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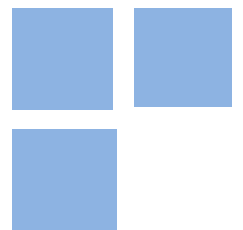# A Reconciliation Proposal of Demand-Driven Growth Models in Open Economies

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# A Reconciliation Proposal of Demand-driven Growth Models in Open Economies

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**Abstract:**

This paper seeks to contribute to the literature on demand-driven Keynesian models of growth in open economies by developing a formal model that combines Dixon and Thirlwall's (1975) export-led growth model and Thirlwall's (1979) balance-of-payments constrained growth models into a more general specification. Then, we built upon the model developed in this paper in order to analyse more broadly some important issues concerning the net impact of a currency depreciation on the short-run growth rate. We demonstrate that this impact can go either way, depending on some conditions such as the average share of imported intermediate inputs in prime costs of domestic firms and the institutional capacity of trade unions to set wages through the bargaining process. The model also shows that the effectiveness of a competitive real exchange rate to promote growth is higher in countries where the share of labour in domestic income is also higher. Lastly, we run some simulations to illustrate the impact of a devaluation on short-term growth rate in different scenarios. The simulations demonstrate how the pattern of convergence of the actual growth rate towards the equilibrium growth rate is intrinsically linked to the institutional framework of the economy.

# A reconciliation proposal of demand-driven growth models in open economies

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## Abstract

*This paper seeks to contribute to the literature on demand-driven Keynesian models of growth in open economies by developing a formal model that combines Dixon and Thirlwall's (1975) export-led growth model and Thirlwall's (1979) balance-of-payments constrained growth models into a more general specification. Then, we built upon the model developed in this paper in order to analyse more broadly some important issues concerning the net impact of a currency depreciation on the short-run growth rate. We demonstrate that this impact can go either way, depending on some conditions such as the average share of imported intermediate inputs in prime costs of domestic firms and the institutional capacity of trade unions to set wages through the bargaining process. The model also shows that the effectiveness of a competitive real exchange rate to promote growth is higher in countries where the share of labour in domestic income is also higher. Lastly, we run some simulations to illustrate the impact of a devaluation on short-term growth rate in different scenarios. The simulations demonstrate how the pattern of convergence of the actual growth rate towards the equilibrium growth rate is intrinsically linked to the institutional framework of the economy.*

*Keywords balance-of-payments constraint, exchange rate, income distribution.*
*JEL classification – O40, O33, E25*

## 1. Introduction

The modern literature on demand-driven Keynesian growth in open economies can be traced back to the pioneering contributions of Dixon and Thirlwall's (1975) export-led growth model and Thirlwall's (1979) balance-of-payment constrained growth model.

The export-led growth model developed by Dixon and Thirlwall (1975) posits the possibility that some countries may be able to reach a 'virtuous circle' of growth in the long run. Drawing upon the work of Kaldor (1966), this formal framework highlights the concept of endogenous technological progress driven by demand via the Verdoorn coefficient[1]. This is the widely known Verdoorn's Law. This law is the statistical relationship between the growth of labour productivity and manufacturing output; empirical evidence for the same relationship between these two variables seems to be very weak for the other sectors of the economy (McCombie and Thirlwall, 1994). The existence of a positive Verdoorn coefficient enables a country or a region to achieve a virtuous circle of growth, as higher rates of growth in manufacturing increase learning, technical change and hence labour productivity, which boosts manufacturing output once again and so on. Moreover, Kaldor argues that, for open, mature economies, exports markets expand faster than domestic markets, thus allowing a country to grow at higher rates. Accordingly, increased manufacturing exports induce to a faster growth of domestic output and so spur technological innovations, the stock of knowledge and competitiveness. The problem here is, when this model is put to test, the growth rates predicted are usually overestimated in relation to the actual growth rates (Dixon and Thirlwall, 1975).

On the other hand, the balance-of-payments constrained growth model, originally developed by Thirlwall (1979), is a dynamic version of the Harrod foreign trade multiplier or, more generally, the Hicks super-multiplier (McCombie, 1985). It is based on the proposition that 'for most countries the major constraint on the rate of growth of output is likely to be the balance-of-payments position because this sets the limit to the growth of demand to which supply can adapt' (Thirlwall & Hussain, 1982, p. 1). In other words, a country's growth rate in the long run must be the rate compatible with the equilibrium in the balance of payments. Therefore, assuming the existence of an external constraint, a likely path of a virtuous circle of growth, as stated by the model of export-led growth, is likely to be restricted. That is to say, in Thirlwall's (1979) model the Verdoorn coefficient plays no role in the explanation of the long-run growth rate.

Thirlwall and Dixon (1979) tried to reconcile these two models in order to obtain, in one single theoretical framework, the cumulative causation effect, given by the Verdoorn

coefficient, and the balance-of-payments constraint. However, the problem with their model is that only by a fluke the new specification of the long-run growth rate is equal to the equilibrium growth rate from the standard balance-of-payments constrained growth model of Thirlwall (1979). It means that this model incurs in the same problems of the export-led growth model discussed above. In order to overcome this problem and find convergence between these two growth rates the authors must assume that either the Marshall-Lerner condition is not satisfied or that there is no cumulative causation effect, that is, the Verdoorn coefficient is negligible, so that the growth rate predicted by the model can converge towards the equilibrium growth rate. More recently, Blecker (2013) extends Thirlwall and Dixon's (1979) model by allowing for capital flows. He shows that, even though in the long run both the export-led and balance-of-payments constrained growth models are incompatible, in the medium run, the balance-of-payments constrained growth rate presents cumulative causation effects via Verdoorn coefficient. In this theoretical framework, even though it lacks a formal mechanism of relative price adjustment, the author states that domestic expenditures, nominal exchange rate and money wages must adjust in order to bring the medium run growth rate into equality with its balance-of-payments constrained growth rate. Razmi (2013) also proposes a framework combining both models. By imposing a balance-of-payments constraint on the export-led growth model, he demonstrates how output growth, mark-up and exchange rate endogenously accommodate external and internal shocks in different exchange rate regimes and degrees of capital mobility.

This paper proposes an alternative analytical framework that combines Dixon and Thirlwall's (1975) export-led growth model and Thirlwall's (1979) balance-of-payments constrained growth models into a more general formal specification. Our model adds to these previous ones by incorporating some salient stylised features of open economies, such as the utilisation of imported intermediate inputs by domestic firms. We also endogenise the mark-up set by domestic firms as a positive function of the real exchange rate (RER). These extensions are particularly relevant as they allow us to analyse different possible scenarios following the impact of a currency devaluation on short-term growth. The theoretical framework presented below enables us to define the conditions under which a currency devaluation could propel or curb short-run growth.

The impact of exchange rate variations on growth has never been in the centre of the discussions within the demand-driven Keynesian growth literature. However, a few contributions can be pointed out. Even though the Dixon and Thirlwall's (1975) export-led growth model is mostly used in the analysis of regional growth for regions under a common

currency, the theoretical framework of the model suggests that a currency devaluation boosts exports and labour productivity through the Verdoorn effect, thus creating a virtuous circle of growth over time; according to this model a permanent increase in the growth rate of the RER also permanently raises the growth rate of output. With respect to the standard Thirlwall (1979) balance-of-payments constrained growth model with no capital flows, it is theoretically possible for a policymaker to keep the balance-of-payments constrained growth rate above the long-run equilibrium growth rate indefinitely only by constantly depreciating the RER. However, in practice, such a possibility does not seem to be feasible, as gains from trade would be mitigated over time by adjustments in the mark-up factor, unit labour costs and unit imported intermediate costs. Increasing domestic inflation caused by constant currency devaluations would also engender dissatisfaction within the domestic economy and pressures for a change on a political level. Currency wars in international affairs might also emerge as a consequence of such a policy of permanent currency devaluation. Thirlwall's (1979) model allows for relative prices into the trade demand functions, but assumes that price-competition factors have very little, or no effect, on the balance-of-payments equilibrium growth rate. Pugno (1998) extends the standard Thirlwall (1979) model by making explicit the mechanism of relative price adjustment and hence explains the stability of the equilibrium growth rate. In his model, the RER adjustment happens through wages and prices. Assuming a short-run relationship between wages and employment (Phillips curve), and given a constant growth rate of labour supply, he demonstrates that any initial gains of price competitiveness due to a devalued currency is mitigated over time as increased growth rates of exports and income lead to a tighter labour market; the reduction in unemployment induces a raise in wages, thus worsening the initial competitiveness of domestic firms. In the same spirit, Porcile and Lima (2010) also claim that RER adjustments are intrinsically related to the degree of accommodation of the labour market. They add to Pugno's (1998) model by endogeneising the labour supply as a function of the wage gap between the modern and the subsistence sector. In their model, a higher elasticity of labour supply implies that a raise in the wages of the modern sector will attract more labour from the subsistence sector, which reduces the employment rate and hence contains inflationary effects, so that a higher elasticity of labour supply boosts domestic firms' price competitiveness. Blecker (1998) claims that in foreign trade the neoclassical concept of competitiveness, which focuses on RER adjustments, and the Post-Keynesian definition of competitiveness, which highlights non-price factors such as technological capabilities, quality of products and so on, are not incompatible. He, then, develops a general model where he claims that both price and non-price competitiveness can

equilibrate the trade balance, which means that the concept of competitiveness that prevails is ultimately an empirical question.

As we can see, very few contributions to the literature have addressed the issue of price competitiveness within the balance-of-payments constrained growth model. Even though some previous works have shed some light on the working of the RER adjustment (Pugno, 1998; Porcile and Lima, 2010), the analysis of the impact of a currency devaluation on short-run growth has been left unattended in this literature since the development of Thirlwall's (1979) standard model. First, this paper adds to the literature by setting out a growth model that incorporates a mechanism of relative prices self-adjustment over time. An exchange rate pass-through mechanism is incorporated to the standard Thirlwall and Dixon's (1979) growth model. After that, we set up a model where the impact of a currency devaluation on growth can go either way. The analysis of the impact of changes in relative prices on balance-of-payments constrained growth rate allows us to map out the conditions under which a currency devaluation can promote growth. We seek to demonstrate that these conditions are in fact closely linked to the institutional framework of the economy that intermediates the class struggle between workers and capitalists in the wage decision-making process. In short, the purpose of this paper is twofold: (i) to develop a more general theoretical framework by reconciling both the export-led and balance-of-payments growth models; (ii) to investigate, both analytically and through simulation the effectiveness of a currency devaluation to promote growth in the light of the formal model developed in this paper.

## 2. Demand-driven Keynesian growth models in open economies: a brief overview

### 2.1. The export-led growth model

The export-led growth model set forth in Dixon and Thirlwall (1975) follows the Kaldorian tradition in which exports are the main autonomous component of aggregate demand for open economies. 'According to Kaldor, regional growth is fundamentally determined by the growth of demand for exports, to which the rate of growth of investment and consumption adjust' (Dixon and Thirlwall, 1975, p. 491). The equilibrium growth rate is given by (see appendix A.1 for a formalisation of the model)

$$y_{DT} = \frac{\gamma[\rho(w - a_0 + \tau) + \delta(p_f + e) + \varepsilon z]}{1 + \gamma \rho \lambda} \tag{1}$$

where $y_{DT}$ is the rate of growth of output of the Dixon and Thirlwall's (1975) model; $\gamma$ is the constant elasticity of output growth with respect to export growth; $p_f$ is the inflation rate in

the foreign country; $e$ is the rate of change of the nominal exchange rate (i.e. the home price of foreign currency); $z$ is the growth rate of the foreign income; $\rho < 0$ is the price elasticity of demand for exports; $\delta$ is the cross elasticity of demand for exports; $\varepsilon$ is the income elasticity of demand for exports; $w$ is the growth rate of nominal wages; $\tau$ is the growth rate of the mark-up factor; $a_0$ is rate of growth of the autonomous component of labour productivity growth; $\lambda$ is the Verdoorn coefficient. Given $\rho < 0$, the condition for a stable growth is $1 + \gamma\rho\lambda > 0$. Furthermore, if the equilibrium solution in equation (1) is stable, it is easy to see the positive relationship between the Verdoorn coefficient, $\lambda$, and the rate of growth of output. A positive Verdoorn coefficient is precisely what enables a country to achieve a virtuous circle of growth over time.

Lastly, it is worth noting that the Kaldorian model shown above lacks an imports demand function hence failing to impose a balance of payments constraint on growth. In practical terms, it means equation (1) could therefore yield consistently overestimated growth rates through time.

### 2.2. The balance-of-payment constrained growth model

The growth of the economy must be consistent with the equilibrium in the current account. Otherwise the external debt would follow an unsustainable trajectory over time that would have to be corrected by a sharp fall in the growth rate at some point. Accordingly, there is no guarantee *a priori* that the equilibrium growth rate described by equation (1) satisfies the balance-of-payments identity over time. By imposing the external constraint on growth, Thirlwall (1979) develops the canonical balance-of-payments constrained growth model. The growth rate within this framework is given by (see the appendix A.2 for a formal derivation)

$$y_T = \frac{\varepsilon z + (1 + \psi + \eta)\big(p_d - p_f - e\big)}{\pi} \tag{2}$$

where $y_T$ is the growth rate of the Thirlwall's (1979) model. As relative prices do not change in the long run, $\big(p_d - p_f - e = 0\big)$, the balance-of-payments equilibrium growth rate in (2) is determined by the growth rate of the foreign demand and by income elasticities of demand for export and import ratio, that is, $y_T^* = \varepsilon z / \pi$. The equilibrium growth rate is widely known in the literature as the Thirlwall's law. This law states that the domestic growth is directly related to the foreign demand growth rate. It also states that a country's output growth rate depends positively on its existing non-price competition factors, here expressed by the ratio

$\varepsilon/\pi$. This ratio reflects disparities between countries with respect to factors determining the demand for a country's exports and imports, such as technological capabilities, product quality, stock of knowledge, and consumer preferences, for instance.

As one might notice, by comparing equations (1) from the export-led growth model and (2) from the balance-of-payments growth model, it can be seen that the foreign income plays an important role in both theoretical frameworks. Nonetheless, if a balance of payments constraint is imposed on the Kaldorian export-led growth model it seems that a country can no longer achieve a virtuous circle of growth over time since the Verdoorn coefficient is completely suppressed in equation (2), as price-competition effects are mitigated in the long run[2]. On the other hand, if we decide to stick with the cumulative causation effect, expressed by the Verdoorn coefficient, then we must bear in mind that the growth rate we are dealing with may be overpredicted in the long run. Such a dilemma suggests both models are, in principle, incompatible. However, the underlying assumptions within their formal framework are in the core of the Keynesian-Kaldorian tradition. Hence, reconciling these two views in a more general model can be considered a very important problem for theorists of demand-driven Keynesian growth models.

## 2.3. The Export-Led and Balance-Of-Payment Constrained Growth Model

In an attempt to reconcile the export-led and the balance-of-payments constrained growth models, Thirlwall and Dixon (1979) developed a more general framework that takes into account the cumulative causation effect (Verdoorn coefficient) and the external constraint imposed by the balance-of-payments identity. The equilibrium growth rate is given by (see the appendix A.3 for more details)

$$y_{TD} = \frac{\varepsilon z + (1 + \eta + \psi)(w - a_0 + \tau - p_f - e)}{\pi + \lambda(1 + \eta + \psi)} \tag{3}$$

where $y_{TD}$ is the growth of output of the Thirlwall and Dixon's (1979) model and $\pi + \lambda(1 + \eta + \psi) > 0$ is the stability condition. The equation above represents the growth rate of the export-led and balance-of-payment constrained growth model. Equation (3) shows that price and non-price competitiveness factors affect growth. The problem with this specification is that there is no built-in mechanism in this framework that guarantees the convergence of relative prices and output growth towards their respective equilibrium levels. Therefore, the price effect can be suppressed if we assume that the Marshall-Lerner condition does not hold

in the long run (i.e. $1 + \eta + \psi = 0$). This case is known as the "elasticity pessimism". If it does happen, it is easy to see in equation (3) that the basic Thirlwall's law holds, $y_T^* = \varepsilon z / \pi$.

That said, it is worth noting that the framework proposed by Thirlwall and Dixon's (1979) transcends its previous models aforementioned as it simultaneously accounts for virtuous circles effects and balance-of-payments constraints in the short run.

## 3. Reconciling the export-led and the balance-of-payments constrained growth models

In this section we draw upon the work of Thirlwall and Dixon (1979) in order to develop a more general theoretical framework combining the export-led and the balance-of-payments constrained growth models. As pointed out in the previous section, nothing guarantees *a priori* that the growth rate given by the export-led and balance-of-payments constrained growth model will settle at the long-term equilibrium growth rate given by the Thirlwall's law. Thus, in order to obtain this result we must assume that the Marshall-Lerner condition is not satisfied. Our model, however, seeks to overcome these restrictive hypotheses for convergence. That is, we present a model wherein convergence is possible even if the Marshall-Lerner condition holds.

Let us assume the global economy consists of basically two different countries: a richer foreign country and a poorer home country. The foreign country is a large economy that issues the international currency and the home country is a small economy facing a balance-of-payments constraint in the long run. The foreign country is a two-sector economy which produces and exports consumption goods and industrialised intermediate inputs. The home country is a one-sector economy that produces and exports only one sort of consumption good with imperfect substitutability between the foreign and domestic consumption goods. We could also assume, at the expense of simplicity, that the home country is a two-sector economy which produces consumption goods and intermediate inputs; however, the addition of the intermediate input sector in the domestic economy would not change any of the main qualitative conclusions of the model. It is also assumed that the home country imports consumption goods and intermediate inputs from the foreign country. That is to say, the home country imports are disaggregated in two different categories, namely, imported consumption good ($M^c$) and imported intermediate inputs ($M^i$), that is, $M = M^c + M^i$. By doing so, we have now an extended balance-of-payments identity

$$P_d X = E \left( P_f M^c + P_f^i M^i \right) \tag{4}$$

where $P_d$ is the domestic price, $P_f^i$ is the price of imported intermediate inputs in foreign currency, $X$ is the volume of exports and $E$ is the nominal exchange rate. Assuming, for convenience, that the prices of imported consumption goods and the imported intermediate inputs in foreign currency are equal $\left(p_f = p_f^i\right)$, in rates of change we have

$$x = \left(e + p_f - p_a\right) + \theta m^c + (1 - \theta)m^i \tag{5}$$

where the lower case letters represent the growth rates of the levels of variables. Given $M = M^c + M^i$, we have $\theta = M^c/M$, and $(1 - \theta) = M^i/M$.

The next step is to define the exports and imports demand functions. For the formal specification of the exports demand function, please see equation (viii, appendix A.2).

The imports demand functions are given by

$$M^c = M_0^c \left(\frac{EP_f}{P_d}\right)^\psi Y^{\pi_c} \qquad \psi < 0, \pi_c > 0 \tag{6}$$

$$M^i = \mu Y \tag{7}$$

where $M_0^c$ and $\mu$ are constants, $\psi$ is the price elasticity of demand for imported consumption goods, $\pi_c$ is the income elasticity of demand for imported consumption goods, and $Y$ is the domestic income. Equation (6) is a multiplicative demand function for imported consumption goods, whereas equation (7) expresses a linear short-run relationship (one can view it as a linear approximation to a long run multiplicative function at a specific point in time). Moreover, it will be assumed here that demand-driven technological innovations are neutral with respect to the amount of intermediate inputs utilised in the production process, and hence the ratio of imported intermediate inputs to domestic output $\left(M^i/Y\right)$ does not change over time. This means domestic firms have a fixed proportions production function with respect to intermediate inputs. In this scenario, it is reasonable to assume that the coefficient $\mu$ is constant, meaning that changes in the nominal exchange rate will pass through completely into the unit variables costs. This hypothesis can be flexibilised at the expense of simplicity.

In rates of change

$$m^c = \psi\left(e + p_f - p_a\right) + \pi_c y \tag{8}$$

$$m^i = y \tag{9}$$

where $y$ is the growth of output. Now we define the domestic price index. Let us extend the mark-up pricing equation by making domestic prices a function of imported intermediate

inputs. To do so, the unit variable cost must be disaggregated in two parts, namely the unit labour cost and unit imported intermediate inputs cost

$$P_d = T\left(\frac{W}{a} + P_f E\mu\right) \tag{10}$$

where $T$ is the mark-up factor (1 + % mark-up); $P_f E\mu$ is the unit imported intermediate inputs cost in domestic currency; and $W/a$ is the unit labour cost. In growth rates

$$p_d = \tau + \varphi(w - \hat{a}) + (1 - \varphi)(p_f + e) \qquad \varphi \in (0,1) \tag{11}$$

where $\tau$ is the growth of the mark-up factor and $\varphi = (W/a)/\left[(W/a) + P_f E\mu\right]$ is the share of unit labour cost in total prime costs. It is worth noting that the share $\varphi$ is positively related to the degree of bargaining power of workers. Dividing the numerator and denominator of the share $\varphi$ by $P_d$ we have

$$\varphi = \frac{W/P_d a}{(P_f E/P_d)\mu + (W/P_d a)} = \frac{\sigma_W}{\sigma_{M^i} + \sigma_W} \tag{12}$$

where $\sigma_W$ and $\sigma_{M^i}$ are the wage and the imported intermediate inputs shares in income, respectively. If we plausibly argue that workers bargain for a higher wage share in total income, then we can say that, other things being equal, the higher their bargaining power, the higher the share $\varphi$.

Now we must examine the dynamics of relative price adjustment. The exchange rate pass-through mechanism states that an inflationary process over the imported intermediate inputs caused by a nominal currency devaluation feeds through into the rate of change of domestic prices with a lag. Per equation (11), we can point out basically two other transmission channels through which a permanent positive shock in the nominal exchange rate can impact on domestic prices:

- the rate of change of money wage increases with a lag after an increase in the inflation rate of imported intermediate inputs until the rate of change of the real wage matches the rate of change of labour productivity;
- a currency devaluation increases the market share of domestic goods in foreign markets, thus allowing exporters to raise the mark-up and, consequently, domestic prices.

The first transmission channel concerns the process of adjustment of nominal wages in response to changes in relative prices. Herein, we use the methodological strategy of dividing the time frame in two periods: the short run period, when $t_0 < t < t^*$, and the medium- to long-run period, when $t \geq t^*$. It will be assumed here that there is strong nominal wage rigidity in the short run, that is, the growth rate of nominal wages remains constant in this period. This scenario lasts until workers start the wage bargaining process in time $t^*$. From this moment onwards, we assume imperfect nominal wage flexibility. That said, consider that changes in the real wage growth rate are given by the following expressions

$$w = w_0 \qquad \text{when } t_0 < t < t^* \tag{13}$$

$$w = p_d + \alpha \hat{a} \qquad \text{when } t^* \leq t \tag{14}$$

where $w_0$ is the initial nominal wage growth rate, $\alpha = 2/\{1 + exp[\kappa(\varphi^e - \varphi)]\}$, $\kappa > 0$ is an adjustment parameter and $\varphi^e$ is the unit labour cost share of unit variable cost expected by workers in the long run. It is worth noting that workers bargain for a higher wage share, not for a higher $\varphi^e$; however, since $\varphi$ is a function of the wage share, we can say that $\varphi^e$ is also a function of the wage share desired by workers. Here we assume that the path followed by $\alpha$ over time can be proxied by a logit-like function. The specification (14) guarantees that increases in the growth of labour productivity progressively pass through into the growth of nominal wages. An increase in the labour productivity, $\hat{a}$, must reduce the parameter $\alpha$ in order to keep the equality (14). Given $\alpha = 2/\{1 + exp[\kappa(\varphi^e - \varphi)]\}$, an increase in the labour productivity widens the gap $(\varphi^e - \varphi)$, by reducing the wage share, $\sigma_W$, and consequently the share $\varphi$, thus diminishing $\alpha$. It is assumed herein that the share $\varphi^e$ is exogenous. Accordingly, in the long run, as the wage share converges to its equilibrium level expected by workers $(\varphi \rightarrow \varphi^e)$, the adjustment parameter $\alpha$ converges to unity $(\alpha \rightarrow 1)$ and hence the growth of real wages matches the growth of labour productivity $(w - p_d = \hat{a})$.

The second transmission channel concerns the mark-up factor. Following Blecker (1989), let us redefine the mark-up as a function of the RER. As a devaluation increases the market power of domestic firms, it enables them to raise their mark-up. Therefore, if we rewrite the mark-up as a positive function of the RER and take into account the extended mark-up price equation (9), we have

$$T = \delta \left( \frac{EP_f}{P_d} \right) = \delta \left[ \frac{EP_f}{T \left( \frac{W}{a} + P_f E \mu \right)} \right] \tag{15}$$

where $\delta > 0$ is a parameter. In rate of change we have (see appendix A.4)

$$\tau = -(\varphi/2)\big[(w - \hat{a}) - (p_f + e)\big] \tag{16}$$

That said, let us show the formal specification of the inflation rate for both periods.

(i)  Short-run period: substituting equations (16) and (13) into (11), we have the short run inflation rate with endogenous mark-up

$$p_d = (\varphi/2)(w_0 - \hat{a}) + (1 - \varphi/2)(p_f + e) \tag{17}$$

(ii) Medium- to long-run period: substituting equations (16) and (14) instead into (11) and then solving for $p_d$, we obtain

$$p_d = [(\alpha - 1)\hat{a}/(1 - \varphi/2)] + p_f + e \tag{18}$$

which means that, in the long run, given $\alpha = 1$, we have $p_d = e + p_f$, that is, relative prices remain constant . In other words, when the wage share matches the wage share desired by workers in the long run ($\sigma_W \to \sigma_W^e \Rightarrow \varphi \to \varphi^e \Rightarrow \alpha \to 1$), relative prices do not change. It is worth noting the major role played by distributive conflicts between workers and capitalists in this framework. The higher the bargaining power of workers, the faster the share $\varphi$, and, consequently, the relative prices adjust to their equilibrium level.

We also have to define the growth of the labour productivity. Kaldor (1966) persuasively argues that the growth of the labour productivity is an increasing function of the growth of output in the long run (the so-called Verdoorn's law). Verdoorn's law is interpreted as a long-run relationship between demand growth and labour productivity, as a demand increase leads to higher growth of R&D activities, higher investment rate and the consequent acquisition of new and more efficient machines in some future period. In other words, the growth of labour productivity caused by increasing return effects can only occur with a lag. Drawing upon the formalisation of Kaldor's theory proposed by Dixon and Thirlwall (1975), we have the following equation in the long run

$$\hat{a} = a_0 + \lambda y \tag{19}$$

where $a_0$ is rate of autonomous productivity growth; and $\lambda > 0$ is the Verdoorn coefficient.

Now we must show the growth rate equations for both periods:

(i)  Substituting the short-run inflation given by (17), the imports demand functions given by (8) and (9), the growth of exports as shown in (viii, appendix A.2) into the balance-of-

payments identity in (5) and solving for $y$, we find the short-run growth rate of the extended model $(y_{SR})$

$$y_{SR} = \frac{\varepsilon z + (1 + \eta + \theta\psi)(\varphi/2)(w_0 - \hat{a} - p_f - e)}{\pi} \tag{20}$$

where $\pi = \theta\pi_c + (1 - \theta)$. In other words, the income elasticity of demand for total imports $(\pi)$ is given by the weighted average of income elasticities of demand for imported consumption goods $(\pi_c)$ and imported intermediate inputs (which is equal to unity). It is worth noting that the growth of labour productivity in the short run is constant, as the Verdoorn's law is only observed in the long run.

(ii) In the medium-to long-run period, we have to consider the inflation rate for the same period, given by equation (18). Thus, making the same substitutions above yields the medium- to long-run growth rate of the extended model $(y_{M-LR})$

$$y_{M-LR} = \frac{(1 - \varphi/2)\varepsilon z + (1 + \eta + \theta\psi)(\alpha - 1)a_0}{(1 - \varphi/2)\pi - (1 + \eta + \theta\psi)(\alpha - 1)\lambda} \tag{21}$$

Since in the long run relative prices do not change $(\alpha = 1)$, equation (21) yields the Thirlwall's law, that is, $y_T^* = \varepsilon z/\pi$.

To sum up, we extend the Thirlwall and Dixon (1979) model in two ways: First, we include imported intermediate inputs into the prime cost of domestic firms; second, we model the role played by the conflicting claims in the determination of wages and, consequently, in the dynamics of relative price adjustment. These two modifications allow us to add a built-in exchange rate pass-through mechanism. It is precisely such a mechanism that enables the process of convergence of the actual growth rate towards the long-run equilibrium growth rate given by Thirlwall's law. This convergence process in the original Thirlwall and Dixon's (1979) model only takes place if the Marshall-Lerner condition does not hold in the long run, whereas in our model this hypothesis is not necessary. Our model also adds to the discussion of relative price adjustment posed by Pugno (1998) and Porcile and Lima (2010). Relative price adjustment, in their models, is exclusively linked to movements in the demand and supply of labour. Based in our framework, we claim that wage settlements also make an allowance for future values of nominal wages desired by workers. This idea is somehow inspired by the works of Rowthorn (1977) and Dutt (1994), in which the actual values of inflation and nominal wages depend on future values of inflation expected by decision makers. Equation (14) shows that the adjustment of real wages to the growth of labour

productivity depends on the gap $(\varphi^e - \varphi)$, where the expected unit labour cost share of prime costs, $\varphi^e$, is a function of the expected average real wage desired by workers. Expected values of nominal variables can play an important role in this analysis if we consider, for instance, an economy wherein the Central Bank is credibly committed to achieving the inflation target. In this case, a conventional anchor for the expectation of decision makers is created (Lima and Setterfield, 2008), which may or may not affect future prices and consequently $\varphi^e$. Therefore, a simple change in the inflation target, for instance, can affect $\varphi^e$ and hence trigger a process of wages and relative price adjustment without any real interference in the demand and supply of labour. Another example is the case of an economy with high and accelerating inflation. In this scenario, given a constant employment rate set by the equilibrium in the labour market, if nominal wages are only partially indexed to past inflation, then real wages will fall; alternatively, if nominal wages are over-indexed, than real wages will increase. Once again we can have changes in the real wage and income distribution without any change in the employment rate. Certainly the rules of indexation depend on the bargaining power of workers, but not only. Governments more (less) predisposed toward workers can create (withdraw) incentives for over-indexation or even impose new rules of indexation that might change the expected values of nominal variables without necessarily changing the labour market equilibrium. In other words, unlike Pugno (1998) and Porcile and Lima (2010), our model opens a theoretical possibility for relative price adjustments through changes in domestic prices which take place independently from the dynamics of the demand and supply of labour. Lastly, we can discuss Blecker's (1998) findings in the light of our model. The author sets out a general balance-of-payments constrained growth model where an uncompetitive country can balance its trade by quantity and price adjustments. In other words, in his framework, both price and non-price competitiveness bring the current account back to balance, thus implying that the type of adjustment that predominates is basically an empirical question. However, the author only finds this result due to the fact that nothing guarantees in his framework that the mark-up factor stabilises in the long run. Drawing upon his previous work in Blacker (1989) and Pugno (1998), we define the mark-up as a function of the RER, which means that in the long run both relative prices and mark-up will settle in their equilibrium level and hence only non-price competitiveness determines the long-run growth rate. Therefore, Blecker's (1998) general model can only explain the short- to medium-run growth dynamics. However, if we accept that in the long run the mark-up remains constant, then the Thirlwall's law still holds.

Having said that, now we move on to the second part of this paper. The extended model shown in here also offers some particularly helpful policy insights regarding the impact of a currency devaluation on growth.

## 4. Exchange rate and growth: some further theoretical developments and policy implications

In this section, first we analyse the short-run impact of a currency devaluation on growth according to the extended model and discuss some policy implications. Second, we run some simulations in order to illustrate the convergence dynamics of the actual growth rate towards the equilibrium growth rate given by Thirlwall's law.

### 4.1. The impact of the exchange rate on short-run growth

The aim of this section is to show the effect of a currency devaluation on short-term growth according to the extended growth model. But to make some comparisons, we present below the partial effect of a currency devaluation on growth according to the previous models from the literature

$$\frac{\partial y_{DT}}{\partial e} = \frac{\gamma\delta}{1+\gamma\rho\lambda} > 0 \tag{22}$$

$$\frac{\partial y_T}{\partial e} = -\frac{(1+\psi+\eta)}{\pi} > 0 \tag{23}$$

$$\frac{\partial y_{TD}}{\partial e} = -\frac{(1+\eta+\psi)}{\pi+\lambda(1+\eta+\psi)} > 0 \tag{24}$$

Equations (22), (23) and (24) are obtained by partially differentiating equations (1), (2) and (3) respectively. That is, the partial effect of a currency devaluation according to the export-led growth model is given by (22), the balance-of-payments constrained growth model by (23) and the export-led and balance-of-payments growth model by (24). Equations (22)-(24) show that continuous real devaluations are capable of unambiguously raising permanently the growth rate. However, the effect of a currency devaluation on growth cannot be permanent, as a constant real exchange rate devaluation would pass-through into domestic prices, thus increasing wages and mitigating the price-competitiveness effect on growth.

In the extended model, on the other hand, knowing the impact of a real devaluation on short-term growth becomes quite a complex task. Before we show the immediate impact of a currency devaluation on growth according to the extended model, we must note that the share

$\varphi$, by equation (12), is inversely related to the nominal exchange rate. In other words, in order to analyse the impact of a devaluation on short-term growth it must be taken into account the partial effect, not only of $e$, but also of the share $\varphi$. We also assume, for simplicity, that the impact of a currency devaluation on $\theta$ is negligible. Thus, partially differentiating equation (20) with respect to $e$ gives

$$\frac{\partial y_{SR}}{\partial e} = \frac{(1 + \eta + \theta\psi)[(w_0 - \hat{a} - p_f - e)(\partial\varphi/\partial e) - \varphi]}{2\pi} \qquad (25)$$

Let us analyse separately each component of (25):

- $(1 + \eta + \theta\psi)$: is less than zero if the Marshall-Lerner condition holds;
- $(w_0 - \hat{a} - p_f - e)$: in equilibrium, the growth of real wages equals the growth of labour productivity $(w_0 - p_d = \hat{a})$ and relative prices are constant $(p_f - e = p_d)$, thus yielding $(w_0 - \hat{a} - p_f - e) = 0$; however, a nominal exchange rate devaluation, that is, an increase in $e$ gives $(w_0 - \hat{a} - p_f - e) < 0$;
- $(\partial\varphi/\partial e)$: by equation (12), the share $\varphi$ is inversely related to the nominal exchange rate, that is, $\partial\varphi/\partial e < 0$.

Equation (25) differs from the previous models by showing that the partial effect of a currency devaluation is ambiguously signed. Assuming that the economy is initially in equilibrium, $y_T^* = \varepsilon z/\pi$, and that the Marshall-Lerner condition is satisfied, $(1 + \eta + \theta\psi) < 0$, a currency devaluation can only boost growth if we also find $[(w_0 - \hat{a} - p_f - e)(\partial\varphi/\partial e) - \varphi] < 0$ or, alternatively

$$(w_0 - \hat{a} - p_f - e)(\partial\varphi/\partial e) < \varphi \qquad (26)$$

If the inequality (26) is satisfied, then a currency devaluation boosts growth.

We conclude that the higher the bargaining power of workers, that is, the higher the share $\varphi$, the higher the impact of a real devaluation on growth. The rationale behind it is that in a heated labour market the wage share tends to be relatively high. By equation (12), it means that the negative impact of an increase in the price of imported intermediate input due to a currency devaluation on prime costs is relatively low. Therefore, the higher the wage share (or, alternatively, the lower the imported intermediate inputs share of total prime costs), the more effective a currency devaluation is to propel short-term growth. That is to say that the effectiveness of the exchange rate to improve a country's price-competitiveness and so stimulate positive waves of short- to medium-run growth is closely linked to the capacity of

domestic firms to reduce their dependence of imported intermediate inputs in their production process. Countries that stimulate significant technological innovations or that manage to design successful strategies of import substitution industrialisation are more capable of effectively boosting exports and growth in the short run by devaluing the currency. To sum up, we can say that a successful devaluation in this context basically means that the gains from trade after a devaluation outweigh the negative impact of increased prices of imported intermediate inputs on prime costs. In economic terms, what happens is that the depreciation has basically two effects. On the one hand, it raises the foreign demand for domestic goods, hence boosting exports. On the other hand, a devaluation also feeds through into the prices of imported intermediate inputs in domestic currency, thus harming the price-competitiveness of domestic goods.

In short, it is shown in this subsection that the higher the share $\varphi$, which is positively related to the wage share of income (and inversely related to the imported intermediate inputs share of prime costs), the more effective a currency devaluation is to promote growth. This result implies that a currency devaluation is more likely to boost the short-run growth rate in countries where the wage share is higher. However, to the best of our knowledge, this possibility has not been tested in the empirical literature yet.

### 4.2. Relative price adjustment and the process of convergence towards the long-run growth rate: a numerical exercise

In this section we examine the adjustment process of the RER after an exogenous permanent increase in the growth rate of nominal exchange rate. It allows us to explore different patterns of convergence of relative prices and growth that can emerge from the exchange rate pass-through mechanism.

The idea is to examine the adjustment process of the RER after an exogenous permanent increase in the growth rate of nominal exchange rate. For example, assuming foreign prices remain constant for simplicity, in equilibrium, a 2% inflation rate per period implies a 2% nominal currency devaluation per period; the question we ask is: if the monetary authority start devaluing the currency at 4% per period, how long will it take for the domestic inflation to match the nominal exchange rate devaluation at 4% per period and hence stabilise the level of the RER again?

To commence, let us consider equation (21). This equation illustrates the medium-run growth rate according to the extended model. Assuming, for convenience, that the term in the denominator $(1 + \eta + \theta\psi)(\alpha - 1)\lambda$ is negligible, we can rewrite equation (21) as follows

$$y_t = y_T^* + \xi\widehat{\Theta}_t \tag{27}$$

where $\xi = -(1 + \eta + \psi)/\pi > 0$, which means that the Marshall-Lerner condition is satisfied; and $\widehat{\Theta}_t = (\alpha_t - 1)a_0$ accounts for the relative price effect on growth.

Thus, in order to describe the convergence of the actual growth towards the equilibrium growth $(y_t \to y_T^*)$, we must model the trajectory of the price effect over time $(\widehat{\Theta}_t \to 0)$. By equation (14), we know that when the wage share matches the wage share desired by workers in the long run $(\sigma_W \to \sigma_W^e \Rightarrow \varphi \to \varphi^e \Rightarrow \alpha \to 1)$, relative prices do not change. More formally, we have

$$\widehat{\Theta}_t = \kappa(\varphi_t - \varphi^e) \tag{28}$$

where $\varphi_0$ is the initial value of the share $\varphi_t$ after a devaluation, that is, $\varphi_0 < \varphi^e$; and $\kappa < 0$ is a parameter that measures the speed of adjustment of relative prices. That is, when $\varphi_t = \varphi^e$, relative prices also do not change.

If we assume that in the bargaining process firms and unions set wages according to their knowledge of past wages, then we can also assume that the current unit labour cost share of prime costs, $\varphi_t$, depends on its value in the previous period, $\varphi_{t-1}$. The first-order nonhomogeneous linear equation is given by

$$\varphi_t = \beta\varphi_{t-1} + \omega \tag{29}$$

where $\beta$ measures the degree of persistence of past wages; and $\omega > 0$ is positively related to the bargaining power of workers, which is assumed to be constant in this model. Solving equation (29) and then substituting the result into (28) and (27) gives (see appendix A.5)

$$y_t = y_T^* + \xi\kappa\beta^t(\varphi_0 - \varphi^e) \tag{30}$$

Assuming the stability condition, $|\beta| < 1$, as time passes by the impact of the parameter $\beta^t$ reduces and the growth rate converges to its equilibrium level, $y_T^*$. Below, we assign numerical values to the set of parameters of the model and analyse both cases of convergence: case (1) $0 < \beta < 1$; and case (2) $-1 < \beta < 0$.

Assuming $y_T^* = 0.03$, $\xi = 0.01$, $\kappa = -0.2$, $\varphi_0 = 0.6$, $\varphi^e = 0.66$, and $|\beta| = 0.6$, we have two possible growth trajectories over time.

Case 1: $\beta = 0.6$

Figure 1 – The impact of a real exchange rate devaluation on growth for $\beta = 0.6$
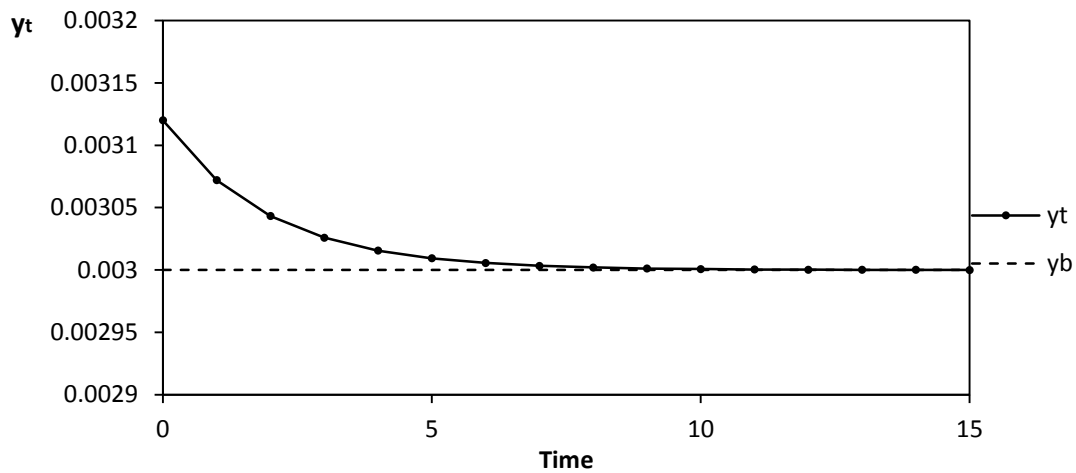


where yt is the actual growth rate and yb is $y_T^*$. Figure 1 illustrates the convergence process of the current balance-of-payments constrained growth rate, represented by the solid line, towards the equilibrium growth rate, represented by the dashed line. In this scenario, after a currency devaluation the share $\varphi_t$ converges asymptotically towards the equilibrium share $\varphi^e$.

Let us now analyse the second case, assuming that the values of the other parameters remain the same.

Case 2: $\beta = -0.6$

Figure 2 – The impact of a real exchange rate devaluation on growth for $\beta = -0.6$
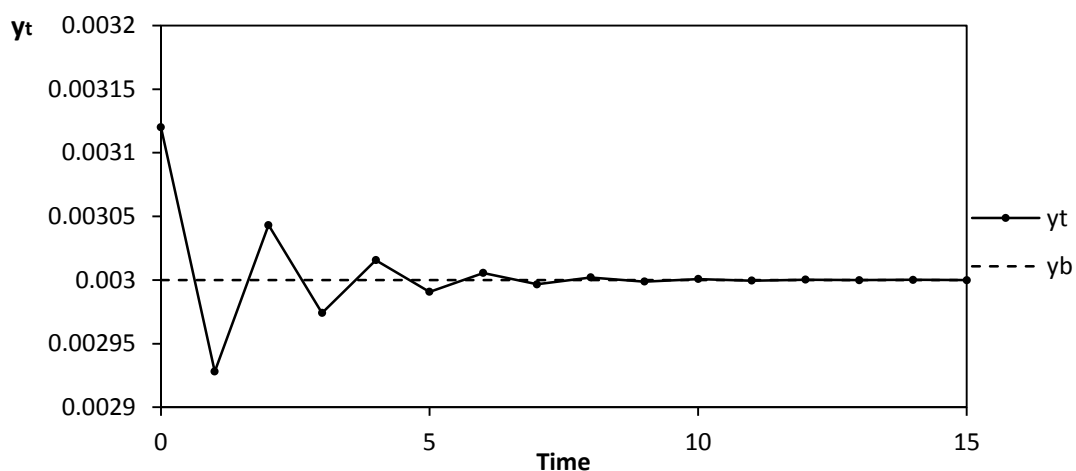


In this scenario, $\beta < 0$ illustrates a more intensified distributive conflict between workers and capitalists. If in the period $t$ workers manage to obtain an increase in the wage share through

the bargaining process, then in the next period capitalists may "strike back" by increasing the mark-up more than proportionally to the initial increase in wages, thus reducing the share of labor in income in the period $t + 1$. The decline in the wage share in $t + 1$ would encourage workers to retaliate by requiring higher wages in $t + 2$; however, the raise in the wage share in $t + 2$ is enough to cover only part of the loss in $t + 1$. In $t + 3$, firms readjust the mark-up more than proportionally again, but still not enough to fully erode the previous gains in the wage share obtained by workers. The stability condition requires that the magnitude of the ups and downs in output fluctuations decrease each period so that, in the long run, the actual growth rate converges towards the equilibrium growth rate.


## 5.  Summary

The purpose of this paper was to build on Thirlwall and Dixon's (1979) model, which purports to reconciliate the export-led and the balance-of-payments constrained growth models. We extend their model in two ways in order to incorporate salient stylised features of open economies: first, by including imported intermediate inputs into the prime cost of domestic firms; and, second, by modeling the role played by the conflicting claims on income in the dynamics of relative price adjustment.

These two modifications are theoretically and empirically relevant because they enable the process of convergence of the actual growth rate towards the long-run equilibrium growth rate given by Thirlwall's law. It is noteworthy that in the original Thirlwall and Dixon's (1979) model this convergence process can only happen if the Marshall-Lerner condition is not satisfied in the long run. Hence, our model contributes to the literature by showing that convergence is possible even if the Marshall-Lerner condition holds.

A second and equally important conclusion that can be drawn from our model relates to the ambiguous impact of a currency devaluation on growth. It is shown that in an economy where firms are highly dependent on imported intermediate inputs, a currency devaluation might reduce the growth rate. Alternatively, a relatively high wage share increases the effectiveness of a RER devaluation for boosting growth. This is an implication that deserves careful consideration in the empirical literature.

Lastly, we run some numerical simulations to illustrate the relative price adjustment and, consequently, the convergence process of the actual growth rate towards the equilibrium growth rate. Depending on the dynamics of the conflicting claims on income in the wage decision-making process, such convergence can be either monotonic or cyclical.

**Notes**

1. As such, it predates the neoclassical endogenous growth models (Romer, 1986).
2. McCombie and Roberts (2002) make the ratio of the income elasticities of demand for exports to the income elasticity of demand for imports a non-linear function of the growth rate of output in the previous period. Therefore, in this framework, the elasticities ratio incorporates the Verdoorn Law, and hence short-term shocks in relative prices might have a permanent long-term effect on the growth rate.

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**Mathematical Appendix**

*A.1 The export-led growth model*

The export-led growth model is given by the following equations

$$y = \gamma x \tag{i}$$

$$x = \rho p_d + \delta(p_f + e) + \varepsilon z \tag{ii}$$

$$p_d = w - \hat{a} + \tau \tag{iii}$$

$$\hat{a} = a_0 + \lambda y \tag{iv}$$

where $y$ is the rate of growth of output; $x$ is the rate of growth of exports; $p_d$ is the domestic inflation rate. The other variables follow their meaning as defined in equation (1).

Solving the system constituted by equations (i)-(iv), we have

$$y_{DT} = \frac{\gamma[\rho(w - a_0 + \tau) + \delta(p_f + e) + \varepsilon z]}{1 + \gamma \rho \lambda} \tag{v}$$

*A.2 The balance-of-payments constrained growth model*

Assuming relative prices remain constant in the long run, the balance-of-payments identity requires that exports, $x$, and imports in domestic currency, $(e + p_f - p_d) + m$, grow at the same rate of change

$$x = (e + p_f - p_d) + m \tag{vi}$$

The import and export demand functions are given by

$$m = \psi(e + p_f - p_d) + \pi y \tag{vii}$$

$$x = \eta(p_d - p_f - e) + \varepsilon z \tag{viii}$$

where $\psi$ and $\eta$ are the price elasticities of demand for imports and exports, respectively; $\pi$ and $\varepsilon$ are the income elasticities of demand for imports and exports, respectively. Solving the system formed by equations (vi)-(viii), we have

$$y_T = \frac{\varepsilon z + (1 + \psi + \eta)(p_d - p_f - e)}{\pi} \tag{ix}$$

*A.3 The export-led and balance-of-payments constrained growth model*

Putting equations (iii) and (iv) into (ix) we obtain

$$y_{TD} = \frac{\varepsilon z + (1 + \eta + \psi)(w - a_0 + \tau - p_f - e)}{\pi + \lambda(1 + \eta + \psi)} \tag{x}$$

*A.4 The growth rate of the mark-up factor*

The RER can be rewritten as $EP_f/P_d = EP_f/T[(W/a) + P_f E\mu] = (1 - \varphi)/\mu$. After rearranging the terms, equation (15) gives

$$T = [(\delta/\mu)(1 - \varphi)]^{1/2} \tag{xi}$$

In rates of change

$$\tau = -\frac{\varphi}{2(1 - \varphi)} \frac{d\varphi}{\varphi} \tag{xii}$$

Now we must find $d\varphi/\varphi$. By definition, we have

$$\varphi = \frac{T(W/a)}{T\left(\dfrac{W}{a} + P_f E\mu\right)}$$

In rates of change

$$\frac{d\varphi}{\varphi} = \frac{d\ln}{dt}\left[\frac{T(W/a)}{T\left(\dfrac{W}{a} + P_f E\mu\right)}\right] = \frac{d}{dt}\left[\ln(W) - \ln(a) - \ln\left(\frac{W}{a} + P_f E\mu\right)\right]$$

$$\frac{d\varphi}{dt} = \varphi(1 - \varphi)\left(w - \hat{a} - p_f + e\right) \tag{xiii}$$

Substituting (xiii) into (xii) gives equation (16)

*A.5 The particular solution of the difference equation*

If we successively calculate equation (29) for previous periods, we obtain

$$\varphi_t = \beta \varphi_{t-1} + \omega$$
$$\varphi_t = \beta(\varphi_{t-2} + \omega)\omega$$
$$\vdots$$

$$\varphi_t$$

$$= \beta^t \varphi_0 + \omega \left( \frac{1 - \beta^t}{1 - \beta} \right) \tag{xiv}$$

In the long run $(t \to \infty)$

$$\varphi^* = \gamma / (1 - \beta) \tag{xv}$$

where $1 - \beta > 0$. Solving equation (xv) for $\omega$ and then substituting into (xiv) yields

$$\varphi_t = \beta^t (\varphi_0 - \varphi^e) + \varphi^e \tag{xvi}$$

Rearranging (xvi) and substituting into (28) and then in (27) yields (30).