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Keywords: Economic Models; Semantic; Tinbergen; Von Neumann.

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TRANSFORMING THE ABSTRACT INTO CONCRETE: THE DUAL SEMANTIC ROOTS OF ABSTRACT ECONOMIC MODELS

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Abstract

The existing connotation of models in economics emerged only in the XXth century, substituting the previous methodological terms: 'scheme', 'diagram' and 'system'. This immature characteristic of economic models provokes important questions: when exactly and why schemes, diagrams, and systems became models? The purpose of the present paper is to look into the history of economic thought, searching for some explanations regarding these changes. The problem will be observed from a semantic point of view, reformulating a broad view of how the term was introduced. Considering this, the paper analyzes how the combination of cartesian and newtonian mathematics with psychological and empirical economics occurred, paving the way for the insertion of the term model into the jargon of economists in two different manners. The review shows that there were two semantic pioneers: Tinbergen (1935) and Von Neumann (1945). Both were the first economists to use the term 'model' for pure abstract reasoning in each of their respective methodologies.

Keywords: Economic Models; Semantic; Tinbergen; Von Neumann

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I. INTRODUCTION

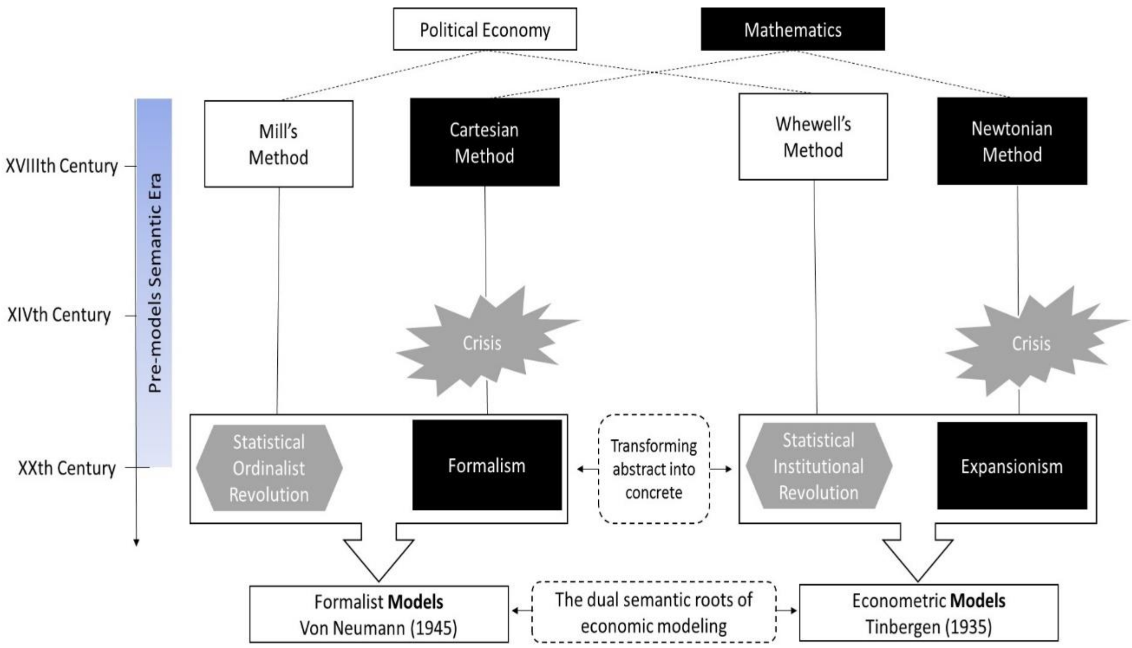
Models are possibly one of the few unanimities of economic thought. From Marxists to Neoclassicists, from Keynesians to Shumpeterians: every modern economist has already used or interpreted an economic model. Somewhat, understanding models is a requirement for contemporary economics. Yet, surprisingly, “economic models” have not always existed. In fact, “model” in economics is an immature term.

The existing connotation of “Economic model” emerged only after the 1930s. Until then, models were almost exclusively a concern of the natural sciences, notably physics. As a consequence, models were far from being purely abstract, having to present a direct relationship with concrete objects or realizable physical theories. Abstract reasoning in economics, back then, received different appellations, such as schemes, systems, and diagrams. Harrod’s (1939) and Domar’s (1946) works, for instance, even though currently known as models, have never used the term to refer to their formulations.

The modern reality is different, and ‘model’ is an essential term of economists’ vocabulary. However, this change provokes important questions: When and why schemes, diagrams, and systems became models? Those are still open quandaries in the history of economic thought and economics methodology. The purpose of the present paper is to look into the history of economic thought, searching for some explanations regarding these changes. The problem will be observed from a semantic point of view. Consequently, the study will be organized in a broad manner, given that languages are created in communities rather than by individuals. In the case of the scientific term ‘model’, this communitarian aspect is even more salient, since the term already existed in different scientific approaches. Therefore, the history behind the insertion of the term into economics is somewhat the history of the import and adaptation of it from a different scientific field.

In this regard, although Duarte and Giraud (2016) affirm that the History of Economic Thought (HET) may be losing space exactly as a result of the use of broad, unspecific methodologies, studying the creation and transformation of vocabulary in archives may not be the adequate solution for the present inquiry. Furthermore, as a semantic analysis has not yet been carried out, it seems plausible to start the research with secondary sources, building a solid basis for future studies. Lastly, this paper supports a philosophical argument: HET, besides being the source of important reflections concerning theory, should also be the source of methodological and philosophical reflections. Hence, it is important to keep in mind that the objective of the paper is to reformulate a broad view of how the term was introduced - and not necessarily find the specificities of the emergence of the new term. Considering this, the prospective historical review concerns several aspects of mathematics, economics and their methodologies, which without some guidance could seem like just an unconnected miscellany of facts. Thus, the following figure intends to work as a guide for the present analysis:

Fig. 1 – Conceptual Timeline of the *Semantic* Roots of Models



Source: elaborated by the author

Following the image, the paper is organized in the subsequent manner: In section 2, after this brief introduction, the division of political economy is observed through the thoughts of Mill and Whewell, demonstrating the dissonance between the psychological proposal of the first and the empirical tendency of the second. Thus, the second section defines the dual economic methodologies behind the origin of the term. In the sequence, complementing the binary economic basis, mathematics is observed as the other foundation of the new connotation. During the period analyzed, Cartesian and Newtonian methods of mathematical analysis were contested, giving rise to two new mathematical aspirations: the expansionist and the formalist. The following sections, then, worry about how these different methodologies of mathematics and economics merged, and what was the semantic impact of this fusion. In the fourth section, it is observed how mathematics' expansionist image and political economy's empirical reasoning merged, transforming the economical domain from abstract into concrete with the aid of the institutional growth of statistics. Therefore, clarifies how the concretization of economics as an empirical field of study allowed the insertion of the term model into the vocabulary of economics. In the fifth section, axiomatic mathematics and economics' deductive reasoning are observed. The advent of ordinalism is demonstrated as the support for the combination of both methodologies and for the succeeding concretization of the economic domain as a formal field of inquiry. Hence, the section describes how the concretization of formal economics added the term model to the jargon of economists. At this point, the pair of semantic pioneers is defined: Tinbergen (1935) and Von Neumann (1945). Both were the first economists to use the term 'model' *for pure abstract reasoning* in each of their methodologies. Until them, only realizable economics had been called models, as is going to be briefly viewed in the examples of Frisch (1933) and William (1934). Finally, in the last section, some final remarks are presented.

II. POLITCAL ECONOMISTS AND THEIR TWO METHODOLOGICAL STANDARDS

The XIXth century and its methodological tensions determined the professionalization of economics. According to Schabas (2005), the period concurrently regulated economics' denaturalization. Until the 1830s, economic facts were thought to be part of nature. *Homo economicus* was viewed as any other animal, whose motives were guided through passions. Therefore, natural laws ruled both the economy and its agents, who were not considered independent social beings: "For the eighteenth century economists, economic regularity stemmed, not from the uniformity of individual reason, but from the cohesive nature of human groupings in conjunction with nature" (Schabas 2005, p. 153). As a result, political economy's methodology was analogous to natural sciences' methodology, following the inductive reasoning. Newton's classical mechanics, thus, was political economy's methodological paragon.

John Stuart Mill had an important role in transferring economists' view from natural phenomena to individuals. Liberty and individuality were central in Mill's arguments, as "the only source of any real progress, and of most of the qualities which make the human race much superior to any herd of animals" (Mill 1871, p. 444). Thus, liberty and individuality distinguished humans, characterizing them as progressive and superior. Progress and superiority, by their turn, were evident in human's control over nature: "Of the features which characterize this progressive economical movement of civilized nations, that which first excites attention, through its intimate connexion (sic) with the phenomena of Production, is the perpetual, and so far as human foresight can extend, the unlimited, growth of man's power over nature." (Mill 1871, p 235)

Yet, Mill's works were not a complete defeat of previews beliefs: "In many respects, Mill may be viewed as putting the capstone on the classical theory of political economy. But, in certain fundamental respects, he ushered in our current conception of a denaturalized economic realm. There is, to be sure, a sense in which economists still believe that they study a physical world, even though, when it comes to explanations of production, concrete definitions of capital and labor are riddled with inconsistencies.". (Schabas, 2005, p. 133). Therefore, for Schabas (2005), even though denaturalized ideas emerged in Mill's suggestions, classical influences were still preventing a total revolution. Nevertheless, the seeds of a new economic thought were sown. Nature and its laws were being left behind, and the human mind was rising as the dominant field of inquiry for economists. "This was a sharp contrast to political economy before 1830, where reason was subordinate to the passions. In the classical theory, the individual mind did not make choices that determined the pricing and distribution of economic goods." (Schabas 2005, p. 140)

From a methodological point of view, the realm of economics became more complex. Agents, instead of merely following nature's laws, acquired laws of their own. Consequently, the economic thought searched for different explanations and methods. As Hausman (1992) points out, once the complexity of the social ambiance increased, political economists started to look for "genuine universal laws of *human nature*" as the foundation of their analysis. Introspective exploration of essential economic motives - deprived from disturbances - was XIXth century economists' option for analyzing agents' psychological laws. In Blaug's (1992, p. 56) words: "we should not take the whole man as he is, staking our claim on correctly predicting how he will actually behave in economic affairs. [...] What Mill says is that we shall abstract certain economic motives, namely, those of maximizing wealth subject to the constraints of a subsistence income and the desire for leisure..." .

For Mill, discovered psychological laws were irreducible fundamental laws, which should be the foundation of any economic study. Therefore, the "genuine universal laws of human nature" were solid "*a priori*" knowledge bolstering posterior deduction of economic laws. Roughly, introspection allowed economists to discover psychological foundations, and deduction permitted them to evince economic rationality. Data, observation, and evidence had little importance considering that the laws of economy resulted from deduction. Those who used observation as a source of information were practical and not theoretical economists, who used the "*a posteriori*" method:

"Suppose, for example, that the question were, whether absolute kings were likely to employ the powers of government for the welfare or for the oppression of their subjects. The practicals would endeavor to determine this question by a direct induction from the conduct of particular despotic monarchs, as testified by history. The theorists would refer the question to be decided by the test not solely of our experience of kings, but of our experience of men. They would contend that an observation of the tendencies which human nature has manifested in the variety of situations in which human beings have been placed, and especially observation of what passes in our own minds, warrants us in inferring that a human being in the situation of a despotic king Will make a bad use of power; and that this conclusion would lose nothing of its certainty even if absolute kings had never existed, or if history furnished us with no information of the manner in which they had conducted themselves. The first of these methods is a method of induction, merely; the last a mixed method of induction and ratiocination. The first may be called the method à posteriori; the latter, the method à priori." (Mill 1844, p. 109)

Even though Mill discussed practical quandaries, De Marchi (1972) contends that Mill did not believe that observation could be a source of review for deductions[3]. Social reality's complexity hampered the analysis of observation: "And here only it is that an element of uncertainty enters into the process— an uncertainty inherent in the nature of these complex phenomena, and arising from the impossibility of being quite sure that all the circumstances of the particular case are known to us sufficiently in detail, and that our attention is not unduly diverted from any of them" (Mill 1844, p. 115). Moreover, the social sciences for Mill were different from the natural sciences. In Mill's point of view, during the XIXth century, data regarding social phenomena was far from being as reliable as natural phenomena's ones (Schabas 1985).

Consequently, from a theorist's understanding, psychological introspection and deduction were not susceptible to being blurred by unreliable and convoluted

[3] Hollander (1985), on the other hand, understands observation as an essential form of reevaluation of economic laws for Mill.

observations. Therefore, Mill's proposals opposed to Newton's paragon of science. Psychology was preferred over mathematics and observation was solely a problem for the practicals, having little or no capacity of modifying economic laws[4]. Thus, the rising denaturalized economics was a science of the human mind and not of the nature.

Obviously, the "*a priori*" method is distant from contemporary econometric models and even from pure mathematical ones. As a result, although logic was an important part of the definition of economic laws, the term "model" was not yet used. For instance, Mill's (1871) classical work does not present the term model, while the terms system and scheme were recurrent in different circumstances. Analogously, Mill (1844) refers to "philosophical systems" as a mode of reasoning. Thus, models did not belong to the realm of Political Economy, underlining that some facts were still due to materialize to allow the term to be imported by economists.

To begin with one of those facts, concurrently to the rise of Mill's ideas, statistics was growing institutionally and practically in urban centers. In the United Kingdom of Mill's time, for instance, between 1830 and 1840, both the F section of the British Association for the Advancement of Science – a section dedicated to the exclusive study of statistics and economics – and the Statistical Society of London (lately renamed as Royal Statistical Society) were created. Among the founders of both societies was William Whewell. Therefore, the author is an interesting figure to contrast the practicals' and theorists' proposals, as well as the rise of statistics – essential facts bolstering the modification of economics vocabulary.

William Whewell, the author of the *History of the inductive sciences* and of *Philosophy of the inductive sciences*, was one of the main opponents of Mill's deductions. Whewell understood that observation could be used beyond the natural sciences (Blum, 1976). Unlike Newton, who believed in the divine characteristics of phenomena, Whewell believed that observation was formed through the reunion of facts, experiences, and perception. Therefore, for Johnson (2011, p. 400), Whewell's method could be divided into four different parts: "(i) the decomposition of facts, (ii) the explication of concepts, (iii) the colligation of facts, and (iv) the verification of the resulting proposition". Hence, observation was part of Whewell's method at the beginning and at the end of the process.

[4] Blaug (1992) and Reiss (2013) assert that this disharmony between deduction and observation may prevent economic laws from being falsified.

Whewell's colligation of facts relied upon scientists' imaginative capacity and when formulated was similar to a new inductive process, which used scientists' experience and perception. As an inductive process, observation was necessarily correlated with his proposed mode of reasoning. Practically, for Whewell, economic laws were discovered by the inductive sum of inductive hypothesis. Consequently, data and evidence were central to Whewell's ideas and approximated economic formulations to current econometric modeling.

Still, even though closer to today's econometric models, the term was not used by the author. In this regard, Whewell (1862) when facing mathematical formulations opted for the term diagram instead of model, and system remained a term used to refer to political and economical modes of organization. Practical's ideas were considered inferior to theorist's, specially in the Victorian age, when Mill's ideas were rapidly spread. Hence, at the end of the XIXth century, although political economy was divided into what concerned its relationship to data and observations, economists' vocabulary was mainly based on psychology and deductive reasoning. Economists preferred diagrams, schemes, and systems as a reference to their deductive vocabulary. At the same time, references to diseases and different medical metaphors reaffirmed their relation to psychology (Besomi 2011).

Both practicals and theorists discussed mathematics and natural sciences, but the application of their mode of thinking was unimportant. However, the outburst of a mathematical crisis at the end of the XIXth century would reach economists in different ways. First, mathematics transformed "political economy" definitely into contemporary "economics", inserting mathematical reasoning in it. Second, after mathematizing economics, the relationship between the natural sciences and the social sciences became greater and economists' language had to be transformed. Its multiple foundations – psychology, mathematics, natural sciences, and deductive reasoning – obliged a modification in the form of communication. Among the alterations, the insertion of the term model occurred.

III. MATHEMATICAL CRISIS AND THE TWO EMERGING ASPIRATIONS

Similarly to economic thought, XIXth century mathematics also had a divided methodology. However, the division was less evident, especially as a result of the hegemony of Newtonian mathematics. Still, as Martins (2012, p. 15) affirms: "since the modern age started there has been a tension between a Cartesian approach to mathematics, and a Newtonian approach to mathematics". The Newtonian approach viewed mathematics as the instrument to discover the truth behind natural phenomena, since geometry was attuned with nature. On the other hand, Cartesian mathematics was an abstract tool used for studying immaterial and infinite concepts.

This mathematical division was, in part, explained through regional divergences during the XVIIth and XIXth centuries. While the European continent was under the influence of Descartes, Victorian England was under the influence of Newton. United Kingdom's opulence and the esteem of Newton's discoveries put Newtonian mathematics in a dominant position. In this ambiance, Newton's methods spread through academic circles and overshadowed alternative methodologies. Along these lines, it is interesting to observe how Newton's ideas spread.

Warwick (1998) highlights the importance of the "Cambridge Tripos" in the propagation of Newtonian mathematics. The Tripos was Cambridge's selection mechanism, distinguishing students apt to pursue higher level studies from those who were not apt. The test started at the beginning of the XVIIth century, gradually increasing its relevance throughout the years. Time not only altered the test importance but also modified its requirements. During XVIIIth century the Tripos was an oral test. The reduced quantity of applicants promoted such structure. However, with the increased number of applicants, testing all orally was unmanageable. As a consequence, the Tripos gradually acquired a paper format. This action directed towards aiding the evaluation of a greater quantity of students had dramatic influences on the formation of Cambridge Students. (Warwick 1998)

The transition from oral tests to the paper format allowed the focus of questions to be transferred. The preferred philosophical content of oral questions progressively turned into technical contents. Therefore, mathematics, which was already part of Cambridge's education, achieved a more important role: "[...] the discipline was especially well suited to a system that sought to discriminate between the performance of large numbers of well prepared students" (Warwick 1998, p. 299). Obviously, the mathematics required was from England's genius: Newton. The depth of knowledge required was astonishing, being a challenge even for the smartest minds of Victorian England (Weintraub, 2002).

Newtonian mathematics was demanded in its most profound technicalities, changing students' minds and even the focus of university classes. Progressively the study of Newtonian mathematics became necessary for every student aspiring to participate in the academy. This burgeoning of mathematical reasoning was disseminated to different fields considering that students pursued a myriad of interests beyond mathematics. Therefore, in the XIXth century, the idea of a mathematical economics was already alive in the mind of some economists (Weintraub, 2002). Still, the natural characteristic of Newtonian mathematics was in disharmony with Mill's hegemonic ideas of a psychological political economy. According to Blum (1976), any metaphysical or non-phenomenological hypothesis could not be part of Newtonian reasoning, opposing political economy and mathematics. Moreover, statistics institutions were still consolidating during the XIXth century. As a consequence, economic data was not yet totally reliable or even available. Newtonian mathematics could not be properly implemented without qualified data.

Furthermore, confirming the methodological frictions, the term model remained an expression pertaining mainly to the natural sciences. As pointed by Boumans (2005), by the end of the XIXth century and beginning of XXth, the concept of a model was already a common topic among scientists, such as Boltzmann, Hertz, and Ehrenfest achieving the right to be present in encyclopedic definitions. Thus, the overt methodological distinctions implied obvious language incongruities. Although economics never became a social physics, at some point similarities had to occur allowing terminology overlaps. In other words, in order to mathematics to be definitely inserted into economics, and models to become a semantic reality for economists, either mathematics or political economy hegemonic methods had to change. And in fact this happened, and the end of XIXth century marked the beginning of the debacle of Newtonian reasoning. According to Weintraub (2002, p. 10), three distinct difficulties of explanation gradually emerged in mathematics: "1) the foundations of geometry, specifically the failures of euclidean geometry to domesticate the non-Euclidean geometries; 2) the failures of set theory made manifest through Georg Cantor's new ideas on "infinity" and 3) paradoxes in the foundations of arithmetic and logic..." .

A crisis was manifested in the mathematical enterprise. Using Corry's (1996) distinction between image and body of mathematics, the crisis not only affected mathematical theorems and definitions (its body), but also affected what mathematics understood as proof, rigor, and its ideals of evolution (its image). More specifically, at least two images of mathematics materialized after the crisis. At the end of XIXth century an expansionist image of mathematics emerged, affirming that mathematics should increase its importance in different scientific fields. At the same time, Cartesian economics evolved into a more sophisticated idea, asserting that mathematics should become self-contained and molded by its inherent logical axioms. The first image developed maintaining an association with the Newtonian method and was bolstered by the evolution of statistics, while the second resulted from Hilbert's works.

Each image evolved following different paths. Yet, both are at the root of the transformation of economics into a mathematical form of reasoning (Weintraub, 2002). Consequently, these images, beyond playing an important role in transforming the method of economics, also allowed the concretization of economics as a field of inquiry, transforming economics methodology and making it closer to the natural sciences. This approximation was added to the rise of practical and theoretical ideas in statistics, and allowed the insertion of a new term into economists' vocabulary, the term model. Therefore, a brief definition of the images mentioned above is necessary before explaining the import and adaptation of the term in the next sections.

The Expansionist Aspirations

The incapacity of Newtonian mathematics of answering the problems presented by the crisis and the ascendance of new physics' theories, such as Einstein's and Planck's, at the beginning of the XXth century, infused in some mathematicians the ideal that mathematics should expand its frontiers to adjust to the new times. In Corry's (1996) terminology, this ideal was the mathematicians image of mathematics. Evidently, this image was based on the application of mathematics and should be supported by the evolution of measurement. According to Weintraub (2002), the expansionist image evolved specially in the figure of Vito Volterra. For Volterra, mathematical rigor was completely correlated with the capacity of experimentation, and as some sciences were still incapable of properly testing their theories, mathematics and measurement should attain that to them.

Volterra was not alone in his aspirations. Francis Ysidro Edgeworth and Felix Klein also aspired to spread mathematics in a similar manner (Weintraub, 2002). The authors believed that sciences should maintain physics as the paragon of rigorous methodology. Therefore, the greater the similarity between a particular science and physics, the better.

Hence, as in classical mechanics, good instruments of measurement were necessary to advance different scientific ideas. “For Volterra, as for Klein, the need in a field like economics was for measurement. For Volterra, as for Edgeworth, concepts had to be developed that would allow exact calculations, for that was the route to a mathematical science like the physics that was the paradigmatic mathematical science” (Weintraub, 2002, p. 34).

Curiously, though, is the fact that the mathematical crisis was a result, partially, of the difficulties of measuring the “new” physical reality. Einstein's and Planck’s ideas could not be measured by the same tools and foundations of the Newtonian mathematics. Perhaps Volterra, Klein, and Edgeworth were unconsciously trying to hide mathematics and physics own flaws by studying different fields. Anyhow, the expansionist image flourished and it was essential in the prospective mathematical economics.

The Formalist Aspirations

The second mathematical image emerging at the end of the XIXth century was the formalist image. Volterra’s image could hardly correct mathematical imperfections, and can be better understood as the simple propagation of malpractices, since Newtonian mathematics was starting to be selectively used. The second image was diametrically opposed, proposing mathematics to be a self-contained device to properly overcome the crisis. Hilbert’s works were at the center of the formalist image.

In order to successfully become an independent tool, mathematics had to separate itself from reality. Therefore, rigor was not to be found outside mathematics, but inside it. Mathematics itself should provide the support for its proofs. In Hilbert’s ideas, the inherent axioms of mathematics should be uncovered and studied. Consequently, data and observation should play no role in pure mathematics. Proofs, then, would be based on the consistency and independence of the axioms. According to Weintraub (2002, p. 87), independence “means that each axiom is neither derivable from, nor can be used to establish or prove, any other axiom”, while consistency means “that there is no contradiction to be produced in the theory by assuming the truth of the set of axioms, such that all members of the set are true under that interpretation or model, for if that is the case, then there is no logical contradiction that can arise, no theory based on those axioms will contain an internal contradiction” (Weintraub 2002, p. 87).

Hilbert's main objective was to prove the consistency of arithmetic. However, Gödel (1931) proved the impossibility of this task. Still, Hilbert's ideals represented mathematics as an instrument for the discovery of scientific truth through logic. Gloria-Palermo (2010) affirms that Hilbert's pupils, then, worked on applying Hilbert's ideas, searching for inconsistencies in different scientific fields. Considering this, unlike Volterra's image, Hilbert's ideals intended to reach different fields in order to validate them, not the other way around.

IV. EMPIRICAL REASONING AND THE INSTITUTIONAL GROWTH OF STATISTICS

In order to consolidate the expansionist ideas of mathematics in economics, solid data was required. The measurement of social phenomena should be similar to the measurement of natural phenomena. Therefore, statistics was the connection point between Volterra's ideals and economics. And, in fact, according to Porter (2001), statistics ascended rapidly after the formation and centralization of states in the XVIIth century. During this period, the measurement of population and wealth became increasingly necessary for governors wishing to understand their political and economical performances. Also, Newton's success over the century permitted the evolution of practical aspects of measurement.

As a result, when the XIXth century began, statistics was reasonably consolidated. Klein (2001), for instance, highlights William Playfair's works as one important representation of the consolidation of statistical reasoning. Graphical and data analysis were already used as instruments of popular persuasion and even took place in some academic circles. Clavin (2014), by its turn, points out that graphic reasoning was essential for the understanding of the increasing abstractness of mathematical ideas. Hence, although Volterra's image of mathematics would appear solely at the end of XIXth century and Mill's methods were hegemonic in economics, Whewell's ideas and statistics were already being incipiently used for mathematical reasoning in economics.

Jevons is the classical example of how mathematical reasoning was already present in XIXth century political economy. During 1850s Jevons acquired scientific training in mathematics and chemistry and years later opted for studying economics. As one of the possible founders of the marginalist school (Birken 1988; Coats 1972), Jevons naturally utilized a different method than other thinkers. Already in the middle of the XIXth century Jevons accepted mathematics and statistics as important instruments of economic thought.

Jevons lived during the institutional emergence of statistics, being part of the Manchester Statistical Society. Therefore, his methodology was more attuned with Whewell's ideas than with Mill's. The sum of statistics, economics, and mathematics can be seen in Jevons' work around the value of gold (Jevons 1863). Collecting data and manipulating it in graphical forms was an essential part of Jevons work in the middle of XIXth century (Mass 2014, Peart 1993). Nevertheless, he persuasively discussed the problem using a price index and calculating the variation of prices.

Still, deductive and psychological analyses were hegemonic in political economy. As the Newtonian technique was still solid, mathematics was not emphatically spreading to different fields. Consequently, even though the differences between natural and social sciences were reduced in Jevons works; neither statistics nor mathematics was yet prepared for a new mathematical economics. As a result, the term model stood as a peculiarity of natural sciences. Jevons applied mainly the term diagram as a reference to his formulations as it is possible to notice in his lecture notes and works (Black 1977, 1981).

Notwithstanding, Jevons played a pivotal role in the modification of economics' language. In his pioneer demeanors, Jevons was one of the first economists to study the problem of economic cycles, which in the subsequent years would be essential for the consolidation of mathematics in economics and the insertion of the term model in economists' vocabulary. Jevons used his previous knowledge of the natural sciences to study economic cycles, trying to conciliate meteorology and prices. Climatic changes and the solar cycle were the bases of his formulations, which unfortunately did not succeed for a myriad of reasons (Morgan 1990).

Starting with Jevons work, business cycle's analysis became more common. Morgan (1990) points out the works of Juglar, Mitchell, Moore, and Pearson following the initial discussions of Jevons. Still, all of them worked before the ascension of Volterra's image and the consolidation of statistics. As a result, their mathematical formulations were schemes rather than models. Yet, gradually economics was aligning its mode of reasoning with mathematics and natural sciences. Also, statistics was being implemented as a reliable source of information for economic analysis.

With the consolidation of statistics and the rise of the expansionist image, the study of business cycles resulted in what came to be known as the economic barometers.

These instruments had the objective of measuring the condition of production and commerce just as a barometer measures air pressures. Evidently, in such instruments the similarity between physics and economics was much closer, impacting the methods of economics. Therefore, a semantic bridge permitting the exchange of scientific terms was being created. This allowed the import and necessary adaptation of terms, as is going to be argued below.

Empirical concreteness: the expansionist image, statistics and empirical economics

Jan Tinbergen was the leader of the Dutch economic barometer project in the second quarter of the XXth century. Similarly to Jevons, Tinbergen had his academic formation in the natural sciences. He had been mentored by Paul Ehrenfest, who supervised Tinbergen's application of concepts from physics into economics. Yet, unlike Jevons, Tinbergen had the opportunity of working and thinking during the mathematical crisis and the consolidation of statistics. Therefore, he was capable of going further than Jevons, using the new images of mathematics and the new statistical instruments.

As the leader of the Dutch project, Tinbergen noticed that the barometer did not have theoretical foundations and, as a result, could not represent causal relationships. Thus, Tinbergen's objective was to transform the barometer into a project serving solely for data recording, becoming the source of information to the study of theoretical problems. Instead of predictive goals, the barometer became a descriptive instrument. (Maas, 2014)

Unpredictable – or not - circumstances created the perfect ambiance for Tinbergen aim at combining statistics and the expansionist image in order to study economic problems. Tinbergen: (1) had a unique background in natural sciences; (2) had the necessary social engagement to study economics; (3) commanded the study of an economic *barometer*; (4) lived through the zenith of the expansionist image; (5) counted on consolidated statistics' institutions. These circumstances also allowed him to elicit the modification of economists' language in a lasting and important way. As a consequence of his unusual position in the middle of different modes of reasoning, his work flourished amidst an exuberant semantic hodgepodge. Tinbergen both as a physicist and as an economist could transit through both languages, finding similarities between the modes of thinking and applying imported terms to his own peculiar forms.

Therefore, Tinbergen's work had its semantic construction borrowing from economics, physics, statistics, the expansionist image of mathematics and obviously his own background.

Initially, though, Tinbergen's works - just like those of other economists - were schemes rather than models. Therefore, according to Boumans (2005), the use of 'model' as the definition of Tinbergen's studies occurred through a gradual change of perception. In physics, models were mathematical analogies connecting abstract mathematical formulations with material objects or realizable physics theories. Evidently, physics as a natural science, and specially Newtonian physics, was concerned only with concrete, materializable problems. Therefore, completely abstract formulations were not a reality in physics. Economics, on the other hand, using its psychological foundations and facing complex causality problems, was certainly more abstract than any natural science, what constituted a semantic conflict.

The first step towards the use of the term was basing economic thought on realizable theories. Therefore, economists applying the term model should understand the necessity of concrete and trustful analogies for the use of the term. Frisch (1933), for instance, used the term model to refer to his analogy between pendulum movements and economic cycles. Williams (1934), on the other hand, used the term to define his formulation of a cardboard production function. The perception of economic problems in a concrete form similar to realizable physics' theories and material objects was necessary to name completely abstract economic formulations as models. In other words, Frisch and Williams were doing Physics camouflaged as economics.

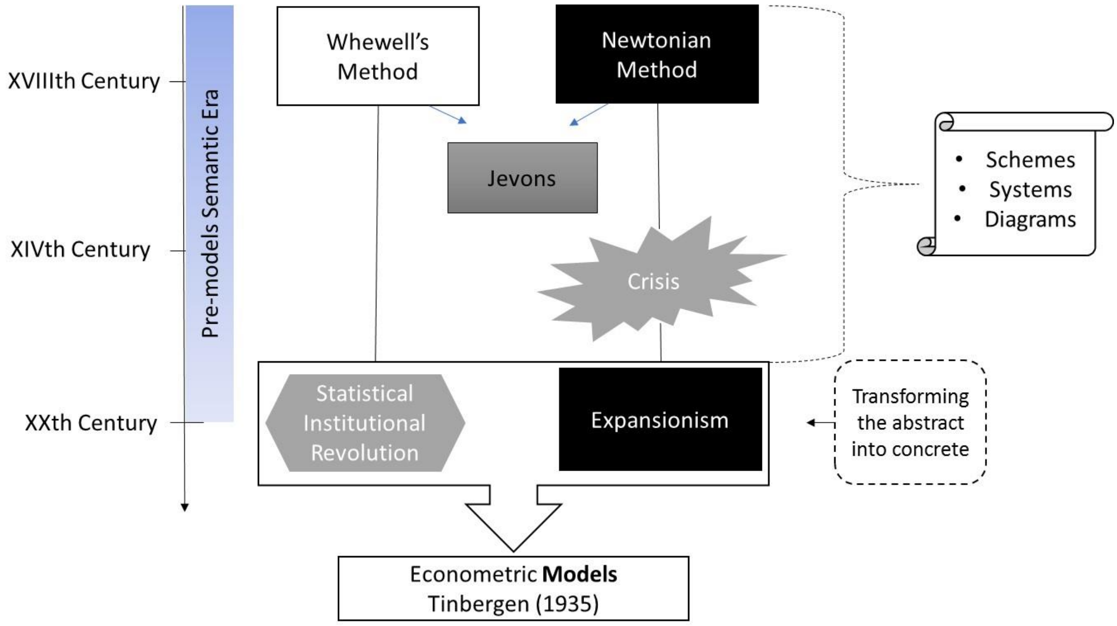
The term, hence, was exclusively used to refer to trustful testable analogies. Economics' lack of concreteness, initially, prevented the use of the term without references to physics. However, the development of statistics and of the expansionist image proposed a different perception of economic problems. The consolidation of statistical institutions guaranteed a higher level of reliability to economic data and the expansionist image proposed the application of classical mechanics to different fields without the necessity of demonstrating clearly, as Frisch (1933) or Williams (1934), that such connection was being made.

Tinbergen and his peculiar circumstances created the perfect setting for him to use the term to refer to complete abstract economic formulations as models. Therefore, with the passage of time, "Tinbergen began to experiment with what he called 'schemata', 'mathematical machines' or 'models'" (Maas 2014, p. 49). According to Boumans

(2005), Tinbergen wrote *Quantitative Fragen der Konjukturpolitik* in 1935: "[This] was the first time an economist used the term model to denote a specific mathematical product of one's empirical research." Tinbergen borrowed the term, but applied an economics identity to it, notably removing the necessity of a material analogy for it.

According to Alberts (1994, p. 300), Tinbergen's notion of models "not only conceptually superseded the notion of applied mathematics, but replaced it in many domains". Tinbergen perceived economic problems and data in a concrete manner, permitting the use of the new term. Economic models were embedded in their own theoretical formulations and based on reliable data. Along these lines, the expansionist image, in unison with Whewell's ideals, allowed the semantic bridge to be created, but statistics' consolidation was essential to the formation of a concrete perception of social reality. Realizable physical theories disappeared from economics, and one type of models was finally created. The following figure can summarize the transition of 'schemes', 'systems' and 'diagrams' to 'models' in the empirical side of economics:

Fig. 2 – Conceptual Timeline of the semantic genesis of econometric models



Source: elaborated by the author

V. LOGICAL REASONING AND THE EMERGENCE OF ORDINALISM

As already stated, the connection between Mill's idea of economics and Newton's mathematics were difficult, given the disharmonies between the psychology of the former and the naturalism of the latter. Notwithstanding, both ideals were hegemonic in the United Kingdom. While it is true that the European isle was dominant in some cultural and scientific aspects, there is no reason to believe that different forms of thinking did not occur, specially in the European continent. In fact, in the continent, French economists were influenced by Cartesian mathematics as well as by Mill's and Newton's notions.

Along these lines, Menard (1980) points out two different perspectives prevalent among French economists. On the one hand, a segment discarded mathematics completely, following Mill's psychological understanding of economics. On the other hand, some tried to apply mathematics to economics. Jean Baptiste Say represented the first attitude towards mathematics and observation. For Say, data and mathematics did not have the capacity of explaining political economy's conundrums. On the other hand, Antoine Augustin Cournot can be seen as an example of the second attitude. In Cournot's point of view, mathematics was a good instrument for studying extreme economic cases. Regrettably for Cournot, his works competed with Mill's and Say's hegemonic mode of thinking and had a difficulty to flourish in the XIXth century.

Yet, Cournot demonstrates that French economists were willing to introduce mathematical reasoning into economics. In this regard, Menard (1980) also pinpoints Léon Walras' works as an important form of mathematical economic thought. Interestingly, Walras' formulation was not completely incompatible with Mill's ideas. For both authors, applied and pure economics were different subjects, and applications could hardly modify pure theoretical constructions. Moreover, Menard (1980) affirms that "the rationality of homo economicus had to do, not with the calculation of averages, but with behavioral psychology" (Menard 1980, p. 535). Consequently, Walras and Mill agreed that economics was a psychological subject. The main difference lied in the fact that Walras accepted mathematical formulations of theoretical economical problems, using Cartesian mathematics in some of his works.

While Mill could have a different approach from Cournot and Walrás in what concerns mathematics, the three authors still wrote in a similar vocabulary. Walras (2005 [1898]; 2010 [1936]; 2014 [1900]), for example, abused of the term system, whereas model was not included in his jargon. Therefore, even though the harmony between hard sciences and political economy was more evident in their works, their vocabularies remained similar, allowing the creation of systems but not yet of models.

Cartesian mathematics was attuned with Hilbert's future image of the field. Yet, during the XIXth century Newtonian mathematics was hegemonic and Walras's formulations, analogously to Cournot's, were not always respected. Furthermore, Walras faced the problem of representing psychological concepts using mathematical formulations. This conundrum could hardly be overcome without the proper evolution of a mathematical and statistical innovation. Until the end of the XIXth century, besides statistics not being institutionally and practically consolidated, measurement was uniquely done throughout cardinal conceptions. Cardinal measurement must present measurement unities, and must be capable of comparing proportions among its measured objects. This perspective is intrinsic in formulations following Newtonian methods, such as Tinbergen's, once the quality of measurement and comparisons are essential. Without a complete seclusion from cardinal measurement, abstract psychological concepts could not be properly represented.

According to Moscati (2013), ordinal measurement emerged at the end of the XIXth century. As a consequence, quantitative proportions between objects became unnecessary. Consequently, previous immeasurable concepts, especially abstract ideas, became measurable. Among these concepts, psychological theories, such as utility and preferences, started to find in numbers a form of being represented. Somewhat, psychological concepts, which were totally abstract until ordinal measurement appeared, found in this innovation a way to become concrete even without being factually material. During the end of the XIXth century and beginning of the XXth century, thus, economic works gradually started to use ordinal understanding for psychological concepts, a benefit Walras did not have.

The incipient Cartesian economics and the rise of ordinal measurement, therefore, created the foundations for the entrance of Hilbert's image in economics. As already seen, Hilbert's mathematics was similar to Cartesian mathematics, and the statistical innovation permitted psychological concepts to be written into mathematics vocabulary. Concreteness was being created also in the most abstract form of economic reasoning. As a result, the connection of Hilbert's image, ordinal measurement and possibly some ideas from physics or the maturation of the term model, allowed formalist economic schemes to fully become models, as is going to be seen in the next subsection.

Logical concreteness: formalist image, ordinalism and logical economics

According to Weintraub (2002), the development of Hilbert's image in economics was greatly affected by Von Neumann's works, who was Hilbert's pupil. Von Neumann was a mathematician working on formalist mathematics and quantum mechanics. Additionally, he had interests in social problems (Von Neumann, 1996). Thus, Von Neumann, beyond being a formalist, had the peculiarities of working on a problem of physics and concerning himself with economic issues. He is usually known as one of the founders of game theory, in which he started working on 1928.

His formalist background allowed Von Neumann to face reality problems from a different manner. The axiomatic method implies the evaluation of pure logical concepts, deprived from any meaning or interpretation. Therefore, his studies of quantum mechanics when reaching a pure logical level, were merely mathematical equations demanding the analysis of consistency and independence. As a result, Von Neumann interest in social reality probably took him to alter the interpretation of one of his formalist expressions. In fact, one of his quantum mechanics formulations really received an economic interpretation and came to be known in 1945 as "A Model of General Economic Equilibrium".

Gloria-Palermo (2010) highlights that the history behind his 1945' model is more extensive than a simple reinterpretation. The first steps of the model happened early during the first formulations of game theory in 1928. However, a version of Von Neumann's economic schemes was presented only in 1931 during a seminar at Princeton. The lack of interest in the new complex mathematical ideas left the paper resting until 1934, when it was discussed again in Karl Menger's seminar in Vienna. The presentation had a positive repercussion and the paper was published in 1937 under the German appellation of "Uber Eines Okonomishes Gleinchungsystem und eine Verallgemeinerung des Brouwerschen Fixpunktsatzes". Notwithstanding that, the term model was not yet part of the paper, not even in a German form. The term model and the consciousness of the modeling formulation emerged in 1945, when Kaldor's interest in the paper granted an English translation for the journal *Review of Economics Studies*. The moment the paper was translated, the title changed to "A Model of General Economic Equilibrium".

Von Neumann's model, hence, had to go through a process similar to Tinbergen's model. Hilbert's mathematics had to find an ordinal measurement innovation and an author related to problems of physics and economics. Consequently, what was once abstract could become concrete and receive the appellation of a model. Therefore, completely abstract psychological concepts were represented in mathematical formulations. However, even though Von Neumann's paper received the name of a model, the name was not defined exactly by the author.

The term model was inserted in the paper solely on its translation. Therefore, although Von Neumann accepted the title, it may not have been him the one to propose the term. As a result, it is probable that, beyond the relationship with physics, the maturation of the label inside the vocabulary of economics played an important role in the use of the term in Von Neumann's paper. During the 1940s, for instance, Marschack (1941) in a discussion about methodology, called Marshall's and Walras's ideas models (authors who have never used the term to define their works!). Also, Tinbergen (1941) had used the term to refer to a work about economic equilibrium "*Unstable and indifferent equilibria in economic systems*". Lastly, Kaldor was the editor of the *Review of Economics Studies'* and, at the time, he had already used the term to refer to some of his works, as in Kaldor (1940) for instance.

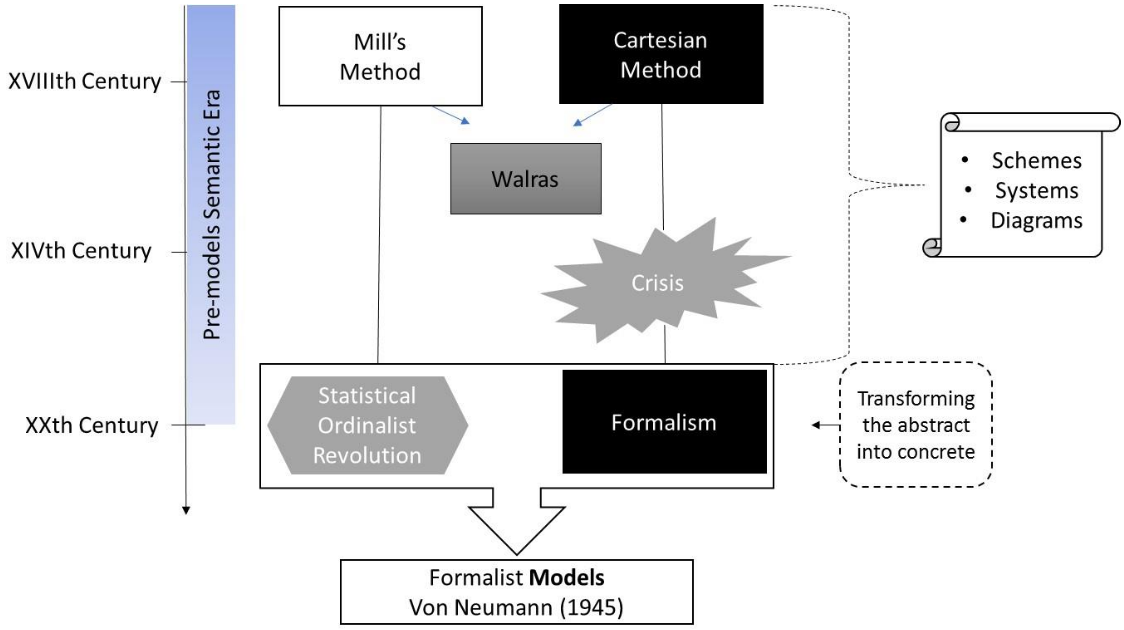
By the same token, a complementary explanation for Von Neumann's paper achieving the semantic status of a model exists. As Mirowski (1989) affirms by extreme propositions, economics at the turn of the century became a social physics, emulating the methods and thus the vocabulary of the natural science. Mirowski's argument in its most extreme understanding may not be real, still it is undeniable that scientists coming from physics were inserted into economics. In the case above, Von Neumann worked closely to physics. As a consequence, this connection with physics may be the foundation of the new appellation. However, such assertion is not taking into consideration the fact that the term in economics received a slightly different meaning than the physics one. While models in physics kept their relation with realizable theories, economic models determined a mathematical analogy with abstract unrealizable ideas[5].

Presumably, both the overlap between the natural sciences and the maturation of the term were behind the new name. Anyhow, the new understanding of the economical domain certainly allowed the removal of physical and material analogies from economic models, transforming abstract concepts into concrete ideas. In Tinbergen's case, the

[5] Remembering Sugden (2002; 2009), economic models are possible worlds and not necessarily real worlds.

expansionist image and the consolidation of statistics were at the center of the new view, while on Von Neumann's case the ordinal innovation and the formalist image transformed economical perception. As already stated, the term was borrowed, but economics identity was inserted into it. This process can be analyzed in the following figure:

Fig. 3 – Conceptual timeline of the semantic genesis of formalist models



Source: elaborated by the author

VI. FINAL REMARKS

Completely abstract economic models, although a universal method of reasoning in contemporary economics, became a reality only after 1930. Previously the term was used for defining analogies with material objects or realizable theories of physics. Therefore, the abstract had to become concrete for the term to be inserted into economists' vocabulary.

XIXth century's economics, both from the point of view of data as the point of view of theory, was an abstract subject. Statistics was not institutionally, practically, or theoretically consolidated. This was a sharp contrast with natural sciences, where data were reliable. On the theory side, economics was being denaturalized and thus was associated with psychology, a completely abstract issue during the XIXth century. In this ambiance, the term model could hardly be used by economists.

Nevertheless, a mathematical crisis by the end of the XIXth century modified the circumstances. Newtonian crisis brought with it the ascension of two different aspirations for mathematics. On the one hand, an expansionist image of mathematics was formed, willing to expand mathematical thought to different fields in order to validate Newtonian mathematical methods. On the other hand, a formalist image of mathematics was formed under the leadership of Hilbert, proposing that mathematics should become a self-contained science. Formalism was supposedly capable of identifying inconsistencies in the logical formulations of mathematical theories. As a result, Hilbert's pupils intended to apply formalist ideas to different fields of knowledge, willing to uncover incongruities in the logical foundations of distinct scientific enterprises.

These two images played an important role in inserting mathematics into the method of economics. Moreover, these images, when combined with the consolidation of statistics, determined the insertion of the term model in economists' vocabulary. On the one hand, the formalist image connected with the psychological ideas of Mill's antique proposals, enjoying the rise of ordinal measurement. The expansionist image, by its turn, connected to Whewell's ideals once the institutionalization of statistics was complete. In the first case, Von Neumann represents the consummation of the different pieces, creating a general equilibrium model. In the second case, Tinbergen united the other pieces, creating the first models of economic cycle.

Both models understood social reality in a concrete manner, not requiring material analogies anymore. Therefore, Von Neumann and Tinbergen's models contrasted with previous formulations such as Frisch (1933) and Williams (1934). This result from the fact that both models were the representation of the appropriation of a term coming from a different science, which in the process of being imported was embedded in economics own idiosyncrasies. Furthermore, it is interesting to notice that both models are methodologically different. While Von Neumann is purely abstract and is mainly deductive, Tinbergen's model uses data to formulate its equations and to validate its conclusions. This characteristic confirms that the term was maturing in economists' vocabulary and transforming the contemporary semantic notion of economic models. This evolution even allows works which never used the term to be considered as so, such as the works of Harrod, Domar, Marshall, Walras and several others.

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