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1. Introduction

The main role of commercial banks is to work as financial intermediaries between depositors and borrowers. One of the reasons for the existence of commercial banks is the presence of asymmetric information, which is the fact that the borrower has more knowledge about his own situation than the lender. In the presence of asymmetric information, it pays the lender to monitor the borrower. However, the monitoring cost may be high for any particular borrower. When many agents perform this monitoring activity, they may find worthwhile to delegate it to a specialized entity to save on monitoring costs. This is one possible theory explaining the origins of banks, according to Douglas W Diamond 1984.

In a fractional reserve bank system, banks are subject to runs. If a significant amount of depositors decides to withdraw their resources from the bank, the bank will run out of its reserves, which may trigger liquidity or solvency problems.

We define bank runs as withdrawals over and above the expected demand for liquidity. When this happens, bank insolvencies may arise. Such withdrawals may occur due to random shocks (Douglas W. Diamond and Dybvig 1983) or may be a result of depositors' perceptions that the bank is facing some difficulties (Calomiris and Mason 2003).

Apart from its historical lessons, the study of bank runs is relevant because banks with good fundamentals may go bust due to a panic crisis triggered by bank runs. The social cost of bank failures may be relevant and policymakers may benefit from a better understanding of how bank runs work.

In this paper, we simulate a bank run triggered from depositors' strategic decisions in a coordination game based on Douglas W. Diamond and Dybvig (1983).

Our model is part of the literature on complex adaptive systems, where agents react to the environment through signals and its internal rules. They have memory and can choose which rule provides a better response, so agents adapt in such a way that they optimize their utility functions in the long term. For details about complex adaptive systems, see Holland (2014).

The original Diamond-Dybvig economy lasts for three periods. We embed this economy in a dynamic simulation so that the three periods of this model repeat in cycles.

Each agent uses data from his memory to estimate what might happen and act to maximize its returns. In addition, banks arise endogenously in the model; any agent can become a banker if proper conditions arise.

Our work is an extension of Grasselli and Ismail (2013). Our paper's main contribution is the consideration of neighborhood influence on the patient agent decision as well as the sequential service constraint. Such conditions lead to a long-term stability, with a high bank concentration. Grasselli and Ismail (2013) also arrive at the same result that there are few established banks in the long run, but they did not measure the number of bank runs in each cycle.

Our most important result is that the number of bank runs decreases with the size of the banks as measured by the number of clients. We do not impose any deposit insurance. The only elements we have in our model are the sequential service constraint and the agent's punishment when he does not receive the amount promised by the bank and decides not to be a customer anymore.

There is a long lasting debate about possible trade-offs between financial stability and bank competition. Our results indicate that such trade-offs are indeed important. The price to pay for a more stable banking sector may be to have a less competitive one.

The structure of the paper is as follows. In section 2, we review the literature. Section 3 presents the methodology, section 4 shows the results and section 5 concludes the paper.

2. Literature

Douglas W. Diamond and Dybvig (1983) model a bank as a mechanism that allows investors to finance illiquid but profitable projects, protecting them from unforeseen shocks that result in anticipated consumption. There are two types of agents, patient and impatient and there are three periods. In the initial period, zero, agents do not know their type and deposit their endowment of one unit of currency in the bank. In period one, the agent learns his type through a random draw. Impatient agents do not derive utility from period two and therefore decide to withdraw in period one. Patient agents derive utility at both periods one and two and therefore they decide whether to withdraw in period one or in period two. Those agents who decide to withdraw in period one receive an amount

$c_1 \geq 1$. In period two, the illiquid asset return is R, such that $R \geq c_1$. However, patient agents receive only $c_2 \leq R$. The decision of patient agents whether to withdraw in period one or two depends on the comparison of c_1 and c_2. If $R \geq c_2 > c_1$ these depositors wait to withdraw in the second period, but if $R \geq c_1 > c_2$ they withdraw in period one. In addition, the bank does not know the type of each depositor. One possible coordination game for patient agents can be seen in table 1.

Table 1: *Example of coordination game*

	Wait	Withdraw
Wait	R, R	c_2, c_1
Withdraw	c_1, c_2	$0, 0$

The row player of table 1 is a patient depositor and the column represents all other agents of this type. Therefore, if everyone waits, they may withdraw R. If he waits and the others withdraw, he only receives c_2. If he withdraws in the first period and the others expect for their reward he receives c_1. However, if all of them withdraw, there may be no amount to receive.

Douglas W. Diamond and Dybvig (1983) conclude that there may be bank runs even for banks with sound finances. The authors study how to pay the depositors and they come up with the idea of sequential service constraint: the depositors withdraw sequentially until the bank reserves run out. In addition, they propose that deposit insurance mechanisms inhibit bank runs and the suspension of convertibility leaves agents with liquidity needs without money.

2.1. Sequential Service Constraint

According to Douglas W. Diamond and Dybvig (1983), the demand deposit contract satisfies sequential service constraint when the amount owed to the depositor depends on his position in the queue and is independent of the state of the agents who are after him in line. Therefore, if many of the patient agents decide to anticipate the service, the bank will serve depositors until its cash is exhausted, leaving the agents that are behind the last to receive with nothing. Some authors show that such a measure guarantees the payment promised to patient depositors in the last period, however, it might happen that people in need of liquidity run out of cash. That is, the solution does not optimize utility. Kelly and

Ó-Gráda (2000) and Ó-Gráda and White (2003) show that the 1854 bank run in New York was triggered by some depositors' fear of not receiving the promised amount by the bank for arriving too late in a possible run.

Sequential services constraint is an assumption often adopted in the bank run literature. Wallace (1988) points out that the hypothesis that people do not communicate in period 1 implies the sequential services constraint. The result is that the returns on early withdrawals depend on the random order of withdraws. Calomiris and Kahn (1991) treat sequential services constraint on a theoretical model, in which bankers can divert customer resources away, and propose a contract to avoid such a situation. Romero (2009) makes explicit this restriction in his simulations. It works like the following: only one agent, chosen at random, decides to withdraw or not, imitating the formation of a queue.

Does sequential service constraint always involve bank run? Green and Lin (2000) say that the answer to this question is no. They construct a theoretical environment in which there is no bank run, even with the sequential service constraint. In order to achieve this equilibrium, depositors would be encouraged to tell the truth. That is, there is not asymmetric information featuring a coordination game. Peck and Shell (2003) criticize the work of Green and Lin (2000) because bank runs are historical facts. To explain the reason for the existence of this phenomenon, they relax two hypotheses of Green and Lin (2000) model. The first is to allow each depositor to have different utility. The second hypothesis is the elimination of the knowledge that the agent would have about their position in the queue to withdraw. Peck and Shell (2003) then conclude that in these cases there is the possibility of bank runs.

The assumption of this restriction may be implicit. The experimental work of Garratt and Keister (2009) does not impose the sequential service constraint to participants. If a bank fails, it splits the available amount of cash among depositors. However, if this restriction is placed, the expected value that a depositor receives is the same as was imposed by Garratt and Keister (2009). Deng, Yu, and Li (2010)'s simulation, on the other hand, makes this constraint implicit on the decision of patient depositors to withdraw or not in the first period.

2.2. Avoiding Bank Runs

How do avoid bank runs? Douglas W. Diamond and Dybvig (1983) exploit the suspension of convertibility and deposit insurance as alternatives. First option, we do not allow anyone in the line withdraws in the first period after reaching a pre-established ratio of depositors.

Under the assumption of sequential service constraint, the authors show that when the proportion of impatient agents is random, bank contracts fail to achieve optimal risk sharing, i.e., to serve all impatient in the first period and all patients in the second one. The use of convertibility suspension in the 1857 bank run is cited by Kelly and Ó-Gráda (2000) and by Ó-Gráda and White (2003). Ennis and Keister (2009) focus on policies that are *ex post* efficient, once the run is underway. The authors show how the anticipation of such intervention can create necessary conditions for a self-fulfilling run to take place in the paradigm of Douglas W. Diamond and Dybvig (1983).

Douglas W. Diamond and Dybvig (1983) propose that contracts of deposit insurance provided by government achieve a unique Nash equilibrium if the ruler imposes an optimal rate to fund the deposit insurance. Douglas W. Diamond and Dybvig (1983) 's statement is contested by Wallace (1988), who concluded that the deposit insurance proposed by Douglas W. Diamond and Dybvig (1983) is not feasible, but he leaves open the feasibility of other arrangements. Another drawback, according to Ennis and Keister (2009), is that it is not always feasible for the government to guarantee payment of the full amount of deposits in the advent of a widespread run. Calomiris and Kahn (1991) argue that the bank run is a disciplinary mechanism of the market, because if depositors realize that the bank is diverting money, they withdraw and may start a bank run. Therefore, deposit insurance encourages moral hazard, because depositors can invest in banks taking more risks. Chari and Jagannathan (1988) consider a similar Douglas W. Diamond and Dybvig (1983) model, but introduces a random return on investment and some patient type agents can observe it. If the signal that agents receive indicates poor performance, it induces them to wish to withdraw in period 1.

2.3. Social Network Influence

The depositor's social network influences decision-making. Kelly and Ó-Gráda (2000) discover that during the panics of 1854 and 1857 in New York, the social network in which these depositors belonged was the most active factor in the withdrawal decision. Hong, Kubik, and Stein (2004) point out that Kelly and Ó-Gráda (2000) did not consider "antisocial" agents. Hong, Kubik, and Stein (2004) develop a model for the stock purchase decision and conclude that sociable people are more susceptible to invest in stock markets compared with those who are "antisocial".

Complexity literature makes the hypothesis that social networking matters for bank runs. For example, the simulations of Romero (2009), Deng, Yu, and Li (2010), and Grasselli and Ismail (2013) take into account the influence of the depositor's network of contacts in their decision-making. According to Deng, Yu, and Li (2010), a bank run may occur only by imitation among depositors, even in the absence of exogenous shock.

3. Methodology

We choose Grasselli and Ismail (2013)'s model as our base model. We briefly present this model in the first subsection. We extend the model through the inclusion of the influence of social network and of the sequential service constraint.

3.1. Grasselli and Ismail's Model

There are three periods, as in Douglas W. Diamond and Dybvig (1983) model. In the first one, without banks, each agent i has a random preference parameter from a uniform distribution U_i in $[0, 1]$. Agents with $U_i \leq 1/2$ are the impatient ones, whereas those with $U_i > 1/2$ are the patient ones. In addition to this preference parameter, the agent has the choice of receiving a monetary unit in the second period, if he decides to invest in the liquid asset. In the case of investing in the illiquid asset, he may receive $r < 1$ if he withdraws in the second period or $R > 1$ if he waits until the third period. Let $\bar{U}_i$ be the actual endowment in the first period. There is then an exogenous liquidity preference shock such that his preference in the second period becomes:

$$\rho_i = \bar{U}_i + (-1)^{b_i} \frac{\epsilon_i}{2}$$

where b_i is a random variable with Bernoulli distribution taking values in the set $\{0.1\}$, and $\epsilon_i \in [0, 1]$ has uniform distribution. In other words, agents can change their liquidity preference depending on the size of the shock in $\bar{U}_i$. As before, agents are impatient if $\rho_i \leq 1/2$, and patient otherwise.

The model allows for cycles repeated several times. So, given a cycle k, the draw to determine the preference of each agent and consequently the assets in which he invests is done in period $k(0)$; in period $k(1)$ the preference shock occurs as well as the search for partners to trade, and in period $k(2)$ agents with illiquid assets receive R.

A bank is defined as an agent who owns a contract that pays c_1 in the second period, such that $1 < c_1 < R$, and pays c_2 in the third period, with $c_1 < c_2 < R$. When there is a bank, agents may or may not adhere to this contract according to the rules described later.

This structure allows for memory and learning. Based on his predictions, the agents decide to become or not customers of a bank.

The transition rules for agents are as follows:

> Bargain $B_{U,v}$: agents, who have a positive preference shock, look for a neighbor that was hit by a negative preference shock in order to trade assets.

In this rule, after the preference shock, U, those who have liquid assets, and now wants to wait to receive R can trade with someone in his social network v who has illiquid asset and needs money immediately. This change improves the situation to both; however, it is not always possible to find a partner in his social network, v.

> Become a bank $BK_{c_1,c_2,R}$: the agent estimates the proportion of impatient agents in his neighborhood. Given the parameters c_1, c_2 and R, the agent decides whether to open or not a bank.

By assumption, the first candidates to become bank clients are the bank's eight neighbors and himself, see figure 1, so there are nine potential depositors. The decision to become or not a bank depends on an unknown proportion of impatient agents; for this reason, it is necessary to estimate this number through a random variable w such that:

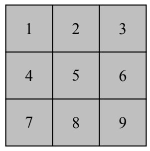

Figure 1: *The number five represents an agent and the others numbers are its eight neighbors in the lattice.*

$$w \in \left\{0, \frac{1}{9}, \frac{2}{9}, ..., \frac{9}{9}\right\}$$

The estimated amount to pay to each impatient customer in the second period, $y_i = w_i c_1$, and to each patient customer in the third period, $Rx_i = (1 - w_i)\, c_2$, must be less than or equal to one for otherwise it is not worth establishing himself as a banker. Then, $y_i + x_i \leq 1$. Manipulating theses expressions, it follows:

$$w_i \leq \frac{R - c_2}{Rc_1 - c_2}$$

We can write the per capita present value amount that the bank has to provide for paying customers in the second and third periods as:

$$f\,(w) = c_1 w + \frac{c_2\,(1 - w)}{R}$$

In cycle k, bank j updates the estimate of w by the following Exponential Moving Average (EMA) formula:

$$w_k^j = w_{k-1}^j + \alpha\left(\bar{w}_k^j - w_{k-1}^j\right)$$

with $\alpha \in [0, 1]$ and $\bar{w}_k^j$ is the realization of the impatient clients proportion at period $k(1)$.

If $w \in [0, 1]$ then $f\,(w) \in [c_2/R, c_1]$. According to Bolzano's theorem, if a function f is continuous in a closed interval $[a, b]$ then $\forall S \in (f\,(a)\,, f\,(b)), \exists c \in (a, b) :$ $f\,(c) = S$. That is, for each value $Q \in (1, c_1)$ there exists $\omega \in [w^*, 1]$ such that f $(\omega) = Q$, where $f\,(w^*) = 1$. In other words, there are realizations of w which discourage the creation of a bank or, if it already exists, the amount collected from customers in a cycle may not be sufficient to honor the contract.

Figure 2 shows the graph of the function $f(w)$.

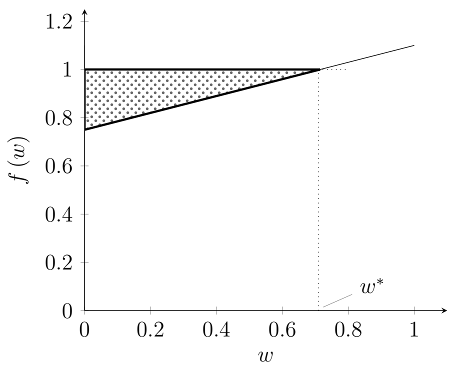

Figure 2: *Amount needed to support withdrawals as function of the proportion of impatient customers*

The parameters used in Figure 2 were $c_1 = 1.1$, $c_2 = 1.5$ and $R = 2$. Note that when w^* is above a certain threshold, deposits are insufficient to honor the contract between the bank and its customers. The shaded area is where $1 - (x + y) \geq 0$ and this difference becomes bank reserves.

The rule described in Grasselli and Ismail (2013) for opening a bank account is:

> Search clients $P_{idd,v,pyf}$: Before moving to the next period, let banks that were established at t_{2k-2} offer their services to new clients in the neighborhood of their existing clients.

In the first period, the bank may have their immediate eight neighbors as potential customers. In the second period, the neighborhood size increases from eight to twenty-four and so on.

The index idd is the age of the bank measured in number of periods. The result of the comparisons of payoffs for an agent to decide to join a bank or not is denoted pyf. Each agent uses seven potential predictors constructed from his memory, which is limited to five periods. This memory contains information on whether the budget constraint does not change after the shock, N; if there was a change but no partner was found, B; or if there was a change someone to bargain was found, G. The forecast of each situation is compared to the actual realization and an array of forces of the predictors, Φ is updated as follows: if the prediction turns out to be correct, one is added to the appropriate element in Φ; otherwise, one is subtracted. The stated predictors are:

1. Next period will be the same as the previous cycle
2. Next period will be the same as two cycles ago
3. Next period will be the same as three cycles ago
4. Next period will be the same as four cycles ago
5. Next period will be the same as five cycles ago
6. Next period will be equal to the mode of the last three previous cycles
7. Next period will be equal to the mode of the last five previous cycles

In the decision to become a customer, the agent can map the return of each predictor from U_k^i to a situation in which he deposits or not his cash in a bank, obtaining respectively the vectors Π_d e Π_n. We use the vector of forces of the predictors, Φ, as weights. So the respective expected returns are $\Pi_d^T \cdot \Phi$ and $\Pi_n^T \cdot \Phi$.

The final rule defines the withdrawal behavior:

> Withdraw $R_{U,atv}$: if $U \leq 1/2$ the agent withdraws in the second period. If the agent is not a bank customer, he can receive one monetary unit if he has liquid assets or he can receive r if he has the illiquid asset. If he is a bank customer, the returns are c_1 and c_2 in the second and third periods, respectively.

Everyone who has endowments below the assumed threshold, $1/2$ in this case, will withdraw in the second period of a cycle.

Grasselli and Ismail (2013) let banks learning as well in order to establish an interbank market. We use the same mechanism of prediction as before using five periods and seven predictors to assess predictors. Banks use both statistical accuracy and their current levels of bank reserves to evaluate if their estimates, updated by EMA processes, are adequate or not. Concerning the level of reserves, banks verify if their own reverses are greater than a threshold R_{min}. In respect to statistical accuracy, bank j tests if the estimate is in the Wilson score confidence interval [see Wilson (1927) for details], that is:

$$w_k^j \in \left[z_k^j - \sigma_k^j, z_k^j + \sigma_k^j\right]$$

where center $\bar{w}_k^j$ is the realized proportion of impatient clients at time $k(1)$ and z_k^j is given by:

$$z_k^j = \frac{\bar{w}_k^j + \frac{1}{2N_k^j} Z_{1-\frac{\alpha}{2}}^2}{1 + \frac{1}{N_k^j} Z_{1-\frac{\alpha}{2}}^2}$$

The $Z_{1-\frac{\alpha}{2}}^2$ is the $100\,(1-\alpha/2)$th percentile of the standard normal distribution, N_k^j is the number of clients of bank j and the half-width σ_k^j is:

$$\sigma_k^j = \frac{Z_{1-\frac{\alpha}{2}}^2 \sqrt{\frac{\bar{w}_k^j\left(1-\bar{w}_k^j\right)}{N_k^j} + \frac{Z_{1-\frac{\alpha}{2}}^2}{4\left(N_k^j\right)^2}}}{1 + \frac{1}{N_k^j} Z_{1-\frac{\alpha}{2}}^2}$$

If bank j reserve is lesser than R_{min} or if $\bar{w}_k^j \notin \left[z_k^j - \sigma_k^j, z_k^j + \sigma_k^j\right]$ then bank j try linking with another bank to receive or make a deposit. Grasselli and Ismail (2013) make an additional assumption, that banks know the realized proportion of impatient agents in the population in the former cycle, $\bar{w}_{k-1}$. If the estimate w_k^j is inadequate, bank j compares it with $\bar{w}_{k-1}$ and we have:

$$\begin{cases} O_k^j & = N_k^j\left(w_k^j - \bar{w}_{k-1}\right) \text{ if } w_k^j > \bar{w}_{k-1} \\ I_k^j & = N_k^j\left(\bar{w}_{k-1} - w_k^j\right) \text{ if } w_k^j \leq \bar{w}_{k-1} \end{cases}$$

where O_k^j is the amount of bank deposit if it concludes that w_k^j is possibly overestimated and I_k^j is how much in deposits a bank can accept if it believes that w_k^j is underestimated.

The contract between banks has the same conditions of a contract between client and bank.

Grasselli and Ismail (2013) build interbank links in the beginning of each cycle, $k(0)$, by an algorithm in which older banks have preference to satisfy their deposit requirements. In the second period of each cycle, $k(1)$, banks compare their estimates, w_k^j

with $\bar{w}_k^j$. Payments are made following the algorithm in list 1. Remaining interbank links are dissolved in $k(2)$. Bank runs can occur as before in $k(1)$ if patient clients believe that banks will not afford their demands in $k(2)$.

```
!if there is a shortfall
if w̄_k^j > w_k^j
then
  !evaluates planned investment
  amount = (1 − x_k^j) N_k^j
  if  amount is enough
  then
    uses part of amount to pay clients
  else
    !evaluates interbank deposits
    amount = amount + O_k^j
    if amount is enough
    then
      uses part of amount to pay clients
    else
      !uses available reserves
      amount = amount + R_k^j
      if amount is enough
      then
        uses part of amount to pay clients
      else
        !evaluates iliquid asset
        amount = amount + r (1 − w_k^j) N_k^j
        uses part of amount to pay clients
!if there is a surplus
if w̄_k^j > w_k^j
then
  !evaluates interbank deposits
  amount = I_k^j
  if amount is enough
  then
    uses part of amount to pay banks
    update amount
    if amount is enough
```

```
then
  pay early clients
else
  sell part of reserves
  update amount
  pay early clients
```

Listing 1: *Dissolving interbank links*

3.2. Grasselli and Ismail's Model Modified

We use only part of Grasselli and Ismail (2013)'s model because we are mainly interested in the effect of suspending the liquidity rule in the end. We also modified the rule of how clients choose banks and this new rule is explained below.

Netlogo, developed by Wilensky (1999), was used to develop the model. Our written Netlogo code is available upon request.

Before proceeding, we note that the rule $P_{idd,v,pyf}$ is not consistent with the own results presented in Grasselli and Ismail (2013). The rule that actually produces the pattern proposed by Grasselli and Ismail (2013) is the following:

> Become client of next-door bank or next-door neighbor's bank $T_{v,pyf}$: If the agent evaluates that is advantageous, he opens an account in a bank in the immediate neighborhood; if there is no one in this condition, he becomes a client of the same bank of one of hers/his neighbors.

Figures 3 to 6 illustrate the differences between the two rules using simulations' snapshots. In such figures, each point is an agent, banks and their respective customers have the same color pattern. Blue color agents are not clients of any financial institution. For example, in the upper part of figure 3 there are two large banks, a yellow one and an orange. We can see, at the bottom, which they take a rectangular shape and four different banks of colors yellow, orange, purple and brown are initially formed. In Figure 4 the yellow bank becomes almost monopolistic.

Figures 3 and 4 describe how the evolution of a world with the rule described by Grasselli-Ismail would be, i.e., when the bank increases its range by a layer of neighbors in every period. The region of a bank's customers tend to follow a rectangular like pattern.

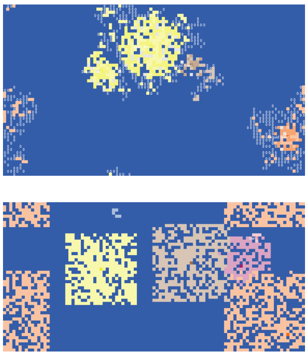

Figure 3: *World at the beginning under Grasselli-Ismail rule*

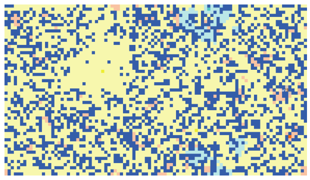

Figure 4: *World in the long run under Grasselli-Ismail rule*

On the other hand, figures 5 and 6 are similar to those shown in the Grasselli-Ismail text and are produced using the next-door rule ($T_{v,pyf}$) instead. At the top screen of figure 5 there are three banks: in the orange region, the bank is the red point near the middle. There are other two green areas, each with its bank, which we can hardly visualize. The large dark blue area are the agents who are neither customers nor banks. Figure 5 presents a setting similar to Grasselli and Ismail (2013). After some periods, the configuration in the bottom panel emerges. Now banks have more customers and there are more banks. As the world has no borders the light green region of the upper left and upper right part belong to the same bank [1]. Figure 4 is a world in which banks cannot find more major regions of agents to prospect. The white dots are patient depositors who decided to withdraw in advance due to the large proportion of people in their social network which are queuing up to withdraw money. Taking the example of the dark blue bank, see that in addition to white customers, customer colors have two shades: the darkest signals that this customer wants to cash out at this point and the lighter color indicates who is willing to wait until the next period. Bank first serves whoever is closer to it, but those who want to withdraw and are located further away may not get money when they get to the front of the line.

The rationale for next-door rule ($T_{v,pyf}$) is to become a client of the same bank from someone from his own social network. We now extend Grasselli and Ismail (2013) model introducing sequential service constraints:

Sequential service SS_d: Closest bank clients withdraw first.

[1]In the Netlogo world there is the option of a world without borders, that is, part of the top is the continuation of the bottom appearing across the world, and the left side is a continuation of the right. One can think in donuts to represent this situation.

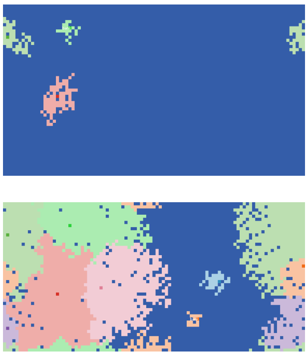

Figure 5: World at the beginning under next-door rule

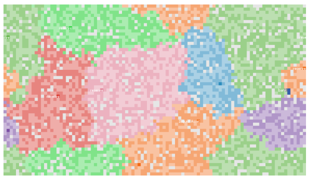

Figure 6: World in the long run under next-door rule

We also introduce an imitation rule:

Type change rule $M_{t,v}$: If $U > \frac{1}{2}$ and more than v neighbors intend to withdraw now, the client decides to withdraw too.

Withdrawals follow the order in a queue where closer clients have priority to withdraw. If there is more than one client at the same distance from the bank, we choose randomly who will receive first. This queue represents the sequential service constraint described by Douglas W. Diamond and Dybvig (1983) and is a key element discussed in the literature.

It may be the case that the amount estimated for the bank to face withdraws of the second period is not sufficient. The bank then uses its reserves to serve the customers. If the reserves run out, the bank sells the illiquid assets. If the bank exhausts all the resources, it is liquidated.

Fail F: if a bank runs out of resources for paying customers, it fails and its clients are released.

After the bank liquidation, the remaining customers return to their original state and decide whether to join another bank.

4. Results

In this section, we present some simulations from our model. The simulations' parameters are $c_1 = 1.1$, $c_2 = 1.5$, $r = 0.8$ and $R = 2$. And

$$U \in \{0, .1, .2, .3, .4, .5, .6, .7, .8, .9, 1\}$$

.

Figure 7 describes how many bank runs occur in 100 simulations from 3,334 cycles in a world with $97 \times 55 = 5{,}335$ agents in a lattice. The vertical axis is the number of bank runs and the abscissa is the number of cycles of Douglas W. Diamond and Dybvig (1983). The bank serves the customers in the queue in the second period of each cycle until the remaining resources are still sufficient for paying the customers in the third period. Thus, customers receive c_2 in the second period. This rule would prevent withdrawals from patient agents; the simulation, however, allows them to follow the imitation rule.

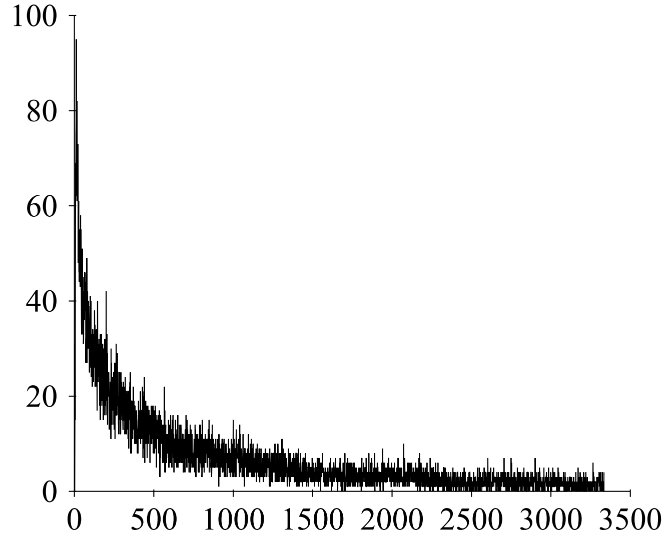

Figure 7: *Number of bank runs by cycle*

Smaller banks lose depositors during the process. These lost clients end up joining a larger financial institution until the presence of small banks becomes unfeasible. Due to this feature, the pattern in the simulations show the survival of few banks after a long time. For an example, figure 8 shows a world with about thousand cycles. In this world there are four banks represented by darker spots. Although being of different sizes, they all have a large number of customers.

The sharp decay of bank runs in figure 7 is due to the increase in banking concentration; at the end of the simulations, there are only big banks. We also measured the number of customers who cannot withdraw in the first period and therefore get nothing. In Figure 9 we depict the evolution of this metric, where the ordinate represents the num-

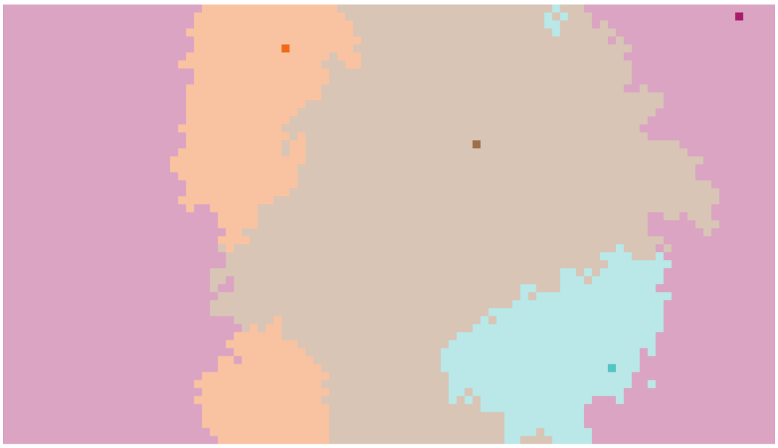

Figure 8: *Long run world configuration*

ber of customers who stood in queue and do not get money. The number of simulations remains at 100, but it may happen that more than one customer does not receive during a cycle, so there are cycles with more than 100 cashless customers.

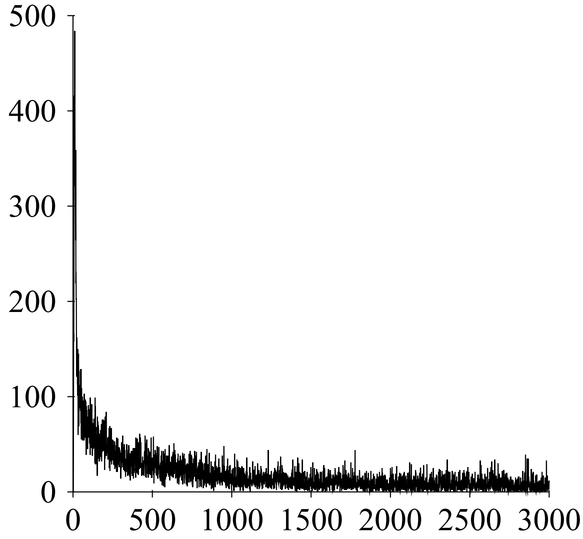

Figure 9: *Number of depositors that tried to withdraw and do not get money*

At the beginning of these worlds, there are no banks and the first viable bank appears after about 30 cycles. There are more bank runs at first, possibly because banks have few customers. Bank runs decrease with the size of banks, as seen in figure 7. Since there could be the question of whether a run in a larger bank could hurt more customers, figure 9 shows that a greater bank concentration also means a smaller number of clients harmed by a possible run. The number of frequently runs at the beginning of the simulations may be due to the law of small numbers cited by Kahneman (2012). According to the author, people tend to generalize from small samples; for example, assume that in the case of bank runs the likelihood of being impatient is 40%. A patient agent considers only the behavior of his eight immediate neighbors so that if four or more are impatient he decides to withdraw earlier. The probability of misinterpretation becomes:

$$\sum_{k=4}^{8} \binom{8}{k} \cdot 0.4^{k} \cdot 0.6^{(8-k)} = 40.59\%$$

That is, due to a small sample size, the probability that half or more of his neighbors are impatient is 40.59%.

5. Conclusion

The results in our simulations indicate that long-term bank runs are rare due to the increased size of banks, even if some clients are locally subject to errors induced by a small sample. Banks calculate the amount to pay in the last period in each cycle depending on the size of the queue and the number of customers who stayed to withdraw.

This paper adds sequential service constraint to the model of Grasselli and Ismail (2013). Sequential service constraints do not stop bank runs, but they discipline the market, because agents are no longer customers of a bank that did not honor the contract and look for another bank instead. In this process, they punish smaller banks and there is increasing bank concentration as a result.

In the context of adaptive complex systems, we could extend the model to allow for random mutations in the rules followed by the agents. We can also allow switching rules between them. Such an approach would assess the effect of the emergence of new rules in the evolution of the financial system. Another possible improvement would be to allow the emergence of a lender of last resort for banks and to measure the number of bank runs in such environment. In our paper, we do not implement the case of an informed patient agent who has knowledge about her bank's situation. We also do not consider deposit insurance in this paper, which we can implement in a future work.

References

Calomiris, Charles W. and Charles M Kahn (1991). "The Role of demandable debt in structuring optimal banking arrangements". In: *The American Economic Review*, pp. 497–513.

Calomiris, Charles W. and Joseph R. Mason (2003). "Fundamentals, panics, and bank distress during the Depression". In: *The American Economic Review* 93, pp. 1615–1647.

Chari, Varadarajan V and Ravi Jagannathan (1988). "Banking panics, information, and rational expectations equilibrium". In: *The Journal of Finance* 43.3, pp. 749–761.

Deng, Jing, Tongkui Yu, and Honggang Li (2010). "Bank runs in a local interaction model". In: *Physics Procedia* 3.5, pp. 1687–1697.

Diamond, Douglas W (1984). "Financial intermediation and delegated monitoring". In: *The Review of Economic Studies* 51.3, pp. 393–414.

Diamond, Douglas W. and Philip H. Dybvig (1983). "Bank runs, deposit insurance, and liquidity". In: *Journal of Political Economy* 91, pp. 401–419.

Ennis, Huberto M and Todd Keister (2009). "Bank runs and institutions: The perils of intervention". In: *The American Economic Review* 99.4, pp. 1588–1607.

Garratt, Rod and Todd Keister (2009). "Bank runs as coordination failures: An experimental study". In: *Journal of Economic Behavior & Organization* 71.2, pp. 300–317.

Grasselli, Matheus R and Omneia RH Ismail (2013). "An agent-based computational model for bank formation and interbank networks". In: *Handbook on Systemic Risk*. Ed. by Jean-Pierre Fouque and Joseph A. Langsam. Cambridge: Cambridge University Press.

Green, Edward J and Ping Lin (2000). "Diamond and Dybvig's classic theory of financial intermediation: What's missing?" In: *Federal Reserve Bank of Minneapolis Quarterly Review* 24.1, pp. 3–13.

Holland, John Henry (2014). *Complexity: A very short introduction*. Nova York, NY: Oxford University Press.

Hong, Harrison, Jeffrey D Kubik, and Jeremy C Stein (2004). "Social interaction and stock-market participation". In: *The journal of Finance* 59.1, pp. 137–163.

Kahneman, Daniel (2012). *Rápido e devagar: Duas formas de pensar*. Rio de Janeiro - RJ: Editora Objetiva.

Kelly, Morgan and Cormac Ó-Gráda (2000). "Market contagion: Evidence from the panics of 1854 and 1857". In: *The American Economic Review* 90.5, pp. 1110–1124.

Ó-Gráda, Cormac and Eugene N White (2003). "The panics of 1854 and 1857: A view from the Emigrant Industrial Savings Bank". In: *The Journal of Economic History* 63.1, pp. 213–240.

Peck, James and Karl Shell (2003). "Equilibrium bank runs". In: *Journal of Political Economy* 111.1, pp. 103–123.

Romero, Pedro P (2009). "Essays on banking and capital: An agent-based investigation". PhD thesis. George Mason University. URL: `http://hdl.handle.net/1920/4565` (visited on 07/09/2013).

Wallace, Neil (1988). "Another attempt to explain an illiquid banking system: The Diamond and Dybvig model with sequential service taken seriously". In: *Federal Reserve Bank of Minneapolis Quarterly Review* 12.4, pp. 3–16.

Wilensky, Uri (1999). *NetLogo*. `http://ccl.northwestern.edu/netlogo/`. Version 5.1.0. Evanston, IL: Center for Connected Learning and Computer-Based Modeling, Northwestern University.

Wilson, Edwin Bidwell (1927). "Probable inference, the law of succession, and statistical inference". In: *Journal of the American Statistical Association* 22.158, pp. 209–212. DOI: `10.1080/01621459.1927.10502953`.