdots & \cdots & B_{m(n-m)} \end{bmatrix} \begin{bmatrix} v_{\beta 1} \\ v_{\beta 2} \\ \vdots \\ v_{\beta (n-m)} \end{bmatrix}$$

It can also be written as:

$$\begin{bmatrix} v_{\alpha 1} \\ v_{\alpha 2} \\ \vdots \\ v_{\alpha m} \end{bmatrix} = \begin{bmatrix} B_{11} \\ B_{21} \\ \vdots \\ B_{m1} \end{bmatrix} v_{\beta 1} + \begin{bmatrix} B_{12} \\ B_{22} \\ \vdots \\ B_{m2} \end{bmatrix} v_{\beta 2} + \cdots + \begin{bmatrix} B_{1(n-m)} \\ B_{2(n-m)} \\ \vdots \\ B_{m(n-m)} \end{bmatrix} v_{\beta (n-m)} \tag{3'}$$

Representation (3') allows us to see directly two properties we will employ. First, when considering a shock to a single exogenous variable, the effect on v_α will be proportional to the vector in (3') multiplying this variable. Second, when evaluating a multidimensional shock, the total impact on v_α can be computed as a sum of effects of shocks to individual exogenous variables.

[4] Up to this point we have followed closely the notation and exposition of the Johansen approach presented in Dixon et al. (1992).

2.2. Target

Suppose there is a size k subset of the elements of v_α for which we wish to establish a target. We build it as vector $t_{(kx1)}$. Assume further that we have identified a size j subset of v_β containing exogenous variables regarded as relevant to impact variables in t.[5] Therefore, to analyze what is relevant for achieving a result close to the target, we may reduce the system in (3') by simply disregarding the elements of every vector in the rows not corresponding to a variable in t, and setting to zero the variation of every exogenous variable regarded as not relevant. We will obtain a system of the same form as (3'), although with (much) smaller dimensions. We identify the elements obtained from such procedure with "^":

$$\begin{bmatrix}\hat{v}_{\alpha 1}\\ \hat{v}_{\alpha 2}\\ \vdots\\ \hat{v}_{\alpha k}\end{bmatrix} = \begin{bmatrix}\hat{B}_{11}\\ \hat{B}_{21}\\ \vdots\\ \hat{B}_{k1}\end{bmatrix}\hat{v}_{\beta 1} + \begin{bmatrix}\hat{B}_{12}\\ \hat{B}_{22}\\ \vdots\\ \hat{B}_{k2}\end{bmatrix}\hat{v}_{\beta 2} + \cdots + \begin{bmatrix}\hat{B}_{1j}\\ \hat{B}_{2j}\\ \vdots\\ \hat{B}_{kj}\end{bmatrix}\hat{v}_{\beta j} \tag{4}$$

Or in a notation similar to (3):

$$\hat{v}_\alpha = \hat{B}\hat{v}_\beta \tag{5}$$

From (4) we see that $\hat{v}_\alpha$ depends on the choice of the values of the exogenous variables in $\hat{v}_\beta$. Thus all we need is a norm to evaluate the distance between $\hat{v}_\alpha$ and t, in order to treat such a choice as an optimization problem aiming to find the best choice for $\hat{v}_\beta$. Generally, we have:

$$min_{\hat{v}_\beta}\left\| \hat{v}_\alpha - t \right\| \tag{6}$$

In our approach, we use the square of the Euclidean norm, i.e., the sum of the squares of the difference of each element in $\hat{v}_\alpha$ and the corresponding element in t. This norm, widely used in economics, will show itself to be very convenient when we will need to re-optimize (6) several times to perform the sensitivity analysis.

[5] Identifying which of the exogenous variables belong in this subset relies on the knowledge of economic relationships captured by the CGE model being used.

We call the solution of (6) $\hat{v}_\beta^*$, and the resulting vector of targeted endogenous variables $\hat{v}_\alpha^*$. These results may be scrutinized in two ways. First, $\hat{v}_\beta^*$ should be composed of variations of exogenous variables consistent with the economic relationships that resulted in their pre-selection for vector $\hat{v}_\beta$. Second, depending on the source of data/model used for the construction of *t*, we may evaluate whether the procedure resulted in a $\hat{v}_\alpha^*$ that is "close enough" to the target variables. A $\hat{v}_\alpha^*$ considered too far from *t* may indicate that the target is not consistent with the structure or the data used in the CGE model, or the set of exogenous variables pre-selected to be part of $\hat{v}_\beta$.

2.3. Reconstruction of Results and Caveats

The limitations of this procedure stem either from those of the Johansen approach itself or from numerical issues related to the migration of data between different models. Regarding the first type of problem, results accuracy will drop as the original system becomes less linear (or log-linear) and as variations in the exogenous variables (size of shocks) increase in absolute value. As for the second type, the truncation of the numerical results from the CGE software used to calculate $\widehat{B}$ may also reduce accuracy in the mapping of $\hat{v}_\alpha^*$ into *t*, since both calculations are carried in different computational environments.[6]

From this point, we may proceed in two ways. We may introduce $\hat{v}_\beta^*$ shocks in the CGE software or simply reconstruct all the endogenous results v_α using (3) or (3'). While the first alternative may be interesting to use built-in options to minimize problems resulting from the Johansen procedure, the second one is more convenient to perform sensitivity analysis.

2.4. Sensitivity Analysis

Given the intrinsic uncertainty in the shock magnitudes (and parameter values), sensitivity tests have been advocated as an important step in the more formal evaluation of the robustness of CGE analysis. In our context, to conduct systematic sensitivity analysis of the

[6] The CGE simulations are carried out using the software GEMPACK (Harrison and Pearson, 2002) while the optimization procedure to generate the mapping of the target into the CGE variables is done using MATLAB®.

results, one may want to consider the possibility that the target used is stochastic. This is particularly interesting when *t* is built using outputs of econometrics or statistics applications, which may provide it with a distribution.[7]

In this case, it is possible to simulate N random draws for *t*, but each of them will require an optimization in the fashion explained before. This is where the norm we employed becomes very convenient. Let t_i denote each draw from the distribution of *t*, and $\hat{v}^*_{\beta i}$ the corresponding optimal choice of shocks. Then $\hat{v}^*_{\beta i}$ is simply given by:

$$\hat{v}^*_{\beta i} = \left(\hat{B}'\hat{B}\right)^{-1}\hat{B}'t_i \quad (7)$$[8]

Now, let $\hat{V}^*_\beta$ be a (jxN) matrix in which the columns are the $\hat{v}^*_{\beta i}$s, with i varying from 1 to N. We denote T a (kxN) matrix constructed in the same way, using the t_i as columns. Then, because $\hat{B}$ is invariant across simulations, equation (7) can also be written for these matrices, and all $\hat{v}^*_{\beta i}$s may be calculated simultaneously in a single step:

$$\hat{V}^*_\beta = \left(\hat{B}'\hat{B}\right)^{-1}\hat{B}'T \quad (8)$$

Each row of $\hat{V}^*_\beta$ displays, for the shocks to each relevant variable, the N optimal values consistent with values drawn for the t_i. We denote $\hat{v}^*_{\alpha i}$ as the optimal vector of targeted variables associated with $\hat{v}^*_{\beta i}$, and $\hat{V}^*_\alpha$, as a (kxN) matrix, constructed with these vectors as columns. Then we have, from (5), (7) and (8):

$$\hat{v}^*_{\alpha i} = \hat{B}\left(\hat{B}'\hat{B}\right)^{-1}\hat{B}'t_i \quad (9)$$

[7] The Gaussian Quadrature (GQ) approach (Arndt, 1996; DeVuyst and Preckel, 1997) was proposed to evaluate CGE model results' sensitivity to parameters and exogenous shocks. This approach views key exogenous variables (shocks or parameters) as random variables with associated distributions. Due to the randomness in the exogenous variables, the endogenous results are also random; the GQ approach produces estimates of the mean and standard deviations of the endogenous model results, thus providing an approximation of the true distribution associated with the results.

[8] This can be easily checked reinterpreting the columns of $\hat{B}$ as a set of observations of different variables. Then the form of the problem is equal to that of a simple regression. Obviously this is not an econometric application, but the ordinary least squares solution form still applies as a mathematical result.

and

$$\widehat{\mathrm{V}}_{\alpha}^{*} = \hat{B}\left(\hat{B}'\hat{B}\right)^{-1}\hat{B}'T \tag{10}$$

$\widehat{\mathrm{V}}_{\alpha}^{*}$ may be used for sensitivity analysis considering each targeted variable, by taking each of its rows separately, as well as the observed co-movement.

If the sensitivity analysis is to be performed on variables that do not belong to the vector of targeted variables, all we have to do is to resort once more to equations (3) and (3'). Take matrix B and delete all columns which do not correspond to an exogenous variable in $\hat{v}_{\beta}^{*}$. We call the resulting matrix $\widetilde{B}$. Then all[9] endogenous variables may be reconstructed by:

$$\hat{v}_{\alpha i}^{*} = \widetilde{B}\hat{v}_{\beta i}^{*} \qquad (11)^{10}$$

3. Application: Regional Effects of a Monetary Policy Shock

We now apply the proposed methodology simulating a monetary policy shock in the Brazilian economy, integrating econometric estimates of macro models to an interregional CGE model. We simulate an increase of 100 basis points in the Brazilian policy interest rate, Selic. Given the structure of the equations we use to build the macro targets, we restrict to analyzing the immediate (short run) impacts of such an increase, supposing the shock is permanent and has no effect on expectations, at least in this time span.

The econometric estimates provide the macro effects of the monetary shocks which are to be used to construct the targets for the subset of endogenous variables in the CGE model. The use of the CGE model is intended to generate further information on the effects of the rise in the policy interest rate by consistently disaggregating the macro effects across regions and sectors. As each of the models is issue-specific, we are able to amend the CGE model by

[9] If computational resources are a relevant constraint, one can proceed in the same way, excluding some rows of $\widetilde{B}$ one is not interested in, or breaking this matrix in two or more subsets of rows, and reconstructing the results separately.

[10] Or for a matrix V_{α}^{*} colecting all $v_{\alpha i}$: $V_{\alpha}=\widetilde{B}\widehat{V}_{\beta}^{*}$

linking both systems in order to account quantitatively for those channels that are deemed important in determining how monetary policies affect the regional structures in the short run. Thus, while the econometric model provides the channels through which the monetary shock affects macro aggregates, the CGE model allows for a more detailed structural perspective of such results.

We use the B-MARIA model, developed by Haddad (1999), for illustrative purposes. The B-MARIA model – and its extensions – has been widely used for assessing regional impacts of economic policies in Brazil. Since the publication of the reference text, various studies have been undertaken using, as the basic analytical tool, variations of the original model.[11] Moreover, critical reviews of the model can be found in the Journal of Regional Science (Polenske, 2002), Economic Systems Research (Siriwardana, 2001), and Papers in Regional Science (Azzoni, 2001). The theoretical structure of the B-MARIA model is well documented.[12] Its mathematical structure is based on the MONASH-MRF model for the Australian economy (Peter et al., 1996). It qualifies as a Johansen-type model in that the solutions are obtained by solving the system of linearized equations of the model. In this paper, we use a simplified version of the core equations of the B-MARIA model specified for five macro-regions in Brazil, 23 products and 16 sectors. In order to capture the immediate effects of an interest rate shock, the simulations are carried out under a standard short run closure.

3.1. Macro Results as the Target

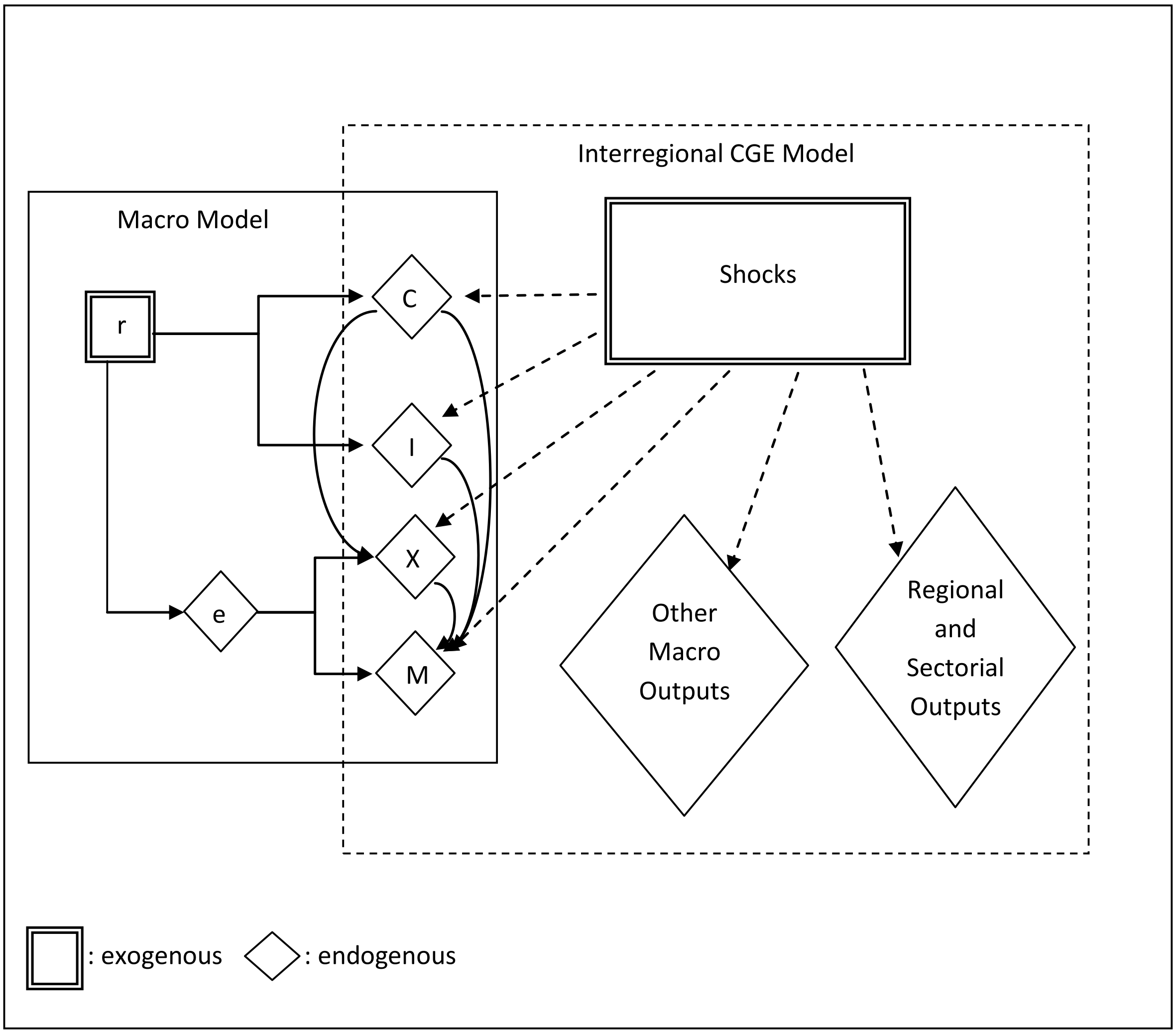
In order to map the effects of the monetary shock[13] into the CGE model structure, we draw heavily upon Minella and Souza-Sobrinho (2009). In that paper, the authors explain the main monetary policy channels and quantify them for Brazil. The traditional interest rate channels, according to them, are the household interest rate channel, which results in a change in consumption[14], and the firm interest rate channel, which affects investment decisions.

[11] Among them, five doctoral dissertations: Domingues (2002), Perobelli (2004), Porsse (2005), Ferraz (2010), and Santos (2010).
[12] See Haddad (1999), and Haddad and Hewings (2005).
[13] The version of the B-MARIA model employs data for 2007. In that year, Selic averaged around 12% a year across months. A rise to 13% would represent an increase of 0.89% on the gross rate.
[14] We understand this would affect especially consumption of luxury goods.

Additionally, there is a channel capturing the impact of interest rate on aggregate demand components through the exchange rate, called the exchange rate channel, and an expectations channel.

Figure 1 depicts such channels (except for the one regarding expectations) and their macro effects to be used as inputs/targets for the CGE model: *r* stands for the policy interest rate (Selic); *e* stands for the real exchange rate; and *C*, *I*, *X* and *M*, stand for the national account aggregates, consumption, investment, exports and imports, respectively. The B-MARIA may then be employed to generate estimates of disaggregated regional effects as well as other macro and sectoral effects, working with shocks in an integrated fashion.

Figure 1. Monetary Policy Channels and the Regional Model



Working through the equations estimated by Minella and Souza-Sobrinho (2009)[15], we obtain the macroeconomic scenario (Table 1), which also includes the associated GDP change, computed from changes in its components. Now, notice that the figures in Table 1 are exactly the components of our *t* vector.[16]

Table 1. Macroeconomic Scenario (Target)

Variable	% Variation
C	-0.14
I	-0.51
X	-0.05
M	-0.23
GDP	-0.15

3.2. Linkage Variables from the CGE Core

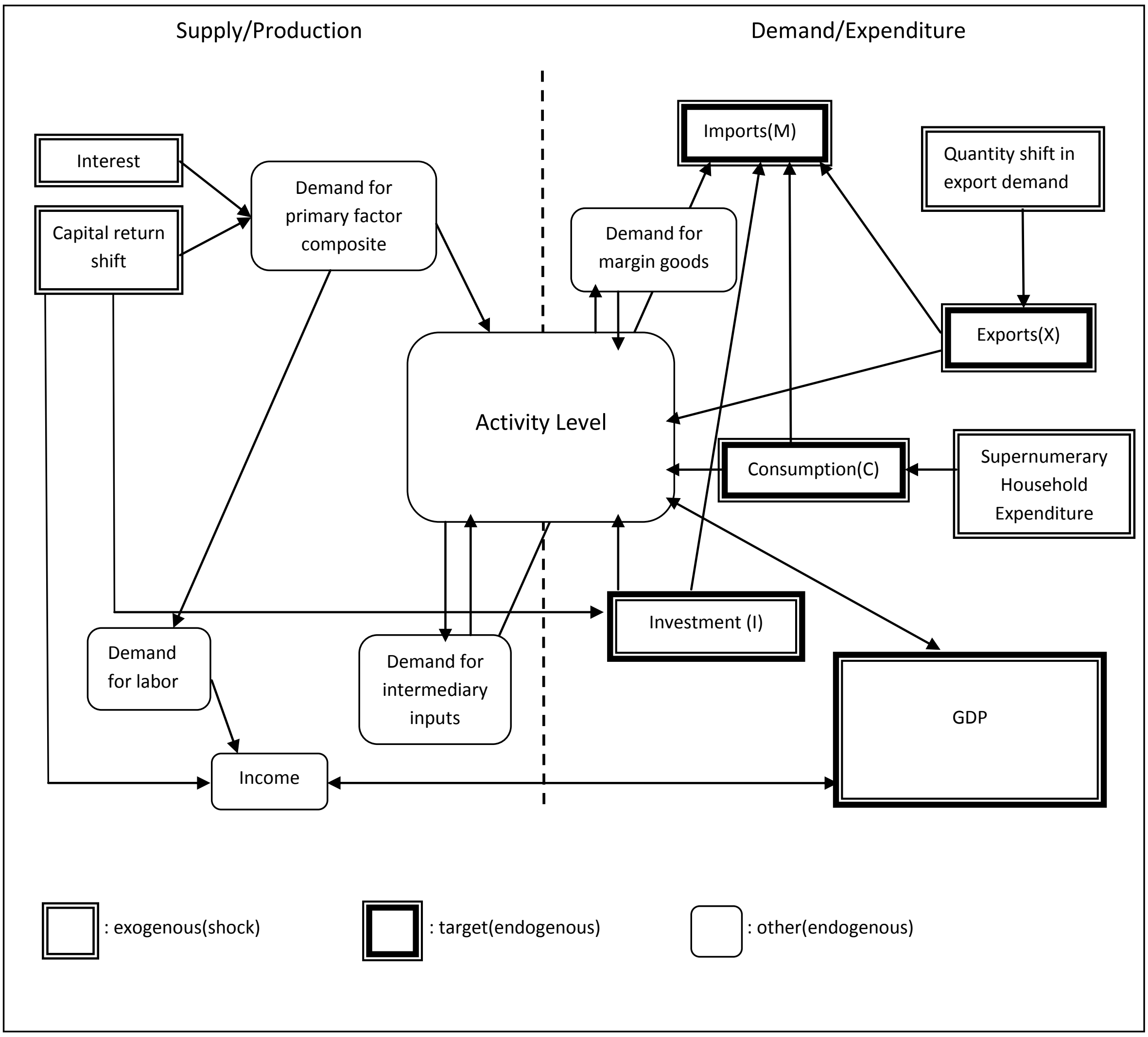
After defining both the analytical setting and the target, we should find a way to reproduce *t* within the B-MARIA model. Since we are interested in assessing the effects of monetary policy through its main transmission channels, we choose to work with the standard short run closure, with one single difference. In the standard short run closure, capital stocks are held fixed, so that it is assumed that, on the demand side, investment expenditures are fixed exogenously and firms cannot reevaluate their investment decisions. To capture the aforementioned firm interest rate channel, we swap the previously exogenous current capital stocks (in each sector and region) with the regional-industry-specific capital shift terms, in order to capture investment reactions to changes in the interest rate.

Further inspection of the CGE model structure is needed to better select the remaining variables that should be part of vector $\hat{v}_\beta$. The borrowing interest rate variable, which in the CGE model affects only demand for capital, and the capital return shift terms are the natural choices to capture responses to policy interest rate changes (although none of them represents it directly). We also select the variable reflecting the overall quantity shift term in the export demand to capture the adjustment to the corresponding national aggregate target. Finally, to

[15] Detailed explanation of steps taken to use such equations may be found in the Appendix.
[16] As for the exact variables in the code, *t* contains: national real household consumption, national real investment, national export volume, national import volume, and real GDP.

adjust for the impact on consumption, we shock the total supernumerary household expenditure since in this model we expect the "luxury" part of the consumption expenditure by households to respond to interest rate movements, given that non-luxury (subsistence) consumption is mainly related to the number of households in each region. Given the lack of further information, all the shocks are to be regarded as uniform across the respective dimensions of the variables. The main propagation channels of these shocks are represented in Figure 2.

Figure 2. Shock Propagation Channels in B-MARIA



3.3. Note on the Nominal Exchange Rate

In the B-MARIA model, the nominal exchange rate works as a nominal anchor (*numéraire*).[17] This means that all nominal results we obtain may only be used in a comparative context, always having in mind that the CGE model is not specified for looking at the *levels* of prices. We assume that the actual immediate impact of a policy interest rate increase on all price indices is negative, although in the simulations prices outcomes may be either positive or negative, depending on their relative position to the *numéraire*. Thus, assuming, for instance, a small decrease in the nominal exchange would be enough to make all these model outcomes negative, without altering their ranking. However, since we do not have a target for a price index in the same time span and we are not particularly interested in this issue, we simply analyze the results keeping the potential effect of the use of the nominal exchange rate as *numéraire* in mind.

3.4. Numerical Example: Relevant Matrices

As mentioned above, our *t* vector is as follows:

$$t = \begin{pmatrix} -0.14 \\ -0.51 \\ -0.05 \\ -0.23 \\ -0.15 \end{pmatrix}$$

Initially, we obtain matrix $\widehat{B}$ by setting all the shocks to each of the selected variables equal to unity.

$$\widehat{B} = \begin{pmatrix} -0.992991 & 0.197482 & -0.005304 & 0.488849 \\ -2.104161 & -0.766600 & 0.116376 & 0.437469 \\ -3.106364 & 0.709317 & 0.613895 & -0.034329 \\ 0.079847 & -0.367360 & 0.166054 & 0.410547 \\ 0.606312 & 0.131552 & 0.117398 & 0.350270 \end{pmatrix}$$

[17] This is clearly noted, for instance, when we perform the homogeneity test with the model.

The columns of matrix $\widehat{B}$ correspond, from left to right, to the specific effects of a 1% change in the CGE variables specifying capital shift terms, borrowing interest rate, overall shift in the export demand schedule, and the supernumerary household expenditure.[18]

Using equation (7), we obtain[19]:

$$\hat{v}_{\beta}^{*} = \begin{pmatrix} 0.070812 \\ 0.305804 \\ -0.090708 \\ -0.264302 \end{pmatrix}$$

The sign of the elements of $\hat{v}_{\beta}^{*}$ are in line with underlying economic theory and stylized facts.

Using equation (5):

$$\hat{v}_{\alpha} = \begin{pmatrix} -0{,}138647 \\ -0{,}509609 \\ -0{,}049667 \\ -0{,}230257 \\ -0{,}152262 \end{pmatrix}$$

If instead of using (5) we use the information of $\hat{v}_{\beta i}^{*}$ to run a simulation in GEMPACK[20], we obtain:

$$\hat{v}_{\alpha} = \begin{pmatrix} -0{,}138996 \\ -0{,}512468 \\ -0{,}052066 \\ -0{,}232161 \\ -0{,}153338 \end{pmatrix}$$

As we can see, differences between results using an improved approximation method and the reconstructed vector are not substantial. In cases when we are not satisfied with such

[18] It is important to use the maximum number of decimal places available from GEMPACK calculations for accuracy purposes.
[19] $\hat{v}_{\beta i}^{*}$ was rounded to 6 decimal places, but when transferred from MATLAB to GEMPACK, maximum possible precision was used (16 decimal places).
[20] We use the following solution method: Gragg 2-4-6 steps with extrapolation.

differences, a possible course of action would be to proceed iteratively, recalculating $\widehat{B}$ with individual shocks closer to the ones we intend to use.

3.5. Outcomes and Sensitivity Analysis

There are few studies about the effects of an aggregate-based monetary policy shock on different regions. For regional effects inside one country, there are the works with SVAR by Carlino and DeFina (1999) and Di Giacinto (2003) for the US; De Lucio and Izquierdo (1999) that also uses SVAR to estimate the effects of monetary policy on Spanish regions; and Beenstock and Felsenstein (2005) that develops a spatial VAR and uses it for Israel.

In this section we look at other outcomes of the CGE model and conduct an exercise of robustness check. As for comparison and heuristic validation of our results, we draw upon two previous studies that employed econometrics to quantify the regional effects of monetary policy in Brazil (Araújo, 2004; and Fonseca and Vasconcellos, 2003). While Araújo (2004) estimated vector error correction (VEC) models using monthly data on industrial production index for some states of Northeastern and Southern Brazil looking at price effects of interest rate shocks, Fonseca and Vasconcelos (2003) analyzed the impact of the Selic interest rate on industrial production on Brazilian macro regions. Two stylized facts emerged from the fact that richer regions are better prepared to face growing demand than poorer regions, since they host most dynamic sectors in the production structure in the country, their production mix is more diversified with faster response to credit , etc.(Azzoni, 2001).

Stylized fact 1. Regional prices tend to adjust faster to monetary shocks in the more dynamic areas.

Araújo (2004) found that monetary policy effects on price indices[21] are weaker in the less developed region of Northeast region than in the richer South. Although the time span and the technique are different than the ones used in this paper, it is interesting to check how regional price indices behave in our simulations as compared to those results. The estimates are displayed in Table 3A and 3B. As explained before, positive variations would be understood

[21] In that case IPCA and IGP-M are used.

as an attenuation of the negative impact of the nominal exchange rate (set exogenously and kept unchanged).

Table 3A. GDP Deflator Results

% Variation	Reconstructed	GEMPACK
NORTH	-0.024	-0.026
NORTHEAST	-0.009	-0.012
SOUTHEAST	-0.005	-0.008
SOUTH	-0.013	-0.016
CENTERWEST	-0.007	-0.011
BRAZIL	-0.006	-0.010

Table 3B. CPI Results

% Variation	Reconstructed	GEMPACK
NORTH	-0.009	-0.011
NORTHEAST	0.011	0.009
SOUTHEAST	0.007	0.005
SOUTH	-0.001	-0.003
CENTERWEST	0.003	0.006
BRAZIL	0.005	0.002

Comparing the values in these tables, we find smaller values for the South than for the Northeast, what is in line with the stylized fact.

Stylized fact 2. Regional output in more developed areas is more sensitive to monetary shocks.

Fonseca and Vasconcelos (2003) found that the output of the Northeast region would be less affected by a rise in interest rates than the rest of the country. They used a manufacturing output index, analyzing a time span of 6 months. Once more, we use this stylized fact as a comparison for our results. Table 4 contains the CGE impacts on aggregate output (value added weights) for the five regions and Brazil. As can be seen, the stylized fact is also revealed.

Table 4. Aggregate Output Results

% Variation	Reconstructed	GEMPACK
NORTH	-0.244	-0.248
NORTHEAST	-0.130	-0.130
SOUTHEAST	-0.189	-0.187
SOUTH	-0.193	-0.195
CENTERWEST	-0.187	-0.189
BRAZIL	-0.181	-0.183

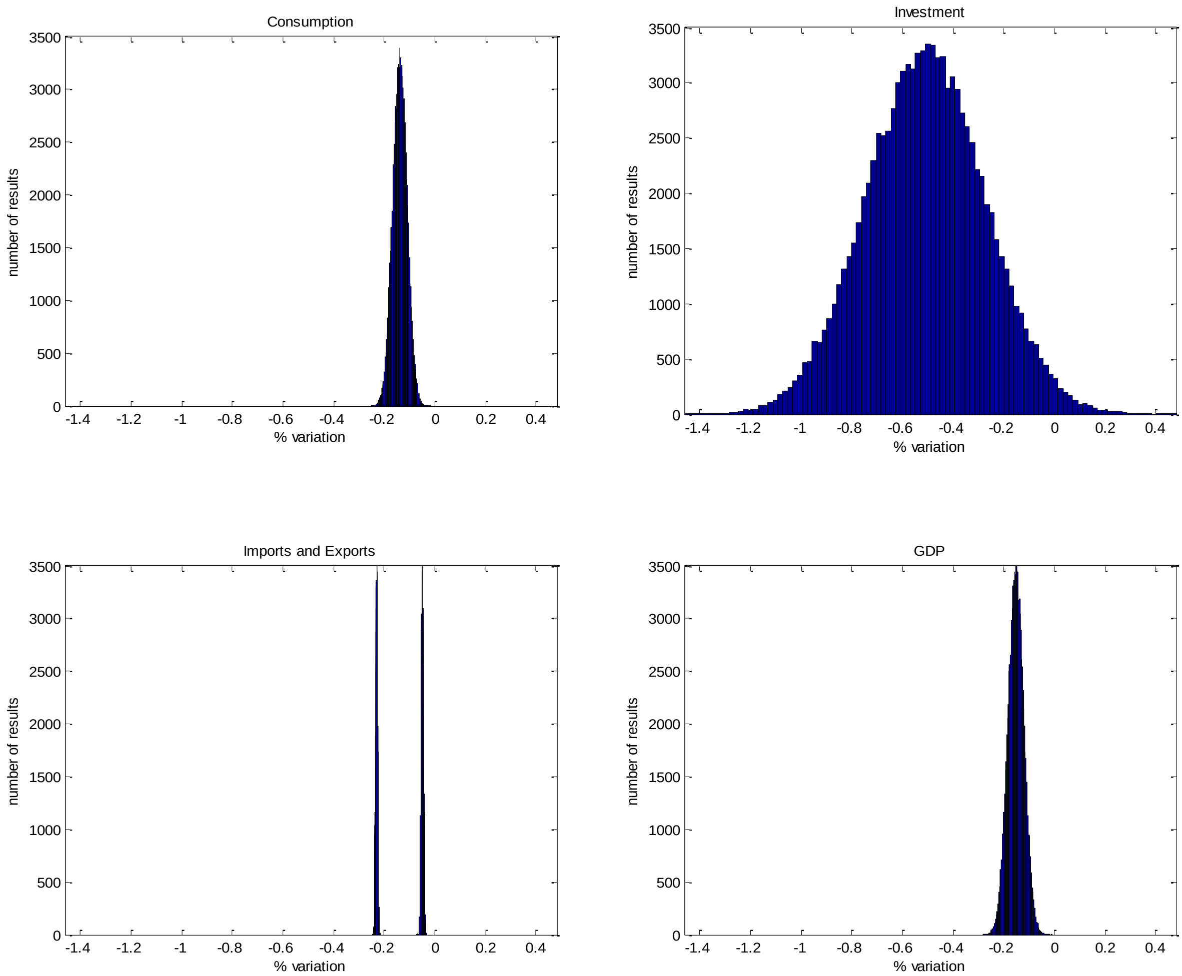
Let us now check the robustness of these two sets of results. We assume, in particular, that uncertainty stems from the targets in Table 1. According to Minella and Souza-Sobrinho (2009), when the expectations channel is not considered, the household interest rate and the firms interest rate channels account for about 86% of the GDP dynamics after a monetary policy shock (one-year horizon). So, to keep matters simple, we suppose that the consumption and investment targets are random and that their variability comes only from one of the coefficients used to estimate them. Other targets are assumed to be fixed. Since we do not have any information about coefficient correlations, we assume them to be i.i.d. and obtain a standard deviation of 0.03 for the consumption target and of 0.24 for the investment target.[22]

We proceed by simulating a random sample of 100,000 draws of our target vector. The consumption and investment targets are assumed to be normally distributed. For each of these vectors, a new optimal shock is calculated using equation (8) and all values for $v_{\alpha i}$ and Tables 3A, 3B and 4 are reconstructed 100,000 times using equation (11).

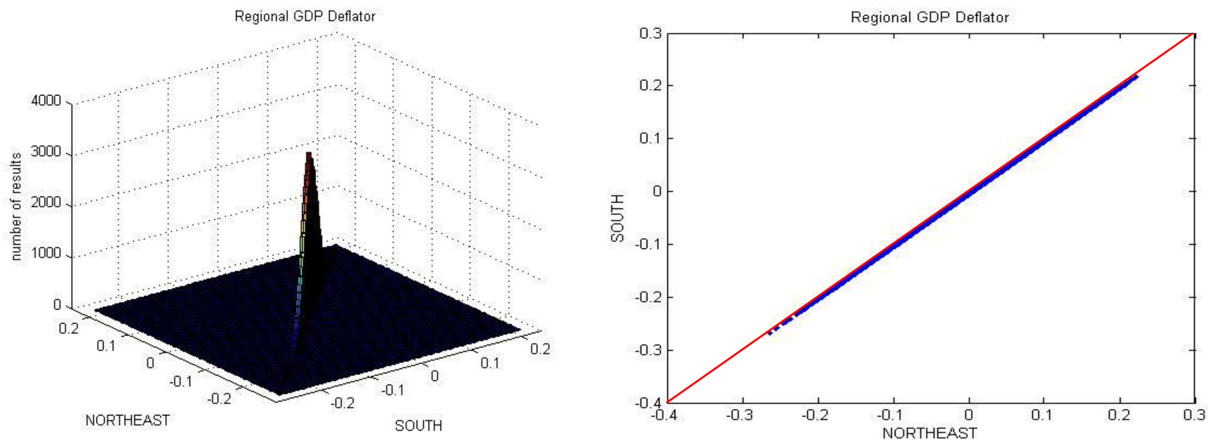
Panel 1 shows the distributions of the resulting $v_{\alpha i}$. As expected, the distribution of consumption and investment are more disperse. As the targets for these two variables change, projections for the others targets also vary.

[22] Appendix I contains the calculations.

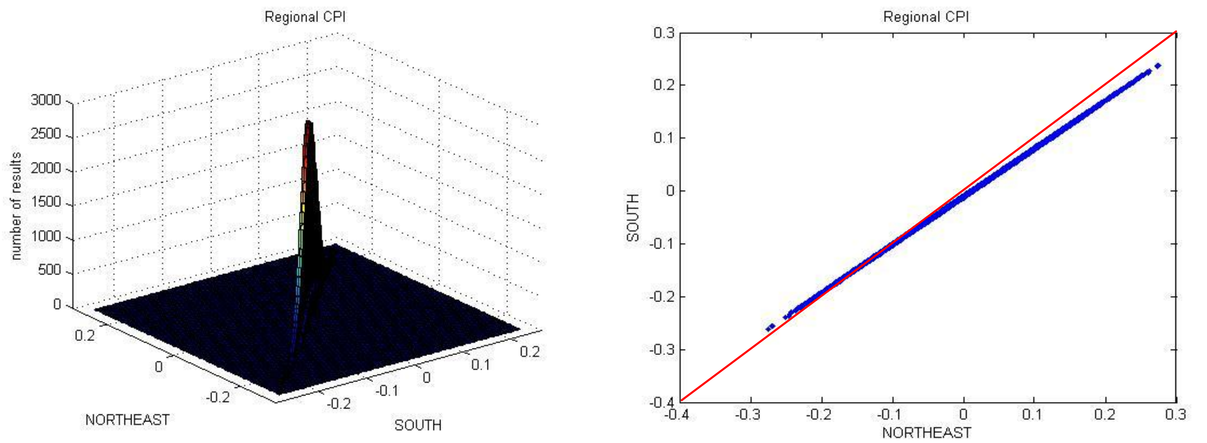
Panel 1. Simulated Target Variables Dispersion



Panel 2A. Robustness Test – Regional GDP Deflator



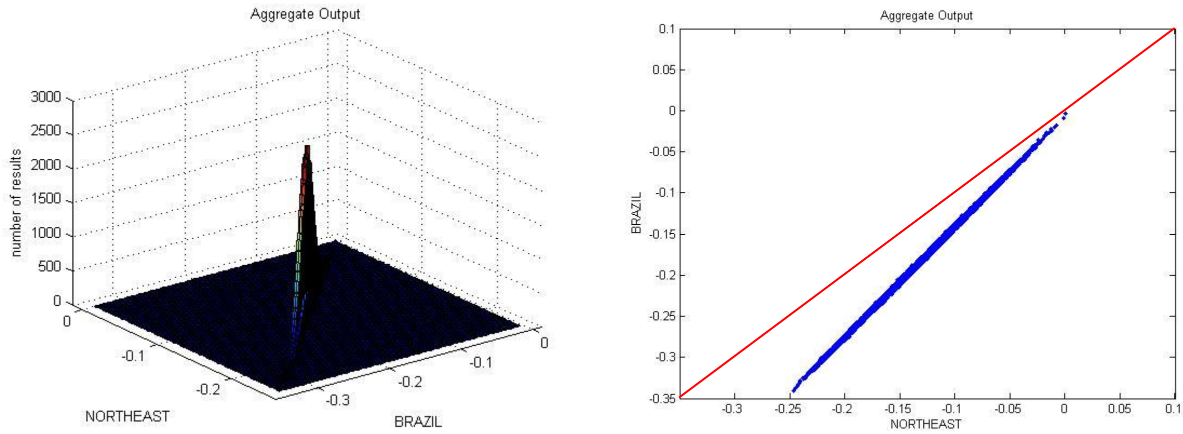
Panel 2B. Robustness Test – Regional CPI



Panels 2A and 2B show robustness tests for the stylized facts observed in Tables 3A and 3B. The red line on each of the graphs on the left marks where values on both axes are equal. We can thus infer that while the stylized fact 1 is very robust as far as regional GDP deflator is considered, it is not so robust for regional CPI.

Panel 3 shows the results for a similar exercise considering the stylized fact 2 observed in Table 4. We can conclude that results for GDP are quite robust to the distribution of the shocks, as it persists across almost the whole range of simulations.

Panel 3. Robustness Test – Regional Aggregate Output



4. Final Remarks

We conclude that the proposed methodology for target fitting is a simple extension of the Johansen approach. The combination of a convenient norm to measure distance from such target with some basic matrix algebra makes it straightforward to compute model outcomes and also to extend it to perform sensitivity/robustness analysis.

To illustrate the analytical capability of the proposed modeling linkage strategy, an exercise using Brazilian data was undertaken, in which the short run regional effects of monetary policy were assessed. In the Brazilian case, the 'lack of data' issue is evident at the regional level, and the use of traditional econometric techniques based on time series data becomes inappropriate. Because vector regressions are often used, the equations system becomes unfeasible as we increase the number of variables included. Therefore, in order to consider all Brazilian states over the post monetary stabilization period (1994 onwards), for instance, we would need at least monthly GDP series for every region to pursue proper estimation. The scarce availability of data with these features has forced authors to work with different specifications and regional aggregation methods in order to have enough degrees of freedom. This problem often implies a poorer analysis based on models with insufficient number of lags or methods that do not include enough spatial disaggregation of the country inside just one model (see Bertanha and Haddad, 2008).

At the regional level, the use of CGE models is appealing for policy makers. Data availability has always been of great concern to regional scientists, and regional econometric models often encounter severe problems in their specification and implementation. First, reliable time-series data for sufficiently long periods are not available at the regional level, and, when available, the data often present inconsistencies, which affect econometric estimation procedures. Secondly, regional structural changes appear to be very dynamic, which call for different structural models, thereby reducing the time span available for hypothesis testing with a selected econometric model (see De Melo, 1988). However, CGE models are not without their limitations – especially their limited ability to handle dynamics. Hence, they should be viewed as complement to existing models rather than as replacement. In this sense, modeling integration/linkage becomes a major goal to be pursued.

This is especially important in integration of models for which macro-consistency is needed. Given their many virtues, if adequately addressed, CGE models are the main candidates for the core subsystem in integrated/linked systems.

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Appendix. Calibration of Macro Scenario

We suppose an increase of 0.89% on the gross policy interest rate, r, and that the new level will be maintained from a given quarter on. Equation and Table numbers in this appendix refer to those in Minella and Souza-Sobrinho (2009).

Real Exchange Rate

Equation (18) specifies the uncovered interest parity (UIP) condition, with a direct interest rate elasticity of real exchange equal to -1. However, the lead and lag structure requires iteration of this equation, which results in an estimate of -1.43. For the variation of 0.89% in r, we would expect a variation of -1.27% in q.

Consumption

From equation (16), an increase of 0.89% in r results in a variation of 0.30% in r^h (average lending rate to households).

From (4), the main impact on consumption comes from r^h, with a coefficient of -0.54, implying, through equation (16), a percentage variation of -0.16 in consumption. Additionally, exchange rate movement would imply a percentage variation of 0.03, resulting in a total of approximate -0.14.

Standard deviation: the r^h coefficient has a standard deviation of 0.09. Considering this as the only source of randomness in the consumption target, we obtain a standard deviation of approximately 0.03.

Investment

Equation (17) models the swap interest rate, r^s, as depending on the policy rate in a forward looking fashion. Since in equation (5) investment depends on the lag of r^s, we may estimate its variation assuming perfect foresight. Thus, the percentage variation of the lag of r^s is estimated at 0.37, and the percentage variation of investment at -0.51.

Standard deviation: the r^s coefficient has a standard deviation of 0.66. Considering this as the only source of randomness in the investment target, we obtain a standard deviation of approximately 0.24.

Exports

Exports, from equation (7), depend on imports from the rest of the world and past domestic absorption. We regard these as constant; however, we also consider the change in past exchange rate, given the forward looking part of equation (18). Therefore, lagged real exchange rate would present a variation of -0.46% and the percentage change on exports would be estimated at around -0.05.

Imports/GDP

Imports, from equation (8), depend on lagged real exchange rate and on contemporaneous real GDP.

Following the same path as that of exports, the real exchange rate channel would imply a percentage variation of 0.08 on imports.

Using this variation and those of consumption, investment and exports, together with initial values in the B-MARIA database, we would obtain a -0.19% variation in GDP, which would imply a percentage variation of -0.37 on imports. However, we may not use this result directly, given the simultaneity of GDP and imports. That is to say, this reduction in imports dampens the initial impact on GDP, which in turn attenuates the effect of GDP on imports. We solve this by iterating variations in GDP and in imports. The result is a variation of -0.30% in imports and of -0.15% in GDP. Together with the initial impact from exchange rate, the percentage variation of imports amounts to -0.23%.