Department of Economics- FEA/USP

A Path Through the Wilderness: Time Discounting in Growth Models

PEDRO GARCIA DUARTE

WORKING PAPER SERIES Nº 2013-18

DEPARTMENT OF ECONOMICS, FEA-USP
WORKING PAPER Nº 2013-18

A Path Through the Wilderness: Time Discounting in Growth Models

Pedro Garcia Duarte (pgduarte@usp.br)

Abstract:

Although economists have recognized long ago that "time enters into all economic questions", the ways they treated and modeled time has varied substantially in the last century. While in the 1930s there was a distinctive Cambridge tradition against discounting utilities of future generations, to which Frank Ramsey subscribed, postwar neoclassical growth economists (of the "Ramsey-Cass-Koopmans model") applied the discount factor either to individual's or to social planner's decision-making as a technical requirement of dynamic general equilibrium models. My goal in this article is to shed some historical light on how a practice that was condemned as ethically indefensible when applied to intergenerational comparisons became a technical requirement in dynamic models of either a consumer or a planner deciding the intertemporal allocation of resources.

Keywords: time discount, growth models, Ramsey-Cass-Koopmans model, economic dynamics

JEL Codes: B22, B23, E32

A Path Through the Wilderness: Time Discounting in Growth Models[1]

Pedro Garcia Duarte (pgduarte@usp.br)
Department of Economics (FEA/USP), University of São Paulo, Brazil

Introduction

Time is a pervasive element of many economic activities. As Jevons (1888, 64-65) once wrote:

> Quantity of supply must necessarily be estimated by the number of units of commodity divided by the number of units in the time over which it is to be expended. … Consumption of commodity must have the same dimensions. For goods must be consumed in time… .
>
> Following out this course of thought we shall arrive at the conclusion that time enters into all economic questions. We live in time, and think and act in time; we are in fact altogether the creatures of time.

Even though economic decision-making can be seen as an intrinsically intertemporal process, the ways economists treated and modeled time has varied substantially in the last century. More specifically, from the 1930s to the postwar period, economists discussing issues of economic growth in a utilitarian framework changed in important ways their modeling strategies with respect to discounting the future. In Cambridge in the 1930s, economists like Arthur Cecil Pigou (1912, [1920] 1924) and Frank Ramsey (1928) condemned the use of a discount factor to the utility of future generations. However, for postwar neoclassical growth economists, who consider Ramsey to be one of their patron saints, the discount factor, applied either to individual's or to social planner's decision-making, is a technical requirement of dynamic general equilibrium models.[2]

In present-day macroeconomics, discounting future utilities at a constant rate is one way of assuring the intertemporal maximization problem is well-defined and has finite solution.

[1] I am grateful to participants at the HOPE lunch group at Duke University for comments and suggestions when the ideas I explore here were at an incipient stage. I thank also participants at the 2009 HISRECO (Antwerp, Belgium) for comments on an earlier draft. I thank FAPESP and CNPq (Brazil) for financial support.

[2] In most present-day graduate macroeconomics textbooks the discount factor is treated as a technically indispensable hypothesis not only of growth models but also of dynamic general equilibrium models more generally: see, for

Therefore, economists can rely on the so-called contraction-mapping theorem that gives an easy recursive solution to their problems (Stokey and Lucas 1989). On the other hand, the idea that consumers are impatient with respect to future utilities became a common interpretation of these models either when applied to a representative agent's or to a social planner's maximization problem.

The stabilization of time discounting in postwar economics serves as a case study to understand better different aspects of the mathematization and the dominance of neoclassicism in modern economics (Morgan and Rutherford 1998, Mirowski 2002, and Weintraub 1991, 2002). My goal here is to analyze this episode in more details, by looking at how economists working with what we now call neoclassical growth models, operating in particular communities, assumed that individuals and generations discount the future with a constant rate.

One way to start this narrative is to look at the early uses of time discounting in the literature on economic dynamics developed from the 1920s to the 1950s. Although for the most part these models did not discuss economic growth, they do illustrate how economists and mathematicians then working with formal intertemporal models and using calculus of variations had no common understanding about whether or not to discount the future. This does not mean that they were ignorant of ideas that goods or payments differed in time are in fact different things from today's perspective. To the contrary, despite knowing this they chose to use or not time discounting in their models depending on the problems they tackled.

In the 1950s and 1960s the burgeoning literature on economic growth discussed more explicitly the convergence conditions that guarantee the existence of solutions to intertemporal utilitarian models. Nonetheless, different notions of convergence (with the use of a discount factor being just one among them) were used at that time and it was even possible to see economists presenting their models with and without time discounting, as it was the case, for instance, of Edmund Phelps (1967) in discussing welfare-maximizing economic policies by borrowing some ideas from Ramsey (1928) and from the growth literature of David Cass and Tjalling Koopmans. Roughly from the 1970s macroeconomists treated time discounting more and more as a technical requirement of their models.

In pondering the different moves of this narrative, there are broad changes occurring in economics that are relevant to it. The fact that time discounting became a technical requirement in

---

instance, Blanchard and Fischer (1989), Stokey and Lucas (1989), Obstfeld and Rogoff (1996), Barro and Sala-i-Martin (2003), Woodford (2003), and Ljungqvist and Sargent (2012).

economics relates to the dominance of formal Walrasian general-equilibrium models in the last half of the last century.[3] A crucial element here is the axiomatization of time preferences that economists like Koopmans worked out: it transformed the concept of impatience previously introduced by Irving Fisher (1930, ch. 4) into the element defining an ordinal utility function; and such function in turn defines time preferences either of an individual consumer, or of an aggregate of such individuals, or finally choices in a centrally planned economy (Koopmans 1960, 288). Add to this the increasing use of a representative agent in macroeconomic models in general, and in growth models in particular, and you get the overall picture of the current understanding that a time discount factor is a technical requirement of either a decentralized or a social planner's problem.

Moreover, the use of time discounting as a technical component of growth models goes hand in hand with the emergence of the so-called "Ramsey-Cass-Koopmans (neoclassical) model," in which a representative agent discounts the future at a constant rate.[4] In this respect it is also important to note the changes in method that happened in the growth literature with the Hamiltonian formalism (Wulwick 1995).

As one can see, the narrative of the use of a time discount rate in growth models is no straightforward account. My goal in this article is to highlight some major elements that help us understand how a practice that was condemned as ethically indefensible when applied to intergenerational comparisons became a technical requirement in dynamic models of either a consumer or a planner deciding the intertemporal allocation of resources.

1. An ethically indefensible practice

In the first half of the twentieth century there was in Cambridge a tradition against discounting utilities of future generations. For instance, Arthur Cecil Pigou (1912, [1920] 1924) argued, following Sidgwick's utilitarianism, that "there was no philosophical basis for not treating future people just like present people. But in practice, because of a 'defective telescopic faculty' and the vagaries of time and blood … future people would actually count for less" (Collard 1996, 587). Pigou argued that because the present generations have a defective telescopic faculty – their

---

[3] See De Vroey (2004) for a history of the macroeconomics after Keynes as a general movement from Marshallian to Walrasian general equilibrium models.

members see things distant in time as being smaller than current ones – and also as a consequence of mortality and weak linkages over time, they invest relatively few resources today, especially in human capital. Therefore, according to Pigou, the government could intervene to guarantee a fairer generational justice. Collard (1996) analyzed in details this issue and showed that there was "a strong Cambridge tradition (Mill-Sidgwick-Marshall-Pigou-Ramsey) against discounting future utilities" (585). For Pigou discounting future utilities was a mistake because it led to sub-optimal intertemporal allocation of consumption.

Frank P. Ramsey (1928), the Cambridge mathematician who had Pigou as a mentor and supporter in economics (Duarte 2009a), took a slightly stronger stand than the latter and stated that the use of a discount rate when one discusses the intertemporal allocation of resources by the society as a whole is ethically indefensible because generations ought to be treated equally. In this context he "assumed that we do not discount later enjoyments in comparison to earlier ones, a practice which is ethically indefensible and arises merely from weakness of the imagination" (Ramsey 1928, 543).

Then how did Ramsey guarantee that his intertemporal maximization problem would have a solution, when no discount rate is used and, thus, the integral of current and future utilities may be improper? He did not choose to introduce a finite horizon for utility maximization and used instead a trick that became well known after the publication of his 1928 paper: he assumed that the total utility of consumption net of the disutility of labor has an upper bound, given either by satiation of utility or by technological restrictions (so that a greater stock of capital would increase neither income nor leisure), which he called "Bliss." He then represented his intertemporal maximization problem (in continuous time) as the minimization of the integral, from the present to the indefinite future, of the deviations of current net utility from the bliss level:

$$\underset{x_t, a_t}{Min} \int_{t=0}^{\infty} \{B - [U(x_t) - V(a_t)]\} \cdot dt \qquad s.t.: \ \frac{dc_t}{dt} + x_t = f(a_t, c_t)$$

where B is the bliss level; x_t, a_t, and c_t denote respectively, in Ramsey's notation, consumption, labor, and the stock of capital; $U(x_t) - V(a_t)$ is the "net enjoyment per unit of time" (the difference between the utility of consumption, $U(x_t)$, and the disutility of labor, $V(a_t)$); $f(a_t, c_t)$ is the production function that uses labor and capital as inputs; and the constraint to this minimization

---

4 Duarte (2009c) discusses how Ramsey's contribution was received over time and became central to the economic growth literature of the 1960s. See also several papers in Boianovsky and Hoover (2009) for an analysis of different

problem is the economy's resource constraint that states that investment plus consumption should equal total output.

It should be stressed here that Ramsey did present economic arguments to justify his assumption of the existence of a bliss, no matter the fact that it was "just" a mathematical trick he used to guarantee that the integral of lifetime utility does converge and, therefore, that the intertemporal problem has a finite solution whose properties could be characterized. He stated that the net utility that solves the above problem is an increasing function of the capital stock "since with more capital we can obtain more enjoyment" (544). However, this utility ceases to increase for either of the following two reasons: (1) from the production side of the economy, further increases in capital "would not enable us to increase either our income or our leisure"; (2) from the demand side of the economy (consumers), the net utility reaches a maximum conceivable level. In either case, there exists a finite level of capital that is associated with the maximum rate of net utility economically obtainable (whether or not it corresponds to the conceivable maximum).

Ramsey (1928) then considered a more general case: one in which the net utility always increases with increments to the capital stock. Here he analyzed two sub-cases: first, one in which net utility approaches asymptotically a certain finite limit (a finite supremum, in mathematical terms that Ramsey did not use in the article), and second, one in which net utility increases without bound. Clearly, if the latter were possible, Ramsey could not go on with a notion like "bliss." He dismissed this mathematical possibility with a brief economic argument: "we shall dismiss [the possibility of the net enjoyment increasing unboundedly] on the ground that economic causes alone could never give us more than a certain finite rate of enjoyment (called … the maximum conceivable rate)" (545).

Besides discussing in economic terms his mathematical approach, in the final parts of the article Ramsey (1928) also extended his results and analysis to the case that right in the first page he warned against on ethical grounds: an intertemporal allocation of consumption by either one individual or a community that discounts future utilities at a constant rate. Ramsey showed how the formulas obtained in the first part could be adapted to the discounting case. We see Ramsey using time discounting both in the published article and the notes in which he not only sketched the mathematical calculations for this article but also analyzed the case of taxing savings (Duarte

aspects of the development of this literature.

2009b).[5] The reason for using a discount rate is that from the beginning Ramsey (1928) implicitly considered his approach applicable either to an individual or to a "community that goes on forever without changing either in numbers or in its capacity of enjoyment or in its aversion to labour" (543).[6] In going from one case to another, from the individual to the community, Ramsey is not explicit about whose utility function he analyzed, whether of a social planner (community) or of one individual who would represent the utility of all others. As explored by Duarte (2010, 126-130), there is some evidence that Ramsey may have had in mind a notion of a representative agent similar to what became widely used macroeconomics decades later. If this is indeed the case, his ethical criticisms to discounting the utilities of future generations would disappear in this representative agent world because generations do not differ in any substantial manner (i.e., in their utility of consumption and disutility of labor). Therefore, such notion of a representative agent may have been lying behind Ramsey's use of time discounting by a community in the final parts of his article.

2. Time discounting in economic dynamics, 1920s-1940s

In this section I want to show a brief panorama of uses of a discount rate by mathematicians and economists working on economic dynamics from the 1920s to the 1940s (with more emphasis on the 1930s and the 1940s). The goal is to look at a literature similar to the optimal growth literature that emerged in the 1950s in a crucial dimension: works in which there is an explicit intertemporal optimization problem solved with calculus of variations, no matter whether or not they employed time discounting in the objective function. Therefore, this is a branch of economic dynamics related to names like the mathematicians Griffith Evans (1887-1973) and his advisee Charles F. Roos (1901-1958), who was one of the founders of the

---

[5] Keynes asked Ramsey to cut this section on taxation out of the article published in 1928 in the *Economic Journal*, as discussed by Duarte (2009a).

[6] In the first page of the article, immediately after stating that discounting the utility of future generations is ethically indefensible, Ramsey (1928, 543) says that he "shall … in Section II include such a rate of discount in some of our investigations." But even in the first section, where he analyzes the case of a community, after presenting Keynes's intuition for his main result, he mentions that the set of equations derived can be extended to the case of discounting future utilities while Keynes's intuition cannot – thus, discounting could be used to the case of a community. Then, in section II, when extending his previous analysis to the discounting case, Ramsey (1928, 553) was only concerned with not contradicting his initial hypothesis that "successive generations are actuated by the same system of preferences." For this, he assumed that the rate of discount is constant, but this does not mean that "it is the same for all individuals, since we are at present only concerned with one individual or community." In the set of notes now published (Duarte 2009b), we see that he struggled also with the case of a time varying rate of discount.

Econometric Society in 1930 and its first secretary, and the economists Jan Tinbergen (1903-1994), Albert Gailord Hart (1909-1997) and Gerhard Tintner (1907-1983).[7]

But this branch of dynamic economics is not related to the *Foundations of Economics* of Paul Samuelson, a key contribution to economic dynamics (Weintraub 1991, ch. 3). The dynamics expounded by Samuelson ([1947] 1983), and reaffirmed by him in a survey of this field for the *American Economic Association* (Samuelson 1948), is that of casting economic phenomena in terms either of difference or of differential equations and then analyzing the dynamics that emerge after imposing restrictions (generally in terms of parameters) that make comparative-static exercises render sensible results, given initial conditions. Here, as in the dynamics of Evans, Roos and the others, "there are functional relationships between economic variables and their rates of change, their 'velocities,' 'accelerations,' or higher 'derivatives of derivatives'" (Samuelson 1948, 354). However, contrary to Evans, Roos, Tinbergen, Hart and Tintner, there is no explicit optimization problem being solved. Therefore, Samuelson did not analyze the issue of time discounting in this context notwithstanding his earlier use of discounting when discussing either the measurement of utility (where he postulated that individuals maximize the discounted integral of future utilities over a finite time interval) or capital theory (where he proposed that firms ought to maximize the present value of income stream) (Samuelson 1937a, b).

2.1. The mathematicians: Evans and Roos

Griffith Evans and his student Charles Roos were mathematicians who, together with other economists like Tintner, were concerned with dynamic econometric models of demand in the 1920s and 1930s, as Mary Morgan (1990, 152-8) discusses it.[8] The main feature of Evans's and

---

[7] Roos and Evans were among the first Fellows of the Society elected in 1933. This meant that they were considered to be outstanding econometricians by their peers, someone acquainted with economic theory, having a mathematical foundation and some knowledge of statistics, and have done original work in economic theory, as Ragnar Frisch described to Irving Fisher (Louçã 2007, 30). Both Roos and Tintner were members of the Cowles Commission in the 1930s: Roos helped found Cowles and was its research director from 1934 to 1937 (he resigned as director to take a job at the Econometric Institute Inc.); Tintner was a research fellow for a bit less than a year, 1936-1937 (he became a Fellow of the *Econometric Society* from 1940s onwards) (Louçã 2007, xvii, xxix).

[8] Griffith Evans earned his Ph.D. at Harvard in 1910 and subsequently was a postdoctoral student of Vito Volterra at the University of Rome (1910-12), which "was to be the marker event of his intellectual life" (Weintraub 2002, 42). From 1912 to 1933 Evans was a teacher at the Rice Institute (nowadays Rice University), when he moved to Berkeley. His papers published in the 1920s in mathematical journals, which led up to his 1930 book, all "called attention of mathematicians to interesting problems in an applied discipline" (Weintraub 2002, 57). Charles Roos obtained a BA degree in 1921, received his MA in 1924 and his Ph.D. (under Evans) in 1926, all at Rice University. From 1926 to 1928 he was a National Research Fellow first at the University of Chicago for one year and then at Princeton University for another; from 1928 to 1931 he was an assistant professor of mathematics at Cornell University (Davis 1958).

Roos's research agendas that is of interest here is that they introduced the rate of price changes as an argument of the demand function.[9] As a consequence, they turned the problems of competition and monopoly into problems in the calculus of variations (Roos 1927, 635) in which firms choose prices that maximize profits over an interval of time, subject to a demand function (time is continuous in their models). It is in expressing this integral of profits that they do or do not take into account a discount factor.

Evans (1924, 1925, 1930 chaps. 14-15) considers for simplicity, and avoiding further behavioral assumptions,[10] the case of a quadratic cost function and a demand function that is a linear differential equation without having an explicit term involving time. He also mentions the two additional cases. The first is of a linear demand function with a time argument, an idea that Irving Fisher suggested to him. Here, the volume of traded goods at time t depends on the rate of change of the price index at a previous time. Evans considered also a second case: that of a law of demand ("integral demand law", Evans 1930, 46) in which the quantity demanded depends on a weighted average of either past prices (Evans 1925, 108) or past quantities (Evans 1930, 158-62) – with a weighting function that decreases with the time elapsed from the past to the present.[11] In all these cases, Evans assumed that the monopolistic firm chooses a function $p(t)$ that maximizes the integral of profits over the interval (t_1, t_2) (subject to a demand function):

$$\underset{p(t)}{Max} \quad \Pi = \int_{t_1}^{t_2} \pi(p, p', t) \cdot dt$$

where $p' \equiv dp/dt$. The issue is that Evans provides no discussion on discounting. If $(\partial\pi/\partial t) = 0$ there is no discounting, but $(\partial\pi/\partial t) = -\rho \cdot \pi(p, p')$ makes the case for a constant discounting (with the rate of time discount being $\rho \geq 0$). Evans assumed (implicitly) that t_2 is finite and when he specified particular functional forms for $\pi(p, p', t)$ he made clear that there was no discounting in his analysis (as, for example, in Evans 1924, 78; 1925, 164; 1930, 144; 1931, 63; 1934, 42-43).

---

9 As Roos (1927, 635) pointed out, "a number of the Department of Agriculture economists have been" introducing the time element explicitly into the demand functions "in the last few years."

10 Weintraub 2002, ch. 2, argued that Evans, "reprising Volterra's critique of Pareto more than a quarter century earlier" understood that "in mathematical economics one should not be so concerned with the behavioral theories themselves" (69). Economic theory should derive implications "testable either empirically through data analysis or through common sense" (70).

11 Evans (1930, 158 n. 1) explains that Volterra was the first to discuss this type of law in problems in physics and that Roos, his student at Rice, applied it to the theory of competition in his MA thesis and in his 1925 article.

From a mathematical standpoint there is no problem in assuming away time discounting because the above integral of (undiscounted) profits is well defined over a finite interval of time.

After the publication of his book in 1930 and after the contributions of his student Charles Roos to economic dynamics in which time discounting was introduced (see below), Evans (1931, 1934) followed the same approach as before and assumed that the objective function that firms maximize is the undiscounted integral of profits over a finite interval of time. Two peculiarities are relevant in these two works: first, the 1934 article was published in an economics journal (in contrast to all his previous works cited here, which were published in mathematics journals) and, second, Henry Schultz (at the University of Chicago department of economics) discussed the 1931 piece. In assessing Evans's theory of economic dynamics Schultz did refer to Roos (1930) (in which firms maximize the integral of discounted profits) but did not mention whether or not it is more appropriate to include a discount factor in the firm's maximization problem. Thus, we can infer that time discounting was a practice that was interpreted differently by members of this community of scientists working on economic dynamics, and it was not understood to be a mathematical necessity.

For Roos, in his very first article published (Roos 1925), he followed Evans's (1924) approach of a monopoly and extended it to the case of a duopoly. He considered two problems of competition among two firms: (1) given the price at the initial period, firm 1 chooses a quantity to be supplied to the market that maximizes its integral of undiscounted profits over the interval (t_1, t_2) regarding the quantity supplied by firm 2 as not subject to variation, and at the same time firm 2 chooses a quantity to be supplied that maximizes its integral of profits over the same interval taking the quantity supplied by firm 1 as not subject to variation; (2) given the price level at the initial and end periods, t_1 and t_2, firms choose prices, as a function of time, that maximizes its own integral of profits considering that only its quantity supplied varies with the price chosen. As pointed out by Roos (1925, 164), both of these are problems that "do not seem to reduce strictly to problems in the calculus of variations, but solutions, however, are given" in the article. Here Roos assumed, as Evans, not only that the cost functions of the firms are quadratic and that the demand function they face is linear in the level and the rate of change of prices, but also that firms maximize an integral of undiscounted profits over an interval of time (implicitly assumed to be finite). In the last sections of the article Roos extended his analysis to the case of several producers and the case of, using Evans's terms, the "integral demand law" that Evans (1925) had already referred to.

In a set of future publications we observe that Roos moved over time from problems with no discounting to those in which future profits are discounted. While he was a national research fellow in mathematics at the University of Chicago, Roos published an article in 1927 in an important economics journal, the *Journal of Political Economy*, in which he extended both his previous analysis and that of Evans (1924, 1925): he discussed "the phenomena of competition, monopoly, and cooperation for general functions of demand and cost" (Roos 1927, 635). Throughout the paper he assumed, as before, that firms maximize an integral of undiscounted profits over a finite interval of time.[12]

Two years later, stimulated by the concerns of bankers and businessmen on whether or not the prosperity observed in the second half of the 1920s was going to continue and whether it was "possible to scientifically forecast business conditions" (Roos 1929, 186), Roos (1929) discussed issues of business forecasting just a few months prior to the Great Depression. However, in contrast to his previous works, now he considered that firms maximize an integral of discounted profits over a finite interval of time without discussing why discounting was necessary: "each producer attempts to maximize his net profit over an interval of time t_1 to t_2 discounted to the time t_1" (subject to a demand function) (Roos 1929, 187):

$$\underset{u(t),p(t)}{Max} \quad \Pi = \int_{t_1}^{t_2} [p \cdot u - Q(u, u', p, p', t)] \cdot E(t_1, t) \cdot dt$$

where primes denote derivatives with respect to time; u and p are "the rate of production" (quantity supplied) and the price level, respectively; $Q(\circ)$ is the cost function; and "$E(t_1, t)$ is the discount factor $\exp\left\{-\int_{t_1}^{t} \delta(r) \cdot dr\right\}$ and $\delta(r)$ is the force of interest" (Roos 1929, 188).[13]

In another article published in the *Journal of Political Economy* in 1930, Roos systematized his dynamic approach to economics according to which firms maximize an integral of discounted profits subject to a linear demand function that depends on the level and the rate of change of prices. An interesting aspect of this essay is that Roos (1930) tried to connect the theory of business cycle that existed at the time – which, he argued, collected statistical observations, not

[12] It is worth pointing out that Roos had at the time contact with economists such as Henry Schultz (who discussed the 1931 article by Evans, as mentioned above), whom he thanks in the paper for supplying him much of the bibliography and for patiently discussing with him much of the paper.

all consistent with one another, about the price level oscillation, its periodicity and characteristic phases – with a dynamic economic theory that was in fact the mathematical theory that he had been developing over the years. According to this theory changes of parameters could generate oscillations around a trend. Thus the cycle was not a consequence of either sunspots or irrational behavior of individuals. Despite being an article published in an economics journal, he again did not explain why a discount factor is necessary or economically meaningful.

Roos (1930) started discussing the simpler case of monopoly but introducing new elements when compared to his previous analysis of the problem: he considered that the goods produced and demanded at time t may not be equal or, what amounts to the same thing, that a part of the current production "will go into stocks on hand" (Roos 1930, 503). Therefore he presented a general model that encapsulated Evans's analysis as a particular case: one in which there is no discounting, the cost function is quadratic and supply and demand are equal at every period. He concluded that "these restrictive assumptions prevented [Evans] from obtaining solutions typical of our economy" (505). One interesting case analyzed by Roos (1930) is a periodic solution that is obtained in the case of no discounting and a general cost function.

In the book he published in 1934 as the first Cowles Commission monograph, Roos (1934) gave a clear economic explanation for assuming that firms discount future profits.[14] In fact, the first time that a discount factor was introduced in the book was in chapter nine when he considered that firms face several risks that should be accounted as costs, from "destruction of plant and equipment by fire or storm" to "technological improvements and new discoveries" (156). However:

> Since a remote risk is less important than an immediate one (conditions may change to eliminate the risk and, also, return of capital occurs), to obtain the present (time *0*) value of a future (time t) risk it is justifiable to multiply [the total risk] by a discount factor...

---

13 In presenting again such discount factor, Roos (1930, 504) defines the "force of interest" as "the rate of increase of an invested sum S divided by S" and cites a book by the mathematician Lloyd Leroy Smail of 1925 titled *Mathematics of Finance*.

14 As Gerard Debreu argued (quoted in Dimand and Veloce 2007, 520), Roos's monograph "is not a genuine Cowles product ... since it was completed by the time its author ... joined the Commission and became its research director in September 1934." Dimand and Veloce (2007, 528) stated that Roos's book consisted "largely [of] papers collected at [Harold] Davis's suggestion."

(Roos 1934, 157)[15]

A few pages later in this same chapter, with arguments similar to those of the time preference theory as we understand it nowadays, Roos (1934, 160) explained with economic arguments why future profits should be discounted in the firm's objective function (the integral of profits over a time interval):

> It is almost universally true that producers prefer early profits to remote or deferred ones. Waiting is an element of cost as truly as effort is, and it should be taken into account. This does not mean, however, that a producer is unwilling to forego present profits in order to obtain greater ones in the future, but it does mean that the expectation of future profits will have to be greater than actual present ones.

In summary, the mathematicians working on economics dynamics in the 1920s and 1930s differed in their use of time discounting. While Griffith Evans kept formalizing the objective function of a firm as the integral of undiscounted profits over a finite interval of time, his student Charles Roos moved from an integral of undiscounted to one of discounted profits over such interval. Initially Roos did not explain why assuming a discount factor was a sensible hypothesis, but in his 1934 book he provided a clear explanation in economic terms for this strategy. However, contrary to the Cambridge tradition, in Evans's and Roos's models time discounting was an issue related to the behavior of the firms, not of the consumers: both assumed given demand functions that the firms face and did not derive it from any sort of utility maximization problem in which discounting future utilities would appear. Evans was dismissive of the subjective theory of value, as argued by Weintraub (2002, ch. 2), and Roos followed closely his professor in avoiding behavioral assumption about the consumers.[16] By considering firms that each maximize its own integral of profits over a finite time interval, a discount factor was not a mathematical necessity

---

15 Interestingly, Paul Rosenstein-Rodan (1934) used Knightian uncertainty (about tastes and income) to justify that future wants are less foreseeable and, thus, would count less from today's perspective.

16 While Evans was dismissive of the utilitarian theory of value, Roos was not much in favor of general-equilibrium theory. As Philip Mirowski and D. Wade Hands (1998, 274) point out, although Roos was the first research director of the Cowles Commission, an institution that later, in the 1940s, became the "standard bearer for general-equilibrium theory" and was an "avid supporter of mathematical economics," he was not "enamored of Walras or Pareto."

and, thus, it could be used by the mathematicians without acquiring a similar ethical dimension as that of Ramsey's intergenerational utility maximization problem.

2.2. The economists: Tinbergen, Hart and Tintner

A few economists received well the mathematical approach proposed by Evans and Roos to solve some economic problems: Jan Tinbergen, Albert Gailord Hart, and Gerhard Tintner.[17] As discussed by Marcel Boumans (1993) from the beginning of his career Tinbergen wanted to bring economics to a more developed stage, more akin to the natural sciences, in which it would base scientific policymaking. He looked for a utilitarian dynamic framework for discussing business cycle with the aid of Hamiltonian formalism that was familiar to him due to his physics background. Hart became known for his discussions about the implications of uncertainty to businessmen and mainly policymakers, with an important book published in 1940 based on his Ph.D. dissertation (Hart [1940] 1951). In an article published in the *Quarterly Journal of Economics* in 1937, also based on his dissertation, he explained that "even before the onset of the great depression, there was a strong feeling among economists that the chief problem before them was that of business fluctuations [(as it is clear in Roos 1929)]; and this feeling has been intensified by the experience of the last few years" (Hart 1937, 273). As for Tintner, he was a great enthusiast of the work of Evans and Roos and further developed their approach to economic dynamics, among other contributions he made to economics (see Fox 2008). In contrast to Hart, Tintner did employ extensively calculus of variations to the problems he studied.

Tinbergen studied mathematics and physics at the Netherlands with the physicist Paul Ehrenfest (1880-1933), who taught him the works of such mathematical economists as "[Arthur] Bowley, Wicksell, Pareto, Barone and Roos" (1930 letter from Tinbergen to Ehrenfest quoted by Boumans 1993, 138).[18] Tinbergen used his training in physics to translate discussions on economic policies into the optimal control problems he was familiar with. As Boumans (1993) argues, Tinbergen's transition to economics started with his PhD thesis on "Minimum Problems in Physics

---

[17] There was also a group of Paretians in Italy trying to build a dynamic general equilibrium theory who knew well and were receptive to the works of Evans and Roos. However, they were mainly concerned with microeconomic models and, as the latters, had no significant impact on the economic dynamics produced in the postwar period in the US (see Pomini and Tusset 2009). Interestingly, among the Italian economists one finds a similar ambiguity about using a discount factor either to utility or to profit functions that we see in the interwar literature discussed here.

[18] Arthur Lyon Bowley (1869-1957) was a Cambridge mathematician who studied economics under Alfred Marshall and worked on economic statistics (Dale and Kotz 2011, 7-8). His 1924 book *The Mathematical Groundwork of Economics* was one of the earliest book published in English that systematized the mathematical treatment of economics.

and Economics". For him, the aim of economics is "the desire to know the implications of certain changes in the social mechanism or in the conditions under which that mechanism works" (Tinbergen 1933 quoted by Boumans 1993, 137). His intention was to turn policymaking into a science, with economists being the experts who would objectively instruct politicians which instruments to use and the consequences associated with any given scenario. However, as discussed by van den Bogaard (1999, 29), "the future had to be defined by the politicians."

Boumans (1993, 140) presents Tinbergen's early works (mostly published in German and Dutch), and shows that Tinbergen had an appendix in his thesis discussing economic problems that was published in 1929. Here, among other things, he discussed the same dynamic problems of competition and monopoly that occupied Evans and Roos.[19] To this end he considered a firm, or a combined group of firms, that choose how much to produce, $q(t)$, in order to maximize the integral (because he also considers time to be continuous) of undiscounted flow of profits over a finite interval of time (Boumans 1993, 144):

$$\underset{q(t)}{Max} \quad G = \int_0^T (p-a) \cdot q \cdot dt$$

where p is the commodity price, which is a function of the quantity, q, and a is the cost price (assumed not to depend on the quantity).

Tinbergen further developed his dynamic model in three subsequent articles in the early 1930s, including one published in English in the newly created journal *Econometrica*. In it, Tinbergen opened his analysis by stating that dynamic problems that he was interested in ought to be analyzed in a normative framework, that of maximizing an "ophelimity function" (a social welfare function).[20] Moreover, supply and demand schedules for each moment depend not only on current prices, but also price expectations. He then presented an argument that expectations of the near future should count more than those of a remote future, which would justify the use of a discount rate, but, as a first approximation, no discounting would be employed:

> In a theory of economic dynamics, the ophelimity function of individuals must be supposed to depend on the quantities of goods consumed and the sacrifices brought, not only at the moment considered, but also at later moments.

---

[19] See also Morgan (1990, 154-55).

[20] In fact, as Boumans (1993, 143) analyzes, the presence of an ophelimity function was a central condition to translate economic problems into variational problems that abound in mechanics.

> Their offer and demand schemes for each moment then depend not only on the prices governing at that moment, but also on the *price expectances* the individuals have for the future. Among those expectances, those relating to the near future will be of more importance than those relating to a further period. As a first approximation it might be supposed that only the expectances relating to a certain time period (the "*horizon*") are of importance, and all of the same importance.
> (Tinbergen 1933, 247)

In fact, Tinbergen's model of supply had no room for time discounting, as we can see in the reconstruction made by Boumans (1993, 150-54). Two years later, Tinbergen (1935, 306) again proposed that discussions on optimal economic policies ought to be cast in terms of the maximization of a "general ophelimity function." However, he considered that "practical calculations of this sort are impossible." Decades later he would characterize the intertemporal consumption allocation as the one that maximizes discounted lifetime utility function over an infinite horizon, mentioning that the discount factor "may be called the 'psychological discount rate,' comparable to an interest rate" (Tinbergen 1956, 605). Interestingly, Tinbergen here equated the problem of a nation (his academic interest) and of an individual, where discounting would apply: "There need not be, in principle, any difference between the choice an individual makes and the choice to be made for a nation as a whole" (604).

In the 1960s he characterized the problem of the optimal savings rate of a nation going back to Ramsey's ethical stance against discounting future generations. Tinbergen (1960, 481) reports on "an unsuccessful attempt to find a simple solution to the problem of optimum savings" in a context of no discounting and a satiation level (and also a subsistence level in consumption). Then, he would be able to develop fully his normative approach in a co-authored book on growth economics (Tinbergen and Bos 1962, see especially pp. 2-3, 24-31), repeating his argument in the 1960 article against discounting (in the book he made no mention to Ramsey (1928), but in the article he did):

> No *discount* for future consumption was applied in the belief that for a country's planning, future generations should count as much as present generations. According to this philosophy, a discount may be realistic for the individual's plans but not necessarily for a nation's. It is not difficult to introduce

> discounts for future consumption when so desired, but the question then arises at what level the discount should be put. Instead of a discount, a finite horizon *T* may be introduced; a similar question then comes up about its length.
> (Tinbergen and Bos 1962, 25-6)

It is important to emphasize that Tinbergen's attitude towards discounting was very much in line with his goal of simplifying the planner's problem, which was necessarily very complex. He wanted to give his analysis practical applicability and this pragmatic approach, if I may put this way, was very present from the beginning of his works in economics. From this follows his insistence on practical calculations (which also required having numerical values for the parameters, such as the discount rate) and on using simplified models as a first approximation to complex problems at hand.

Another important economist working on dynamics in the 1930s and 1940s was Albert G. Hart. He received his BA from Harvard in 1930 and his Ph.D. in economics from the University of Chicago in 1936, under Henry Schultz, Frank H. Knight, Jacob Viner, and Theodore O. Yntema (the latter was then a professor of statistics at that university).[21] A decade after obtaining his Ph.D. he became a professor in economics at Columbia University where he stayed until his retirement in 1979 (Earl 2008).[22] Before going to Columbia he spent a few years at Iowa State College where he became Tintner's colleague.

The issue of anticipations became central to Hart's analysis of fluctuations, in which there was a crucial difference between risk (when the holding of anticipations constitute a probability distribution with known parameters) and uncertainty (when the parameters of the distribution of the holding of anticipations are themselves not single valued, i.e. each one has a probability distribution). He then "urges that theorists concentrate their attention on uncertainty rather than on risk" (Hart 1942, 110) and advocates a dynamic approach to economics as the econometric work that Tinbergen and the Cowles Commission were developing (Hart 1945, 544).

Hart plays a central role in the present narrative because he stressed "the time elements of the firm's planning, with special emphasis on anticipations" (Hart [1940] 1951, xi) and brought

---

[21] From the acknowledgements in Hart's Ph.D. dissertation, from the article he published based on it (Hart 1937), and from the preface of his 1940 book (Hart [1940] 1951, vi n. 8) we can not know exactly whom from the four members of his committee was his advisor (in case there was just one).

[22] Hart's obituary in the Columbia University Record, vol. 23, no. 5 (October 3, 1997), registers that he was a visiting professor in 1946, becoming a professor at the department of economics in 1947 and that he retired in 1977.

anticipations of change and uncertainty to bear on business cycle problems. As a consequence, he made time discounting a central piece of his theory of firm's behavior and cycles, as he clearly presented in his 1940 book. In it, he combined in his analysis different strands of understanding of a discount factor. Hart ([1940] 1951, v) explained in the 1951 preface to his book, "as the events of the 1930's made fluctuations our chief focus of interest, a number of Anglo-American economists (as well as the Swedes who had pioneered this field), found themselves forced to reformulate their micro-economics in terms of anticipations." His monograph was part, according to himself, of "the rather substantial literature" on business fluctuations and dynamic problems, and was shaped by Frank W. Taussig's ([1911] 2007, ch. 52) concept of discounted marginal productivity that he had learned as an undergraduate student. The genesis of Hart's book is so important to show how he familiarized himself with those different understandings of time discounting that it is worth quoting the preface to the 1951 edition of his book at length:

> The problem [of anticipations] became acute for me in the winter of 1930-31, in Vienna, when I was forced to think hard about the questions raised in Hayek's 1931 London lectures – which were published as *Prices and Production*. I had been indoctrinated with the anticipations standpoint in my undergraduate days by F. W. Taussig's insistence that the lag between input and corresponding output made it necessary to handle imputation problems in terms of *discounted* marginal productivity.[23] This approach was still more natural by the concentration of the "trio seminar" of Haberler, Hayek and Morgenstern in Vienna in the autumn of 1930 upon Fisher's *Theory of Interest*. The insistence of the students in the seminar on reformulating Fisher in terms of Böhm-Bawerk (!) gave plenty of exercise in input-output lags. When confronted with Hayek's business cycle theory, I found I could not be either for it or against it without new constructions. …
>
> These questions were a focus of discussion among the graduate students at Chicago in 1931-34, and not unnaturally led me to a dissertation topic. … In the autumn of 1934, when I arrived at the London School of Economics with a rough draft under my arm (written just after landing in England), I found a lively

[23] Earlier, Hart (1937, 273 n. 1) had already pointed out that Taussig's concept of "'discounted marginal productivity' led [the former] to a study of anticipations."

> discussion in progress.[24] J. R. Hicks was just preparing the lectures which became his article on "Wages and Interest – the Dynamic Problem." Shortly, there was circulated a mimeograph from Erik Lindahl, anticipating a key section of his later *Money and Capital*. In this atmosphere, I rounded off in December 1934 my first reasonably complete draft of the material which went into this monograph. After various vicissitudes, my dissertation was accepted at the University of Chicago in the spring of 1936. Various offshoots of it got into print during 1936-37. This monograph, published in 1940, was a revision of the parts of the dissertation dealing most specifically with the firm.
>
> Hart [1940] 1951, v-vii

From Hart's account of his book a decade after its first edition, it is clear that he was well acquainted with the work of a variety of economists that were connected to his own enterprise – other relevant names cited in the book and not mentioned in the passage quoted above are Gunnar Myrdal, George L. S. Shackle, Frank Knight, Jacob Marschak, and Gerhard Tintner. In all these works the idea that something at different points of time should be treated as different things was clearly understood. Moreover, Hart ([1940] 1951, v n. 1) claimed that, in the United States, Roos "published in 1925-1934 a series of writings which he asserts 'pioneered and abandoned' the expectational approach" to which Hart was contributing.[25] However, he seems not to cite Roos's works in the chapters of his 1940 book (and in his 1937 article, for that matter).

In this literature, the appropriate objective function that firms maximize (if discounted or undiscounted flow of profits or any other variable) was an open question. Hart (1937, 278 n. 5) justified his use of discounted profits (discounted "net receipts") as the firm's objective "as the most natural extension of the traditional 'maximum net receipts' [used in static problems]" and that it has been "used by several of the writers in English on anticipations" such as Harold

---

24 In the 1940 preface to the book Hart [1940] 1951, xii) thanks the University of Chicago "for a generous grant of leave in the academic year 1934-35 which enabled him greatly to extend his contacts with economists interested in these questions and which provided leisure for a first attempt to analyze them at length."

25 Hart ([1940] 1951, v n.1) cited Roos's "most accessible of his early papers": those published in the *Journal of Political Economy* in 1927 and 1930, and noted that Roos "described the firm as maximizing discounted profits" (as we discussed in section 2.1). Later in the preface Hart cites Samuelson's literature review on dynamic economics to the American Economic Association (Samuelson 1948) which was "an essay on the logic and applications of differential equations, with no discussion of anticipations anywhere visible" (viii).

Hotelling (1931) and John R. Hicks (1935).[26] However, continued Hart, Griffith Evans (1930) and Kenneth Boulding (1935) proposed that firms maximize "undiscounted net receipts" and the "average rate of return on capital over the life of the enterprise," respectively.[27] Boulding (1942) himself noted that time discounting was not a common practice among economists working with the theory of the firm. In his review of the literature of the 1930s to the early 1940s he refers to Hart, Hicks, Hotelling, and Tintner, but Evans or Roos are to be found nowhere.[28]

Therefore, Hart's contributions to anticipations and the business cycle are important for bringing together a diverse set of works on economics dealing with these issues, including, to a lesser extent, Evans and Roos. His discussions on the use of a discount factor to the firm's objective function shows that a diverse understanding about this practice existed up to the 1940s. Tintner not only contributed to this literature but, as Hotelling, also considered discounting future utilities and employed heavily calculus of variations to analyze intertemporal problems related to monopoly, distribution of income, business fluctuations, utility maximization and choice theory.

Gerhard Tintner, born in Germany in 1907 and educated in economics, statistics and law in Vienna (he obtained his doctorate in 1929 at the University of Vienna), became in the mid-1930s an enthusiast of the work of Evans and Roos on economic dynamics.[29] In contrast to Hart, as already noted, here we see him explicitly using calculus of variations and extending Evans's and

---

26 Hotelling (1931) not only modeled firms as maximizers of the present value of their profits over time but also discounted future utilities at a constant rate: he was one of the first to cite Ramsey's 1930 article (Duarte 2009c) but, differing from the latter, he applied a discount factor to future utilities even in the case of integrating them over a finite time interval (Hotelling 1931, 143). See also Hotelling's argument against the criticism of discounting future utilities for the case of mining he analyzed (145-146).

27 Boulding (1942, 793) explained that the understanding shared by those assuming that firms maximize the discounted "net revenue" was that discounting was necessary in order to take into account the opportunity cost of investment: "The discounting presumably is to be done for each period of time at that rate of interest which represents the alternative cost of employing capital in the occupation in question; that is, at the rate which the entrepreneur could obtain in other investments."

28 It is interesting, in this respect, that Lawrence Klein's attempt to provide a mathematical derivation of Keynes's *General Theory* considered both a generic intertemporal utility function (where we cannot see whether there is time discounting) and firms that maximized the integral of discounted profit streams over a finite time interval (Klein [1947] 1961, technical appendix).

29 In the early 1930s Tintner was in the *Institut fuer Konjunkturforschung*, in Vienna. I do not know whether or not he met Hart there, in the winter of 1930-31. In 1936 he accepted an invitation to become a fellow of the Cowles Commission – Roos was then the research director of Cowles –, but resigned less than a year thereafter, in September 1937, to join the faculty of Iowa State College. Hart eventually joined the faculty of this college and became Tintner's colleague. Tintner was a Fellow of the *Econometric Society* from 1940 onwards, member of the editorial board and associate editor of *Econometrica* (Louçã 2007, xxix). See also Morgan (1990, ch. 8) for a discussion of Haavelmo's probability model that was accepted by Tintner in his econometric work.

Roos's works.[30] Despite Tintner's use of a time discount factor in many problems, going further than Hart when he employed it to the utilities of individuals at different points in time (but not in the same way that modern growth economists do), we observe a similar ambiguity with respect to such use as Roos's.

From the mathematical notation to the problems analyzed to the references cited and to the mathematical tools, it is clear from all this that in the set of papers that Tintner published from the mid-1930s to the 1940s, most of them in *Econometrica*, he followed Evans and Roos closely.[31] For instance, in order to discuss income distribution over time Tintner (1936a) applied to the utility function the same idea that Evans and Roos used to the demand function faced by the firms: he assumed that the utility function depended not only on the quantities of the n goods of the economy, "but also on their flow in time" (i.e., their derivatives with respect to time). In this article Tintner did not write explicitly an intertemporal utility function in which we would observe whether he introduced time discounting: he just stated that it was a function of the level and the rate of change of the quantity of goods available over time. From the budget constraint of the intertemporal utility-maximization problem (p. 63) we observe that he did not compute the present value of future expenditures, which would suggest that he did not consider discounting future utilities.

From the other related papers it is clear that Tintner wished to relate the dynamic theory of the firm expounded by Evans and Roos to a dynamic theory of choice: if the utility function depended on expected prices, which in turn depend on price tendencies, the time derivatives of prices would be arguments both of the utility function (which he called "dynamic utility function") and the demand function of the individuals and, thus, monopoly firms would maximize profits subject to a demand function that is a differential equation, as in Evans's and Roos's works. In such integrated framework issues on "expectations, economic horizon, and the role of time in economic life in general" would play a significant role (Tintner 1937, 161). However, only in a paper published in 1942 that Tintner turned his attention to a dynamic theory of choice.

With respect to the use of a discount factor, Tintner incorporated it in his analysis only from 1938 onward. Before this, in his 1937 essay he stated, as Evans, that monopoly firms

---

30 Tintner's enthusiasm with the work of Roos appears also in his favorable review of Roos's 1934 book for the *Journal of Political Economy* in 1936.

31 The same applies to Tintner's (1942b) attempt to discuss business fluctuations in a simple linear model that echoes both Evans (1931) and Roos (1930), but much more the former than the latter.

maximize the undiscounted integral of profits over an implicitly assumed finite interval of time (the notation is exactly the same as that of Evans or Roos):

$$\underset{p(t)}{Max} \quad \Pi = \int_{t_0}^{t_1} \pi(p, p') \cdot dt$$

In the three papers published in 1938 and 1939 Tintner adopted time discounting in the intertemporal utility maximization and demand problems he studied. As in the 1936 piece (Tintner 1936a), the use of a discount factor here can be inferred only from the budget constraint or the expenditure function, since Tintner simply assumed that the utility function depends on quantities which the individual expects to consume over time (in other words, he did not write the intertemporal utility function as the integral over time of instantaneous utility functions – discounted or not). Tintner closed his 1939 article relating his discussion about the influence of income, prices and interest rates upon expenditures at different points in time to that of Irving Fisher and the theory of time preference:

> The difference between our approach and the theory of Fisher is the following: We assume explicitly that the utility function or indifference map of the individual depends not merely upon the undiscounted expenditure stream … (Fisher's income stream), but upon all the quantities of all commodities which the individual plans to consume at all points in time … The time preference, if any, should be expressed in the form of the utility function.
> (Tintner 1939, 270)

In this passage it is clear that Tintner, as Roos, was somewhat reluctant to build his "dynamic utility function" upon explicit behavioral assumptions derived from a theory of time preference. This led him to discuss his dynamic theory of choice under uncertainty – here understood exactly in the way of Hart, who was then his colleague at Iowa State College –, in which time discounting was important but not a salient feature of his analysis (Tintner 1942a, c, d). This set of papers constituted the last step towards his dynamic approach to economics in which expectations, time, and anticipations were central components.

In conclusion, in this section I analyzed the works of mathematicians and economists who were among the first to apply calculus of variations to economic problems. These scientists understood that the technique developed by Euler and Lagrange in the eighteenth century – which "deals with problems of finding a function or a path that maximizes some criterion" (Kamien 2008) – was applicable to many problems in economics involving optimal decisions through time, whose solution was a path of actions rather than a single decision.

While Ramsey (1928) condemned on ethical grounds the use of a discount factor in problems of allocating resources among different generations in a community, the mathematicians and economists working on economic dynamics from the 1920s to the 1940s had a diverse understanding about the use of such factor, but not because of ethical considerations. Even before the Great Depression, they became interested in problems of business fluctuations. By discussing these problems they took stands on the proper objective of a firm and, therefore, on whether future profits ought to be discounted. Griffith Evans and Jan Tinbergen postulated that firms maximize the undiscounted integral of profits over a finite interval of time. Charles Roos and Gerhard Tintner adopted time discounting after some point, but in their first papers in this literature firms maximize the undiscounted integral of profits over time. Albert Hart, in contrast, saw his postulate that firms maximize the integral of discounted profits as a natural extension to economic dynamics of the static principle of profit maximization. In any circumstances, Hart ([1940] 1951) and Boulding (1942) provide evidence that this lack of consensus about time discounting characterized the literature on the dynamic behavior of firms.

From all these names, in the 1930s and 1940s Tintner was the only one to extend his dynamic analysis to the behavior of consumers, as Tinbergen eventually did later (Tinbergen 1956, 1960; Tinbergen and Bos 1962). Nonetheless, the ethical problem raised by Ramsey did not turn up in Tintner's analysis: he avoided writing his intertemporal utility function as an integral over time of instantaneous utility functions, as Ramsey did; he just postulated the existence of a utility function that had as arguments the quantities which the individual expects to consume over time (which includes Ramsey's integral of instantaneous utility functions as a special case). Therefore, he made no explicit statement about discounting future utilities, keeping his analysis safe from ethical judgments on the value of the utility of future generations relative to that of the living generation.

The question of discounting future utilities, be that of an individual or of a generation, was addressed by economists working on optimal neoclassical growth theory in the 1960s, who employed optimal control and dynamic programming methods that are generalizations of the calculus of variations. A clear axiomatization of time preferences and the consolidation of representative agent models (together with the increasing use of recursive methods in economics) set the stage for turning discounting future utilities into a rather technical question, gradually deprived of an ethical content.

3. A technically indispensable practice

In their entry to the *New Palgrave* Christopher Chabris, David Laibson and Jonathon Schultz (2008) provide an overview of the recent literature on intertemporal choices and on time discounting. They state that “the theory of discounted utility is the most widely used framework for analyzing intertemporal choices” both in descriptive (positive economics) and in prescriptive (normative economics) terms. They continue: “descriptive discounting models capture the property that most economic agents prefer current rewards to delayed rewards of similar magnitude. … Normative intertemporal choice models divide into two approaches. The first approach accepts discounting as a valid normative construct, using revealed preference as a guiding principle. The second approach asserts that discounting is a normative mistake … [and] adopts zero discounting … as the normative benchmark.” They then assert that the “most widely used discounting model” is that of Ramsey (1928).

Although it is true, as discussed in section 1, that Ramsey (1928) did consider the case of discounting future utilities of individuals, he is in fact mostly remembered nowadays for condemning the discounting of utilities of future generations (and, thus, postulating the existence of a bliss).[32] This modern association was already present in the 1950s before the “Ramsey-Cass-Koopmans” became a workhorse model in the economic growth literature, as we can see in Samuelson and Solow (1956, 538). Therefore, what Chabris, Laibson and Schultz (2008) most probably had in mind is the “Ramsey-Cass-Koopmans model” (also known as the “Ramsey model”), in which a representative agent discounts future utilities at a constant rate.[33]

The controversy with respect to the use of a discount factor in the modern literature reflects the ethical problems raised by Ramsey in his analysis of the intergenerational allocation of resources. Such a tension also played an important role in the unfolding discussion of optimal

---

[32] For example, after stating the intertemporal utility function of the Ramsey-Cass-Koopmans model, Blanchard and Fischer (1989, 81-2 n.4) recognized that because they assumed a positive rate of time preference they “depart from Ramsey who, interpreting the maximization problem as the problem solved by a central planner, argued that there was no ethical case for discounting the future.”

[33] The discount factor is employed precisely because in this model utility and production functions are assumed not to have any kind of “satiation levels” as Ramsey’s bliss. See Blanchard and Fischer (1989, ch. 2), Romer (1996, ch. 2, part A) and Barro and Sala-i-Martin (2003, ch. 2).

neoclassical growth models in the 1960s and the 1970s. However, it did not prevent the stabilization of a particular normative framework for economic growth that features discounting of future utilities prominently: the Ramsey-Cass-Koopmans model in which the representative agent blends together the social and the individual equilibra. It is precisely in reaction to such stabilization that present-day economists try to go beyond this dominant neoclassical framework.

As we shall see now, it is not only the interconnections of optimal social and individual analyses that shed historical light on the issue of time discounting. This issue was also intertwined with a discussion on the time horizon that agents (or the social planner) consider relevant for making their decisions. On the one hand, with a finite time horizon one may avoid the use of a discount factor and have a way of selecting an optimal growth path. But then the question is how one would select such an arbitrary finite horizon? What happens after the terminal period? Agents would select reaching this terminal period with no capital and thus, will starve to death afterward. On the other hand, an infinite horizon would solve this arbitrariness of pinpointing a finite terminal period and it also was instrumental to bringing nice mathematical properties with a recursive formulation to complex dynamic optimization problems. Time discounting and infinite horizon became different sides of the same mathematical coin. Not without compounding even further the difficulties: with an infinite horizon, one would have to deal with uncertainty, as the usual assumption that future technology (and possibly preferences) throughout the planning horizon are known at its beginning would no longer be reasonable. But economists could claim that they would maintain the assumption of certainty in an infinite horizon context because "analytical difficulties must be taken one by one:" "there is no reason to suppose that the recognition of uncertainty will provide an automatic cut-off point to the horizon," and we shall make "the first study of an indefinitely extended horizon on the simpler basis of a model that presupposes complete certainty about the future" (Koopmans 1957, 107).

3.1 Normative analyses of economic growth

With the surge of interest in problems of economic growth and development that started in the 1940s, simultaneous to a great interest in activity analysis and linear programing, macroeconomists did not wait long to bring an explicit normative analysis to growth. In the mid-1940s, John von Neumann's 1937 growth model was translated into English (von Neumann [1937]

1945-46) and different economists tried to simplify its proofs and to extend it.[34] His input-output (linear) model determined the activity level of the different sectors and the rate of expansion of the economy. It originally had no role for consumption outside the production process and, thus, no utilitarian normative analysis. Dorfman, Samuelson and Solow (1958, ch. 12) brought to the table the "turnpike theorem" for selecting one among the many equilibrium paths: the balanced-growth path (known at the time as the von Neumann growth path) that maximizes the growth rate of the economy.[35] But only in the late 1960s David Gale (1967, 1971) and Lionel McKenzie (1968) would demonstrate a turnpike theorem in which the lifetime utility (instead of output or consumption growth) over an infinite horizon is maximized – this is a problem equivalent to that of the Ramsey-Cass-Koopmans model, but in the context of von Neumann's multi-sectorial linear model.

Gale (1967, 1) was very explicit, in the opening paragraph of his article, that intertemporal consumption allocation is the "chief problem in the theory of dynamic economic planning" and that "there is no completely rational way to attack this problem without considering development programmes over an infinite time horizon." After all, finite horizons are arbitrary and there is no rational way of setting targets for capital accumulation for the final period given that any comparison of benefits and costs of accumulating for future consumption would require taking into account benefits that will accrue after the terminal period (however long it may be). He then placed his contribution in the strand of the literature that does not discount future consumptions, "pioneered by Ramsey [1928]," and argues for the infinite time horizon as an important reference point:

> To describe the situation figuratively, one is guiding a ship on a long journey by keeping it lined up with a point on the horizon even though one knows that long before that point is reached the weather will change (but in an unpredictable way) and it will be necessary to pick a new course with a new reference point, again on the horizon rather than just a short distance ahead.

---

[34] See Loomis (1946), Georgescu-Roegen (1951), Kemeny, Morgenstern, and Thompson (1956), and Dorfman, Samuelson, and Solow (1958), among others.

[35] Dorfman, Samuelson and Solow (1958, 408-16) did extend their analysis of productive efficiency, proposed by Koopmans (1951), to a Pareto-optimum context looking at the consumers' preferences. They argued that they were mostly interested in the production side of the model and that they would propose "a loose and intuitive treatment" of household behavior instead of taking themselves "too far afield" by formulating explicitly the household behavior (which would "multiply the number of variables by too high a factor") (408).

(Gale 1967, 2)

But how did Gale guaranteed that he had a well-defined intertemporal utility (social welfare) maximization problem in this context of no discounting and infinite time horizon? Not willing to resort to Ramsey's bliss, he brought to his analysis the so-called overtaking criterion proposed by Atsumi (1965) and von Weizsäcker (1965): a consumption path, $\{c_1(t)\}_{t=0}^{\infty}$, is said to be better than another path, $\{c_2(t)\}_{t=0}^{\infty}$, if there exists an instant T such that the (undiscounted) lifetime utility of the first path is greater than that of the latter path for every point in time after that T (even if the utility of the path $c_1(t)$ is lower than that of $c_2(t)$ before T, the former is optimal if it overtakes the latter in terms of lifetime utility after T):

$$\int_0^t u(c_1(\tau))\cdot d\tau > \int_0^t u(c_2(\tau))\cdot d\tau \qquad \forall t \geq T$$

Note that in order to choose the best among two alternative consumption paths, according to this criterion, one needs to compute integrals over finite horizons. Therefore, the problem of an eventually unbounded integral like $\int_0^{\infty} u(c_1(t))\cdot dt$ can no longer exist. However, in contrast to the strategy of maximizing the discounted lifetime utility (or minimizing the lifetime distance of the utility to the bliss level), the overtaking criterion does not choose between every conceivable pair of paths, as pointed out by Koopmans (1967, 5). Nonetheless, he continues, "the partial ordering defined by [this] criterion suffices for determining a unique optimal path in the circumstances assumed by Ramsey [(1928)]," which are basically the same as those of the Ramsey-Cass-Koopmans model of a single good.[36]

However, a utilitarian analysis in a von Neumann framework was not the only way to make a normative discussion in the economic growth literature of the 1950s and 1960s. Another way came from the activity analysis literature and the effort to extend the general-equilibrium theory to the case of dated commodities – which brought the question of treating interest rate as an additional price. So, the key issue was to extend the theory of competitive Pareto efficient equilibrium reached through the price system to a dynamic case, recognizing that the same commodity in different points in time are in fact different goods.

---

[36] McKenzie (1968) extended Gale (1967) also using the overtaking criterion and placing Ramsey (1928) as the father of studies of optimal (utilitarian) programs of economic growth.

Growth economists usually refer to the work of Edmond Malinvaud (1953) as the crucial contribution in the context of their models, and he was working exactly with this issue of a dynamic general-equilibrium theory.[37] Malinvaud (1953) discussed both the cases of finite and of infinite horizons, and recognized that he was following the steps of his advisor, Maurice Allais, who made important contributions, in French, to this literature in two books published in the 1940s (*A la recherche d'une discipline économique* of 1943 and *Economie et intérêt* of 1947).[38] It was not uncommon here, as it also happened in the turnpike literature, to define an optimal path as that which maximized consumption, say, in the terminal period of a finite horizon problem. While Malinvaud did bring a clear Paretian principle to his normative analysis, he could concentrate on consumption comparisons because he imposed that only the preferences of present individual consumers matter, and referred to his advisor's work for a less restrictive criterion:

> One might think the Pareto principle is still too restrictive as soon as choices involving time are concerned. … Clearly, only present individual preferences are considered in this paper. Each consumer is supposed fully to appreciate the relative urgency of his present and future needs. However, should this hypothesis be rejected, it would still be possible to introduce a weaker principle for social choices. One may say that C is better than C^1 if it is preferred by all consumers now, and will still be preferred by them given all their future preference patterns. The latter concept has been used extensively by M. Allais…
> (Malinvaud 1953, 242 fn. 14)[39]

So the issue of intergenerational justice that haunted Ramsey and led him to avoid using a time discount factor (thus needing a bliss point in order to have a well-defined optimization

---

[37] See, for instance, how Koopmans (1957, ch. 1, especially pages 105-126) places Malinvaud (1953) centrally when summarizing the developments in the dynamic general-equilibrium theory.

[38] Malinvaud (1953) thanked staff members and guests of the Cowles Commission and, besides Allais, Koopmans and Gerard Debreu for helping him with his analysis.

[39] Koopmans (1957, ch. 1) preferred to discuss these developments associating optimality with "production efficiency," i.e., paths that maximized output and consumption over time. In the end of the chapter he justified his option of skirting utilitarian analysis: "So far our discussion has been held entirely in terms of the price implications of productive efficiency, without regard to further problems of efficiency in the use of outputs to satisfy consumer's preferences. It will be clear, however, that one can … introduce each consumer's preferences through a convex preference ordering on a suitable consumption set… There are, of course, conceptual difficulties in the notion of preference for present goods if it is applied to periods further apart than the life expectation of the economic individual. To explore these further would lead us beyond the scope of this already lengthy essay" (125).

problem) was set aside by Malinvaud and those working in the turnpike literature of maximal growth rate. Yet, there was an additional way of making normative discussion even in growth models *à la* Solow, which consisted in choosing a saving rate (thus sacrificing current consumption) in order to achieve the stationary equilibrium with the highest consumption level. This result became known as the "golden rule" of capital accumulation, and had Phelps (1961, 1965) as an important contributor.

Phelps (1961) wrote his "fable for growthmen," taking place in the "Kingdom of Solovia," and clearly cast the problem of choosing the optimal stationary equilibrium in terms of selecting among simple policies: picking the saving ratio that maximizes each generation's consumption. In order to skirt intergenerational justice, Phelps (1961) concentrated his analysis into the economies which lack "a definite beginning" and are in a stationary equilibrium (he called this a "boundless golden age"): the problem does not require infinite horizon, but indefinite beginning. With this, he proved that the choice of the optimal saving ratio, *s*, is "independent of the 'generation' whose consumption we choose to maximize. The *s* which is optimal for one generation [i.e., that maximizes its own consumption,] in a natural boundless golden age is optimal for all" (641).

Pearce (1962) and Samuelson (1962) criticized Phelps's normative analysis arguing that he had not proven that a golden rule path maximizes intertemporal utility. Moreover, when one takes this into consideration, they argued, the rate of time preference may make the utility maximizing path different from the golden rule one. Curiously, Pearce (1962) made the argument by maximizing the undiscounted integral of instantaneous utility over a finite time horizon. Phelps (1965) accepted the criticism but insisted that his golden-rule analysis could still have a normative content, as it indicated that we should avoid what he called dynamic inefficiencies: an economy that accumulates more capital than the golden-rule level "by at least some constant amount" is dynamic inefficient because it could choose another path that implied more consumption "at least some of the time and never less consumption" (794). Phelps thus framed the normative discussion of growth in terms of maximizing the consumption level, although in a clever Paretian argument that dynamic inefficient paths are dominated by a path with lower saving ratio that implied higher consumption level at some point and no lower level than that of the dynamic inefficient path.

What we see here is that there were alternative (and not necessarily antagonistic) ways of making normative analyses of economic growth in the 1950s and the 1960s, at the time when Cass (1965) and Koopmans (1965) made their contributions. One could associate optimality to paths that maximize either output growth or utility in a multisectoral model, or to paths that maximize consumption (over time or in a particular time period). The Ramsey-Cass-Koopmans one sector model brought to the fore the optimality criterion of maximizing discounted lifetime utility of a representative agent over the indefinite future.

3.2 Axiomatizing Time Preferences, Recursiveness, and Resisting Discounting

A crucial element in the developments of intertemporal models of growing economies is the axiomatization of time preferences done by economists like Koopmans (1960, 1964, 1972), Koopmans, Peter Diamond and Richard Williamson (1964), and Diamond (1965). What these authors did was to transform Fisher's concept of impatience (that individuals facing the alternatives of having a given reward today or in the future opt for current reward) into a central element defining an ordinal intertemporal utility function.

Koopmans (1960) argued that "simple postulates" about the utility function (concerning its continuity, stationarity and other properties – but nothing related to timing preference), which has as argument consumption paths over the infinite future, logically imply impatience, i.e., that agents prefer current than future utilities of the same magnitude.[40] He then modified slightly one of his postulates and showed that this had the drastic implication that the intertemporal utility function is now cardinal and exhibits "period independence": it is "a discounted sum of all future one-period utilities, with a constant discount factor α," $\alpha \in (0,1)$

$$U = \sum_{t=1}^{\infty} \alpha^{t-1} \cdot u(x_t)$$

where x_t is the vector of the goods in the economy at time t, u is the instantaneous utility function, and U the lifetime utility. In other words, a discount factor representing impatience is a key element that extends the comparison of utilities within the same period to that of utilities in different periods. As Koopmans (1972, 97) put this: "a complete and continuous preference ordering of consumption programs for an infinite future necessarily gives a decreasing, or eventually decreasing, weight to consumption in a more distant future." If one wishes to consider the no discounting case, for ethical or other reasons, then one must employ the overtaking criterion instead of this intertemporal utility function.

[40] He justified the infinite horizon as follows: "To avoid complications connected with the advancing age and finite life span of the individual consumer, these postulates were set up for a (continuous) utility function of a consumption program extending over an infinite future period" (Koopmans 1960, 287).

Koopmans (1960, 288) calls attention to the fact that his theory is flexible and can be interpreted as defining a function that represents time preferences either of an individual consumer, or of an aggregate of such individuals, or, finally, choices in a centrally planned economy. However, "in each of these interpretations further modifications and refinements may be called for." More and more extensions of Koopmans's work treated discounting as an abstract and formal issue that delivered well-behaved intertemporal utility functions which would represent generic preferences (i.e., there is no discussion if these preferences were of individuals or of a social planner).[41] Here, Ramsey's objective function (undiscounted lifetime utility of generations) became just one possible criterion, but one that "lacks generality in the nature of the evaluation [of streams that extends over an infinite future] and fails to define a sensitive ordering in parts of the program space" (Diamond 1965, 170).

As a consequence of this axiomatization, economists now knew exactly how to do what Ramsey (1928) had done and what Tintner was reluctant to do: to write the lifetime utility function as the integral over time of the instantaneous utility function, and, going beyond Ramsey, to work with it in an Arrow-Debreu general equilibrium framework. Another consequence was that economists had a way of doing what Ramsey did not do explicitly: to write the lifetime utility function as the integral of discounted instantaneous utilities:

$$\underset{c_t, k_t}{Max} \quad U_0 = \int_0^{\infty} e^{-\theta \cdot t} u(c_t) \cdot dt \qquad s.t.: \begin{cases} f(k_t) = c_t + \dfrac{dk_t}{dt} + (\delta + n) \cdot k_t \\ k_t, c_t \geq 0 \quad \forall t; k_0 \quad given \end{cases}$$

where c_t and k_t are per capita consumption and capital stock, n and δ are the rates of population growth and of depreciation, respectively, and θ the discount rate. This problem of the Ramsey-Cass-Koopmans model is that of a benevolent social planner who, at time zero, maximizes social welfare (which is the discounted lifetime utility of the representative household). It is analogous to Ramsey's problem presented in section 1, with the substantial difference that the use of a discount factor allowed economists not to use Ramsey's trick of the existence of a bliss.

Cass (1965), who was aware of Koopman's work (available in 1963 as a Cowles Foundation discussion paper), placed his own contribution as an elaboration of Ramsey's discussion of the optimum saving problem. Cass (1965, 234) postulated a centralized economy that

[41] See, for instance, Koopmans, Diamond and Williamson (1964), and Diamond (1965). The same spirit of generalizing mathematical results, using discounting as a technical necessity, can be seen in papers considering that intertemporal utility function is not additive (the literature on intertemporally dependent preferences). See Marcel Boyer (1975) for an early discussion of this issue.

maximizes a discounted welfare function that was justified with a brief politically pragmatic argument:

> … along with population growth, the central planning authority recognizes that consumption tomorrow is not the same thing as consumption today. For this reason, it takes the politically pragmatic view that its planning obligation is stronger to present and near future generations than to far removed future generations. This view is implemented in practice by discounting future welfare at a positive rate greater than the population growth rate … rather than by short term planning …

Koopmans (1965, 228-9) presented an "eclectic model" with a twofold purpose: first, "to illustrate the usefulness of the tools and concepts of mathematical programming in relation to the problem of optimal economic growth"; second, "to argue against the complete separation of the ethical or political choice of an objective function from the investigation of the set of technically feasible paths." Very much in line with his second purpose, Koopmans (1965) takes an agnostic view of considering different types of lifetime utility to be maximized, undiscounted and discounted, over finite and infinite horizons. Building on his previous work that axiomatized intertemporal utility functions, he went on and argued that the existence of an optimum path depends critically on the criterion adopted. He started by admitting Ramsey's stance of "an ethical preference for neutrality between the welfare of different generations" in per capita terms (Koopmans 1965, 239), to then argue that no utility function of all consumption paths exists in this case. This technical difficulty can be circumvented either by adopting Ramsey's bliss point, or by taking the ethical stance of discounting the utility of future generations. Koopmans (1965) did not suppose a bliss, but recognized that with a steady population increase "the golden rule path can take the place of Ramsey's state of bliss in defining eligibility" (240): so one could minimize the distance between the flow of utility and the utility of the golden rule path. Coupled with mathematical simplicity of finite horizon problems, Koopmans (1965, 241) proved the following proposition:

> There is a number $\overline{U}$ such that

$$U_T = \int_0^T \left(u(x_t) - \hat{u}\right)dt \le \overline{U}$$

for all feasible paths (x_t, z_t) and for all horizons *T*. [where $\hat{u}$ is the utility of the golden rule equilibrium]

He then argues that for every feasible path either the $\lim_{T\to\infty} U_T$ exists (i.e., the limit is finite) or U_T diverges as *T* tends to infinity. When the limit is finite Koopmans calls this path "eligible," and when it is not, the path is "ineligible." Finally, the best among the eligible paths is the one that maximizes the following utility function:

$$U = \int_0^\infty \left(u(x_t) - \hat{u}\right)dt$$

After characterizing the optimal path for the Ramsey case of no discounting, Koopmans (1965) then argues that analogous results are obtained for the integral over an infinite horizon of the discounted utility, and proceeded with his analysis of this case without justifying further the use of a discount factor.

Another important development in economics after World War II relevant for understanding the stabilization of time discounting was the dynamic programming methods advanced by Richard Bellman (1953, 1957), David Blackwell (1965), and others. These methods provided recursive solutions to dynamic problems that were easier to obtain relative to the standard calculus of variations approach and became a staple in present-day macroeconomic analysis. Among the many situations studied in the dynamic programming literature, there is the special case of maximizing a payoff (objective) function over the infinite future, subject to some constraints. The infinite horizon brings to economists the possibility of obtaining a time-invariant functional solution: it is easily obtainable when one explores the recursiveness of the problem. Recursiveness refers here to the fact that decisions over the indefinite future are just a decision on an immediate action as a function of the current situation (the "state" of the economy) and a continuation value (e.g. the utility or the payoff of future periods) in a problem whose structure recurs each period (Stokey and Lucas, 1989, 5).[42]

---

42 Bellman (1957, 230) referred to this time invariance of the functional solution introduced by the infinite horizon as "homogeneity": "The infinite problem is, as usual, simpler than the finite case because of the homogeneity introduced by infinite time; after any initial actions, we are confronted by a problem of the same type, with different initial values." The same idea was present in the optimal growth literature as summarized by Koopmans (1967, 2).

However, the problem of maximizing payoffs over the indefinite future is that a solution may imply infinite payoffs. In order to avoid this problem of the inexistence of a finite solution, time discounting is introduced: under certain conditions, maximizing discounted payoffs has a finite solution, as stated by the contraction mapping theorem[43] – which is "an extremely simple and powerful fixed point theorem" (Stokey and Lucas, 1989, 49). This mathematical usefulness of discounting payoffs was clearly articulated by Bellman and Blackwell. In discussing the problem of a firm that minimizes costs from today to the indefinite future (without referring to the earlier literature on maximizing profits over time analyzed in section 2), Bellman (1957, 156) wrote:

> If we wish to consider an unbounded period of time over which this [cost minimization] process operates, we must introduce some device to prevent infinite costs from entering.
>
> The most natural such device is that of discounting the future costs, using a fixed discount ratio … for each period. This possesses a certain amount of economic justification and a great deal of mathematical virtue, particularly in its invariant aspect.

Blackwell (1965, 226), in his turn, treated time discounting from a purely technical perspective:

> This total reward may well be infinite… We shall avoid this problem by introducing a discount factor β, $0 \leq \beta < 1$, so that unit reward on the *n*th day is worth only β^{n-1}, and shall try to maximize the total expected reward over the infinite future.

The effort economists made to construct a behavioral basis of an ordinal intertemporal utility function that characterized either the behavior of an individual or of a social planner did not prevent serious criticisms either to employing a social discount factor or to treating individual and social decisions in the same way. In the discussion about the economic principles of socialism (i.e., of allocating scarce resources in planned economies) of the late 1930s and 1940s, Pigou's

---

43 See Stokey and Lucas (1989, 49-55).

defective telescopic faculty of consumers was used to support "that the socialist Board must disregards consumers' preferences on the question of savings and observe instead the principle that future satisfactions be valued equally with equivalent present satisfactions" (Bergson 1948, 415). Maurice Dobb and Oskar Lange advocated that consumer sovereignty would not apply under socialism, as individuals would undervalue future satisfactions relative to equivalent present ones and would not invest enough from the social standpoint.[44] Years later a similar debate would emerge in the context of market (mixed) economies, which was concerned with the relationship between individual and social decisions. Dobb (1960, ch. 2) criticized, using ideas proposed by Amartya Sen (1957), an individualist approach to social decisions: that of equating social decisions a sum of independent individual decisions. This is even more problematic for intertemporal decisions, as Sen (1960, 18-19) described:

> …individuals' choices over time are notoriously irrational … [There] is no good reason why the State, *qua* custodian of future generations as well as the present, should adopt this irrationality as its own. The irrationality consists in discounting the future *solely* because the passage of time (i.e. apart from differences in income over time or uncertainty regarding it). If one is likely to be the same person five years hence, and to have roughly the same real income, the gift of a certain enjoyment … will add the same amount to the pleasures of a lifetime whether it is promised in five years' time or today. If one places a premium on having it today, this can only be a sign of weakness of will or of temperament – a defect of the 'telescopic faculty' (as Pigou so aptly put it). In our rational moments we surely would not want our planners to imitate this defect. … We cannot derive any investment criterion from individual savings-decisions, whether registered on a market or in some other way.

Sen (1957) criticized Tinbergen's (1956) discussion of the optimum saving rate – where Tinbergen used a discounted lifetime utility function over an infinite horizon as the criterion to be maximized – even after accepting his "assumptions about measurability, addibility and interpersonal comparability of utility as well as the identity of utility with welfare, which are not difficult to criticise" (Sen 1957, 745). One of the points he criticized was exactly the use of a

[44] See Bergson (1948) for a survey of this literature and the references therein.

discount rate for the social planner's problem, notwithstanding his view that using it for individual decisions is not "entirely irrational" (due to individuals' mortality, income uncertainties, and greater weight to his own satisfaction relative to that of his heirs):

> Surely the "psychological discount" of future income arising from the so-called defective telescopic psychology is not very relevant from the point of view of planning. We are interested in tomorrow's satisfaction as such, not in today's assessment of tomorrow's satisfaction. The "psychological discount rate" … which Tinbergen uses in this connection, appears to be not quite applicable in planning problems.
>
> Sen (1957, 746)[45]

Many others followed suit. Sen (1961) and Marglin (1963) also criticized the idea of applying consumers' sovereignty to the problem of intertemporal allocation of resources.[46] An important debate then was whether individual decisions by the present generation through the market mechanism give the same allocation as the case in which they all vote collectively on this matter. This brought the issue of the appropriateness of intertemporal discount rates that may make sense at the individual level but implied that the present generation attaches lower weight to future generations. This in turn brought a series of difficulties related to the political mechanisms of aggregating isolated individual decisions into a social decision.[47]

Although the narrative of economic dynamics (and of optimal economic growth) can be seen as a struggle to find ways of dealing with time preferences, the axiomatization proposed by Koopmans and others did not mean that a consensus would soon emerge in the growth literature. Alternative concepts of convergence were developed to guarantee that maximization of intertemporal utility was a well-defined mathematical problem, and the discussion on a social discount rate generated heat and light. Still in the 1970s, we see economists in the optimal growth literature considering versions of their models with and without discounting and infinite horizon,

---

[45] See also Sen (1961) for an overall view of this literature and suggestions for venues to be explored in which intertemporal judgments can be avoided.

[46] Marglin (1963) was criticized by Tullock (1964) and Lind (1964).

[47] Sen (1967) argued that all this literature essentially analyzed problems of the type of the "prisioners' dilemma" in game theory and that there were additional difficulties to be faced, which implied that social and private discount rates are different.

thus using, when needed, the overtaking criterion.[48] Over time, however, economists increasingly adopted one way of modeling intertemporal growth problems, in which time discounting became a technical requirement. Clearly by the late 1980s, this was the predominant view, as exemplified by the books by Stokey and Lucas (1989) and Blanchard and Fischer (1989).[49]

4. Concluding remarks

This is a narrative in which a technical aspect of economic models had its meaning and use negotiated among different economists (and mathematicians) over time. Time discounting was controversial not only because it acquired an ethical dimension when applied to future utilities of generations. Even the mathematicians and economists working on the dynamic behavior of firms understood the necessity of a discount factor in different ways, and such varied understandings did not arise from an ignorance of ideas about preferences over time, as impatience.

In the 1960s the optimal neoclassical growth literature that emerged postulated individuals (or a planner) maximizing an objective function subject to restrictions (in contrast to the earlier growth literature *à la* Solow that assumed *ad hoc* consumption or saving functions). In it, there were alternative criteria of convergence that allowed economists to specify variants of their model with and without time discounting, despite the contributions of economists like Koopmans and Diamond who found explicit behavioral basis for a discounted intertemporal utility functions. These contributions, together with the development of dynamic programming methods paved the way for the use of a time discount rate to become the most widely used framework for analyzing intertemporal choices, to use the words of Chabris, Laibson and Schultz (2008).

However, all these technical developments did not imply that economists would immediately be comfortable with using a discount rate to a planner's problem of intertemporal allocation of resources. Economists like Dobb, Sen, and Marglin dissented from this practice and resisted to treating individual and social discounting equally as technical requirements. The modeling strategies of intertemporal discounted utility maximization were intertwined not only

[48] See, for example, Brock (1973) and Brock and Mirman (1973). As already noted, both cases were also considered by Phelps (1967) in his critique to the Phillips curve.

[49] Not many years before these books were published, Sargent (1987, ch. 1) presented dynamic programming in general terms and applied it to the Ramsey-Cass-Koopmans model without discussing the appropriateness of discounting future utilities. In appendix A.7 he briefly discussed the "discounted dynamic programming" and referred to the forthcoming book by Stokey and Lucas, so that discounting was also a technical issue.

with normative issues of intergenerational comparison, but also with issues like the planning horizon (if finite or infinite) and (un)certainty about the future.

Over time, with the spread of the representative agent growth model known as the Ramsey-Cass-Koopmans model, discounting the utilities of future generations had less and less an ethical dimension to it. Economists working on economic growth (and on intertemporal macroeconomics models more generally) increasingly treated the use of a discount factor as a technical requirement that guarantee the existence of finite solutions, which are easy to obtain with recursive techniques. From an ethically indefensible to a technically indispensable practice, there is a path – clearly one not straightforward – leading economists through the wild understandings about the use of a time discount factor.

References:

Atsumi, H. 1965. Neoclassical Growth and the Efficient Program of Capital Accumulation. *Review of Economic Studies*, 32 (2): 127-136.

Barro, R., and X. Sala-i-Martin. 2003. *Economic Growth*. 2nd ed. New York: McGraw-Hill.

Bellman, R. 1953. *An Introduction to the Theory of Dynamic Programming*. Santa Monica: RAND Corporation.

__________. 1957. *Dynamic Programming*. Princeton: Princeton University Press.

Bergson, A. 1948. Socialist Economics. In: H. S. Ellis (ed.), *A Survey of Contemporary Economics*, vol. 1. Homewood, Illinois: Richard D. Irwin.

Blackwell, D. 1965. Discounted Dynamic Programming. *Annals of Mathematical Statistics*, 36 (1): 226-35.

Blanchard, O. J., and S. Fischer. 1989. *Lectures on Macroeconomics*. MIT Press.

Boianovsky, M., and K. D. Hoover (eds.). 2009. *Robert Solow and the Development of Growth Economics*, *HOPE* 41 (supplement).

Boulding, K. E. 1935. The Theory of a Single Investment. *Quarterly Journal of Economics*, 49 (3): 475-94.

__________. 1942. The Theory of the Firm in the Last Ten Years. *American Economic Review*, 32 (4): 791-802.

Boumans, M. 1993. Paul Ehrenfest and Jan Tinbergen: A Case of Limited Physics Transfer. In: N. de Marchi (ed.), *Non-Natural Social Science: Reflecting on the Enterprise of 'More Heat than Light'*, *HOPE* 25 (supplement): 131-56.

Boyer, M. 1975. An Optimal Growth Model with Stationary Non-Additive Utilities. *Canadian Journal of Economics*, 8 (2): 216-37.

Brock, W. A. 1973. Some Results on the Uniqueness of Steady States in Multisector Models of Optimum Growth When Future Utilities are Discounted. *International Economic Review*, 14 (3): 535-59.

Brock, W. A. and L. J. Mirman. 1973. Optimal Economic Growth and Uncertainty: The No Discounting Case. *International Economic Review*, 14 (3): 560-73.

Cass, D. 1965. Optimum growth in an aggregative model of capital accumulation. *Review of Economic Studies*, 32 (3): 233-40.

Chabris, C. F., D. I. Laibson and J. P. Schuldt. 2008. Intertemporal Choice. In: S. N. Durlauf and L. E. Blume (eds.), *The New Palgrave Dictionary of Economics*. 2nd ed. Palgrave Macmillan. Available at: http://www.dictionaryofeconomics.com/article?id=pde2008_P000365, accessed on 06 February 2009.

Collard, D. A. 1996. Pigou and Future Generations: a Cambridge tradition. *Cambridge Journal of Economics*, 20 (5): 585-97.

Dale, A. I., and S. Kotz. 2011. *Arthur L. Bowley – a pioneer in modern statistics and economics*. Singapore: World Scientific Publishing.

Davis, H. T. 1958. Charles Frederick Roos. *Econometrica*, 26 (4): 580-89.

De Vroey, M. 2004. The History of Macroeconomics Viewed Against the Background of the Marshall-Walras Divide. In: K. D. Hoover and M. De Vroey (eds.), *The IS-LM Model: Its Rise, Fall, and Strange Persistence*, *HOPE* 36 (supplement): 57-91.

Diamond, P. 1965. The Evaluation of Infinite Utility Streams. *Econometrica*, 33 (1): 170-7.

Dimand, R., and W. Veloce. 2007. Charles F. Roos, Harold T. Davis and the Quantitative Approach to Business Cycle Analysis at the Cowles Commission in the 1930s and early 1940s. *European Journal of the History of Economic Thought*, 14 (3): 519-42.

Dobb, M. 1960. *An Essay on Economic Growth and Planning*. London: Routledge & Kegan Paul.

Dorfman, R., Samuelson, P. A., and R. Solow. 1958. *Linear Programming and Economic Analysis*. New York: McGraw-Hill.

Duarte, P. G. 2009a. Frank P. Ramsey: A Cambridge Economist. *HOPE*, 41 (3): 445-70.

__________. 2009b (ed.). Frank Ramsey's Notes on Saving and Taxation. *HOPE*, 41 (3): 471-89.

__________. 2009c. The Growing of Ramsey's Growth Model. In: M. Boianovsky and K. D. Hoover (eds.), *Robert Solow and the Development of Growth Economics*, *HOPE*, 41 (supplement): 161-81.

__________. 2010. Beyond Samuelson's Chapter on Ramsey. *History of Economic Ideas*, 18 (3): 121-59.

Earl, P. 2008. Hart, Albert Gailord (1909–1997). In: S. N. Durlauf and L. E. Blume (eds), *The New Palgrave Dictionary of Economics*. 2nd ed. Palgrave Macmillan. Available at: http://www.dictionaryofeconomics.com/article?id=pde2008_H000026, accessed on 06 February 2009.

Evans, G. C. 1924. The Dynamics of Monopoly. *American Mathematical Monthly*, 31 (2): 77-83.

__________. 1925. A Mathematical Theory of Economics. *American Mathematical Monthly*, 32 (3): 104-10.

__________. 1930. *Mathematical Introduction to Economics*. New York: McGraw-Hill.

__________. 1931. A Simple Theory of Economic Crises. *Journal of the American Statistical Association*, 26 (173, supplement): 61-8.

__________. 1934. Maximum Production Studied in a Simplified Economic System. *Econometrica*, 2 (1):37-50.

Fisher, I. 1930. *The Theory of Interest*. New York: Macmillan.

Fox, K. A. 2008. Tintner, Gerhard (1907–1983). In: S. N. Durlauf and L. E. Blume (eds.), *The New Palgrave Dictionary of Economics*. 2nd ed. Palgrave Macmillan. Available at: http://www.dictionaryofeconomics.com/article?id=pde2008_T000066, accessed on 07 April 2009.

Gale, D. 1967. On Optimal Development in a Multi-Sector Economy. *Review of Economic Studies*, 34 (1): 1-18.

__________. 1971. Correction to "On Optimal Development in a Multi-Sector Economy". *Review of Economic Studies*, 38 (3): 384.

Georgescu-Roegen, N. 1951. The Aggregate Linear Production Function and Its Applications to von Neumann's Economic Model. In: T. Koopmans (ed.), *Activity Analysis of Production and Allocation*. New York: Wiley.

Hart, A. G. 1937. Anticipations, Business Planning, and the Cycle. *Quarterly Journal of Economics*, 51 (2): 273-97.

_________. [1940] 1951. *Anticipations, Uncertainty, and Dynamic Planning*. New York: Augustus M. Kelley.

__________. 1942. Risk, Uncertainty, and the Unprofitability of Compounding Probabilities. In: O. Lange, F. McIntyre, and T. O. Yntema (eds.), *Studies in Mathematical Economics and Econometrics—in memory of Henry Schultz*. Chicago: The University of Chicago Press.

__________. 1945. "Model-Building" and Fiscal Policy. *American Economic Review*, 35 (4): 531-58.

Hicks, J. R. 1935. Wages and Interest, the Dynamic Problem. *Economic Journal*, 45 (179): 456-68.

Hotelling, H. 1931. The economics of exhaustible resources. *Journal of Political Economy*, 39 (2): 137-75.

Jevons, W. S. 1888. *The Theory of Political Economy*. 3rd ed. London: Macmillan.

Kamien, M. I. 2008. Calculus of Variations. In: S. N. Durlauf and L. E. Blume (eds.), *The New Palgrave Dictionary of Economics*. 2nd ed. Palgrave Macmillan. Available at: http://www.dictionaryofeconomics.com/article?id=pde2008_C000003, accessed on 07 April 2009.

Kemeny, J. G., Morgenstern, O., and G. L. Thompson. 1956. A Generalization of the von Neumann Model of an Expanding Economy. *Econometrica*, 24 (2): 115-35.

Klein, L. R. [1947] 1961. *The Keynesian Revolution*. New York: Macmillan.

Koopmans, T. C. 1951. Efficient Allocation of Resources. *Econometrica*, 19 (4): 455-65.

__________. 1957. *Three Essays on the State of Economic Science*. New York: McGraw-Hill.

__________. 1960. Stationary Ordinal Utility and Impatience. *Econometrica*, 28 (2): 287-309.

__________. 1964. On Flexibility of Future Preference. In: M. W. Shelly and G. L. Bryan (eds.), *Human Judgments and Optimality*. New York: John Wiley & Sons.

__________. 1965. On the concept of optimal economic growth. In: *The Econometric Approach to Development Planning*. First Part. Città del Vaticano: Pontificia Academia Scientiarvm.

__________. 1967. Objectives, Constraints, and Outcomes in Optimal Growth Models. *Econometrica*, 35 (1): 1-15.

__________. 1972. Representation of Preference Orderings Over Time. In: C. B. McGuire and R. Radner (eds.), *Decision and Organization: a volume in honor of Jacob Marschak*. Amsterdam: North-Holland.

Koopmans, T., P. A. Diamond and R. E. Williamson. 1964. Stationary Utility and Time Perspective. *Econometrica*, 32 (1-2): 82-100.

Lind, R. C. 1964. The Social Rate of Discount and the Optimal Rate of Investment: further comment. *Quarterly Journal of Economics*, 78 (2): 336-45.

Ljungqvist, L., and T. Sargent. 2012. *Recursive Macroeconomic Theory*. 3rd ed. Cambridge, Massachusetts: MIT Press.

Loomis, L. H. 1946. On a Theorem of von Neumann. *Proceedings of National Academy of Science*, 32 (8): 213-15.

Louçã, F. 2007. *The Years of High Econometrics – a short history of the generation that reinvented econometrics*. London: Routledge.

Marglin, S. 1963. The Social Rate of Discount and the Optimal Rate of Investment. *Quarterly Journal of Economics*, 77 (1): 95-111.

McKenzie, L. 1968. Accumulation Programs of Maximum Utility and the Von Neumann Facet. In: J. N. Wolfe (ed.), *Value, Capital, and Growth: Essays in Honor of Sir John Hicks*. Edinburgh: Edinburgh University Press, pp. 353-383.

Mirowski, P. 2002. *Machine Dreams: Economics Becomes a Cyborg Science*. Cambridge: Cambridge University Press.

Mirowski, P. and D. W. Hands. 1998. A Paradox of Budgets: The Postwar Stabilization of American Neoclassical Demand Theory. In: M. Morgan and M. Rutherford (eds.), *From Interwar Pluralism to Postwar Neoclassicism*, *HOPE* 30 (supplement): 260-92.

Morgan, M. 1990. *The History of Econometric Ideas*. Cambridge: Cambridge University Press.

Morgan, M., and M. Rutherford, eds. 1998. From Interwar Pluralism to Post-war Neoclassicism, *HOPE* 30 (suplement).

Obstfeld, M., and K. Rogoff. 1996. *Foundations of International Macroeconomics*. Cambridge, Massachusetts: MIT Press.

Pearce, I. F. 1962. The End of the Golden Age in Solovia: a further fable for growthmen hoping to be ‘one up’ on Oiko. *American Economic Review*, 52 (5): 1088-97.

Phelps, E. S. 1961. The Golden Rule of Accumulation: a fable for growthmen. *American Economic Review*, 51 (4): 638-43.

__________. 1965. Second Essay on the Golden Rule of Accumulation. *American Economic Review*, 55 (4): 793-814.

__________. 1967. Phillips Curves, Expectations of Inflation and Optimal Unemployment over Time. *Economica*, 34 (135): 254-81.

Pigou, A. C. 1912. *Wealth and Welfare*. London: Macmillan.

__________. [1920] 1924. *The Economics of Welfare*. London: Macmillan.

Pomini, M. and G. Tusset. 2009. Habits and Expectations: dynamic general equilibrium in the Italian Paretian school. *HOPE*, 41 (2): 311-42.

Ramsey, F. P. 1928. A Mathematical Theory of Saving. *Economic Journal*, 38 (152): 543-59.

Romer, D. 1996. *Advanced Macroeconomics*. New York: McGraw-Hill.

Roos, C. F. 1925. A Mathematical Theory of Competition. *American Journal of Mathematics*, 47 (3): 163-75.

__________. 1927. A Dynamical Theory of Economics. *Journal of Political Economy*, 35 (5): 632-56.

__________. 1929. Some Problems of Business Forecasting. *Proceedings of the National Academy of Sciences*, 15 (3): 186-91.

__________. 1930. A Mathematical Theory of Price and Production Fluctuations and Economic Crises. *Journal of Political Economy*, 38 (5): 501-22.

__________. 1934. *Dynamic Economics: theoretical and statistical studies of demand, production, and prices*. Bloomington: The Principia Press.

Rosenstein-Rodan, P. N. 1934. The Rôle of Time in Economic Theory. *Economica*, new series, 1 (1): 77-97.

Samuelson, P. A. 1937a. A Note on Measurement of Utility. *Review of Economic Studies*, 4 (2): 155-61.

__________. 1937b. Some Aspects of the Pure Theory of Capital. *Quarterly Journal of Economics*, 51 (3): 469-96.

__________. [1947] 1983. *Foundations of Economic Analysis*. Enlarged Edition. Cambridge, Massachusetts: Harvard University Press.

__________. 1948. Dynamic Process Analysis. In: H. S. Ellis (ed.), *A Survey of Contemporary Economics*, vol. 1. Homewood, Illinois: Richard D. Irwin.

__________. 1962. Comment. *Review of Economic Studies*, 29 (3): 251-54.

Samuelson, P. A. and R. Solow. 1956. A Complete Capital Model involving Heterogeneous Capital Goods. *Quarterly Journal of Economics*, 70 (4): 537-62.

Sen, A. K. 1957. A Note on Tinbergen on the Optimum Rate on Saving. *Economic Journal*, 67 (268): 745-748.

__________. 1961. On Optimising the Rate of Saving. *Economic Journal*, 71 (283): 479-496.

__________. 1967. Isolation, Assurance, and the Social Rate of Discount. *Quarterly Journal of Economics*, 81 (1): 112-24.

Stokey, N. L., and R. E. Lucas. 1989. *Recursive Methods in Economic Dynamics*. Cambridge, Massachusetts: Harvard University Press.

Taussig, F. W. [1911] 2007. *Principles of Economics*. Vol. 2. New York: Cosimo Classics.

Tinbergen, J. 1933. The Notions of Horizon and Expectancy in Dynamic Economics. *Econometrica*, 1 (3): 247-64.

__________. 1935. Annual Survey: Suggestions on Quantitative Business Cycle Theory. *Econometrica*, 3 (3): 241-308.

__________. 1956. The Optimum Rate of Saving. *Economic Journal*, 66 (264): 603-9.

__________. 1960. Optimum Savings and Utility Maximization Over Time. *Econometrica*, 28 (2): 481-89.

Tinbergen, J. and H. C. Bos. 1962. *Mathematical Models of Economic Growth*. New York: McGraw-Hill.

Tintner, G. 1936a. A Note on Distribution of Income Over Time. *Econometrica*, 4 (1): 60-6.

__________. 1936b. Review of "Dynamic Economics" by Charles Frederick Roos. *Journal of Political Economy*, 44 (3): 404-9.

__________. 1937. Monopoly Over Time. *Econometrica*, 5 (2): 160-70.

__________. 1938a. The Maximization of Utility Over Time. *Econometrica*, 6 (2): 154-58.

__________. 1938b. The Theoretical Derivation of Dynamic Demand Curves. *Econometrica*, 6 (4): 375-80.

__________. 1939. Elasticities of Expenditure in the Dynamic Theory of Demand. *Econometrica*, 7 (3): 266-70.

__________. 1942a. A Contribution to the Non-Static Theory of Choice. *Quarterly Journal of Economics*, 56 (2): 274-306.

__________. 1942b. A "Simple" Theory of Business Fluctuations. *Econometrica*, 10 (3/4): 317-20.

__________. 1942c. The Theory of Production Under Nonstatic Conditions. *Journal of Political Economy*, 50 (5): 645-667.

__________. 1942d. A Contribution to the Nonstatic Theory of Production. In: O. Lange, F. McIntyre, and T. O. Yntema (eds.), *Studies in Mathematical Economics and Econometrics – in memory of Henry Schultz*. Chicago: The University of Chicago Press.

Tullock, G. 1964. The Social Rate of Discount and the Optimal Rate of Investment: comment. *Quarterly Journal of Economics*, 78 (2): 331-36.

van den Bogaard, A. 1999. *Configuring the Economy: The Emergence of a Modeling Practice in the Netherlands, 1920–1955*. Amsterdam: Thela Thesis.

von Neumann, J. [1937] 1945-46. A Model of General Economic Equilibrium. *Review of Economic Studies*, 13 (1): 1-9.

von Weizsäcker, C. C. 1965. Existence of Optimal Programs of Accumulation for an Infinite Time Horizon. *Review of Economic Studies*, 32 (2): 85-104.

Weintraub, E. R. 1991. *Stabilizing Dynamics: constructing economic knowledge*. Cambridge: Cambridge University Press.

__________. 2002. *How Economics Became a Mathematical Science*. Durham: Duke University Press.

Woodford, M. 2003. *Interest and Prices: foundations of a theory of monetary policy*. Princeton: Princeton University Press.

Wulwick, N. J. 1995. The Hamiltonian Formalism and Optimal Growth Theory. In: I. H. Rima (ed.), *Measurement, Quantification and Economic Analysis – Numeracy in Economics*. London: Routledge.