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Keywords: Profit sharing; income distribution; economic growth; evolutionary dynamics.

JEL Codes: E11; E25; J33; O41.

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1. Introduction

Employee profit sharing has experienced an increasing (even if fluctuating) popularity in several advanced economies in the last few decades (D'Art and Turner, 2004, Kruse et al., 2010). Meanwhile, from a longer-term perspective, there have been rising and falling waves of interest in employee profit-sharing schemes since their inception in the 19th century (Mitchell et al.,1990, D'Art and Turner, 2006, Blasi, et al., 2013).

The main motivation behind a firm's offering of profit sharing to workers is that connecting workers' earnings to the profit performance of the firm is believed in theory to induce workers to increase commitment, effort and other attitudes leading to their higher productivity. Therefore, by reducing shirking behavior, profit sharing may reduce supervision costs (Fang, 2016). Indeed, attitude surveys find that both employers and employees usually believe that profit sharing helps improve firm performance in several dimensions (Weitzman and Kruse, 1990, and Blasi et al., 2010).

Some other compensation mechanisms through which employees' earnings depend on the performance of the firm (or work group) are gain sharing, employee ownership and stock options. Meanwhile, profit-sharing plans themselves vary considerably, and some major ways in which they differ concern what is shared (e.g., total profits or profits above some target or threshold level), how and when profit sharing is made (e.g., in cash or company stocks, in a deferred or non-deferred way) and with whom profit sharing is made (e.g., directly to workers or to some workers' retirement or pension plan). Nonetheless, there is robust survey evidence that the combination of non-deferred and in cash profit sharing is ranked first by employees as a motivation device (Blasi et al., 2010).

There is considerable empirical evidence that profit sharing raises labor productivity. Although the estimated magnitude of the productivity gain varies across empirical studies, it is frequently non-negligible. In fact, Weitzman and Kruse (1990) apply meta-analysis to sixteen studies to find that the productivity gain of profit sharing is positive at infinitesimal significance levels. Doucouliagos (1995) applies meta-analysis to forty-three studies to find that profit sharing is positively associated with productivity. Cahuc and Dormont (1997) use

French data to find that profit sharing firms outperform other firms in both productivity and profitability. Conyon and Freeman (2004), using UK data, find that profit-sharing firms tend to outperform other firms in both productivity and financial performance. D'Art and Turner (2004) use data for 11 European countries to find that profit sharing is positively associated with both productivity and profitability, while Kim (1998), using U.S. data, finds that albeit profit sharing raises labor productivity, profits do not increase, given that gains from profit sharing are cancelled out by raised labor costs. Meanwhile, in a field, quasi-experimental investigation, Peterson and Luthans (2006) randomly assigned profit-sharing plans at three of twenty-one establishments within a U.S. firm, finding that labor productivity and profits rose in the profit-sharing establishments relative to the control group.

Against the background of the above evidence, this paper sets forth a classical model of economic growth, in which the distribution of income can feature profit sharing with workers. In attempting to extract labor from labor power more effectively, firms can behave heterogeneously as regards the choice of compensation strategy. Firms periodically choose to compensate workers with either solely a conventional base wage or a share of profits on top of this conventional base wage. In accordance with the empirical evidence reported above, workers in profit-sharing firms have a higher productivity than workers in non-sharing firms. Moreover, any observed heterogeneity in labor-extraction compensation strategies across firms and the resulting additional labor productivity gain that accrues to profit-sharing firms are not parametric constants, but instead co-evolve endogenously towards the long-run as driven by evolutionarily satisficing imitation dynamics. It turns out that our model is well fitted to explore, for instance, possible explanations for evidence such as that U.K. firms do switch modes of employee compensation frequently, with the gross changes in modes (which include profit sharing) being far more numerous than the net changes (which in turn suggests that firms seem to have trouble optimizing and that the transaction costs of mode switching are relatively low) (Bryson and Freeman, 2010).

Moreover, we explore the implications of the coupled dynamics of the distribution of labor-extraction compensation strategies across firms and the additional productivity gain that accrues to profit-sharing firms for the distribution of aggregate income between profits and wages and thereby capital accumulation and economic growth. Arguably, given the essential role of the functional distribution of income in the classical tradition, it is fitting to explore the implications of profit sharing for capital accumulation and economic growth in the long-run in a model that conforms to essential tenets of this tradition.[1]

The remainder of this paper is organized as follows. Section 2 describes the structure of the model and investigates its behavior in the short run. Section 3 explores the coupled evolutionary dynamics of the frequency distribution of labor-extraction compensation modes across firms and the additional productivity gain that accrues to profit-sharing firms. This section also investigates the implications of such coupled evolutionary dynamics for income distribution and thereby capital accumulation and economic growth in the long run. The final section summarizes the main conclusions reached along the way.

2. Structure of the model and its behavior in the short run

The model economy is closed and without government activities, producing a single (homogeneous) good for both investment and consumption purposes. Production is carried out by a large (and fixed) population of firms, which combine two (physically homogeneous) factors of production, capital and labor, by means of a fixed-coefficient technology. Firms produce (and hire labor) without facing realization (or effective demand) constraints, thereby being able to sell profitably all its output production at the prevailing price level. However, the model is cast in real terms.

In attempting to extract labor from labor power more effectively, an individual firm chooses periodically between two worker compensation strategies: it compensates workers

[1] From a mainstream (of New Keynesian variety) perspective, Weitzman (1985) claims that profit sharing can deliver full employment with low inflation. If part of workers' total compensation is shared profits, so that the base wage is lower, the marginal cost of labor is lower and firms will be willing to hire more workers. As the marked up price is lower, a real balance effect creates a higher aggregate demand and hence a higher desired output. Therefore, Weitzman (1985) sees involuntary unemployment as due to downward wage inflexibility (Davidson, 1986-87, Rothschild, 1986-87). In our model, the fraction of profit-sharing firms and the average labor productivity are endogenously time-varying according to a satisficing imitation mechanism. Also, we explore implications of profit sharing for growth performance in a classical model, in which the functional distribution of income plays a key role.

with either solely a conventional (or subsistence) base wage $v \in \mathbb{R}_{++}$ (*non-sharing labor-extraction compensation strategy*) or instead with a conventional base wage plus a share $\delta \in (0,1) \subset \mathbb{R}$ on profits (*profit-sharing labor-extraction compensation strategy*). In a given period there is a proportion $\lambda \in [0,1] \subset \mathbb{R}$ of profit-sharing (or type *s*) firms, while the remaining proportion, $1-\lambda$, is composed by non-sharing (or type *n*) firms. In accordance with the empirical evidence on profit sharing reported in the preceding section, a profit-sharing firm is willing to play such labor-extraction compensation strategy because it believes that the resulting labor productivity gain will be sufficiently higher than otherwise. Labor productivity is homogeneous across workers in firms playing a given worker compensation strategy. Since the conventional base wage is assumed to be the same under both available labor-extraction compensation strategies, a worker hired by a profit-sharing firm receive a higher total compensation than a worker hired by a non-sharing firm along the economically meaningful domain given by strictly positive profits for sharing firms.[2]

To keep focus on the dynamics of the distribution of labor-extraction compensation strategies and its implications for the functional distribution of income and thereby economic growth, we simplify matters by assuming that the conventional base wage, v, and the profit-sharing coefficient, δ, both remain constant over time. The distribution of labor-extraction compensation strategies across firms, $(\lambda, 1-\lambda)$, which is given in the short run as a result from previous dynamics, changes beyond the short run according to an evolutionary dynamic. In the short run, for given values of the conventional base wage, profit-sharing coefficient, productivity differential, distribution of labor-extraction strategies and, therefore, individual and average profit shares, the short-run value of the rate of economic growth is determined. As the economy evolves towards the long-run, the co-evolution of the frequency distribution

[2] Empirical evidence shows that profit sharing has a meaningful effect on worker total compensation (Kruse et al., 2010; Long and Fang, 2012). Capelli and Neumark (2004), for instance, find that total labor costs exclusive of the sharing component do not fall significantly in pre/post comparisons of firms that adopt profit sharing. This suggests that profit sharing tends to come on top of, rather than in place of, a base wage. Meanwhile, Long and Fang (2012) also find that, although adoption of profit sharing increases fluctuations in employee earnings, it also increases the growth of these earnings in the longer term.

of labor-extraction strategies and the labor productivity differential, by leading to changes in the average share of profits, causes changes in the short-run value of the rate of economic growth.

Formally, we define the productivity differential as:

(1) $$\alpha \equiv \frac{a_s}{a_n},$$

where $a_\tau = X_\tau^i / L_\tau^i$ denotes the labor productivity of the i-th firm of type $\tau = s, n$, X_τ^i is the total output of the i-th firm of type $\tau = s, n$, and L_τ^i is the total employment of the i-th firm of type $\tau = s, n$. While labor effort (and hence productivity) is homogeneous across firms of a given type, labor effort (and productivity) is heterogeneous across the two types of firms. For simplicity, we normalize labor productivity in non-sharing firms, a_n, to one, so that we obtain $a_s = \alpha > 1$ in (1). As it turns out, profit-sharing firms can be intuitively described as firms willing to bet on the prospect of attaining a productivity differential, α, which is high enough to allow them to have a lower unit labor cost than non-sharing firms. At the end of the day a sharing firm expects such productivity differential to be high enough to allow it to have a higher *net* profit rate (i.e., net of shared profits) than a non-sharing firm. However, as explored in the next section, the resulting labor productivity differential may fall short of the level required for the profit-sharing bet to prove successful.

Using (1), the total real profit of the *i*-th firm of type τ is given by:

(2) $$R_\tau^i \equiv X_\tau^i - vL_\tau^i = \begin{cases} \left(1 - \frac{v}{\alpha}\right) X_\tau^i, & \text{if } \tau = s, \\ (1-v) X_\tau^i, & \text{if } \tau = n, \end{cases}$$

which requires further assuming that $v < a_n = 1 < a_s = \alpha$.

Considering (2), the profit share of the *i*-th firm of type τ is given by:

(3) $$\pi_\tau^i \equiv \frac{R_\tau^i}{X_\tau^i} = \begin{cases} 1 - \frac{v}{\alpha} \equiv \pi_s, & \text{if } \tau = s, \\ 1 - v \equiv \pi_n, & \text{if } \tau = n. \end{cases}$$

The first profit share expression in (3) denotes the proportion of gross profits in the output of sharing firms, since an exogenously given fraction of such profits, given by $\delta \in (0,1) \subset \mathbb{R}$, is shared with workers. Thus, using (3), the net profit share of each sharing firm in the short run is given by:

(4) $$\pi_s^{c,i} \equiv \frac{(1-\delta)R_s^i}{X_{s,i}} = (1-\delta)\left(1-\frac{v}{\alpha}\right) = (1-\delta)\pi_s \equiv \pi_s^c .$$

Of course, by assumption, the net profit share of each non-sharing firm in the short run is equal to the (gross) profit share π_n, as defined in (3).

The conditional expected value of the net profit share π^c given the type τ is simply:

(5) $$\mathrm{E}\left(\pi^c \middle| \tau\right) = \begin{cases} \pi_s^c \ , \ \text{if } \tau = s, \\ \pi_n \ , \ \text{if } \tau = n. \end{cases}$$

Based on the law of iterated expectations (see, e.g., the simplified version in Wooldridge, 2010, Property CE.2, p. 31) and the conditional expectation (5), we can establish the short-run average profit share $\bar{\pi}^c$ as the expected net profit share in the short run for a given frequency distribution of labor-extraction compensation strategies across firms $(\lambda, 1-\lambda)$:

(6) $$\bar{\pi}^c(\alpha,\lambda) \equiv \mathrm{E}(\pi^c) = \mathrm{E}\left[\mathrm{E}\left(\pi^c \middle| \tau\right)\right] = \lambda \mathrm{E}\left(\pi^c \middle| \tau = s\right) + (1-\lambda)\mathrm{E}\left(\pi^c \middle| \tau = n\right) = \lambda \pi_s^c + (1-\lambda)\pi_n .$$

Let $k^i \equiv X_\tau^i / K_\tau^i$ be the output to capital ratio of the *i*-th firm of type $\tau = s, n$, where K_τ^i is the respective capital stock. We assume that these individual ratios remain constant when firms switch labor-extraction compensation strategy. Using (2)-(3), the (gross) profit rate of the *i*-th type τ firm in the short run can be expressed as follows:

(7) $$r_\tau^i \equiv \frac{R_\tau^i}{K_\tau^i} = \begin{cases} \pi_s k^i \ , \ \text{if } \tau = s, \\ \pi_n k^i \ , \ \text{if } \tau = n. \end{cases}.$$

Based on (7), the net profit rate of the *i*-th sharing firm in the short run is given by:

(8) $$r_s^{c,i} \equiv \frac{(1-\delta)R_s^i}{K_s^i} = (1-\delta)\left(1-\frac{v}{\alpha}\right)k_i = \pi_s^c k^i .$$

Evidently, by assumption, the net profit rate of the *i*-th non-sharing firm in the short run is equal to $\pi_n k^i$.

We further assume that the individual output to capital ratios given by $k^i \equiv X_\tau^i / K_\tau^i$ are randomly distributed across the population of firms around the average value given by $k \in \mathbb{R}_{++}$, which is taken to be an exogenously given constant. Thus, the conditional expected value of the net profit rate r^c given the type τ is simply:

(9) $$\mathrm{E}\left(r^c \middle| \tau\right) = \begin{cases} \pi_s^c k \ , \ \text{if } \tau = s, \\ \pi_n k \ , \ \text{if } \tau = n. \end{cases}$$

Based on the law of iterated expectations and the conditional expectation in (9), we can then define the short-run average profit rate $\bar{r}^c$ as the expected net profit rate in the short run for a given frequency distribution of employee compensation strategies across firms $(\lambda, 1-\lambda)$:

(10) $$\bar{r}^c \equiv \mathrm{E}(r^c) = \mathrm{E}\left[\mathrm{E}\left(r^c \middle| \tau\right)\right] = \lambda \mathrm{E}\left(r^c \middle| \tau = s\right) + (1-\lambda)\mathrm{E}\left(r^c \middle| \tau = n\right) = \left[\lambda \pi_s^c + (1-\lambda)\pi_n\right] k \, .$$

Therefore, a comparison between (6) and (10) shows that in the short run the average net profit rate is a multiple of the average net profit share, so that these two profitability measures always move in the same direction.

As assumed earlier, the population of firms, the conventional base wage, v, the labor productivity in non-sharing firms, a_n, the profit-sharing coefficient, δ, the individual output to capital ratios, $\{k^i\}_{i \in [0,1] \subset \mathbb{R}}$, and the average output to capital ratio, k, all remain constant over time. The short run period t is defined as a time frame in which the aggregate capital stock, K_t, the aggregate labor supply, N_t, the productivity differential of profit-sharing firms, $a_{s,t} = \alpha_t$, the frequency distribution of labor-extraction compensation strategies across firms, λ_t, and therefore the distribution of income as measured by the average net profit share, $\bar{\pi}_t^c$, can all be taken as predetermined by previous dynamics.[3]

[3] As in the next section we explore the behavior of the economy in the transition from the short to the long run, thereafter we attach a subscript t to the short-run value of all variables (be they endogenous or predetermined).

The economy is inhabited by two classes, capitalists who own the firms and workers. Workers provide labor and earn solely a conventional base wage income, when they work for non-sharing firms. Workers in sharing firms also receive a share of the latter's profit income, which is the entire surplus over the respective base wage bill. We assume that workers' total compensation is all spent on consumption, while capitalists homogeneously save a fraction, $\gamma \in (0,1) \subset \mathbb{R}$, of their net profit income. As we further assume that capitalists save in order to fully finance their investment decisions, it turns out that all net profit income not consumed is automatically invested. Therefore, assuming for simplicity that individual capital stocks do not depreciate, the growth rate of the capital stock (and output) of the *i*-th firm of type τ can be expressed as follows:

(11) $$g^i_{\tau,t} \equiv \frac{S^i_{\tau,t}}{K^i_{\tau,t}} = \begin{cases} \dfrac{\gamma(1-\delta)R^i_{\tau,t}}{K^i_{\tau,t}} = \gamma(1-\delta)\left(1-\dfrac{\nu}{\alpha_t}\right)k^i = \gamma\pi^c_{s,t}k^i \text{ , if } \tau = s, \\ \dfrac{\gamma R^i_{\tau,t}}{K^i_{\tau,t}} = \gamma(1-\nu)k^i = \gamma\pi_n k^i \text{ , if } \tau = n, \end{cases}$$

where $S^i_{\tau,t}$ stands for the net savings of the *i*-th firm of type τ.

Given that the individual output to capital ratios are randomly distributed across firms around the average value k, it follows that the conditional expected value of the growth rate g_t at the period *t* given the type τ is simply:

(12) $$\mathrm{E}\left(g_t \middle| \tau\right) = \begin{cases} \gamma\pi^c_{s,t}k \text{ , if } \tau = s, \\ \gamma\pi_n k \text{ , if } \tau = n. \end{cases}$$

Using again the law of iterated expectations and considering the individual growth rates in (11), we specify the short-run average growth rate $\bar{g}_t$ at each period *t* as the expected growth rate in the short run for a given frequency distribution of employee compensation strategies across firms $(\lambda_t, 1-\lambda_t)$:

(13) $$\bar{g}(\alpha_t,\lambda_t) \equiv \mathrm{E}(g_t) = \mathrm{E}\left[\mathrm{E}\left(g_t \middle| \tau\right)\right] = \lambda_t \mathrm{E}\left(g_t \middle| \tau = s\right) + (1-\lambda_t)\mathrm{E}\left(g_t \middle| \tau = n\right) = \gamma\left[\lambda_t\pi^c_{s,t} + (1-\lambda_t)\pi_n\right]k,$$

which can be re-written based on (6) as follows:

(13-a) $\bar{g}(\alpha_t, \lambda_t) = \gamma k \bar{\pi}^c(\alpha_t, \lambda_t)$.

Note that the short-run average growth rate in (13-a) depends on parametric constants along with the labor productivity differential, α_t, and the frequency distribution of labor-extraction compensation strategies, λ_t, which are predetermined in the short run and co-evolve in the transition from the short to the long run (as described in the next section). Expectedly, in light of the classical nature of the macroeconomic dynamic, the short-run average growth rate varies positively with the (homogeneous) saving propensity of capitalists and the average net profit share in aggregate income. The intuition for this result is that economic growth is driven by capital accumulation from the aggregate saving of capitalists.

3. Behavior of the model in the long run

The economy moves toward the long run due to changes in the aggregate stock of capital, K, coming from changes in the individual stocks of capital, $\{K^i\}_{i\in[0,1]\subset\mathbb{R}}$, the supply of available labor, N, the labor productivity differential, α, and the frequency distribution of labor-extraction compensation strategies, λ. However, to sharpen focus on the coupled dynamics of the frequency distribution of labor-extraction compensation strategies and the productivity differential as state variables, we further assume that the aggregate labor force grows endogenously at a rate equal to the average growth rate of the capital stock, as defined in (13).[4]

Let us then start by deriving the dynamics of the productivity differential. At a given short-run period t there is a fraction $\lambda_t \in [0,1] \subset \mathbb{R}$ of the population of firms, which may

[4] Consequently, the constancy of the productivity differential and the distribution of labor-extraction compensation strategies in the long-run equilibrium guarantee the constancy of the employment rate as well. In fact, the assumed constancy of the individual output to capital ratios implies that, in the long-run equilibrium, the growth rate of the employment level of the i-th firm of type τ is given as in (11). Therefore, in the long-run equilibrium, the conditional expected value of the growth rate of the individual employment level given the type τ is determined just as in (12). As a result, we can specify the average growth rate of the employment level in the long-run equilibrium as the expected growth rate of the employment level in the long-run equilibrium for a given distribution $(\lambda, 1-\lambda)$, as defined in (13). Thus, our assumption that the aggregate labor force grows endogenously at a rate equal to average growth rate of the capital stock implies that the aggregate labor force and the employment level grow at the same rate in the long-run equilibrium.

vary from one period to the next one, adopting the profit-sharing labor-extraction strategy. The complementary fraction $1-\lambda_t$ is made up of firms that pay only the conventional base wage. Let $\bar{y}_t \equiv \lambda_t y_{s,t} + (1-\lambda_t) y_n$ be the average earnings of workers at period t, where $y_{s,t} \equiv v + \delta R_{s,t} / L_{s,t}$ and $y_n \equiv v$ are the earnings of a worker in a profit-sharing firm and a worker in a non-sharing firm in period t, respectively. Therefore, the differential between the higher earnings and the average earnings can be written as $y_{s,t} - \bar{y}_t = (1-\lambda_t)\delta R_{s,t} / L_{s,t}$ for all $\lambda_t \in [0,1] \subset \mathbb{R}$. In accordance with the empirical evidence reported earlier, we assume that the extent to which productivity in profit-sharing firms is greater than productivity in non-sharing firms varies positively with the relative earnings differential given by $y_{s,t} - \bar{y}_t$. Formally, we consider the following labor-extraction differential function:

(14) $$\alpha_{t+1} = f(y_{s,t} - \bar{y}_t) = f\left((1-\lambda_t)\delta R_{s,t} / L_{s,t}\right),$$

where $f'(\cdot) > 0$ and $f''(\cdot) < 0$ for all earnings differential $(y_{s,t} - \bar{y}_t) \subset \mathbb{R}$. Besides, we assume that $\lim_{(y_s - \bar{y}) \to \infty} f'(y_s - \bar{y}) = 0$. Therefore, in accordance with the empirical evidence that the labor productivity gains arising from profit sharing are not unlimited, the labor-extraction differential in (14) not only rises at a decreasing rate, but also tends to become insignificant for very large values of the relative earnings differential.[5] We can use (2) to re-write (14) as follows:

(15) $$\alpha_{t+1} = f\left(\delta(1-\lambda_t)(\alpha_t - v)\right).$$

As re-written in (15), it follows that the labor-extraction differential function has quite intuitive an interpretation. Given that α_t is the output per worker of a profit-sharing firm and v is the corresponding unit wage cost of labor, it follows that $\alpha_t - v$ is the profit per worker of a profit-sharing firm and $\delta(\alpha_t - v)$ is the amount of profit per worker of an individual profit-sharing firm which is shared with its employees. Thus, given α_t and λ_t, it follows that

[5] The meta-analyses conducted in Weitzman and Kruse (1990) and Doucouliagos (1995), for instance, find that the size of the estimated effect of profit sharing on labor productivity is usually on the order of 3 to 7 percent.

the next-period labor-extraction differential, α_{t+1}, varies positively with the profit-sharing coefficient and negatively with the conventional base wage. Meanwhile, given α_t, v and δ, the next-period labor-extraction differential increases with $1-\lambda_t$, which is the proportion of firms playing the non-sharing compensation strategy in a given period, which is an indicator of the prospects of not receiving any shared profits in the next period. Since an increase in $1-\lambda_t$ performs as an incentive on workers in profit-sharing firms in the next period to deliver a higher labor productivity differential, it follows that $\delta(1-\lambda_t)(\alpha_t - v)$ can be interpreted as reflecting how valuable it is for an individual worker to be hired by a profit-sharing firm.

In fact, it follows from (15) that $\partial\alpha_{t+1}/\partial\lambda_t = -\delta(\alpha_t - v)f'\left(\delta(1-\lambda_t)(\alpha_t - v)\right) < 0$ for all $\lambda_t \in [0,1] \subset \mathbb{R}$ and for any $\alpha_t > v$ (recall that the latter condition was assumed earlier to ensure strictly positive gross profits for profit-sharing firms in (2)). It follows that the greater the proportion of sharing firms in a given short-run period, the smaller the positive labor-extraction differential between sharing and non-sharing firms in the next period. One firm's decision to play the profit-sharing labor-extraction strategy in a given period, by reducing $\delta(1-\lambda_t)(\alpha_t - v)$ for a given α_t and making it less valuable to workers to be employed by a sharing firm in the next period, has a negative payoff externality on all other sharing firms. Thus, there is strategic substitutability in the firms' choice of labor-extraction compensation mechanism. Meanwhile, if all firms choose to play the profit-sharing labor-extraction strategy $(\lambda_t = 1)$, the relative earnings differential represented by $y_{s,t} - \bar{y}_t = \delta(1-\lambda_t)(\alpha_t - v)$ vanishes. In this case with all firms playing the profit-sharing strategy, given that labor productivity becomes uniform across all firms, and should be higher than the average labor productivity when all firms instead pay only the conventional base wage (recall that the productivity in non-sharing firms was normalized to one), we further assume that $f(0) > 1$. Conversely, if all firms choose to play the non-sharing compensation strategy $(\lambda_t = 0)$, the potential relative earnings differential given by $y_{s,t} - \bar{y}_t = \delta(\alpha_t - v)$ takes its maximum value. In this case, a non-sharing firm which switches compensation to the profit-sharing mode is able to reap the largest possible additional labor-extraction gain, since $f\left(\delta(\alpha_t - v)\right) > f\left(\delta(1-\lambda_t)(\alpha_t - v)\right)$ for all $\lambda_t \in (0,1] \subset \mathbb{R}$ and for any $\alpha_t > v$. In order to better convey the substance of all these properties of the labor-extraction differential function (15), they are all represented in Figure 1.

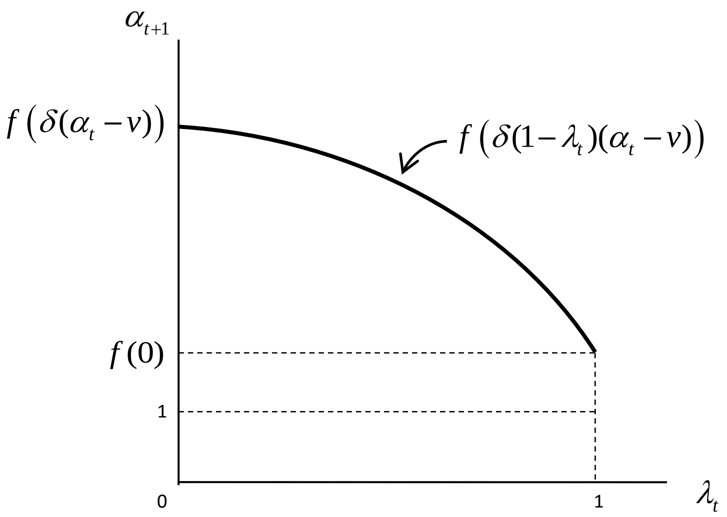


Figure 1. *Labor-extraction differential function*

The following intuitive rationales can be proposed for the additional labor-extraction gain in (14). First, the average earnings can be seen by a worker as a conventional estimate of her outside option or fallback position. Consequently, workers who receive a share of profits in addition to a conventional base wage generate an extra productivity gain (relatively to the productivity gain they would offer if remunerated with only a conventional base wage) which increases with the excess of the higher earnings over their outside option or fallback position. Second, the average earnings can be perceived by a worker as a norm-based reference point against which a compensation package featuring a conventional base wage and shared profits should be compared when she is deciding how much above-normal productivity to provide in return. Therefore, above-average earnings could be seen by workers as warranting the offer of above-normal levels of productivity. In fact, Blasi, Kruse and Freeman (2010) propose an interesting rationale for profit-sharing compensation based on reciprocity and gift exchange (as these notions are suggestively articulated in Akerlof (1982)): a "gift" of higher worker compensation through profit sharing raises worker morale, and workers then reciprocate with a "gift" of greater productivity. More generally, a "gift" of profit-sharing compensation on top of a conventional base wage can be interpreted as helping to create and reinforce a sense of shared interests and the value of a reciprocal relationship. Alternatively, the norm-based reference point provided by the average earnings can be intuitively interpreted as reflecting workers' earnings expectation under unavoidable conditions of uncertainty. As a result, an employee compensation package featuring a conventional base wage complemented with some shared profits is greeted as a quite pleasant surprise which warrants the reciprocation of above-normal levels of labor productivity.

Let us now describe the evolutionarily satisficing imitation dynamics which yield the law of motion of the proportion of sharing firms, λ_t, following the suggestive contributions of Herbert Simon.[6] As elaborated by Simon (1955, 1956), satisficing is a theory of choice centered on the process through which available alternatives are examined and evaluated. By conceiving of choice as intending to meet an acceptability threshold rather than to select the best of all possible alternatives, satisficing theory openly contrasts with optimization theory. As imaginatively suggested by Simon, this contrast is analogous to 'looking for the sharpest needle in the haystack' (i.e., optimizing) versus 'looking for a needle sharp enough to sew with' (i.e., satisficing) (Simon, 1987, p. 244).

A sharing firm *i* takes her current net profit rate given by (8) and compares it with the net profit rate she considers *acceptable*, which is denoted by ρ^i. Taking t as the *current* period, if $\pi^c_{s,t} k^i \geq \rho^i$, this sharing firm *i* does not even consider changing her labor-extraction compensation strategy in $t+1$. Otherwise, if $\rho^i / k^i > \pi^c_{s,t}$, the sharing firm *i* in question then becomes a strategy reviser. The net profit rate that is acceptable to a given firm depends, inter alia, on idiosyncratic features, and we assume that the acceptable net profit rate is randomly and independently determined across firms and over time.

[6] The formal derivation of this evolutionarily satisficing imitation dynamic is based on Vega-Redondo (1996, p. 91).

We further assume that the ratio given by ρ^i/k^i is a random variable with cumulative distribution function $F:\mathbb{R}_+ \rightarrow [0,1] \subset \mathbb{R}$ which is continuously differentiable and strictly increasing. As a result, the probability of randomly choosing a firm *i* in the subpopulation of sharing firms who considers the current net profit rate $\pi^c_{s,t}k^i$ as unacceptable is given by:

(16) $$\Pr(\rho^i > r^{c,i}_{s,t}) = \Pr(\rho^i/k^i > \pi^c_{s,t}) = 1 - F(\pi^c_{s,t}).$$

Following Vega-Redondo (1996, p. 91), let us now suppose that when such satisficing behavior transforms a sharing firm into a potential strategy reviser she will switch to the other labor-extraction compensation strategy with probability given by the fraction of firms who have previously adopted the alternative strategy. This is an imitation effect, which can be associated with the idea of rule-of-thumb behavior in the present setting of choice of worker compensation strategy triggered by satisficing. Under this premise and further assuming that the random variables related to the satisficing and imitation effects are independent from each other, the measure of sharing firms who become non-sharing firms is then given by:

(17) $$\lambda_t[1 - F(\pi^c_{s,t})](1-\lambda_t).$$

Analogously, based on (7), the efflux from the population of non-sharing firms who becomes sharing firms is given by:

(18) $$(1-\lambda_t)\Pr(\rho^i > r^i_n)\lambda_t = (1-\lambda_t)\Pr(\rho^i/k^i > \pi_n)\lambda_t = (1-\lambda_t)[1 - F(\pi_n)]\lambda_t.$$

Therefore, subtracting (17) from (18) yields the following evolutionarily satisficing imitation dynamic:

(19) $$\lambda_{t+1} - \lambda_t = \lambda_t(1-\lambda_t)[F(\pi^c_{s,t}) - F(\pi_n)].$$

As it turns out, given that $F(\cdot)$ is a strictly increasing function, a rise in the relative net profit rate associated with the profit-sharing labor-extraction compensation strategy in the current period leads to an increase in the proportion of firms playing this strategy in the next period, whilst the opposite takes place when the relative net profit rate associated with the non-sharing labor-extraction compensation strategy rises. Thus, the evolutionary dynamics in (19) reflects the operation of a selection mechanism according to which the proportion of firms playing a given labor-extraction employee compensation strategy varies positively with the relative fitness or net profit rate of such strategy.

Consequently, the state transition of the economy is driven by the system of difference equations in (15) and (19), whose state space is $\Theta = \{(\alpha_t, \lambda_t) \in \mathbb{R}_+^2 : \alpha_t > v, 0 \le \lambda_t \le 1\}$, as represented by the shaded area in each panel in Figure 2.

We will show that the dynamic system represented by (15) and (19) has two long-run equilibria featuring survival of only one labor-extraction compensation strategy in each of them. These monomorphic (pure-strategy) equilibria are denoted by E_1 and E_2 in the three panels included in Figure 2. Moreover, we will show the possible existence of a third long-run equilibrium (denoted by E_3 in panel (b) in Figure 2), now featuring the survival of both labor-extraction compensation strategies (polymorphic equilibrium).

Note that $\lambda_{t+1} = \lambda_t = 0$ for any $t \in \{0,1,2,...\}$ readily satisfies (19) for any state $(\alpha_t, 0) \in \Theta$. Moreover, let $\alpha_{t+1} = \alpha_t = \bar{\alpha} \in (v, \infty) \subset \mathbb{R}$ for any $t \in \{0,1,2,...\}$. In this case, the difference equation in (15) is satisfied for any $t \in \{0,1,2,...\}$ if the following condition holds:

(20) $$\bar{\alpha} = f\left(\delta(\bar{\alpha} - v)\right).$$

We demonstrate in Appendix 1 that $\bar{\alpha} \in (v, \infty) \subset \mathbb{R}$ exists and is unique. Therefore, one of the two monomorphic long-run equilibria of the system, E_1 in Figure 2, is given by the state $(\bar{\alpha}, 0) \subset \Theta$, which features the non-sharing labor-extraction compensation strategy as the only survivor.

Meanwhile, if $\lambda_{t+1} = \lambda_t = 1$ for any $t \in \{0,1,2,...\}$, the difference equation in (19) is satisfied for any state $(\alpha_t, 1) \in \Theta$ and, given (15), it follows that $\alpha_{t+1} = \alpha_t = f\left(0\right)$ for all $t \in \{0,1,2,...\}$. Therefore, the other monomorphic long-run equilibrium of the system, E_2 in Figure 2, is represented by the state $\left(f(0), 1\right) \subset \Theta$, which features the profit-sharing labor-extraction compensation strategy as the only survivor.

Finally, if $\lambda_{t+1} = \lambda_t = \lambda^* \in (0,1) \subset \mathbb{R}$ for any $t \in \{0,1,2,...\}$, the difference equation in (19) is satisfied, as $F(\cdot)$ is continuous and strictly increasing, if the individual profit shares π_n and $\pi_{s,t}^c$ defined in (3) and (4) become equalized for any $t \in \{0,1,2,...\}$. Even though the stationarity condition given by $\lambda_{t+1} = \lambda_t = \lambda^* \in (0,1) \subset \mathbb{R}$ for any $t \in \{0,1,2,...\}$ does not imply equalization of individual net profit rates due the heterogeneity in output to capital ratio across firms, the conditional expected value of the net profit rate r^c given the type τ in (9) and the short-run average profit rate $\bar{r}^c$ in (10) nonetheless become equalized. In other words, the expected profitability associated with each strategy becomes equalized with the average profitability. Interestingly, therefore, although there is not equalization of individual actual net profit rates (due to the heterogeneity in output to capital ratio across firms), there is nonetheless equalization of expected net profitability across firms.

Given that the labor-extraction differential (which is equal to the labor-extraction in profit-sharing firms) is the only adjusting variable amongst the determinants of the individual profit shares in (3) and (4), the latter become equalized when the long-run equilibrium value of the labor-extraction differential is given by:

(21) $$\alpha^* = \frac{(1-\delta)v}{v-\delta},$$

where we assume that $v > \delta$. Meanwhile, given that $\alpha_{t+1} = \alpha_t = \alpha^*$ for all $t \in \{0,1,2,...\}$, the difference equation in (15) is readily satisfied if the following condition holds:

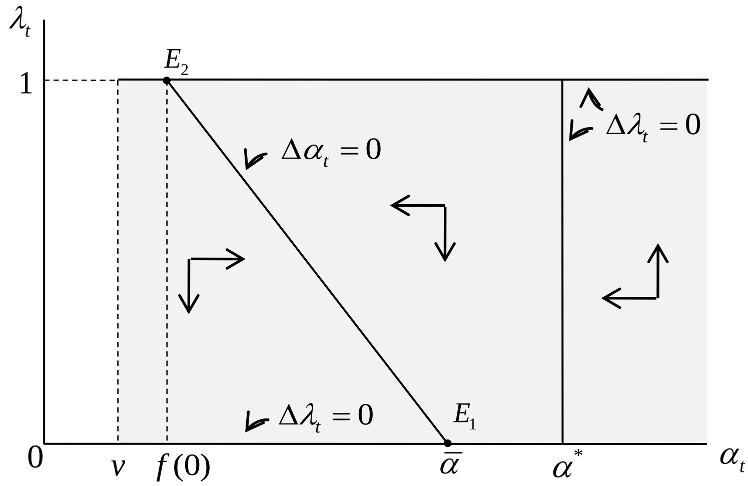
(22) $$\alpha^* = f\left(\delta(1-\lambda^*)(\alpha^* - v)\right).$$

As demonstrated in Appendix 2, there is a unique $\lambda^* \in (f(0), \bar{\alpha}) \subset \mathbb{R}$ which satisfies (19) if the following necessary and sufficient condition is satisfied:

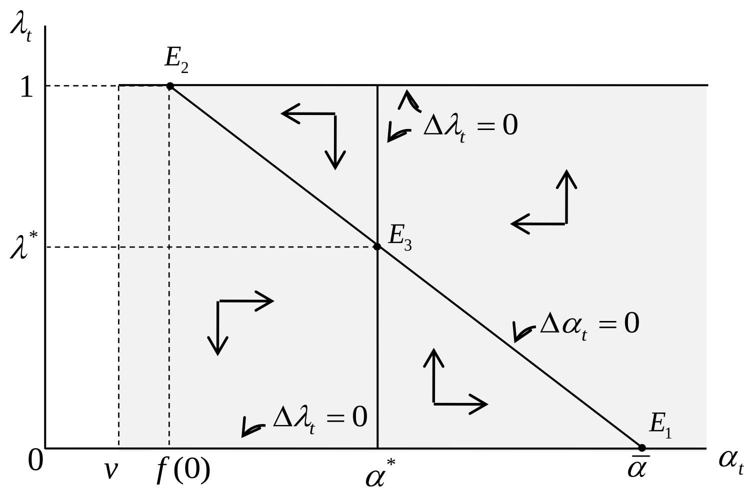
(23) $$f(0) < \alpha^* < \bar{\alpha}.$$

Therefore, if the condition in (23) is satisfied, there exists a third long-run equilibrium given by the state $(\alpha^*, \lambda^*) \subset \Theta$ (and denoted by E_3 in panel (b) in Figure 2), which features the survival of both labor-extraction compensation strategies in the long run.

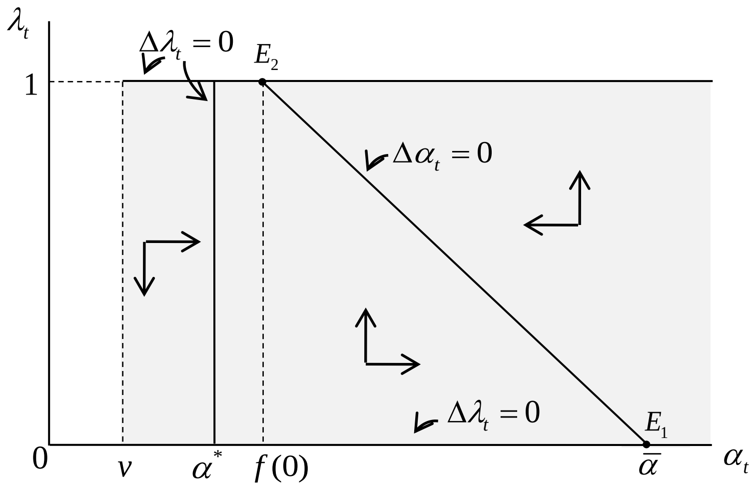
The well-defined ordering in (23), which implies and is implied by the existence and uniqueness of the polymorphic equilibrium, E_3, can be interpreted intuitively with recourse to Figure 3, which plots the next-period labor-extraction differential as a function of its current-period value. Note that, *ceteris paribus*, the function in (15) shifts down with an increase in the proportion of profit-sharing firms at period t. More precisely, this function rotates clockwise around the point $\left(v, f(0)\right)$ as λ_t increases, due to the resulting squeeze in the relative earnings differential given by $y_{s,t} - \bar{y}_t$ for every $\alpha_t > v$.



(a) *Relatively weak labor-extraction differential:* $f(0) < \bar{\alpha} < \alpha^*$



(b) *Relatively moderate labor-extraction differential:* $f(0) < \alpha^* < \bar{\alpha}$



(c) *Relatively strong labor-extraction differential:* $\alpha^* < f(0) < \bar{\alpha}$

Figure 2. *Phase diagram for different magnitudes of the labor-extraction differential*

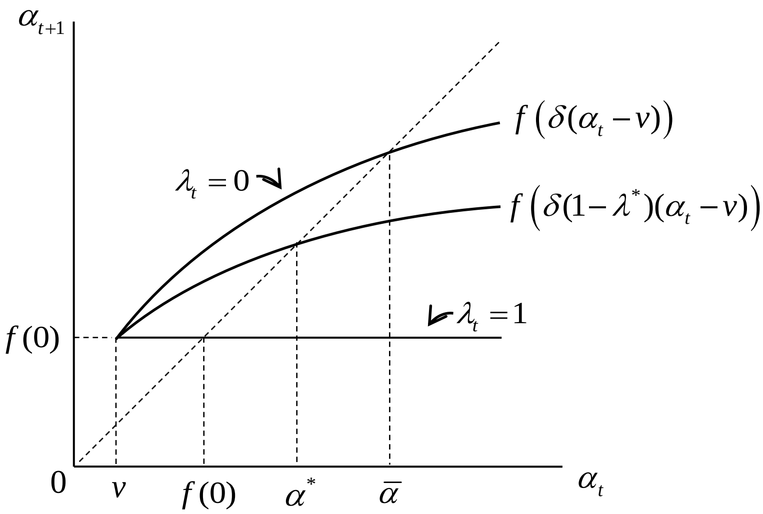


Figure 3. *Ordering of labor-extraction differentials when there is a polymorphic long-run equilibrium*

Let us now explore the dynamics of the system towards the long run as governed by the motion equations in (15) and (19). The $\Delta\alpha_t \equiv \alpha_{t+1} - \alpha_t = 0$ isocline is the locus of all the states of the set given by $\left\{(\alpha_t, \lambda_t) \in \Theta : f\left(\delta(1-\lambda_t)(\alpha_t - v)\right) - \alpha_t = 0\right\}$. Therefore, this isocline connects the equilibrium solutions E_1 and E_2, as depicted in Figure 2. In order to know more about the $\Delta\alpha_t = 0$ isocline, we can use (15) to compute the following derivative:

(24) $$\left.\frac{d\lambda_t}{d\alpha_t}\right|_{\Delta\alpha_t=0} = \frac{\delta(1-\lambda_t)f'\big(\delta(1-\lambda_t)(\alpha_t - v)\big)-1}{\delta(\alpha_t - v)f'\big(\delta(1-\lambda_t)(\alpha_t - v)\big)}.$$

The sign of (24) in the neighborhood of both monomorphic long-run equilibria ($\lambda^* = 0$ and $\lambda^* = 1$) can be determined by taking the limit of (24) as the state of the system approaches each of these equilibria. These limits are given by:

(25) $$\lim_{(\alpha_t,\lambda_t)\to(f(0),1)} \left.\frac{d\lambda_t}{d\alpha_t}\right|_{\Delta\alpha_t=0} = \frac{-1}{\delta(f(0)-v)f'(0)} < 0 \text{ and } \lim_{(\alpha_t,\lambda_t)\to(\bar{\alpha},0)} \left.\frac{d\lambda_t}{d\alpha_t}\right|_{\Delta\alpha_t=0} = \frac{\delta f'\big(\delta(\bar{\alpha}-v)\big)-1}{\delta(\bar{\alpha}-v)f'\big(\delta(\bar{\alpha}-v)\big)} < 0,$$

with the sign of the second limit in (25) coming from (A-2.2) in Appendix 2. The reason why in the vicinity of the extinction of one of the existing labor-extraction strategies, the higher is the labor-extraction differential, the lower is the proportion of profit-sharing firms, is that a higher relative earnings differential is then necessary to generate a higher labor-extraction differential (recall that because there is strategic substitutability in the firms' choice of labor-extraction compensation mode, the labor-extraction differential in (14) rises at a falling rate).

Meanwhile, the $\Delta\lambda_t \equiv \lambda_{t+1} - \lambda_t = 0$ isocline is the locus of all the states of the set given by $\{(\alpha_t,\lambda_t)\in\Theta : \lambda_t = 0\} \cup \{(\alpha_t,\lambda_t)\in\Theta : \lambda_t = 1\} \cup \{(\alpha_t,\lambda_t)\in\Theta : \alpha_t = \alpha^*\}$. As depicted in Figure 2, this set is represented by a semi-inverted H-shaped isocline.

From the local stability analysis carried out in Appendices 3-5, the following results emerge. In the configuration depicted in panel (a) in Figure 2, the additional labor-extraction gain for sharing firms returned by a given relative earnings differential is relatively low. In this case, there are only two monomorphic long-run equilibria, which are $E_1 = (\bar{\alpha}, 0)$, with no firm playing the profit-sharing labor-extraction strategy, and $E_2 = (f(0),1)$, with all firms rather playing the profit-sharing labor-extraction strategy. As shown in Appendices 3 and 4, $E_2 = (f(0),1)$ is a repulsor, while $E_1 = (\bar{\alpha}, 0)$ is an attractor.

Meanwhile, in the situation depicted in panel (c) in Figure 2, the additional labor-extraction gain for profit-sharing firms ensured by a given relative earnings differential is relatively high. Therefore, there are only two monomorphic long-run equilibria, which are

$E_1 = (\bar{\alpha}, 0)$, with all firms playing the non-sharing labor-extraction compensation strategy, and $E_2 = (f(0), 1)$, with all firms playing the sharing labor-extraction strategy. As shown in Appendices 3 and 4, while $E_2 = (f(0), 1)$ is an attractor, $E_1 = (\bar{\alpha}, 0)$ is a repulsor.

Finally, in the configuration depicted in panel (b) in Figure 2, the additional labor-extraction gain accruing to profit-sharing firms is relatively moderate. In this case, there are again two monomorphic long-run equilibria, $E_1 = (\bar{\alpha}, 0)$ and $E_2 = (f(0), 1)$, but now also one polymorphic long-run equilibrium, $E_3 = (\alpha^*, \lambda^*)$, which is characterized by heterogeneity in labor-extraction compensation strategies in the population of firms. As shown in Appendices 2-5, $E_1 = (\bar{\alpha}, 0)$ and $E_2 = (f(0), 1)$ are repulsors, whilst $E_3 = (\alpha^*, \lambda^*)$ is an attractor provided condition (A-5.4) in Appendix 5 is satisfied.

Nonetheless, since the state space of the system is positively invariant (as shown in Appendix 6), in this configuration heterogeneity in labor-extraction compensation strategies does persist in the long run even if the polymorphic long-run equilibrium is not an attractor. This positive invariance implies that, if the long-run equilibrium with coexistence of both labor-extraction compensation strategies is not an attractor, the system keeps undergoing endogenous, self-sustaining and persistent fluctuations in the frequency distribution of labor-extraction compensation strategies and the average productivity, with the average levels of the income shares and economic growth persistently fluctuating as well. In fact, Mitchell et al. (1990) and D'Art and Turner (2006) find that the adoption of profit sharing schemes has tended to be cyclical in nature in many advanced countries since the origin of these schemes in the 19th century.

It is worth computing the long-run equilibrium values of the functional distribution of income and economic growth in each one of these three configurations as regards the relative magnitude of the additional labor-extraction gain which accrues to profit-sharing firms, as depicted in Figure 2. We can use (3), (4), and (6) to establish that:

(26) $\bar{\pi}^c(\bar{\alpha}, 0) = \bar{\pi}^c(\alpha^*, \lambda^*) = \pi_n = 1 - v$,

and:

(27) $\bar{\pi}^c\left(f(0),1\right) = \pi_s^c\left(f(0)\right) = (1-\delta)\left[1 - \frac{v}{f(0)}\right]$,

where the first equality in (26) follows from the fact that $E_3 = (\alpha^*, \lambda^*)$ is defined implicitly by the condition given by $\pi_{s,t}^c(\alpha_t, \lambda_t) - \pi_n = 0$ (recall that the profit share of non-sharing firms is an exogenously given constant). In panel (a), which features $\bar{\alpha} > f(0)$, we obtain $\bar{\pi}^c(\bar{\alpha}, 0) > \bar{\pi}^c\left(f(0),1\right)$ and, consequently, using (13-a), $\bar{g}^c(\bar{\alpha}, 0) > \bar{g}^c(f(0),1)$. In panel (b), which features $\alpha^* > f(0)$, we obtain $\bar{\pi}^c(\bar{\alpha}, 0) = \bar{\pi}^c(\alpha^*, \lambda^*) > \bar{\pi}^c\left(f(0),1\right)$, with (13-a) then yielding $\bar{g}^c(\bar{\alpha}, 0) = \bar{g}^c(\alpha^*, \lambda^*) > \bar{g}^c(f(0),1)$. Meanwhile, in panel (c), in which $\alpha^* < f(0)$, we obtain $\bar{\pi}^c(\bar{\alpha}, 0) < \bar{\pi}^c\left(f(0),1\right)$, it following from (13-a) that $\bar{g}^c(\bar{\alpha}, 0) < \bar{g}^c(f(0),1)$. Thus, if the economy converges to a long-run equilibrium (which may not occur in panel (b), as shown in Appendix 5), the coupled evolutionary dynamics of profit-sharing adoption and labor-extraction gain takes the economy to a configuration in which the average values of the net profit share, net profit rate and rate of economic growth are all at their highest possible long-run equilibrium levels.

Finally, note that despite the possibility of a stable long-run equilibrium outcome with strategic dualism, the evolutionary dynamic in (15) ensures that the expected (net) rates of profit associated with the coexisting labor-extraction strategies become equalized. However, in our one-sector model, equalization across individual expected profit rates is brought about not by classical competition with capital mobility across different sectors, but by evolutionary competition with firms' (and their capital stock) mobility across alternative labor-extraction compensation strategies.

4. Conclusions

This paper is motivated by several pieces of suggestive empirical evidence. First, historically, there has been a persistent heterogeneity in worker compensation strategies across firms, and profit-sharing schemes have experienced a fluctuating popularity. Second, profit sharing raises labor productivity in the firm, and surveys find that both employers and workers usually perceive profit sharing as helping to improve firm performance in several dimensions. Third, surveys find that non-deferred and in cash profit sharing is ranked first by workers as motivation device. Fourth, firms switch mechanisms of worker compensation (which include profit sharing) frequently, with the gross changes in mechanisms being more numerous than the net changes. Fifth, profit sharing has a meaningful effect on worker total compensation, which suggests that profit sharing tends to come on top of, rather than in place of, a base wage.

Against this empirical backdrop, this paper formally models firms as periodically choosing to compensate hired workers with either solely a conventional base wage or a share of profits on top of this conventional base wage. As a result, the frequency distributions of labor-extraction compensation strategies and labor productivity in the population of firms are evolutionarily time-varying as driven by satisficing imitation dynamics. Besides, we explore the implications of this coupled evolutionary dynamics for income distribution and economic growth.

When the additional productivity gain for profit-sharing firms is relatively low, there are two long-run equilibria, one featuring all firms sharing profits and the other with no firm sharing profits. While the former is a repulsor, the latter is an attractor. When the additional productivity gain for profit-sharing is relatively high, the economy has the same two long-run equilibria. However, the long-run equilibrium with all firms sharing profits is an attractor, while the long-run equilibrium with no firm sharing profits is a repulsor. When the additional productivity gain for profit-sharing firms is relatively moderate, there is another long-run equilibrium, now one that features heterogeneity in worker compensation modes across firms. In this case, the long-run equilibria with survival of only one strategy are repulsors, while the long-run equilibrium with coexistence of both strategies can be an attractor. However, even if the long-run equilibrium configuration with coexistence of both labor-extraction strategies is not an attractor, the economy keeps undergoing endogenous, self-sustaining and persistent fluctuations in the distribution of labor-extraction compensation strategies and average labor productivity (which accords with the empirical evidence), with functional income distribution and economic growth persistently fluctuating as well.

Meanwhile, when there is convergence to some of the three long-run equilibria, the average net profit rate (which excludes shared profits) and the rates of capital accumulation and economic growth are both at their highest possible long-run equilibrium values. Besides, individual expected (net) profit rates become equalized across firms (and hence compensation strategies) even in the long-run, evolutionary equilibrium configuration with coexisting labor-extraction strategies. However, such profit rates equalization is brought about by evolutionary competition with firms' mobility across labor-extraction employee compensation strategies and not by classical competition with capital mobility across sectors.

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Appendix 1 - Existence and uniqueness of a long-run equilibrium with all firms playing the non-sharing labor-extraction strategy

Let $x = \alpha - v$ and $h(x) = f(\delta x) - v$. In order to show the existence and uniqueness of a given $\bar{\alpha} \in (v, \infty) \subset \mathbb{R}$ which satisfies (20) we have to show that the function h has a unique strictly positive fixed point, i.e, there is a unique real value $\bar{x} = \bar{\alpha} - v$ such that $\bar{x} = h(\bar{x})$.

We can accomplish this by using Theorem 3 in Kennan (2001, pp. 895), which makes it possible to ascertain that h has a unique $\bar{x} \in \mathbb{R}_{++}$, if the following conditions are satisfied: (i) h is increasing; (ii) h is strictly concave; (iii) $h(0) \geq 0$; (iv) $h(a) > a$ for some $a > 0$; and (v) $h(b) < b$ for some $b > a$.

Conditions (i) and (ii) are satisfied. Since $f'(\cdot) > 0$ and $f''(\cdot) < 0$ in the entire domain of f, it follows that for all $x \in \mathbb{R}_+$ we have:

(A-1.2) $$h'(x) = \delta f'(\delta x) > 0 \text{ and } h''(x) = \delta^2 f''(\delta x) < 0.$$

Therefore, the function h is strictly increasing and strictly concave.

Condition (iii) is also satisfied. Since $f(0) > 1 > v$, it follows that $h(0) = f(0) - v > 0$.

Condition (iv) is satisfied as well. Given that $0 < v < 1$, it follows that $0 < 1 - v < 1$. Besides, since h is strictly increasing and $f(0) > 1$, it follows that there is a given $a = 1 - v$ such that $h(1-v) = f\left(\delta(1-v)\right) - v > f(0) - v > 1 - v$.

Condition (v) is also satisfied. Recall that we have assumed that $\lim_{(y_s - \bar{y}) \to \infty} f'(y_s - \bar{y}) = 0$. Therefore, since $y_s - \bar{y} = \delta(\alpha - v) = \delta x$ for $\lambda = 0$, it follows that $\lim_{x \to \infty}(y_s - \bar{y}) = \lim_{x \to \infty} \delta x = \infty$. For $\lambda = 0$, we can deduce that $\lim_{x \to \infty} \frac{h(x)}{x} = \lim_{x \to \infty} \frac{f(\delta x) - v}{x} = \lim_{x \to \infty} \delta f'(\delta x) = 0$, where in the last equality we have used L'Hôpital's rule. Thus, we can assert that for every $\varepsilon > 0$ there is some $M > 0$ such that for all $x > M$ we have $\left|\frac{h(x)}{x}\right| < \varepsilon$. This last inequality can be re-written as $h(x) < \varepsilon x$, since $h(x) = f(\delta x) - v > f(0) - v > 0$ for all $x > 0$. We can set $\varepsilon = 1$. In this case, there is some $M>0$ such that for all $x>M$ it follows that $h(x)<x$. Thus, it is enough to choose any $x=b>Max\{M,a\}$ to obtain $h(b)<b$ and $b>a=1-v$.

Appendix 2 - Existence and uniqueness of a polymorphic long-run equilibrium (both labor-extraction compensation strategies are played across the population of firms)

Let $\phi(\alpha,\lambda)\equiv\alpha-f\left(\delta(1-\lambda)(\alpha-v)\right)$. Condition (22) is satisfied if, and only if, $\phi(\alpha^*,\lambda^*)=0$. Let us show that, given α^*, if condition (23) is satisfied, there is a unique $\lambda^*\in(0,1)\subset\mathbb{R}$ such that $\phi(\alpha^*,\lambda^*)=0$.

Given the existence and uniqueness of the fixed point $\bar{\alpha}$ shown in Appendix 1, we can use the Index Theorem (Kehoe, 1987, p. 52) to write:

(A-2.1) $$index(\bar{\alpha})=\operatorname{sgn}\left(1-\frac{\partial f\left(\delta(\bar{\alpha}-v)\right)}{\partial\alpha}\right)=1,$$

where $\operatorname{sgn}(\cdot)$ stands for the sign function. Based on this function, we can establish that:

(A-2.2) $$\frac{\partial\phi(\bar{\alpha},\lambda)}{\partial\alpha}=1-\delta f'\left(\delta(\bar{\alpha}-v)\right)>0.$$

Therefore, given the existence and uniqueness of $\bar{\alpha}$, the graph of $\phi(\bar{\alpha},\lambda)$ in the plane given by $\{(\alpha,\lambda)\in\mathbb{R}^2_+\}$ always crosses the 45-degree line only once and from above, as shown in Figure 3. Since it follows from (20) that $\phi(\bar{\alpha},0)=\bar{\alpha}-f\left(\delta(\bar{\alpha}-v)\right)=0$, we can conclude that $\phi(\alpha,0)=\alpha-f\left(\delta(\alpha-v)\right)<0$ for all $\alpha\in(v,\bar{\alpha})\subset\mathbb{R}$. For all $\alpha^*<\bar{\alpha}$, then, we can infer that:

(A-2.3) $$\phi(\alpha^*,0)=\alpha^*-f\left(\delta(\alpha^*-v)\right)<0.$$

Moreover, it is straightforward that (23) implies that:

(A-2.4) $$\phi(\alpha^*,1)=\alpha^*-f(0)>0.$$

Since $\phi(\alpha^*,0)<0$, $\phi(\alpha^*,1)>0$ and ϕ is continuous along the domain, we can then apply the intermediate value theorem to readily conclude that there is some $\lambda^*\in(0,1)\subset\mathbb{R}$ such that $\phi(\alpha^*,\lambda^*)=0$. Moreover, given that $f'(\cdot)>0$ for all $\lambda\in[0,1]\subset\mathbb{R}$, we have:

(A-2.5) $$\frac{\partial\phi(\alpha^*,\lambda)}{\partial\lambda}=\delta(\alpha^*-v)\,f'\big(\delta(1-\lambda)(\alpha^*-v)\big)>0\,,$$

for all $\lambda\in[0,1]\subset\mathbb{R}$. As a result, since the function in (A.2-5) is continuous in the closed interval $[0,1]\subset\mathbb{R}$, there is only one $\lambda^*\in(0,1)\subset\mathbb{R}$ such that $\phi(\alpha^*,\lambda^*)=0$.

Appendix 3 - Local stability of the monomorphic long-run equilibrium with all firms playing the non-sharing labor-extraction strategy

The Jacobian matrix evaluated around the equilibrium $(\bar{\alpha},0)\subset\Theta$ is given by:

(A-3.1) $$J(\bar{\alpha},0)=\left[\begin{array}{c|c}\delta f'\big(\delta(\bar{\alpha}-v)\big) & -\delta(\bar{\alpha}-v)\,f'\big(\delta(\bar{\alpha}-v)\big)\\ \hline 0 & 1+F(\pi_s^c(\bar{\alpha}))-F(\pi_n)\end{array}\right],$$

where $\pi_s^c(\bar{\alpha})=(1-\delta)\big(1-v/\bar{\alpha}\big)$ and $\pi_n=1-v$.

Let ξ be an eigenvalue of the Jacobian matrix (A-3.1). We can set the characteristic equation of the linearization around the equilibrium:

(A-3.2) $$\left|J-\xi I\right|=\left|\begin{array}{c|c}a-\xi & -b\\ \hline 0 & c-\xi\end{array}\right|=\xi^2-(a+c)\xi+ac=0\,,$$

where $a\equiv\delta f'\big(\delta(\bar{\alpha}-v)\big)>0$, $b\equiv\delta(\bar{\alpha}-v)\,f'\big(\delta(\bar{\alpha}-v)\big)>0$, and $c\equiv1+F(\pi_s^c(\bar{\alpha}))-F(\pi_n)$. The solutions of (A-3.2) are the eigenvalues of the Jacobian matrix (A-3.1), which are given by:

(A-3.3) $$\xi_1=\delta f'\big(\delta(\bar{\alpha}-v)\big)\quad\text{and}\quad\xi_2=1+F(\pi_s^c(\bar{\alpha}))-F(\pi_n).$$

Let us investigate the absolute value of ξ_1. Given that $\delta>0$ and $f'(\cdot)>0$ along the domain, it follows that $\xi_1=\delta f'\big(\delta(\bar{\alpha}-v)\big)>0$. Also, it follows from (A-2.2) that $\xi_1=\delta f'\big(\delta(\bar{\alpha}-v)\big)<1$. Hence, it follows that $|\xi_1|<1$.

Let us check the absolute value of ξ_2. We want to find under what condition(s) it follows that $-1<\xi_2<1$. Per (A-3.3), it then readily follows that $-1<\xi_2<1$ obtains if, and only if, $-2<F(\pi_s^c(\bar{\alpha}))-F(\pi_n)<0$.

Since $0 < F(\pi_s^c(\bar{\alpha})) < 1$ and $0 < F(\pi_n) < 1$, we obtain that $-1 < F(\pi_s^c(\bar{\alpha})) - F(\pi_n) < 1$. It then follows that $F(\pi_s^c(\bar{\alpha})) - F(\pi_n) > -2$. As $F(\cdot)$ is continuously differentiable and strictly increasing, $F(\pi_s^c(\alpha)) - F(\pi_n)$ is increasing in α. Given (21), we have $\pi_s^c(\alpha^*) - \pi_n = 0$ and, then that $F(\pi_s^c(\alpha^*)) - F(\pi_n) = 0$. Therefore, if $\bar{\alpha} < \alpha^*$ (panel (a) in Figure 2), we find that $\pi_s^c(\bar{\alpha}) - \pi_n < 0$, and hence that $F(\pi_s^c(\bar{\alpha})) - F(\pi_n) < 0$, which means that $|\xi_2| < 1$ if $\bar{\alpha} < \alpha^*$. But if $\bar{\alpha} > \alpha^*$ (panels (b) and (c) in Figure 2), it readily follows that $\pi_s^c(\bar{\alpha}) - \pi_n > 0$, and hence that $F(\pi_s^c(\bar{\alpha})) - F(\pi_n) > 0$, so that $|\xi_2| > 1$ if $\bar{\alpha} > \alpha^*$. This completes the demonstration that the long-run equilibrium with no firm playing the profit-sharing strategy, $(\bar{\alpha}, 0) \subset \Theta$, is an attractor if $\bar{\alpha} < \alpha^*$ (panel (a) in Figure 2) and a repulsor if $\bar{\alpha} > \alpha^*$ (panels (b) and (c) in Figure 2).

Appendix 4 - Local stability of the monomorphic long-run equilibrium with all firms playing the profit-sharing labor-extraction compensation strategy

The Jacobian matrix evaluated around the equilibrium $\left(f(0), 1\right) \subset \Theta$ is given by:

(A-4.1) $$J(f(0),1) = \left[\begin{array}{c|c} 0 & -\delta\left(f(0) - v\right) f'(0) \\ \hline 0 & 1 - F\left(\pi_s^c(f(0))\right) + F(\pi_n) \end{array}\right],$$

where $\pi_s^c(f(0)) = (1-\delta)\left[(1 - v / f(0)\right]$ and $\pi_n = 1 - v$.

Let ξ be an eigenvalue of the Jacobian matrix (A-4.1). We can set the characteristic equation of the linearization around this equilibrium:

(A-4.2) $$|J - \xi I| = \left|\begin{array}{c|c} -\xi & -a \\ \hline 0 & b - \xi \end{array}\right| = \xi(\xi - b) = 0,$$

with $a \equiv \delta\left[f(0) - v\right] f'(0) > 0$ and $b \equiv 1 - F\left(\pi_s^c(f(0))\right) + F(\pi_n)$.

In this case, the eigenvalues of the Jacobian matrix (A.4-1) are easily computed from (A-4.2):

(A-4.3) $$\xi_1 = 0 \quad \text{and} \quad \xi_2 = b = 1 - F\left(\pi_s^c(f(0))\right) + F(\pi_n).$$

Therefore, the local stability of $\left(f(0),1\right)\subset\Theta$ depends on ξ_2. Given (A-4.3), it follows that $-1<\xi_2<1$ obtains if, and only if, $0<F\left(\pi_s^c(f(0))\right)-F(\pi_n)<2$.

Since $0<F\left(\pi_s^c(f(0))\right)<1$ and $0<F(\pi_n)<1$, it follows that $-1<F\left(\pi_s^c(f(0))\right)-F(\pi_n)<1$. Consequently, we can readily infer that $F\left(\pi_s^c(f(0))\right)-F(\pi_n)<2$.

As $F(\cdot)$ is continuously differentiable and strictly increasing, it can be concluded that $F(\pi_s^c(\alpha))-F(\pi_n)$ is increasing in α. Given (21), we obtain that $\pi_s^c(\alpha^*)-\pi_n=0$ and then that $F(\pi_s^c(\alpha^*))-F(\pi_n)=0$. Therefore, if $f(0)<\alpha^*$ (panels (a) and (b) in Figure 2), we obtain that $\pi_s^c(f(0))-\pi_n<0$, and consequently, that $F\left(\pi_s^c(f(0))\right)-F(\pi_n)<0$, which means that $|\xi_2|>1$ if $f(0)<\alpha^*$. Meanwhile, if $f(0)>\alpha^*$ (panel (c) in Figure 2), we obtain that $\pi_s^c\left(f(0)\right)-\pi_n>0$, and hence that $F\left(\pi_s^c(f(0))\right)-F(\pi_n)>0$. As a result, it follows that $|\xi_2|<1$ if $f(0)>\alpha^*$. This completes the demonstration that the long-run equilibrium with all firms playing the profit-sharing labor-extraction strategy, $\left(f(0),1\right)\subset\Theta$, is a repulsor if $f(0)<\alpha^*$ (panels (a) and (b) in Figure 2) and an attractor if $f(0)>\alpha^*$ (panel (c) in Figure 2).

Appendix 5 - Local stability of the long-run equilibrium with heterogeneity in labor-extraction compensation strategies across firms

The Jacobian matrix evaluated around the equilibrium $(\alpha^*,\lambda^*)\subset\Theta$ is given by:

$$\text{(A-5.1)}\quad J(\alpha^*,\lambda^*)=\left[\begin{array}{c|c}\delta(1-\lambda^*)f'\left(\delta(1-\lambda^*)(\alpha^*-v)\right) & -\delta(\alpha^*-v)f'\left(\delta(1-\lambda^*)(\alpha^*-v)\right)\\ \hline \lambda^*(1-\lambda^*)F'(\pi_s^c(\alpha^*))(1-\delta)\dfrac{v}{(\alpha^*)^2} & 1\end{array}\right],$$

where $\pi_s^c(\alpha^*)=(1-\delta)\left[(1-v)/\alpha^*\right]$.

Let ξ be an eigenvalue of the Jacobian matrix (A-5.1). We can set the characteristic equation of the linearization around the equilibrium:

(A-5.2) $$\left|J-\xi I\right| = \left|\begin{array}{c|c} a-\xi & -b \\ \hline c & 1-\xi \end{array}\right| = \xi^2-(a+1)\xi+(a+bc)=0,$$

where $a \equiv \delta(1-\lambda^*)f'\left(\delta(1-\lambda^*)(\alpha^*-v)\right)>0$, $b \equiv \delta(\alpha^*-v)f'\left(\delta(1-\lambda^*)(\alpha^*-v)\right)>0$, and $c \equiv \lambda^*(1-\lambda^*)F'(\pi_s^c(\alpha^*))(1-\delta)\frac{v}{(\alpha^*)^2}$.

We can use the Samuelson stability conditions for a second order characteristic equation to determine under what conditions the two eigenvalues are inside the unit circle. Based on Farebrother (1973, p. 396, inequalities 2.4 and 2.5), we can establish the following set of simplified Samuelson conditions for the quadratic polynomial in (A-5.2):

(A-5.3) $$1+a+bc>\left|-(a+1)\right|=a+1 \quad \text{and} \quad a+bc<1.$$

Let us prove that these conditions are satisfied if $a<1$.

First, note that $1+a+bc>a+1$ simplifies to $bc>0$, which is trivially satisfied given that $b>0$ and $c>0$.

Meanwhile, the second inequality, $a+bc<1$, can be expressed as follows:

(A-5.4) $$a+bc=\delta(1-\lambda^*)f'(\cdot)\left\{1+(\alpha^*-v)\lambda^*F'(\pi_s^c(\alpha^*))(1-\delta)\frac{v}{(\alpha^*)^2}\right\}<1.$$

Appendix 6 - Positive invariance of the state space

We want to show that $(\alpha_t,\lambda_t)\in\Theta$ for all $t\in\{1,2...\}$ and initial condition $(\alpha_0,\lambda_0)\in\Theta$.

Let us first demonstrate that $\alpha_{t+1}>v$ for all $t\in\{0,1,2...\}$ and $(\alpha_0,\lambda_0)\in\Theta$. Given (15) and the assumptions that $f(0)>1>v$ and $f(.)$ is strictly increasing, we can then establish that $\alpha_{t+1}=f\left(\delta(1-\lambda_t)(\alpha_t-v)\right)\geq f(0)>v$ for any $\alpha_t>v$ and $0\leq\lambda_t\leq 1$. By induction, we can conclude that $\alpha_{t+1}>v$ for all $t\in\{0,1,2...\}$ and $(\alpha_0,\lambda_0)\in\Theta$.

Next, let us prove that $0\leq\lambda_{t+1}\leq 1$ for all $t\in\{0,1,2...\}$ and $(\alpha_0,\lambda_0)\in\Theta$. Let us first show that $\lambda_{t+1}\geq 0$ for all $(\alpha_t,\lambda_t)\in\Theta$. As $\lambda_t\geq 0$ for any $(\alpha_t,\lambda_t)\in\Theta$ and given (19), in order to show that $\lambda_{t+1}\geq 0$ for all $(\alpha_t,\lambda_t)\in\Theta$, we need to show that for all $(\alpha_t,\lambda_t)\in\Theta$ we have:

(A-6.1) $(1-\lambda_t)[F(\pi_{s,t}^c)-F(\pi_{n,t})] \geq -1$.

As $F(\cdot)$ is continuously differentiable and strictly increasing and $0 < F(\pi_{s,t}^c) \leq F(1-\delta) < 1$ for any $(\alpha_t, \lambda_t) \in \Theta$, we can set up the following lower and upper bounds for the probability differential for any $\alpha_t > v$:

(A-6.2) $-F(\pi_{n,t}) < F(\pi_{s,t}^c) - F(\pi_{n,t}) \leq F(1-\delta) - F(\pi_{n,t})$.

From (A-6.2) and the fact that $0 < F(\pi_{n,t}) < 1$, we can write:

(A-6.3) $(1-\lambda_t)[F(\pi_{s,t}^c)-F(\pi_{n,t})] \geq -(1-\lambda_t)F(\pi_{n,t}) > -1$

for any $(\alpha_t, \lambda_t) \in \Theta$. Therefore, by induction, $\lambda_{t+1} \geq 0$ for all $t \in \{0,1,2...\}$ and $(\alpha_0, \lambda_0) \in \Theta$.

Finally, let us demonstrate that $\lambda_{t+1} \leq 1$ for all $(\alpha_t, \lambda_t) \in \Theta$. Given (19), in order to establish that $\lambda_{t+1} \leq 1$ for all $(\alpha_t, \lambda_t) \in \Theta$, we need to demonstrate that:

(A-6.4) $\lambda_t \left\{1+(1-\lambda_t)[F(\pi_{s,t}^c)-F(\pi_n)]\right\} \leq 1$.

This inequality is trivially satisfied for $\lambda_t = 0$. For any $\lambda_t > 0$, we can re-write (A-6.4) as:

(A-6.5) $F(\pi_{s,t}^c) - F(\pi_n) \leq \dfrac{1}{\lambda_t}$.

Since $0 < F(\pi_{s,t}^c) < 1$ and $0 < F(\pi_n) < 1$, we can again make use of (A-6.2) to conclude that for any $\lambda_t > 0$, we have:

(A-6.6) $F(\pi_{s,t}^c) - F(\pi_n) \leq F(1-\delta) - F(\pi_n) < 1 \leq \dfrac{1}{\lambda_t}$.

This completes the proof that the state space Θ is positively invariant, so that $(\alpha_t, \lambda_t) \in \Theta$ for all $t \in \{1,2...\}$ and initial condition $(\alpha_0, \lambda_0) \in \Theta$.