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Abstract:

Frank P. Ramsey (b. 1903 – d. 1930) was a Cambridge mathematician who interacted closely to leading economists of his time such as Arthur Cecil Pigou, John Maynard Keynes and Roy Harrod. In the 1920s he was considered by many as a brilliant student who was clearly integrated to the elite group of Cambridge and who knew and was friend of such luminaries as G. E. Moore, Bertrand Russell, Ludwig Wittgenstein, and also Lytton and James Strachey, Virginia Woolf, Lionel Penrose, Kingsley Martin, Richard Braithwaite, I. A. Richards, and C. K. Ogden. Despite being few in numbers, his contributions to mathematics, logic, philosophy and economics are considered by practitioners in these areas as the most profound and original work in the first half of the twentieth century. This canonization was initiated soon after the untimely death of Ramsey, before completing 27 years of age. Economists portray Ramsey as a sleeping giant, someone with almost no impact until the 1950s, when they finally learned the mathematical tools necessary to apprehend his ideas. Building on my previous works on Ramsey this article surveys his life and work, focusing on economics, and shows how this provides interesting historical windows both to the Cambridge milieu of the 1920s and to the important transformations of economics after World War II.

Keywords: Frank Ramsey, University of Cambridge, Arthur Cecil Pigou, John Maynard Keynes, Paul Samuelson

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Frank Ramsey[1]

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An enthusiast for the public welfare and for the discovery of mathematical truth. With these words the twenty-years old Frank Ramsey described himself in a talk he delivered on January 1924 (Ramsey, 1924, p. 311). While certainly indicating a self-image and identity that are historically constructed, these words serve the purpose here of indicating the wide range of Ramsey's interests and contributions. Despite Ramsey's short publishing career, that lasted for only eight years, 'in a series of brilliant essays [he] laid the foundations of several new and flourishing theories in philosophy, mathematics and economics,' as Nils-Eric Sahlin (1990, p. ix) wrote.[2] From logic to the foundations of mathematics, to combinatorics, to philosophy and probability, and to economics, Ramsey is venerated as having produced profound and original contributions.

In economics, Frank Ramsey published two major papers in the *Economic Journal*, on taxation in 1927 and optimal savings in 1928, under John Maynard Keynes's editorship. After World War II, Ramsey became a sacred predecessor in four important fields of economics (Duarte, 2009a, pp. 445-6; 2009b, pp. 165-9). First, his criticism to Keynes's *Treatise on Probability* (1921), published in 1922 and developed further in 1926 (Ramsey, 1922b, 1926), put forward ideas on subjective probability that in the 1950s became important to the subjective decision theory and expected utility literature.[3] Second, his 1928 paper received growing attention in the economic growth literature of the 1960s, with the works of David Cass and Tjalling Koopmans, leading to the emergence of the so-called 'Ramsey-Cass-Koopmans model' of optimal growth and later related to the economics of conservation and exhaustible resources (cf. Duarte, 2009b; Erreygers, 2009). Third, his 1927 contribution to taxation became a central reference in the public finance literature. William Baumol and David Bradford (1970, pp. 277-80) borrowed a general storyline from Paul Samuelson (1964, p. 95) which placed Ramsey's mathematical proof as a critical development in the history of the literature on departures from marginal cost pricing. This interpretation made its way into Peter Diamond and James Mirrlees's (1970a,b) articles which, according to many, using Samuelson's (1982, p. 179, fn. 9) words, are 'the spring from which all modern discussion flow.'[4] Fourth, with public finance ideas later becoming important to

[1] This chapter is to appear in *The Palgrave Companion to Cambridge Economics* edited by Robert Cord. It draws on my previous incursions on Ramsey's life and work (Duarte 2009a,b; 2010) and on new material made available recently, in particular in the memoir written by his sister (Paul 2012), as indicated in the text. I thank CNPq (Brazil) for financial support.

[2] See similar words by D. H. Mellor in Ramsey (1990, p. xi).

[3] See Heukelom (2014, ch. 2) for a historical discussion of the development of this literature. See also Soifer (2009, pp. 284-6). Mellor (2005) discusses whether the subjective decision theory that incorporated Ramsey's ideas should be interpreted as a positive or a normative theory.

[4] See Duarte (2010) for a historical discussion on the canonization of Ramsey's 1927 article in the public finance literature of the postwar period.

discussions on optimal economic policies, Ramsey (1927) assumed also the role of a distinctive precursor in monetary economics, with economists nowadays having adopted the language of solving a 'Ramsey problem' to characterize an optimal monetary policy.

This canonization of Frank Ramsey in the post-World War II economics came together with a view that he employed a mathematics too advanced for his peers to be able to appreciate his contributions (Duarte, 2009b, 2010). Therefore, the story goes, roughly three decades had to elapse for economists to have the appropriate mathematical knowledge to finally apprehend Ramsey's ideas. Such narrative has a Romantic mold typical of the historical accounts of mathematicians from the nineteenth century: contrary to eighteenth century mathematicians, who were important public figures in a time when mathematics aimed at understanding the natural world, mathematicians of the following century were seen as lonely individuals worried about logical rigor and internal consistency of their field (Alexander, 2006). The tragic, Romantic narratives of mathematicians, as Alexander (2006) discussed, are well represented in the popular book *Men of Mathematics* published in 1937 by the mathematician and science-fiction writer E. T. Bell.

While the mythical view on Ramsey certainly poses important historical questions of how, and with whose help, his contributions secured a reputable place in postwar economics, the view of Ramsey as someone ahead of his time and an outsider to economics was clearly articulated by Keynes in the obituary he published in 1930 in the *Economic Journal* after the death of his young friend:

> When he did descend from his accustomed stony heights, he still lived without effort in a rarer atmosphere than most economists care to breathe, and handled the technical apparatus of our science with the easy grace of one accustomed to something far more difficult. … [His 1928 paper] is, I think, one of the most remarkable contributions to mathematical economics ever made... The article is terribly difficult reading for an economist, but it is not difficult to appreciate how scientific and aesthetic qualities are combined in it together. Keynes ([1933] 1972, pp. 335-6)

The philosopher G. (George) E. (Edward) Moore, much admired by Ramsey (Duarte, 2009a, p. 455, fn. 24), also praised Ramsey as someone who

> seemed to me to combine very exceptional brilliance with very great soundness of judgment in philosophy. He was an extraordinarily clear thinker. … He had, moreover, an exceptional power of drawing conclusions from a complicated set of facts. (in Ramsey, [1931] 1960, pp. vii-viii)

Roy Harrod was at Oxford and went to the University of Cambridge in the Fall of 1922 to interact with Keynes when he became a close friend of Ramsey. Harrod (1951) described Ramsey very affectionately, as 'a man of extreme brilliance and precocity' (p. 320) and the one who, 'more than any others of the post-war vintage, seemed to embody

the intellectual and personal ideas that were cherished in Cambridge at the opening of the [twentieth] century' (p. 398).[5]

It is true that the early and sudden death of Frank Ramsey, roughly a month prior his 27th birthday, 'make[s] it hard to disentangle what he was really like [and what was his relationship with other people] from the shocked exclamations of grief and praise that naturally followed his death', as his sister wrote in her memoir (Paul, 2012, p. 223). Indeed, Keynes had expressed in two letters to his wife Lydia Lopokova immediately after learning about Ramsey's death that 'it is so terrible when a young person dies', and that Frank 'was in his way the greatest genius in [King's] college, and such a dear creature besides' (quoted in Paul, 2012, p. 268). Nonetheless, during his lifetime Ramsey was admired by and friend of such Cambridge luminaries as Moore, Keynes, Bertrand Russell, Ludwig Wittgenstein, Piero Sraffa, the geneticist, mathematician and later leading psychiatrist Lionel Penrose, the literary critic I. (Ivor) A. (Armstrong) Richards, the English linguist and polymath C. (Charles) K. (Kay) Ogden, and the mathematician turned philosopher Richard Braithwaite, a close friend of Ramsey who edited the first posthumous collection of Ramsey's papers (Ramsey, [1931] 1960). Braithwaite wrote in an obituary for *The Cambridge Review* (cited in Paul, 2012, p. 273):

> By his second year [Frank] was accepted as the arbiter of good reasoning on every subject. For eight years, if an abstruse point arose in philosophy, psychology or economics, the question was: 'What does Frank Ramsey think of it?'

Ramsey was also friend of James and Alix Strachey, who later published the English translation of Sigmund Freud's complete works. James was the youngest brother of Lytton Strachey, an eminent biographer and one of the founders of the Bloomsbury group – which congregated intellectuals and artists, including Keynes, and had an important impact in the British culture of the first decades of the twentieth century. Several members of this group were former Cambridge students and some were members of one of the most distinguished societies in Cambridge's intellectual life, the Cambridge Conversazione Society, known as the Apostles. As Donald Moggridge (1992, p. 66) argued, the importance of this society was due to its longevity and 'the intellectual importance of many, although far from all, of its members. A significant proportion of the reformers of Victorian Cambridge were members'. Keynes, Russell, Moore, Wittgenstein, Braithwaite, Lytton Strachey, E. M. Forster, Leonard Woolf, Roger Fry, Lionel Penrose and Frank Ramsey were some of the distinguished members

[5] Goldsworthy Lowes Dickinson, a Kingsman, also saw Ramsey as a typical Cambridge man, someone who is: 'unworldly without being saintly, unambitious without being inactive, warm-hearted without being sentimental. Through good report and ill such men work on, following the light of truth as they see it; able to be sceptical without being paralyzed; content to know what is noble and to reserve judgement on what is not. The world could never be driven by such men, for the springs of action lie deep in ignorance and madness. But it is they who are the beacon in the tempest and they are more, not less, needed now than ever before' (cited in Martin, [1966] 1969, p. 109, and used by Keynes in the obituary of C. P. Sanger he published in 1930 in the *Economic Journal*).

of the Apostles.[6] Ramsey interacted with the Strachey brothers and was acquainted with other Bloomsbury figures such as Virginia Woolf, E. M. Forster, Roger Fry, and also Frances Partridge and Dora Carrington (Paul, 2012, p. 112, p. 140). Virginia described Ramsey as 'something like a Darwin, broad, thick, powerful, and a great mathematician, and clumsy to boot. Honest I should say, a true Apostle' (Duarte, 2009a, p. 454).

Frank Ramsey was thus inarguably immersed in the Cambridge milieu of the 1920s. He interacted closely to leading economists of the first half of the twentieth century, such as Keynes, Arthur Cecil Pigou and Harrod (Duarte, 2009a), and he was canonized immediately after his death by Cambridge notables like Keynes and Moore. However, his two major contributions to economics only secured him a central place in this field after World War II, when Ramsey's ideas crossed the Atlantic and were nurtured by economists in the United States.[7] Therefore, Frank Ramsey and his works provide an interesting window both to the Cambridge environment of the 1920s and to the transformations of economics in the postwar period. In the first section I discuss in details Ramsey's life and Cambridge in the 1920s, to then summarize Ramsey's contributions to economics in section 2, and briefly mention elements of the rediscovery of Ramsey in postwar American economics in section 3.

1 – The Cambridge road to genialness

Frank Plumpton Ramsey was the first child of Arthur Stanley Ramsey and Agnes Mary Wilson, born yellow with jaundice on 22 February 1903. The family was completed with the birth of Michael (1904), Bridget (1907), and Margaret (1918). His brother Michael occupied the highest post in the Church of England, being the Archbishop of Canterbury from 1961 to 1974, and Margaret became an economist at St. Hilda's College and also a Fellow of Lady Margaret Hall, Oxford.[8]

Agnes was born in 1875 and married Arthur in 1902. She went in 1896 to St. Hugh's Hall, Oxford, to study history. From there she went to teach history at East Putney High School, giving up teaching after two years. She nurtured strong interests in socialism and feminism since her high school years in Bristol. Before World War I she was highly interested in the women's movement, in its peaceful wing, working for the 'suffragists', having the support of Arthur. She then opposed the war and did social work to improve the lot of the unemployed, being also instrumental in organizing free milk in British schools. She had a permanent association with the Labour Party, a

[6] The society was founded in 1820, and Paul Levy ([1979] 1989, 300-11) listed the members elected up to the First World War. Similarly, William Lubenow (1998, 413-32) provided a biographical directory of the Apostles of this period. See also Richard Deacon (1985).

[7] It is quite telling to the centrality of Ramsey in postwar economics that the University of Cambridge has no professorship named after either Keynes or Marshall or Pigou or other distinguished Cambridge economists, but it does have one named after Frank Ramsey. However, this professorship appeared only in 1994 for the tenure of Sir Partha Dasgupta. In 2011 it was permanently retitled the Frank Ramsey Professorship of Economics (Cambridge University Reporter, 2011).

[8] The detailed information on Frank Ramsey's family background comes mostly from Paul (2012).

political outlook later shared by Frank Ramsey. She died in 1927, two years before her eldest son, when the car driven by Arthur had an accident (Paul, 2012, pp. 14-21). It seems that Frank was over-dependent on his mother, but he was also the child who was most able to put up with Arthur's harsh treatment of Agnes and he developed a 'serene and affectionate relationship' with him, according to Paul (2012, p. 13).

Arthur was born in 1867 and had troubles entering college. After being rejected in entrance exams of four colleges, he got a scholarship to go to Magdalene College, Cambridge, were he read for mathematics and succeeded very well in the mathematical tripos: sixth in the whole university in part I and third in part II. It was common at the time that Cambridge colleges appointed one of its own alumni to its fellowships, but Magdalene had already a mathematics fellow. Arthur then got a job in a Scottish public school, where he spent six years. Then, in 1897, he became a Fellow of Magdalene College being instrumental in changing the then poor state of the college: according to him, the college, which had no intellectual test for admission, 'was a refuge for boys who could not get in elsewhere' (Paul, 2012, p. 6). Arthur was bursar, president (i.e. vice-Master), and acting Master (1917-1920) at Magdalene College, Cambridge, where he stayed until his death in 1954 – but he had stopped teaching in 1934 (Paul, 2012, pp. 5-14). As evidence of the improvement in the College's state during Arthur's lifetime, Paul (2012, p. 11) wrote that '[i]n Arthur's last year, five of his six finalists were placed in the first class. Several of his pupils obtained distinctions in the tripos (a rare honour) and themselves became university teachers of mathematics.'

Arthur was apparently a good teacher and he specialized in applied mathematics (i.e. classical mechanics), writing several highly praised and used textbooks. In 1894 he wrote with George Richardson a book for the 'Association for the improvement of geometrical teaching', entitled *Modern Plane Geometry*. In 1913 he became co-author of *A Treatise on Hydromechanics* originally published in 1859 by the Cambridge mathematician William Henry Besant as a single volume; Besant turned it into two volumes in 1882 but did not manage to publish the second volume, thus inviting Arthur in 1904 to be responsible for Part II on 'Hydrodynamics'.[9] However, as Arthur wrote in the preface to the first edition (of the two-volume book) he found 'desirable to write a new book *ab initio*…' (Ramsey, [1935] 1960, p. v). In 1914 Arthur published the revised and enlarged fifth edition of Besant's *A Treatise on Dynamics*. More interesting is to note that he also wrote several textbooks aimed at students preparing for Part I of the mathematical tripos at Cambridge, several of them based on his own lecture notes: *Elementary Geometrical Optics* (1914), *Dynamics* (1929), *Statics* (1934), *Hydrostatics* (1936), and *Electricity and Magnetism: an introduction to the mathematical theory*

[9] Besant was a Fellow and mathematical lecturer at St. John's College, Cambridge, and died in 1917. In 1850 he was first in the second part of the mathematical tripos (i.e. Senior Wrangler), and became Fellow of the Royal Astronomical Society in 1871. 'He gained a great reputation as a mathematical coach' and served as examiner for the mathematical tripos in 1856, 1857 and 1885 (Royal Astronomical Society, 1918, p. 241). Based on his experience of preparing Cambridge students for the tripos he wrote in 1863 the textbook *Elementary Hydrostatics* containing mathematical exercises of the kind found in this examination.

(1937).[10] What these last books reveal very clearly is what E. Roy Weintraub (2002, ch. 1) discussed, that the mathematics in Cambridge was disconnected from the developments occurring at the European continent in the field of Analysis: in the nineteenth-century England and still in Arthur's textbooks, 'mathematics was […] defined […] by a set of tricks and details, based on Newton, which were linked to applied physics and mechanics' (p. 14). Arthur Ramsey's books were among the mathematics books read by his son (Paul, 2012, p. 47).[11]

Applied mathematics was taken to task in Cambridge after World War I, and it was understood then as seriously out of date. The Trinity Mathematical Society debated this point and a motion was approved to revise it. Frank Ramsey took part of this discussion, as recorded in the motion of 1922 (Paul, 2012, p. 11):

> That in the opinion of this house applied mathematics should be instantly and radically revised. F. P. Ramsey said that applied mathematics had afforded no stimulus to advance since the middle of eighteenth century. All the recent advances in mathematical physics had been made by pure mathematicians. With regard to applied mathematics as a subject of study, he was of the opinion that it would be better to do away with it in the university, as it was merely a collection of standardized puzzles.

It is not surprising that the young Frank Ramsey would take this position in 1922 as his interests and later works in mathematics came from his familiarity with the 'symbolic school' of mathematics, as defined by the works of Bertrand Russell (in his *The Principles of Mathematics*, of 1903, and *Principia Mathematica*, published with A. N. Whitehead in 1910), following Gottlob Frege, and of Louis Couturat, a French mathematician born in 1868. For these authors, the foundations mathematics lay in logic, and Couturat believed that symbolic logic was the way to advance mathematics and philosophy. In 1918, at the age of fifteen, Frank wanted to expedite his learning of German and read a German translation of Couturat's book (*Les Principes des Mathématiques: avec un appendice sur la philosophie des mathématiques de Kant*, 1905) – which he understood to be 'founded on B. Russell (*Principles of Mathematics, vol. 1*)' (Ramsey in Paul, 2012, p. 46). In Cambridge in 1920-1921, he attended Russell's lectures, travelled to London with his close friend Ogden to talk to Russell, discussed with the latter concepts of *Principia Mathematica*, and attended Moore's lectures including one on logic for which he read the introduction of Russell and Whitehead's *Principia*. He also reported to his father that he read some work of the

[10] His 1929, 1934 and 1936 books had as subtitle 'a text-book for the use of first year students at the universities and for the higher divisions in schools'. Arthur Ramsey also wrote a textbook for Master's students of mathematics entitled *An introduction to the theory of Newtonian attraction* (1940). As evidence of the importance of his books, Paul (2012, p. 10) wrote that Arthur 'used to boast that the 100,000 copies of them that were sold in his lifetime, if stretched out, would have reached to the moon.' Several titles were recently reprinted by Cambridge University Press, featuring in the catalog of 'books of enduring scholarly value', in the series 'Cambridge Library Collection'.

[11] At Winchester, in 1918, Frank even attended his father's lectures on dynamics (Paul, 2012, p. 49).

British mathematician G. H. Hardy and he did not like it much (Paul, 2012, p. 46, pp. 81-2, p. 95, p. 98).[12]

This shows how Frank Ramsey was familiar with a different kind of mathematics, not applied mechanics, that was beginning to appear in England, in which discussions on foundations of mathematics was typical. This was exactly Ramsey's concerns in the work he did for his Fellowship dissertation, which generated the 1925 article 'The Foundations of Mathematics', originally published in the *Proceedings of the London Mathematical Society* (and reprinted in Ramsey, [1931] 1960, ch. 1). In an entry to the fourteenth edition of the *Encyclopedia Britannica*, Ramsey (1929) discussed the paradoxes emerging from Russell and Whitehead's theory of types, and the use they made of the Axiom of Reducibility to circumvent them. Such axiom was 'generally considered unplausible and unsatisfactory' (p. 84), being criticized by the 'intuitionist school' of Brouwer and Weyl, and by the 'formalist school' of Hilbert. Wittgenstein, who interacted with Frege and wanted to study with him, was also critical of *Principia Mathematica* as he developed in his book *Tractatus Logico-Philosophicus* – which Ramsey helped translate into English in 1922. Frank Ramsey wanted to show that 'using Wittgenstein's work the system of Principia Mathematica can be reconstructed so that the unsatisfactory Axiom of Reducibility is no longer required. Thus classical mathematics, interpreted as one with formal logic, may yet be rehabilitated' (Ramsey, 1929, p. 84).[13]

It is clear that Frank Ramsey's work on mathematics is a product of his family background and of the Cambridge circles he was related to. This was also the case for many other aspects of his life. In particular, he had two friends of the family who helped him in many ways, Ogden (b. 1889-d. 1957) and Richards (b. 1893-d. 1979). Both studied at Magdalene College, Cambridge, where Arthur Ramsey worked and taught: Ogden was first class in part I of the classical tripos in 1910, and Richards, who went to study history but turned to moral sciences, was first class in part I of the moral sciences tripos in 1915 – after interrupting his studies several times because of tuberculosis attacks (Scott, [2004] 2009; Storer, [2004] 2008). Ogden became known for his Benthamite principles of linguistic reforms (including a 'Panoptic eliminator' of words) and his scheme of Basic English (an international language comprising 850 words)

[12] Once again, in January of 1924, Frank visited Russell in London to discuss *Principia* with him (Paul, 2012, p. 111), and in 1925 he reviewed the second edition of this book for *Nature* and *Mind*. As Weintraub (2002, pp. 16-17) argued, 'Modern mathematical ideas in England, as shared concerns of the larger world mathematical community, made their appearance with Hardy and Littlewood in the second decade of the twentieth century.'

[13] In 1926 Frank went to Oxford to deliver a paper entitled 'Mathematical Logic' (reprinted in Ramsey, [1931] 1960, ch. 2), having Hardy (who had left Cambridge for Oxford in 1919, returning to his home institution in 1931) in the audience showing to be familiar with his 1925 article. Upon return Ramsey wrote to his wife, Lettice, and explained that he disagreed with Brouwer's denial of the 'law of excluded middle' which states that 'every proposition is either true or false'. He also described his disagreement with Hilbert's proposition that in arithmetic numbers are just 'marks on paper constructed out of the marks 1 and +' (Paul, 2012, pp. 227-8). He wrote: 'I cannot persuade myself that I do not know for certain that the "law of excluded middle" is true'. With respect to Hilbert's position, he said that statements like 'I have two dogs', rendered as 'There are x and y, which are my dogs and are not identical with one another', involve the idea of existence and thus are not 'marks on paper' (p. 228).

developed with his friend Richards (cf. Franke, 2008, pp. 197-201), who was Moore's student. Together, Ogden and Richards published in 1923 the book *The Meaning of Meaning*, unfavorably reviewed by the young Frank Ramsey (Paul, 2012, pp. 80-1).[14]

Ogden (who first met Frank, studying at Winchester, in the Easter of 1920) and Richards were important supporters of Frank: in his second term in Cambridge, 1921, they took him to Moore's lectures on metaphysics and made him member of another important Cambridge society: the Heretics (Paul, 2012, 90).[15] At a time when religion was still important in determining social values and customs in Britain, with chapel attendance being mandatory at Magdalene College (up to World War I no other Cambridge College any longer had it), Ogden was a chief founder of the Cambridge Heretics Society in 1909.[16] While it started as a movement against compulsory college chapel, the goal of this undergraduate society was 'to promote discussion on problems of religion, philosophy, and art', requiring that its members 'reject traditional a priori methods of approaching religious questions', as recorded in the society's laws (Franke, 2008, p. 44).[17] Differently from many other Cambridge societies the Heretics, despite of functioning with membership, opened its meetings to the public and it allowed membership from women colleges, Girton and Newham (the latter being the college of Alix Strachey née Sargant-Florence, Lettice Baker and Frances Partridge). As Franke (2008, p. 20) argued, heresy here meant synthesis, or 'the desire not so much to remain *opposed* to orthodoxy but to *subsume* it. Reconciliation and synthesis were the predominant modes of transgression of the [Edwardian] age.'[18] Before Ramsey's membership, the Heretics' speakers included such notables as Russell, Moore, G. Bernard Shaw, the statistician R. A. Fisher, the Hungarian poet Ferenc Békássy, and the mathematician G. H. Hardy (Franke, 2008, Appendix). As honorary members of the society we have Moore, Hardy, the painter Lowes Dickinson, the historian G. M.

[14] Ogden, Richards, and the artist James Wood had published in 1922 the book *The Foundations of Aesthetics*. Ogden went to become a leading Bentham scholar of his time, delivering in 1932 the Bentham centenary lecture at University College London (Franke, 2008, p. 199). In 1919 Richards was one of the freelance lecturers recruited by the Cambridge English School preparing students for the newly created English tripos. In 1922 he was appointed lecturer of moral sciences and English at Magdalene College, Cambridge, a position he held until 1939 when he moved to Harvard University (returning back to England only in 1974).

[15] Frank became treasurer of the Heretics, succeeding Lettice Cautley Baker. He and Lettice first saw each other in 1921 but had no further contact. In the end of 1924 they would meet again, at a meeting of the Moral Sciences Club (Duarte 2009a, p. 448, fn. 8). In August of 1925 they got married (roughly two weeks after Keynes and Lydia). They had two children, Jane and Sarah. Lettice studied psychology at Cambridge and in the 1930s she became a distinguished photographer, establishing the firm Ramsey & Muspratt that 'became renowned for its portraits of the Cambridge elite' (Stansky and Abrahams, 2012, p. 143).

[16] Compulsory chapel attendance in Magdalene College, Cambridge, was abandoned only in 1921, when students were no longer subjected to sanctions and were only expected to attend at College chapel (Franke, 2008, p. 35).

[17] Those who 'while in sympathy with the general principle of open discussion, are not entirely free' can only be associates, and not regular members. Franke (2008, p. 56) wrote that by '1913 over 200 undergraduates had joined the Heretics, representing between five and ten percent of the student population'.

[18] Franke (2008) goes on to make the case that the Heretics society was a society that 'touched a nerve in the culture of the times' and that it facilitated the dialogue and widened 'the sphere of mutual influence between the Apostles and Bloomsbury' (pp. 28-9).

Trevelyan and Keynes. In 1921 the society inaugurated its economics section that lasted until 1927; according to Philip Sargant Florence it was 'heretical in criticizing theory based entirely on the assumption of a rational economic man' (quoted in Franke, 2008, p. 92).

Ogden was president of the Heretics society from 1911 to 1924, and in 1912 he started *The Cambridge Magazine* turning it into a vehicle for his own ideas (writing under various pseudonyms) and for publishing extracts and reports of society meetings (Franke, 2008, p. 57). His role as editor was important and in 1910 he started as general editor for Kegan Paul of the successful book series 'The international library of psychology, philosophy and scientific method' (Scott, [2004] 2009; Franke, 2008, pp. 78-9).[19] This series became known for publishing important psychological and philosophical books, including many titles of authors associated with the Vienna Circle. It was in this series that Richards published in 1931 the first collection of Ramsey's papers and it was for this series that in the autumn of 1920 Ogden invited Ramsey to translate into English Wittgenstein's *Tractatus Logico-Philosophicus*. It was also for Ogden's *Cambridge Magazine* that in 1922 Ramsey published two important articles: one criticizing Major Douglas's credit theory and another being a review of Keynes's *Treatise on Probability*, which for many scholars was the one carrying most weight with the book's author (see references in Duarte 2009a, p. 455, fn. 25).[20]

Before coming to Cambridge, Frank Ramsey had German lessons at Winchester (a leading British school) in April and May of 1920, and he was reading Ernst Mach 'on the analysis of sensations' and interested in his book *The Science of Mechanics* (Paul, 2012, p. 46). He was encouraged by Ogden to go on with it and expedited his learning by reading German books side by side with their English translations, acquiring knowledge sufficient for enlisting to the German school prize. However, Richards helped spread a story that Ramsey learned German in roughly ten days after reading Mach's *Analysis of Sensations* with the help of a German dictionary and grammar, reinforcing the aura of geniality of the young mathematician. This story made its way into a 1978 BBC radio broadcast by D. H. Mellor, a Cambridge philosopher and editor of several volumes on Ramsey, which was turned later into a series of lectures and an article published in 1995 (Mellor, 1995) – the broadcast was also echoed by Sahlin (1990, pp. 222-3). This story even went into economics with the help of Paul Samuelson, who substituted Kant's *Critique of Pure Reason* for Mach's book in the story (Duarte, 2009a, pp. 450-1).

Ramsey accepted Ogden's invitation and at the age of eighteen, as a student at Trinity College, Cambridge, translated Wittgenstein's *Tractatus* and a year later, after finishing the project, he wrote a review of it for *Mind*, a leading journal of psychology

[19] Ogden edited other book series for Kegan Paul, such as 'The history of civilisation' and 'To-day and to-morrow'. In 1920 he helped found the psychological journal *Psyche* (originally entitled *Psychic Research Quarterly*), becoming its editor in 1922 with the demise of *The Cambridge Magazine*.

[20] Brady and Arthmar (2013) dissent from this view and suggest that Ramsey had a partial reading of the *Treatise on Probability*.

and philosophy edited by Moore. The attempt to publish the *Tractatus* related back to Wittgenstein's years as a student at Cambridge before World War I: it involved Russell and, after being rejected by Cambridge University Press, it also involved Wilhelm Ostwald (who published it in a German periodical) and Ogden in his Kegan Paul library where it was finally published.[21] Ramsey's translation pleased Wittgenstein and it was the starting point of a close and turbulent interaction between them – Ogden was the one to introduce Ramsey to Wittgenstein. In 1923, after finishing his final examinations, Ramsey went to Austria and discussed with Wittgenstein every line of the *Tractatus*. At this time Wittgenstein said to Ramsey that he wanted to get a B.A. degree from Cambridge with his book as his thesis.[22] Ramsey and Keynes started a long campaign to bring Wittgenstein back to Cambridge and make him resume his philosophical work. Only in 1929 Wittgenstein was awarded a Ph.D. with the *Tractatus* as his thesis, having Frank Ramsey as his official advisor and Russell and Moore as examiners (Duarte, 2009a, 452-3).

Richards and Ogden, Russell and Moore, were not the only ones who supported Ramsey in many ways. At the start of his undergraduate life, Frank became a close friend of Kingsley Martin (b. 1897-d. 1969), later journalist and editor of *New Statesman*, who was a Quaker. Through Martin, Ramsey was introduced to a group of Quakers who became very close to him and who 'formed the nucleus of the university Labor Club' (Paul, 2012, p. 90): Lionel Penrose (b. 1898-d. 1972) and Richard Braithwaite (b. 1900-d.1990), both of whom became Apostles as Ramsey.[23] Braithwaite was one year ahead of Ramsey, reading mathematics at King's (not at Trinity as Frank) but mainly interested in philosophy. He knew Keynes since 1919 (Moggridge, 1991, p. 121) and introduced Ramsey to him on January of 1921, when Keynes's *Treatise on Probability* was about to be published. In the winter of this same year Braithwaite proposed Ramsey for membership to the Apostles: in October Frank was elected by unanimity, as required by the society. Election was for life, with the obligation to attend every meeting, but members could 'take wings' and became 'angels', i.e. honorary and not obliged to attend the meetings. In the end of 1925 Ramsey 'took wings' (Paul, 2012, p. 111, p. 221), but remained active in the Apostles until 1929 (Levy, [1979] 1989, p. 65, p. 270). Besides being Apostles, both Richards and Ramsey were invited to Keynes's Political Economy Club, which was not a very exclusive club but it was the one through which 'Keynes came to know the best of each generation of Cambridge economists irrespective of College' (Moggridge, 1992, p. 190).[24]

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[21] See Monk (1990, ch. 9) for a detailed account, and Duarte (2009a, pp. 451-2) for additional references.

[22] In 1914 Wittgenstein, who had already quarreled with Russell, had asked Moore to visit him in Norway when he asked if he could get a B.A. degree with the notes on logic he dictated to the latter. The university requirements mentioned by Moore infuriated Wittgenstein: the two quarreled and only renewed their friendship in 1929 when Wittgenstein returned to Cambridge (Levy, [1979] 1989, p. 272).

[23] Frank shared the political outlook of his mother, Agnes, who was associated with the Labour Party: "While he always remained left-wing, [after his undergraduate days] he … stopped taking an active part in politics" (Paul, 2012, p. 21).

[24] Ramsey liked discussions and took part in several Cambridge societies. In addition to the ones already mentioned, he attended the Moral Sciences Club, the Cambridge University Socialist Society, and Magpie & Stump, a Trinity College debating society.

While the prewar Apostles were much under the spell of Moore's *Principia Ethica*, World War I brought a new intellectual configuration which favored changing the values of prewar society – Martin ([1966] 1969) is an exemplar of this.[25] A postwar vintage of Apostles, including Braithwaite and Ramsey, abandoned Moore and favored a psychological view of ethics (Paul, 2012, pp. 114-6). The postwar era in Cambridge, and in Britain more generally, was a time of two other particular influences: socialism and psychoanalysis.

The sizeable working-class movement that arose in almost all European countries by 1914 witnessed its socialist branch joining governments. In the case of Britain, the labor movement was based on integrating the working class within the capitalist state. After the war the European left was split between reformism and revolutionism, and the working-class unrest and revolutionary potential escalated (Sassoon, [1996] 2010, ch. 2). At Winchester, before entering Cambridge, Frank excelled mathematics and was much interested in socialism and economics, which made his enthusiasm for religion wane. In 1920 he recorded in his diary 45 books he read, with about half of them being in those two subjects: he read Lenin, Kautsky, Marx's *Das Kapital*, Sidney and Beatrice Webb's *A History of Trade Unionism*, J. A. Hobson's *Industrial System*, John Stuart Mill, and Alfred Marshall's *Industry and Trade*. The year before, Frank was most enthusiastic about the Bolsheviks, sympathetic to Guild Socialism, and he even opposed a motion at Winchester for armed intervention in Russia (Paul, 2012, p. 55, p. 64).[26] He wrote an essay on progress where he cited Mill and discussed the two 'reasonable standards' for a 'fair division of wealth': (1) 'men should receive shares according to their social utility', and (2) 'they should receive equally, provided they do their share of the world's work' – he favored the latter. On either standard, it follows that rent and interest ought to be socialized, 'except the interest on a man's own savings, which could be allowed for his lifetime' (Ramsey in Paul, 2012, p. 66).

In July of 1920 Frank Ramsey left Winchester and entered Cambridge, and in his first term he turned his interests to philosophy due Ogden's influence (and later Moore's), who, according to Paul (2012, p. 72), inculcated in him liberal values. Nonetheless, Frank's concern for improving 'the lot of fellow man' accompanied him throughout life and was clearly present in the first article he read to the Apostles, in 1921, in which he criticized Russell's and Hardy's defense of pure mathematics against the charge that it is useless for the 'task of alleviating the suffering of humanity' (Ramsey, 1921, p. 292). The same was the case in another paper to the Apostles which I used to open this chapter (Ramsey, 1924).

At Cambridge, Frank became keenly interested in psychoanalysis. In the Fall of 1922 he was psychoanalyzed by Edward Glover in London and about this time he was

[25] The book's subtitle is 'how a child of Victorian dissent saw the certainties of liberal progress shattered by the 1914 war and found new convictions in socialism and the writings of his radical contemporaries'.

[26] Guild Socialism was a multifarious political movement in Britain, strongest after World War I, associated with G. D. H. Cole and supported by Russell. The idea was that of workers electing, through their factories and industries, a 'guild congress' that would co-exist with parliament. A national guild would control the industry (see Hutchinson and Burkitt, 1997, ch. 1).

reading Freud. Through his close Cambridge friends this interest grew even more: Lionel Penrose, Adrian Bishop, Margaret and Geoffrey Pyke, James and Alix Strachey, and Sebastian Sprott were analyzed (Paul, 2012, p. 189; see also Forrester, 2004).[27] The Stracheys and Penrose went to Vienna to be psychoanalyzed: the Stracheys went in 1920 and were psychoanalyzed by Freud for one year, and Penrose went there in 1923 encouraged by John Rickman, a Quaker who was analyzed by Freud and became his first Cambridge follower (someone whom Frank also met). Glover stimulated Frank to go to Vienna to be analyzed, which happened during the six months Ramsey spent in Austria in 1924 after his graduation as a wrangler, when he received a Fellowship at King's.[28] His analyst was Theodor Reik, the second name suggested by Glover (Paul, 2012, ch. 12). In 1925, Frank, Penrose, Rickman, James Strachey and a few other Cambridge graduates (half of whom were Apostles) set up a short-living little club to discuss psychoanalysis, the Cambridge Psychoanalytic Group (Forrester, 2004, pp. 4-5). Ramsey was so much into this theory that James wrote of one meeting (quoted in Paul, 2012, p. 193):

> [Frank] seemed on the whole to accept [psychoanalysis], but thought the theory very muddled. … He is thinking of devoting himself to laying down the foundations of psychology. All I can say is that if he does we shan't understand them. He seems quite to contemplate, in his curious way, playing the Newton to Freud's Copernicus.

In the paper read to the Apostles in 1924, an imaginary conversation with John Stuart Mill, Frank deprecated Mill's outdated utilitarian psychology and praised Freud and his followers for advancing psychology as scientists 'observing facts and inventing theories to fit them' (Ramsey, 1924, p. 306). Even Keynes, in 1925, published a pseudonymous letter in the journal *Nation and Athenaeum* commenting on a controversy over psychoanalysis, in which he described Freud 'as one of the great disturbing, innovating geniuses of our age, that is to say as a sort of devil' (Keynes in Moggridge, 1982, p. 393).

If the intellectual efflorescence of Cambridge in the 1920s had Frank Ramsey as an important actor, this configuration could no longer continue with the death of the young Cambridge mathematician on 19 January 1930. On November of the preceding year he got flu and later jaundice, but this raised little concern because at the time people understood that jaundice was not life-threatening and it could last long. In early January he went to Guy's Hospital, in London, for an exploratory operation to see if there was any stone that needed to be removed. The operation revealed his liver in a

[27] In 1922 Sprott tried to arrange a lecture for Freud at Cambridge, but the latter never visited that university (Forrester, 2004, pp. 1-2).

[28] Frank Ramsey got the highest class in part II of the mathematical tripos in 1923, making him a wrangler. However, in contrary to what was previously circulated in the literature (including Duarte, 2009a), he was not first, i.e., a senior wrangler (top candidate), to the dismay of his father (Paul, 2012, p. 140). Frank's move from Trinity to King's (something not very common at the time) probably had Keynes's support (cf. Duarte, 2009a, p. 448). Ramsey later became lecturer and director of studies in mathematics at King's.

very serious condition and he did not recover from the surgery, for the stupefaction of his family and Cambridge friends.

2 – Ramsey's economics

Frank Ramsey published only three articles in economics: as an undergraduate student of eighteen years of age he published a critique to Major Douglas's credit theory for Ogden's *Cambridge Magazine* in 1922; then in 1927 and 1928 he published two important articles in the *Economic Journal*, edited by Keynes: the first on taxation and the second on optimal saving.

Major Clifford Hugh Douglas (b. 1879-d. 1952) did not complete his studies at Cambridge and became member of the Institute of Electrical Engineers. Together with Alfred Richard Orage and others, he developed from the late 1910s (a period when the United Kingdom was off the gold standard) a theory of guild socialism known as the 'A+B theorem' in which credit had a central role. Major Douglas saw money as a ticket system that grant the right to its owner to participate in the economy, allowing the goods produced to be distributed to the consumers as they see fit. Additionally, and central to his ideas, he conceived production as a multi-stage process in time, with the firms distributing two types of payments: (A) payments to other firms related to the purchase of raw materials and to other costs; (B) payments to individuals in the form of wages, salaries and dividends (Hutchinson and Burkitt, 1997, ch. 2). The prices of goods reflect both 'A' payments made in the past and 'B' (thus 'A+B'), and in a given period of time 'B' cannot buy 'A+B' (though over time there is no lack of purchasing power). Therefore, extra money in the form of credit should be injected into the economic system every now and then. His argument was also made in terms of selling prices being inferior to cost prices of goods due to the lack of purchasing power.

Apparently Ogden was the one who persuaded Frank to analyze the then discussed Douglas's proposals, as suggested without references by Soifer (2009, p. 293). Douglas's writings are far from very clear and Ramsey opened his article suggesting to those interested in understanding his ideas to read W. Allen Young's *Dividends for All* (Ramsey, 1922a, p. 335). Young suggested a flaw in Douglas's theory, with an argument that in general terms involved the possibility of existing a firm producing intermediary goods and distributing wages and dividends that could absorb the surplus of all other firms. Ramsey set himself the goal of exploring this flaw in Douglas's theory more carefully. Unlike Young's static argument about the existence of purchasing power somewhere in the economy at a given point in time, and contrary to the understanding of Hutchinson and Burkitt (1997, p. 84), Ramsey went on to make a dynamic argument very much in line with Douglas's stress on production as a multi-state process in time. Ramsey (1922a, p. 336) went on to make 'a strong and simple argument for supposing that the [selling price to cost price] ratio does not differ appreciably from unity': this ration is equal to one in a 'stationary state', which is a state 'in which production goes on at an unchanging rate and prices, wages and the national

wealth never alter'. In this state, 'the distribution of purchasing power by all [intermediary] factories proceeds at rate A', which together with wages and dividends through B add up to an aggregate purchasing power of 'A+B' – 'which equals the rate of flow of cost prices of consumable goods' (p. 337).

Finally, Ramsey closes his analysis with two arguments. The first is that 'the present state is not very far from being stationary' thus implying that the ratio of selling to cost prices does not differ greatly from unity. But he understood that this was not a general case. He then used integral calculus, and integration by parts, to study this ratio 'under much wider conditions, which allow for changes in the quantity of production, in the rate of wages, in the productivity of labor, and in the national wealth' (p. 337). With a dynamic analysis, he first showed that under certain conditions the ratio of selling to cost prices is unitary (Ramsey, 1922a, pp.337-9) and then considered even more general conditions. In this case, he showed two possibilities for the ratio to be less than one but they are 'obvious to common sense and are clearly irrelevant to Major Douglas' contention that "just price" is today a quarter of cost price' (p. 340).

The two papers that Frank Ramsey published in the *Economic Journal* were the ones to secure him eventually a distinguished place in the economics profession. Contrary to the view that he was mostly a mathematician distracted by economists to help them with certain mathematical problems, I argued elsewhere that those two papers were part of what we might call Ramsey's research agenda in economics, which emerged from his close interaction with Pigou (Duarte, 2009a). As part of a Cambridge habit of senior Fellows asking other economist Fellows to help them with their research, Pigou involved Frank in his work that culminated with the publication of his 1928 book *A Study in Public Finance*, with a concern about tax exemption on savings.[29] Out of this concern Ramsey first wrote an article setting up a static utilitarian framework for characterizing the optimal tax rates for a government in need of raising revenues at the same time that it does not have lump-sum taxes available. Ramsey (1927) simply postulated that there exists a net utility function of producing and consuming the quantities of the n goods of the economy (denoted by x): $u = F(x_1 \dots x_n)$. He then showed that uniform taxation is generally suboptimal because the tax rates that maximize that function are those that reduce the production of all taxed commodities in the same proportion with respect to the benchmark of prices equal to marginal costs. Not surprisingly, Pigou was the first to cite Ramsey (1927) in his 1928 book, stressing the elasticity version of Ramsey's result already enunciated by the latter: inelastic commodities, either for supply or for demand, should be taxed more than elastic ones.

As a reaction to Pigou's 'treatment of saving as a use of income with its own elasticity of demand' (Ramsey in Duarte, 2009a, p. 463) in the 1928 book, Ramsey (1928) built on his previous article to discuss the taxation of savings in an intertemporal utilitarian framework. He first characterized the choice of savings that maximizes the

[29] Pigou asked Ramsey's help in preparing the third edition of his book *Economics of Welfare*, published in 1929, as he had done earlier with the young Keynes, who helped revise his 1912 book *Wealth and Welfare* (Duarte, 2009a, p. 459). See Gaspard (2003) for a discussion of Ramsey (1928).

utility of consumption net of the disutility of labor subject to the economy's resource constraint (which specifies that expenditures on consumption and investment exhaust aggregate output). In the first part of his article he discussed the savings problem of a society without discounting future utilities on ethical grounds of intergenerational justice. Thus, in order to have a well-defined optimization problem, Ramsey (1928) postulated the existence of an upper bound to net utility and therefore minimized the distance over time of the actual utility to this bliss level:

$$\underset{x_t, a_t}{Min} \int_{t=0}^{\infty} \{B - [U(x_t) - V(a_t)]\} \cdot dt \qquad s.t.: \quad \frac{dc_t}{dt} + x_t = f(a_t, c_t)$$

where B is the bliss level, x_t, a_t, and c_t are respectively consumption, labor, and the stock of capital.

The solution to this problem specifies that 'the saving rate multiplied by marginal utility of consumption should always equal bliss minus actual rate of utility enjoyed' (p. 547).[30] Keynes suggested a marginal reasoning (of equating cost and benefit at the margin of saving one extra unit of money today) as an alternative way of deriving Ramsey's mathematical result, as Ramsey acknowledged in the article – a result nowadays referred to as the 'Keynes-Ramsey rule'. Once again, Pigou was the first to cite Ramsey (1928) in the second edition of his public finance book, published in 1929.

However, the published version of the 1928 paper has no section analyzing taxation. After submitting the article to the *Economic Journal* Ramsey interacted with Keynes, the editor, who asked Ramsey to cut the section on taxation out of the optimal savings article as the mathematical analysis was 'too involved in comparison with the conclusions which were feeble' (Ramsey in Duarte, 2009a, p. 463). Ramsey agreed with Keynes and accepted his suggestion. The draft of this section survived and were published in 2009 (Ramsey, 2009).

What is clear from the few contributions by Ramsey to economics is both how his research agenda emerged out of the Cambridge milieu of his time and how concerned he was in using economics to obtain general results useful to improving human wellbeing. For the two later articles Pigou was instrumental in posing the questions, providing an analysis against which Ramsey reacted, and in citing the articles in an important book of the interwar period. Keynes was important too. He had Ramsey as an interlocutor to try out his ideas and to consult about articles submitted to the *Economic Journal* (Duarte 2009a, pp. 455-6). Moreover, he helped shaping the argument of Ramsey's 1928 article and stressed that rendering sound results should limit the complexity of mathematical analysis in economics.

[30] Later in the article Ramsey considered the optimization problem of an individual who discounts future utilities and showed how this result has to be adapted accordingly.

3 – Crossing the Atlantic

In mathematics some of Frank Ramsey's contributions were acknowledged promptly by Paul Erdös and George Szekeres (Graham and Spencer, 1990, pp. 114-5), leading to the creation of a field in the intersecting areas of combinatorics, number theory, geometry, topology and measure theory, known as 'Ramsey theory' (mathematicians also talk about 'Ramsey numbers'; cf. Soifer, 2009, 2011). In contrast, as I mentioned earlier, in economics Ramsey is presented as a sleeping giant who was awaken only in the 1950s when economists finally acquired the necessary mathematical tools to apprehend Ramsey's ideas – a process that is integral part of the Americanization and dominance of neoclassicism in economics (Morgan and Rutherford, 1998). Claims like this tend to miss important aspects of the canonization of authors in a given community.

Ramsey's 1927 and 1928 articles were discussed before the 1950s to a limited extent when compared to the boom of citations occurred from the mid-1950s onward (Duarte, 2009b, 2010). The 'rediscovery' of Ramsey meant extending his analyses, with new mathematical tools such as Hamiltonian or new theories of welfare economics, and placing them at the core of the neoclassical economics of the postwar period. This is exactly what Michael Intriligator's description, in the late 1960s, of optimal growth literature reveals: this literature 'carries the imprint: "Economics by Ramsey; Mathematics by Pontryagin"' (quoted in Duarte, 2009b, p. 173).

In order to better understand the canonization of Ramsey in postwar economics we have to recognize that in terms of citations to him in economics articles available at JSTOR there were three major waves of 'rediscoveries' (Duarte, 2009b, pp. 168-9): a wave of references to Ramsey in the subjective probability and expected utility literature of the 1950s and early 1960s; another wave on economic growth started in the mid-1950s; and a final wave from the late 1960s on public economics. Furthermore, Paul Samuelson was a leading author citing Ramsey in all of them and an influent teacher of several authors who later contributed either to the optimal growth literature or to the public finance discussion on taxation, or both (as Peter Diamond and Joseph Stiglitz). Samuelson feared dying early and loved telling his students stories about economists, particularly the one that Ramsey learned German in a week, promoting the romantic view of the precocious mathematician who made seminal contributions and died in the prime of life (Duarte, 2010, pp. 150-3).

For postwar economists, several of whom connected to Cambridge, USA, Ramsey is the precursor of a microfounded model of growth in which the lifetime utility of a representative agent is discounted. In public finance, he is also the precursor of a representative agent theory that was unfortunately in the mold of the old welfare economics with interpersonal comparison of utilities.[31] Clearly the mathematical tool used by Ramsey (1928), the calculus of variations, was not the major factor explaining

[31] It is unclear whether Ramsey had a notion of a representative agent, but I argue that he might have had one (Duarte, 2010, pp. 126-30).

its attractiveness to postwar economists: the American mathematicians Griffith Evans and Charles Roos were contemporaries of Ramsey who employed that tool too, but not in a utilitarian framework, and who were marginalized in postwar economics (Weintraub, 2002, ch. 2). Therefore, Frank Ramsey is indeed a very rich historical figure that offers us interesting windows to both the Cambridge economics of the 1920s and to the important transformations of economics after World War II.

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