

Multipliers effects of social protection: a SVAR approach for Brazil

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JEL Codes: E62, I38, H53.

Multiplier effects of social protection: a SVAR approach for Brazil

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1) Introduction

Since the Covid-19 pandemic started to show its effects on the global economy in 2020, the need to address new public health, social and economic challenges enhanced the importance of cash transfers, unemployment insurance, and other social benefits in alleviating the impact of structural transformations in the 21st-century labor market. These efforts find empirical support in a vast literature that shows a significant role of social protection in reducing poverty and inequality¹. However, as governments around the World are adding social protection to their post-pandemic green and inclusive economic recovery plans, the empirical literature on positive short-run macroeconomic effects of social benefits is relatively scarce.

In particular, the most conventional VAR (Vector Autoregressive Model) approach to estimate fiscal multipliers in a country-specific context builds on Blanchard and Perotti (2002) and Perotti (2004), who have removed income transfers from total government revenues in the estimations. Many authors have followed this strategy². Nevertheless, as social security systems have grown substantially in OECD countries after the Second World War, other authors have criticized this approach (Gáldon, 2013; Gechert, Paetz and Villanueva, 2018; Baum and Koester, 2011; Pereira and Wemans, 2013). Hence, while the implementation of the American Recovery and Reinvestment Act (ARRA) in the United States during the Global Financial Crisis has been partially justified in terms of significant multiplier effects of income transfers by the Council of Economic Advisers (2009), only a few studies have focused on estimating the impact of social expenditures, namely income transfers (such as unemployment insurance or cash transfers) and social security.

The literature that began to use the conventional VAR approach of Blanchard and Perotti (2002) shows contradicting results. Some authors have found significant multiplier effects for

¹ See, for instance, Ocampo and Arteaga (2016) and Ulu (2018).

² See Tenhofen, Wolff and Heppke-Falk, 2010; Burriel *et al.*, 2010.

social expenditures (Gechert, Paetz and Villanueva, 2018; Gáldon, 2013; Adams and Wong, 2018), even if non-persistent (Adams and Wong, 2018). Various studies have estimated positive but low multipliers for social transfers. Generally, these studies estimate higher multipliers associated with a cut in direct taxes, a positive shock in government consumption, or, mainly, increases in public investments (Bova and Klyviene, 2019; Pereira and Wemans, 2013; Silva, Carvalho and Ribeiro, 2013). In other cases, the multiplier for social transfers is large in absolute terms, but different types of expenditures show a similar or a higher multiplier effect on output (Pereira and Wemans, 2013; Fatás and Mihov, 2001; Pereira and Sagalés, 2009).

This paper aims to contribute to this literature by adding evidence on the macroeconomic effects of social expenditures in Brazil -- a country with high inequality levels and a relatively well-developed social protection system. Two more aspects add to our interest in focusing on the macroeconomic effects of social benefits in Brazil. First, the country has been facing a deep economic crisis since 2015, but the fiscal consolidation strategy in 2015-2019 has spared several components of social expenditures that are constitutionally mandatory. While advocates of cutting additional budgets have been calling for structural reforms that reduce expenses, these items may have helped prevent an even deeper downturn. Second, Brazil has built on its previous experience with Programa Bolsa Família - the World's most extensive conditional cash transfer program - to provide around 6.5% of GDP in an emergency cash relief program with more than 67 million beneficiaries during the pandemic. In this context, discussions around the role of the program in attenuating the fall in GDP in 2020 and on potential benefits of permanently expanding social transfers in Brazil have attracted renewed interest due to its multiplier effects.

More specifically, our contribution to the empirical literature on fiscal multipliers is twofold: first, instead of subtracting social benefits from revenue, we use the Blanchard and

Perotti's (2002) Structural VAR approach to estimate the macroeconomic effects of social protection. In particular, we provide evidence that social benefits have a high multiplier effect – even comparable to the public investment multiplier. This result barely appears in the literature (see Table III), despite its macroeconomic relevance.

Second, our results contribute to the empirical literature by adding additional evidence on social protection multipliers. In particular, the present study finds considerable differences in the size of social benefits multipliers between the sample that includes Brazil's recent economic recession (2014-2016) when compared to the pre-crisis sample. Our study reveals that a social spending shock triggers a more significant output increase in the sample that includes the recession, thus suggesting an essential counter-cyclical role of social protection.

Moreover, we go beyond existing studies by providing a disaggregated analysis: we estimate social protection multipliers on household consumption and private investment for different types of social benefits expenditures (e.g., cash transfers, unemployment insurance, and pensions). The higher estimated multipliers in the full sample appear in the response of both household consumption and private investment, but with some level of heterogeneity depending on which type of benefit. The response of private investment to a shock in BPC is the component that presents the most significant difference between the two samples. Also, the discrepancy between the response of household consumption in the two samples to a shock in pensions and PBF is the most pronounced.

The article has five sections in addition to this introduction. Section 2 presents the structure and evolution of Brazilian social expenditures. Section 3 reviews the empirical literature on multiplier effects of social protection. Section 4 introduces our methodology, and section 5 analyzes our estimation results. A final section concludes the paper.

2) The Brazilian social protection system

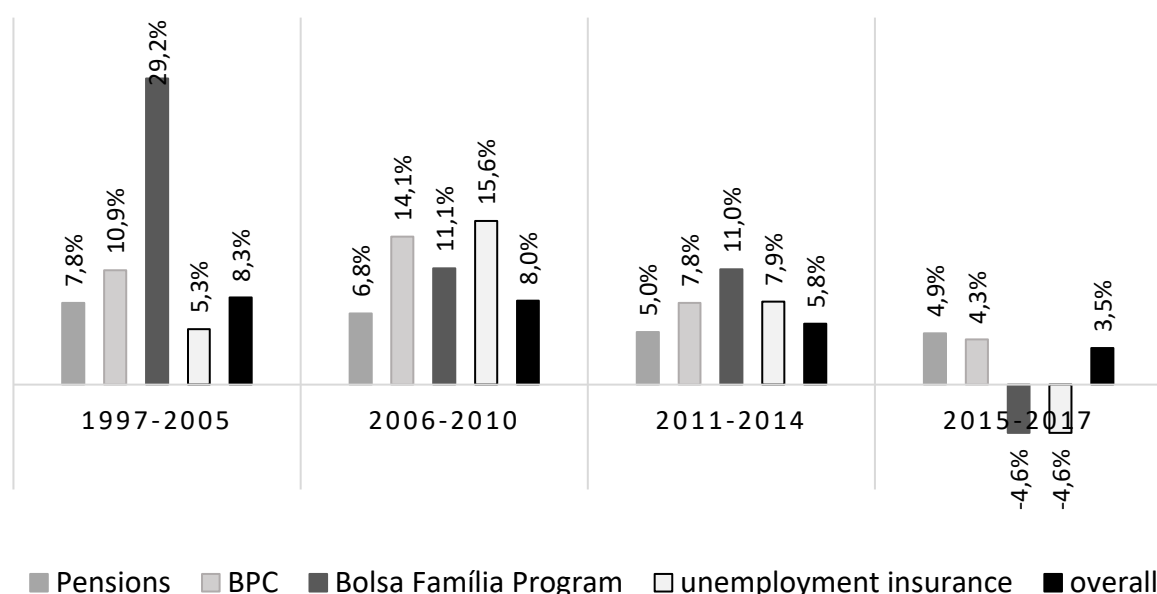
Table I provides a brief description of each item of the Brazilian social protection system. Figure I shows the annualized growth rate of these items of social expenditures between 1997 and 2017. Expenditures on pensions have shown the most stable rise throughout the period. As for the social assistance program aimed at low-income older adults and people with disabilities (BPC), 2006-2010 has been marked by a 14% annual growth in spending, whereas crisis years 2015-2017 have only shown a 4.3% increase.

In 2004, Programa Bolsa Família (PBF) unified the management and implementation of previous federal programs. As suggested by the high growth rate per year in expenditures on PBF observed in Figure I, the program grew substantially since its implementation. The substantial increase in the number of beneficiaries in all states and the rise in the annual per capita value transferred to municipalities in each state explain this expansion (Landim Junior, 2009). While in January 2004, PBF benefited 3.6 million families, in 2010, this number reached 12.8 million. In August 2017, the number of families benefiting from the program was 13.5 million (Carvalho, 2018). However, as shown in Figure I, there is a reduction in expenditures on PBF during the 2015-2017 crisis.

Spending on unemployment insurance (alongside wage allowances) stands out for its sizeable positive variation in 2006-2010, of little more than 15% per year. The role of unemployment insurance expenditures as an automatic stabilizer during the 2008-09 global financial crisis is noteworthy (25% expansion). Nevertheless, its annual average variation was negative during the 2015-2016 crisis. From 2014 to 2015, these items were reduced by 15%. Still, the average yearly growth of total expenditures on social benefits was positive during the 2015-2016 crisis thanks to the maintenance of constitutionally mandatory expenses such as pensions and BPC.

Table II shows each social benefits item presented in Table I as a proportion of GDP. We note the accentuated growth and without much oscillation of the expenses related to the Pensions and BPC. The expenses with unemployment insurance present greater variation over time. The PBF, on the other hand, had considerable growth in its share of GDP since its creation.

Figure I: Annualized growth rates of social benefits in four sub-periods (% per year) in Brazil



Source: Own elaboration. Data from Gobetti and Orair (2017).

Table I: Components of the Brazilian social protection system

Social Protection Component	Description
Programa Bolsa Família (PBF)	PBF is a federal-conditioned cash program targeting families that live in poverty and extreme poverty (monthly income per capita below BRL 154). The transfer is conditional to primary healthcare, child nutrition, and educational attendance criteria.
Pensions	Pension benefits are regimented by Regime Geral de Previdência Social (RGPS) (public pension system). Contributory pensions are calculated based on workers' average salaries. Other pension benefits include survivor's pensions and family and maternity allowances (Boletim estatístico da previdência social, 2015). Rural pensions are non-contributory and consist of a monthly minimum wage paid to individuals who worked in rural activities for at least 15 years (Souza, 2011).

Benefício de Prestação Continuada (BPC)	BPC is a social assistance program established by law that guarantees a monthly minimum wage to older adults aged 65 and over and to people with disabilities. The beneficiaries' monthly family per capita income must be below one-fourth of the minimum wage (Boletim estatístico da previdência social, 2015).
Unemployment Insurance	Besides providing financial assistance to the unemployed worker during a period, the program helps them find a job through integrated orientation, outplacement, and professional training.
Wage allowance	It assures the value of an annual minimum wage to Brazilian workers who receive on average up to two monthly minimum wages from employers who contribute to the Programa de Integração Social (PIS) or the Programa de Formação do Patrimônio do Servidor Público (PASEP).

Source: Own elaboration.

Table II: Social benefits in Brazil (% of GDP)

	Pensions	Benefício de Prestação Continuada (BPC)	Programa Bolsa Família (PBF)	Unemployment insurance + wage allowance
1997	4.78	0.24	n.a	0.45
1998	5.17	0.26	n.a	0.44
1999	5.23	0.28	n.a	0.45
2000	5.35	0.29	n.a	0.39
2001	5.59	0.33	n.a	0.43
2002	5.77	0.34	0.15	0.48
2003	6.13	0.36	0.18	0.48
2004	6.41	0.38	0.28	0.49
2005	6.72	0.43	0.29	0.53
2006	6.87	0.48	0.31	0.61
2007	6.81	0.52	0.32	0.66
2008	6.41	0.52	0.34	0.65
2009	6.75	0.57	0.35	0.81
2010	6.56	0.58	0.35	0.77
2011	6.43	0.58	0.38	0.78
2012	6.57	0.61	0.43	0.82
2013	6.69	0.64	0.45	0.87
2014	6.81	0.67	0.45	0.89

2015	7.26	0.71	0.44	0.79
2016	8.11	0.78	0.44	0.9
2017	8.49	0.82	0.42	0.83

Source: Own elaboration.

3) The empirical literature on social benefits multipliers

The literature on fiscal multipliers has grown significantly since the Global Financial Crisis. In country-specific studies, the most conventional approach has made use of linear VAR models (autoregressive vectors) to estimate the impact of an exogenous shock in public expenditures or government revenues on the level of economic activity, following Blanchard and Perotti (2002). When disaggregating different types of public spending, this literature shows a higher and more persistent multiplier effect of public investment on GDP (Deleidi, Iafrate and Levrero, 2019; Izquierdo et al., 2019; Ilzetzki, Mendoza and Vegh, 2013).

As already mentioned in the introduction, while many papers have focused on the effects of public expenditure and tax shocks, the impact of changes in social benefits has received less attention (Gechert, Paetz and Villanueva, 2018). In this regard, Pereira and Wemans (2013) (p.10) state that: *“Initial studies applying the structural VAR methodology to fiscal policy adopted a very aggregate definition of budgetary variables, considering only taxes net of transfers, on the one hand, and public expenditure (fundamentally consumption and public investment), on the other. These definitions have been used in a great deal of subsequent work in this field. It is, however, plausible that the various headings that make up these aggregates have distinctive influences on economic activity”*. From this standpoint, our study is an extension of Blanchard and Perotti (2002). Instead of subtracting social benefits from revenue, we use the VAR approach to estimate the macroeconomic effects of social protection.

Table III presents studies that apply the conventional VAR methodology and mostly the Structural VAR (SVAR) approach developed by Blanchard and Perotti (2002). In some estimations, the social benefits multiplier is above one: every \$1 of government spending on social benefits would ultimately generate more than \$1 in additional GDP. The study carried out by Hollmayr and Kuckuck (2018) stands out by estimating an even higher multiplier (3.8 after five years).

Some authors have also estimated multipliers using panel techniques for a group of countries (or states/regions of the same country) via VAR or one-equation methods³. In the specific case of social expenditures, the study carried out by Furceri and Zdzienicka (2012) finds a positive accumulated multiplier (despite being smaller than one) for a group of OECD countries, with an emphasis on health expenditures (0.9) and unemployment insurance (2.1). Reeves et al. (2013) estimate a high social protection multiplier for a group of European countries (3). Health expenditures have an even greater multiplier (4.9).

Table III: Multiplier effects of social expenditures in the econometric literature (Linear VAR approach)

Study / Country / Period	Social expenditure	Multiplier	Method
Adams and Wong (2018)- New Zealand (1990-2017)	Transfers	1.53 (impact) and 0.76 (cumulative one year)	SVAR
Gechert, Paetz and Villanueva (2018) - Germany (1974-2013)	Social Security	0.5-1.5 (impact)	SVAR
Pereira and Wemans (2013) -Portugal (1995-2011)	Social transfers in cash	Near 1 (peak); 0.6 (cumulative one year)	SVAR
Bova and Klyviene (2019) -Portugal (1995-2017)	Transfers (old age, unemployment, and disabilities transfers)	-0.27 (impact); 0.1 (cumulative)	SVAR
Silva, Carvalho and Ribeiro (2013) -Euro Area (1998-2008)	Transfers/social expenditures in cash/in kind	-0.118 (impact) and 0.82 (cumulative ten quarters) (recession scenario)	VAR
Pereira and Sagalés	Public transfers	1.88 (impact) and 1.81 (cumulative)	VAR

³ Silva, Carvalho and Ribeiro, 2013; Furceri and Zdzienicka 2012; Reeves et al, 2013; Ilzetzki, Mendoza and Vegh, 2013; Izquierdo et al, 2019; Deleidi, Iafate and Levrero, 2019.

(2009) -Portugal (1980-2005)			
Fatas and Mihov (2001) -United States	Social security, other transfers, and subsidies	A positive and significant impact of transfers on GDP, after eight quarters.	VAR
Romer and Romer (2016) -United States (1952-1991)	Social Security Benefits	Significant response of consumption (mainly impact). However, a tax reduction appears to have the highest and most persistent multiplier effect.	Narrative /VAR
Hollmayr and Kuckuck (2018) -Germany (1993-2017)	Social expenditures (pensions/unemployment)	2 (impact); between 0.3 and 3.8 (after 5 years)	SVAR

Source: Own elaboration.

As can be seen in Table IV, estimations of social benefits cumulative multipliers in Brazil are quite significant, mostly greater than the unity, comparable in magnitude to those measured for public investment - see, for instance, Sanches and Carvalho, 2022; Orair, Siqueira and Gobetti, 2016; and Resende and Pires, 2021. The literature also shows evidence that, in addition to having a high multiplier effect, the impact of social protection expenditure on GDP is more relevant during downturns (Orair, Siqueira and Gobetti, 2016; Sanches and Carvalho, 2022).

Table IV: Multiplier effects of social expenditures in the econometric literature for Brazil

Study/ period	Social Benefits multiplier (impact and peak)	Social Benefits multiplier (cumulative)	Methodology
Orair, Siqueira and Gobetti (2016) (2002-2016)	1.51 (peak) (recessions)	8 (accumulated in four years) (recessions)	STVAR (non-linear VAR)
Resende and Pires (2021) (1997-2018)	0.72 (impact)	4.37 (accumulated over 8 quarters)	VAR
Sanches and Carvalho (2022) (1997-2018)	0.75 (impact) / 1.2 (peak)	2.9 (accumulated in two years) / 2.6 (accumulated in a year)	SVAR
Neri, Vaz, and Souza (2013)	1.78 (PBF), 1.19 (BPC), 1.06 (unemployment insurance/wage allowance), 0.53 (pensions)		input-output methodology
Mostafa, Souza and Vaz (2010)	1.44 (PBF), 1.38 (BPC), 1.23 (pensions)		input-output methodology

Source: Own elaboration.

4) Methodology

As seen in the previous section, the empirical evaluation of fiscal multipliers mainly draws on the estimation of structural vector autoregression (SVAR) models developed by Blanchard and Perotti (2002) (Table III). The identification of exogenous fiscal policy shocks derives from the fact that the model assumes causation from fiscal variables to output. At the same time, there could be reverse causality through automatic stabilizers and discretionary fiscal policy responses of policymakers to output (Perotti, 2007).

Blanchard and Perotti (2002) argue that when using high-frequency data (monthly or quarterly), there is little or no fiscal policy response to unexpected shocks in output within the same month or quarter. GDP does not affect public spending contemporaneously because policymakers take more than a quarter (or a month) to perceive the output shock and decide the next steps in fiscal policy, as well as to present them to the legislature⁴. A more detailed explanation is relegated to Appendix A.

The basic model is estimated⁵ using the vector of endogenous variables, in logarithm: social protection expenditure, total primary tax revenue, GDP. Further, GDP and social spending are replaced, respectively, by other macroeconomic variables (household consumption and private investment) and by each component of social benefits. The central government revenue and expenditure data for 1997-2018 are obtained from Gobetti and Orair (2017). For Gross Domestic Product (GDP), we use data from IBGE (quarterly) and from the

⁴ See, for example, the “test of endogeneity” proposed by Deleidi, Iafrate and Levrero (2019): “[...] we use quarterly data to test whether annual government investment is exogenous by evaluating whether the rate of growth of public investment responds to the rate of growth of GDP within the year” (Deleidi, Iafrate and Levrero, 2019; p.14). Similar tests have been carried out in our study and confirmed that we could consider social benefits exogenous within the quarterly/monthly VAR framework.

⁵ The variables used in this paper are not stationary and hence we use their first difference as indicated by diagnostic tests (Dickey-Fuller, Phillips and Perron, KPSS). The number of lags is chosen based on information criteria and autocorrelation LM test (Matteo et al., 2018) (most models use two lags). All models show stability. The tests for autocorrelation and heteroscedasticity pointed to the absence of these problems in most models.

Central Bank of Brazil (BCB) (monthly interpolated estimated series for GDP). IPCA deflator, household consumption, and total investment data are also extracted from IBGE⁶. All series are in real terms and seasonally adjusted using X-13 arima in Eviews. We also added as control variables the basic interest rate (BCB), a commodity price index (IMF), and a real effective exchange rate index (BCB).

The effect of government spending on economic activity may also differ between different phases of economic cycles (Auerbach and Gorodnichenko, 2012). To examine the existence of this asymmetric effect for social protection spending, we have estimated linear VAR models for the entire sample, covering the years 1997 to 2018 (until June); and for the pre-crisis sample, from 1997 to the first quarter of 2014 (according to the CODACE report, the recession began in the second quarter of 2014). This estimation strategy allows us to evaluate possible changes in social protection multipliers during the crisis (Matheson and Pereira, 2016; Deleidi, Iafrate and Levrero, 2019). Hence, an estimation using a non-linear method (Orair, Siqueira and Gobetti, 2016; Auerbach and Gorodnichenko, 2012) would not allow a specific assessment of the behavior of multipliers during the 2015-2016 crisis.

The empirical literature generally estimates four types of multipliers (Spilimbergo, Symanski and Schindler, 2009): a) the impact multiplier for the analysis of a short-run period; b) the horizon multiplier for a specific period; c) the peak multiplier, which represents the highest value in the period under analysis; d) the accumulated/cumulative multiplier for the analysis of a more extended period. The cumulative multiplier is the most appropriate measure since the economy requires some time to absorb the initial shock (Ilzetki, Mendoza and Vegh, 2013; Ramey and Zubairy, 2018; Restrepo, 2020). Appendix B shows the fiscal multiplier equations.

⁶ Private investment data are obtained by subtracting public investment (from Orair and Gobetti, 2017) from total investment (IBGE). Investment is available from 1997 to 2017.

5) Results

5.1) *The role of economic crisis (aggregated exercise)*

We now estimate the response of GDP to an expansionary shock in social benefits. The following graphs (Figure II and Figure III) display the accumulated response of output to a standard-deviation shock in the total social spending item, using a confidence interval of one and two standard-deviations⁷. Impulse-response functions are significant at 95% at most periods after the initial shock.

As Figures II and III illustrate, the impact of social protection expenditure on GDP in the entire sample turns out to be higher (black line). In particular, the impulse-response functions of the two samples are statistically different from each other when considering a band of one-standard-deviation (Figure III). We have tested several specifications, such as dummy time variables, data in different frequencies (monthly and quarterly), and other deflators. The differences between the two samples do not change⁸.

Table V summarizes the estimated multipliers (impact and cumulative during eight quarters)⁹. Looking at the results, we note there is little difference when comparing the impact multipliers of the two samples. Regardless, a social protection expenditure shock triggers a more prominent output increase in the full sample in accumulated terms, during a period of time.

⁷ For instance, Blanchard and Perotti (2002), Fatàs and Mihov (2001), Tenhofen, Wolff and Heppke-Falk (2010), Perotti (2007), Burriel et al (2010), Izquierdo et al (2019) adopt a one standard-deviation band. Ramey (2011) (p.11-12) claims: "*Although this is a common practice in the government spending literature, it has no theoretical justification*".

⁸ The baseline specification used time dummy variables: 1) dumdate99: assumes value from 1999 Q1, onwards (government's budgetary dynamics have changed); 2) dum08 assumes 1 in 2008Q3-Q4 and 2009Q1 (international crisis), 3) dum09, introduced to capture the post-crisis recovery period, assumes the value 1 in 2009Q3-Q4; 4) dum10 and dum67 (capture outliers in 2010 and 2006/2007). Besides showing at least 10% statistical significance, the inclusion of time dummy variables improved the general model significance. The removal of these variables only changed results marginally.

⁹ The cumulative multiplier is estimated considering its persistence: "*The long-run multiplier is defined as the cumulative multiplier when $J \rightarrow \infty$, but in practice is used the number of periods needed for the multiplier to stabilize at its long-run value*" (Garcia, Lemus and Mrkaic, 2013, p.11).

Overall, our findings align with the range of estimates of short-term multipliers for total government spending on social protection presented in previous empirical studies (Table IV). In particular, our estimated effect of total social benefits on GDP is similar to the evidence provided by Resende and Pires (2021) for Brazil. Their VAR estimations suggest an impact multiplier for total social benefits of 0.75 and an accumulated multiplier effect over eight quarters of 4.37. Our results are also in line with Orair, Siqueira and Gobetti (2016), who estimate more significant social benefits multipliers during economic recessions.

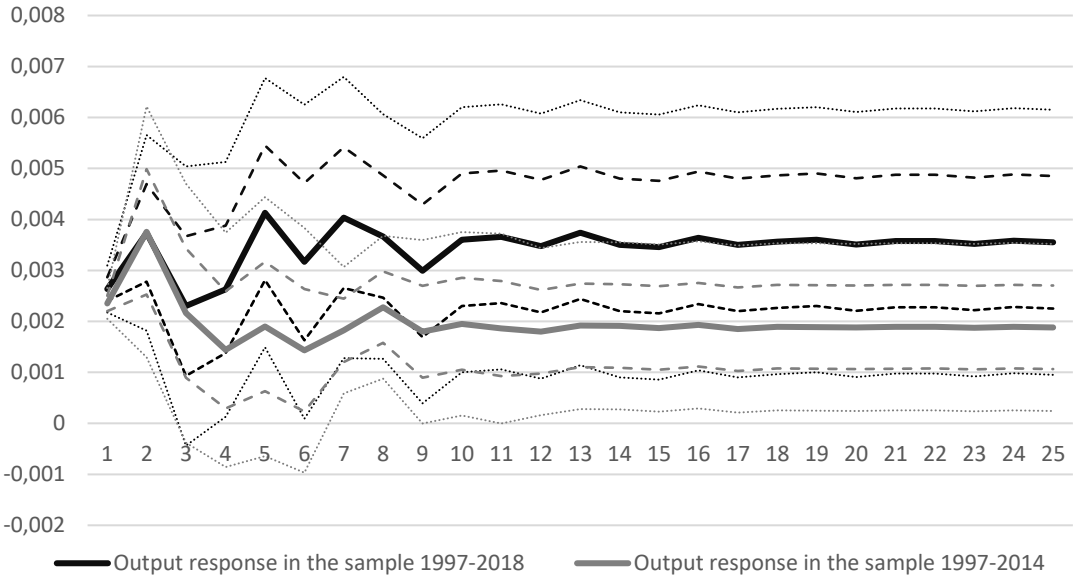
We go beyond Resende and Pires (2021) and Orair et al (2016) by: i) shedding light on the change in the multiplier in the 2014-16 deep economic crisis; ii) disaggregating the different types of social benefits (e.g, pensions, different cash transfer programs, unemployment insurance), and (iii) disaggregating how different components of aggregate demand (household consumption and private investment) respond to different types of social benefits. Our study emphasizes that social protection expenditure performed a significant income stabilizing effect in aggregate demand during this period. As estimated by Sanches and Carvalho (2022), GDP would be 2.53% lower if social benefits had not continued to grow in 2016 and 2017 due to constitutional obligations.

Table V: Social benefits multipliers in the two samples (impact and cumulative in two years)

Sample/Exercise	Monthly exercise-Figure II	Quarterly exercise-Figure III
1997-2018 sample	0.77 (impact) / 2.9 (cumulative)	1.3 (impact) / 4.5 (cumulative)
1997-2014 sample	0.7 (impact) / 1.9 (cumulative)	1.3 (impact) / 3.1 (cumulative)

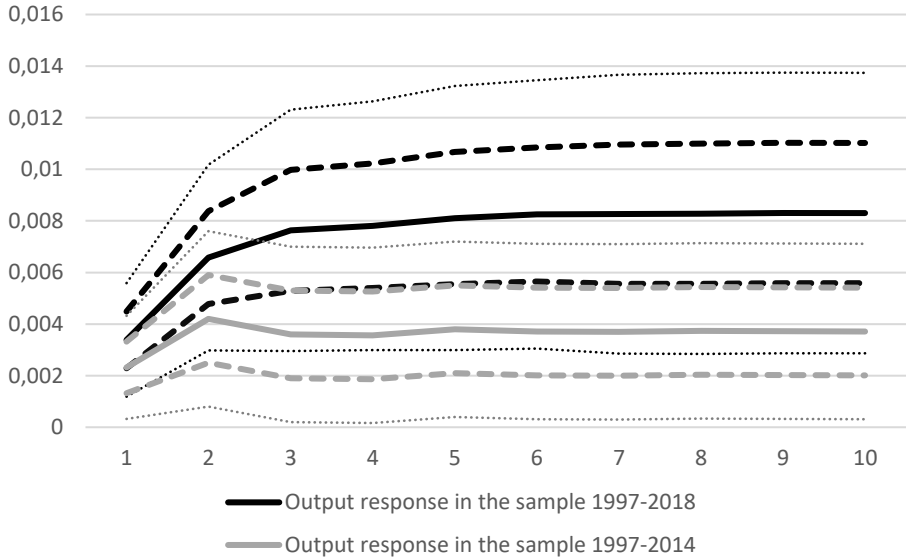
Source: Own elaboration.

Figure II: Accumulated response of output to a shock in social benefits (using monthly data)



Source: Own elaboration. The dashed and dotted lines correspond to the interval of one and two standard-deviations, respectively, namely confidence levels of 68 and 95%.

Figure III: Accumulated response of output to a shock in social benefits (using quarterly data)



Source: Own elaboration. The dashed and dotted lines correspond to the interval of one and two standard-deviations, respectively, namely confidence levels of 68 and 95%.

The cumulative multiplier of 2.9, for example, suggests that spending one unit on social expenditures generates a final change in GDP of almost three after two years. In a simple Keynesian model, the multiplier depends directly on the marginal propensity to consume. Since social benefits tend to be received by households with a higher propensity to consume, these expenditures may boost consumption and raise sales expectations by firms and business investment. Furthermore, our findings suggest that this multiplier effect is more relevant in the sample that includes the recent economic recession. Recipients may consume an even higher share of social benefits when economic activity is weak, in a scenario of falling income.

In other words, social protection multiplier dynamics can be enhanced since people who receive these benefits tend to have a relatively high propensity to consume. In fact, according to the Pesquisa de Orçamento Familiar (POF) data (2017/2018), households from the first income bracket who receive up to R\$ 1.908 monthly have an average propensity to consume larger than one, while the top income bracket with income above R\$ 23.850 has a propensity to consume of 0.43. Moreover, as reported by Ipea (2011), 80 percent of resources transferred by the PBF go to the bottom 40 percent of the distribution. Several studies emphasize the effective targeting of social programs in Brazil to the bottom of the income distribution (Medeiros, Britto and Soares, 2008; Soares, Ribas and Osório, 2007; Denes, Menezes-Filho and Komatsu, 2016; Souza, 2011). For instance, Medeiros, Britto and Soares (2008) estimate that more than 80 percent of the beneficiaries of PBF and BPC programs are in the first four-tenths of the per capita household income distribution.

We have introduced some control variables in the econometric exercises. Table VI summarizes the results when including control variables (quarterly data exercise). Including the interest rate and the exchange rate is standard in the empirical literature on fiscal multipliers (Ilzetzki, 2011; Perotti, 2004; Tenhofen, Wolff and Heppke-Falk, 2010). We also

have controlled for a commodities prices index given that the Brazilian economy is mainly dependent on commodity exports¹⁰. Table VI shows the impact, peak and cumulative multipliers in eight quarters. The period when the peak multiplier occurs is indicated by “t”.

Table VI: Social benefits multipliers– impact, peak and cumulative, respectively

Exercise/Sample	Multipliers complete sample	Multipliers pre-crisis sample
Baseline	1.3, 3.25 (t=7), 4.5	1.3, 2.4 (t=2), 3.1
Controlling for commodities price index	1.1, 2.4 (t=3), 3.5	1, 1.86 (t=2), 2.5
Controlling for interest rate	1.2, 3 (t=8), 4.2	1.2, 2.1 (t=2), 2.78
Controlling for exchange rate	1.19, 3.4 (t=6), 4.4	1.3, 2.5 (t=6), 3.6
Using the three controls	1.1, 2.5 (t=3), 3.6	1.1, 2 (t=3), 2.8

Source: Own elaboration.

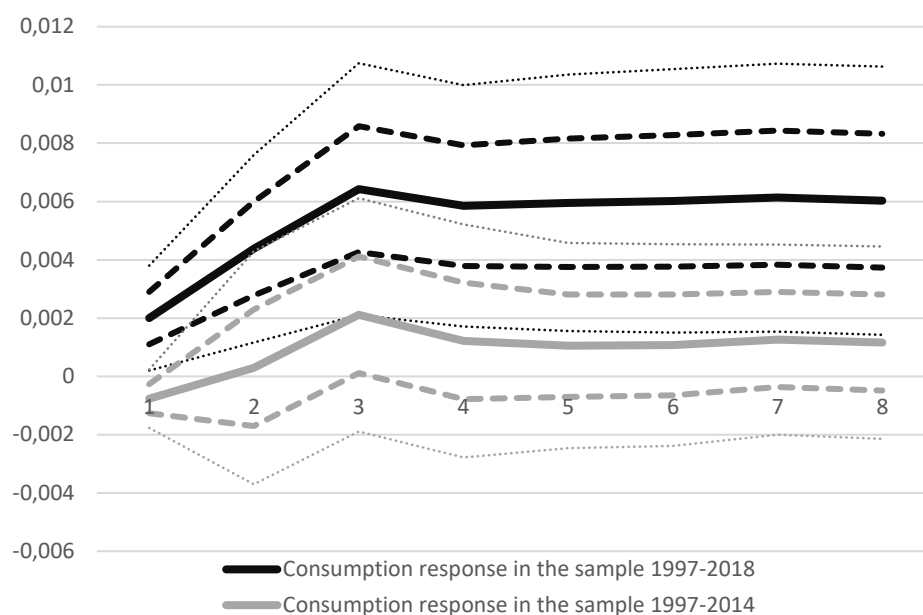
5.2) The effect of social protection expenditures on household consumption and private investment

Given the absence of previous empirical studies focusing on disaggregating the components of demand, we estimate fiscal multipliers for household consumption and private investment in this section. Figure IV considers the accumulated response of household consumption to a shock in social protection expenditure, statistically significant at 95% (complete sample). On the other hand, the impulse-response function estimated using the smaller sample is not statistically significant at 68%.

We find that, when the government spends one unit on social expenditure, it increases household consumption by 2.3 units after two years (baseline exercise). The estimated cumulative multiplier using the pre-crisis sample is 0.54 after two years (Table VII). It means that the multiplier in the full sample is 325% greater.

¹⁰ See Carvalho (2018).

Figure IV: Accumulated response of household consumption to a shock in social benefits



Source: Own elaboration. The dashed and dotted lines correspond to the interval of one and two standard-deviations, respectively, namely confidence levels of 68 and 95%.

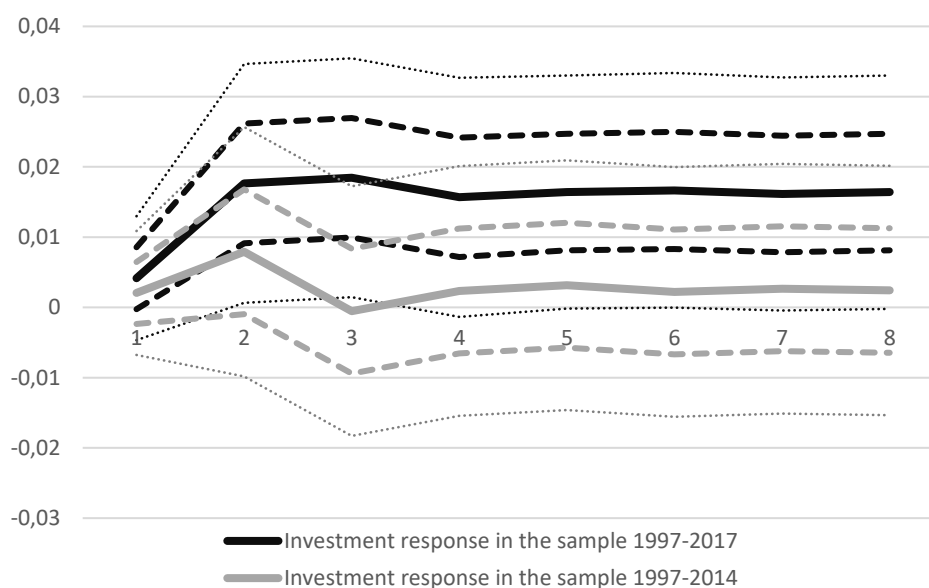
Table VII: Social benefits multipliers (household consumption) – impact, peak and cumulative, respectively

Exercise/Sample	Multipliers complete sample	Multipliers pre-crisis sample
Baseline	0.5, 1.6 (t=3), 2.3	-0.27, 0.75 (t=3), 0.54
Controlling for commodities price index	0.41, 1.22 (t=3), 1.7	-0.5, 0.26 (t=3), 0.08
Controlling for interest rate	0.32, 1.3 (t=3), 1.8	-0.41, 0.45 (t=3), 0.18
Controlling for exchange rate	0.49, 1.6 (t=3), 2.25	-0.26, 0.78 (t=3), 0.57
Using the three controls	0.25, 1 (t=3), 1.4	-0.5, 0.003 (t=3), -0.4

Source: Own elaboration.

Finally, we estimate the response of private investment to a shock in social benefits. As shown in Figure V, private investment is also more responsive in the sample that includes the crisis. Albeit this discrepancy is less significant than in the case of household consumption (Figure IV), the cumulative multiplier estimated for the whole sample is still larger by 1.13 (Table VIII).

Figure V: Accumulated response of private investment to shocks in social benefits



Source: Own elaboration. The dashed and dotted lines correspond to the interval of one and two standard-deviations, respectively, namely confidence levels of 68 and 95%.

Table VIII: Social benefits multipliers (private investment) – impact, peak and cumulative, respectively

Exercise/Sample	Multipliers complete sample	Multipliers pre-crisis sample
Baseline	0.27, 1.22 (t=3), 1.58	0.2, 0.79 (t=2), 0.45
Controlling for commodities price index	0.03, 0.95 (t=3), 1	-0.28, 0.2 (t=2), 0.1
Controlling for interest rate	0.41, 1.66 (t=3), 1.91	0.22, 0.9 (t=2), 1.06
Controlling for exchange rate	0.007, 1.46 (t=3), 1.55	-0.08, 1.15 (t=2), 1.05
Using the three controls	0.2, 1.27 (t=3), 1.4	0.04, 0.7 (t=2), 0.75

Source: Own elaboration.

Overall, expansionary changes in social protection seem to trigger a more robust household consumption response in the full sample than in the pre-crisis sample (the difference is statistically significant - see Figure IV). Regarding the impact of social protection on private investment, the multiplier is 1.13 greater (or 251%) in the sample that includes the 2015-16 recession. Hence, we find that increases in social benefits lead to

significant increases in household consumption, which might drive higher sales expectations by firms and therefore foster the private investment.

Finally, the difference between the samples in terms of multiplier effect is robust to the inclusion of control variables. This evidence reinforces that social protection expenditures played a crucial stabilizing role during the crisis.

5.3) Disaggregation of social benefits

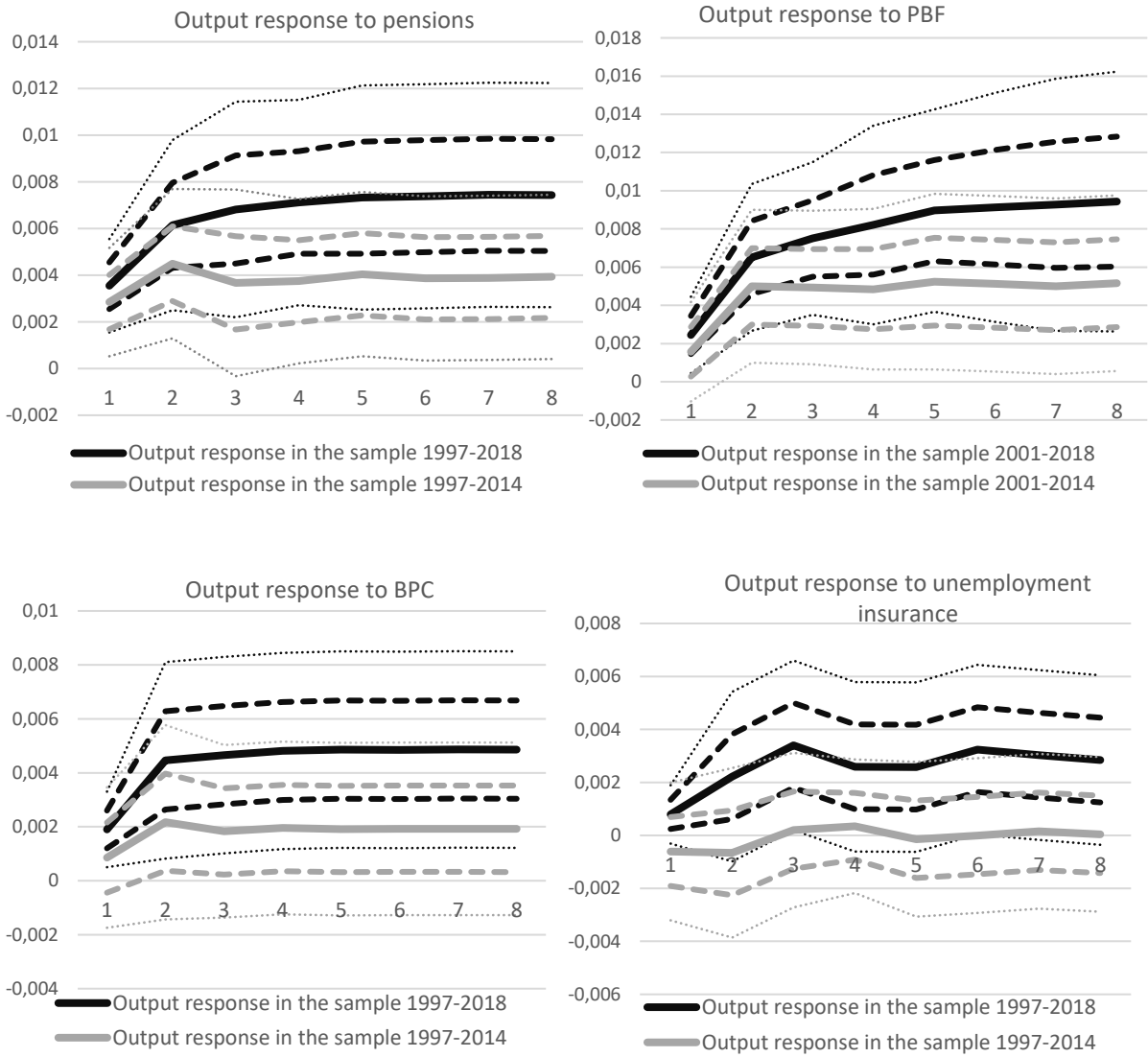
In this section, we disaggregate social benefits expenditures into pensions (social security), Programa Bolsa Família (PBF), Benefício de Prestação Continuada (BPC), and unemployment insurance (alongside wage allowances). The GDP responses to each category (Figure VI) are similar to those observed in the aggregate estimation (Figure III). Looking at sub-components of social benefits expenditure emphasizes that all of them - to a greater or a lesser extent - contribute to the estimated discrepancy in the multiplier effect between the two samples.

Indeed, other studies find a more significant effect of PBF on GDP (Tupy and Toyoshima, 2013; Neri, Vaz and Souza, 2013). In other cases, BPC has the largest multiplier effect (Denes, Menezes-Filho and Komatsu, 2016). Our findings align with Neri, Vaz and Souza (2013), who estimate relevant multiplier effects of PBF, BPC and unemployment insurance. In our study, we also find a significant impact on GDP of pensions.

Figure VII illustrates the household consumption response to a shock in each type of social protection expenditure. It is noteworthy that the discrepancy between the samples is more significant for pensions and PBF. For BPC, on the other hand, it is smaller and less significant. Unlike household consumption, the response of private investment to a shock in BPC is the component that presents the most significant difference between the two samples (Figure VIII). As BPC is established in the Federal Constitution and therefore is not subject to

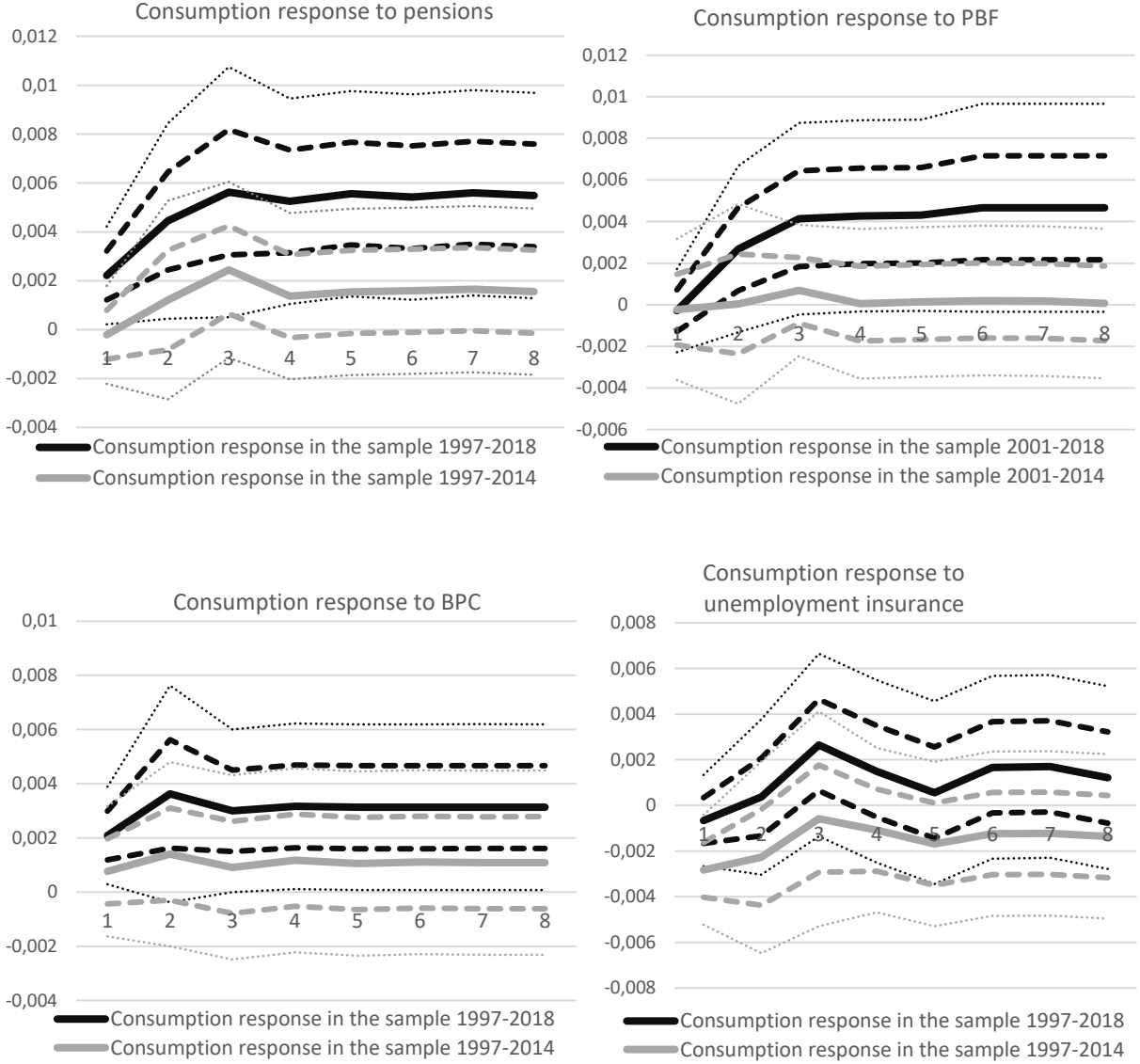
abrupt cuts during fiscal consolidation episodes, it might be more able to stimulate the private investment, for example.

Figure VI: Accumulated response of output to a shock in social benefits components



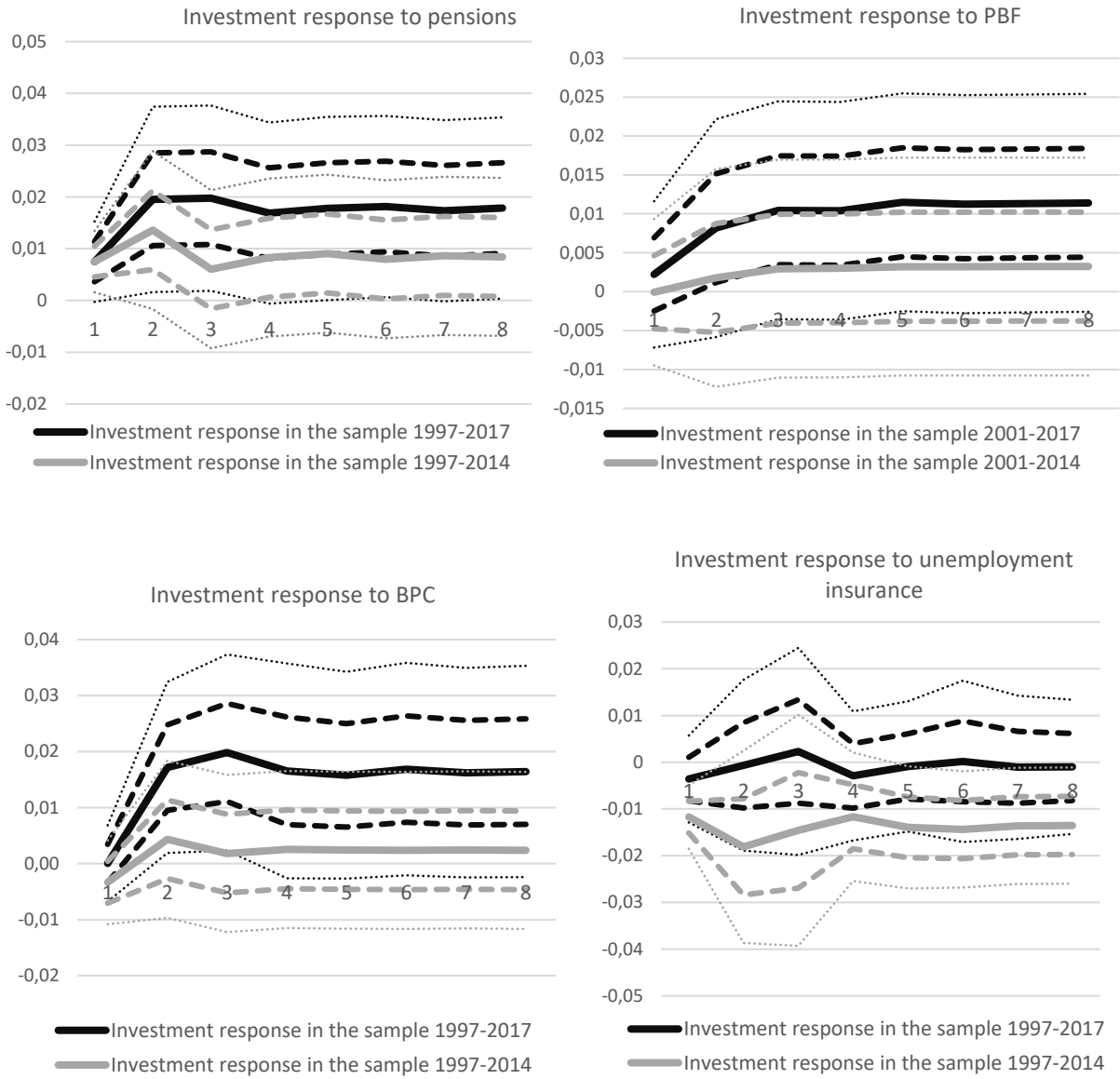
Source: Own elaboration. The dashed and dotted lines correspond to the interval of one and two standard-deviations, respectively, namely confidence levels of 68 and 95%.

Figure VII: Accumulated response of household consumption to a shock in social benefits components



Source: Own elaboration. The dashed and dotted lines correspond to the interval of one and two standard-deviations, respectively, namely confidence levels of 68 and 95%.

Figure VIII: Accumulated response of private investment to a shock in social benefits components



Source: Own elaboration. The dashed and dotted lines correspond to the interval of one and two standard-deviations, respectively, namely confidence levels of 68 and 95%.

6) Concluding remarks

Based on Blanchard and Perotti (2002)'s Structural VAR approach, this paper has analyzed the short-term impact of social spending on economic activity in Brazil for 1997-2018. Our results suggest social benefits have relatively large multiplier effects in Brazil,

comparable in size to those estimated for public investment. In this sense, our findings show the importance of considering social expenditure while applying the Blanchard and Perotti (2002)'s VAR approach to assess multiplier effects of the fiscal policy.

While the high and persistent multipliers of public investment are well-established in the previous literature (Pires, 2011; Pires, 2014; Orair, Siqueira and Gobetti, 2016; Orair, 2016; Resende and Pires, 2021), the result for social benefits had only appeared in the studies carried out by Orair, Siqueira and Gobetti (2016) and by Resende and Pires (2021). Unlike these authors, however, we provide additional evidence on the macroeconomic impact of social protection by focusing on the change in the multiplier effects during the 2014-16 Brazilian deep economic crisis as well as by disaggregating different types of social expenditure and the response of different aggregate demand components in the estimations.

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Appendix A

The vector of endogenous variables is three-dimensional with the series of social expenditure, revenue, and output. According to Perotti (2007), shocks of the reduced form can be seen as linear combinations of three components: a) the automatic response of government spending and revenue to changes in output; b) the discretionary response due to changes in endogenous variables (Perotti (2007) gives the example of tax changes in response to a recession); c) random discretionary shocks: structural shocks, which are uncorrelated and unobservable. So that:

$$u_t^g = \alpha_{gy}u_t^y + \beta_{gt}e_t^t + e_t^g \quad (1A)$$

$$u_t^t = \alpha_{ty}u_t^y + \beta_{tg}e_t^g + e_t^t \quad (2A)$$

$$u_t^y = \gamma_{yt}u_t^t + \gamma_{yg}u_t^g + e_t^y \quad (3A)$$

Where u_t^g, u_t^t, u_t^y are the unexpected movements in the expenditure, revenue taxes and output variables, respectively. Also, e_t^g, e_t^t, e_t^y are the structural shocks. The coefficients α_{ij} reflect the response of variable i to variable j - the components "a" and "b" listed above are captured by the coefficients α . While β_{ij} measures the contemporaneous response of variable i to a structural shock in variable j - that is, the component "c" (Perotti, 2007).

By using high-frequency data, component "b" is removed, which makes the coefficients reflect only the response of the automatic stabilizer: "*it typically takes longer than a quarter for discretionary fiscal policy to respond to, say, an output shock*" (Perotti, 2007, p.176). In other words, given the identification hypothesis, there is no discretionary response of fiscal variables to output, so that¹¹:

$$\alpha_{gy} = 0 \quad (4A)$$

¹¹ Some studies – such as Galton (2013), Adams and Wong (2018), Pereira and Wemans (2013), Baum and Koester (2011), Hollmayr and Kuckuck (2018) - estimate the SVAR considering unemployment insurance as an automatic stabilizer expenditure. They estimate the weighted elasticity of this type of expenditure relative to output. By making this adjustment, however, our results only change marginally.

The elasticity of revenue to output α_{ty} is estimated based on the “IMF method”, as in Andreis (2014), which is a regression of tax revenues on GDP using time dummy variables and a trend control¹². We can capture the cyclically adjusted residuals $u_t^{g,ca}$ and $u_t^{t,ca}$, which are the shocks without the effects of the cycle, to eliminate the automatic stabilizer responses:

$$u_t^{g,ca} = u_t^g - \alpha_{gy}u_t^y = \beta_{gt}e_t^t + e_t^g \quad (5A)$$

$$u_t^{t,ca} = u_t^t - \alpha_{ty}u_t^y = \beta_{tg}e_t^g + e_t^t \quad (6A)$$

Blanchard and Perotti (2002) claim that there is no reason to choose $\beta_{gt} = 0$ or $\beta_{tg} = 0$. After a shock in spending and revenue, there is no theoretical or empirical justification to sustain which variables will react first. However, as the correlation between adjusted residuals is small¹³, Perotti (2007) points out the order does not change the result. By using $\beta_{gt} = 0$ and estimating equation 6A (Burriel et al., 2010)¹⁴, it is possible to obtain estimations for e_t^t and e_t^g (“isolated” from the influence of output since the automatic response component has been removed). Having obtained estimations for the structural shocks, we use them as instruments for the reduced form shocks in the estimation of equation 3A. Structural shocks - which refer to the contemporaneous effects - of spending and revenue are used as instruments because their correlation is low with the structural output shock. The last step is estimating the impulse-response functions of the SVAR model.

¹² We obtained the following estimations of the elasticity of primary revenue to GDP, household consumption, and private investment, respectively: 1.5; 1.4; 0.3 (full sample) and 1.3; 0.96 and 0.2 (pre-crisis sample).

¹³ In most of the models estimated in this work, this correlation is generally smaller than 0.1 (or close to), in modulus.

¹⁴ Models have also been estimated assuming $\beta_{tg} = 0$ and proved to be robust, as is usual in the literature.

Appendix B

To calculate multipliers, we divide the elasticity of GDP to the social benefits variable by the average share of social expenditures in the output (or components). As the variables are in logarithms, impulse-response functions provide the elasticity of the output (Y) related to the fiscal variable that suffered a shock (X):

$$\varepsilon_{Y,X} = \frac{\frac{\Delta Y}{Y}}{\frac{\Delta X}{X}} = \frac{\Delta Y}{Y} \frac{X}{\Delta X} = \frac{\Delta Y}{\Delta X} \frac{X}{Y} \quad (1B)$$

Since $\frac{\Delta Y}{\Delta X}$ is the definition of multiplier (Pires, 2014), we have:

$$\frac{\Delta Y}{\Delta X} = \frac{\varepsilon_{Y,X} \frac{X}{Y}}{1} \quad (2B)$$

The following equations (3B, 4B, 5B and 6B) show the definition of each type of multiplier, respectively: impact, horizon, peak and accumulated/cumulative:

$$\frac{\Delta Y(t)}{\Delta G(t)} \quad (3B)$$

$$\frac{\Delta Y(t+n)}{\Delta G(t)} \quad (4B)$$

$$\max \frac{\Delta Y(t+n)}{\Delta G(t)} \quad (5B)$$

$$\frac{\sum_{i=1}^n \Delta Y(t+i)}{\sum_{i=1}^n \Delta G(t+i)} \quad (6B)$$