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JEL Codes: E03; E31; C62; C73.

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1. Introduction

There is considerable empirical evidence from both survey data and laboratory experiments that inflation expectations are persistently heterogeneous and formed mostly through boundedly rational, rule-of-thumb mechanisms or heuristics (see, e.g., Hommes, 2013, for a detailed overview). Moreover, there is also empirical evidence from survey data that inflation forecast errors must pass some threshold before decision makers abandon their previously selected inflation predictor (Branch, 2004, 2007).

Motivated by these several pieces of empirical evidence (which we report and discuss more thoroughly in the next section), we extend a simple macroeconomic model to explore the implications for macroeconomic stabilization of heterogeneous inflation expectations which vary over time according to *satisficing evolutionary dynamics* (with and without noise). From a methodological perspective, the intended contribution of this paper lies in particular in an alternative modeling of the dynamics of inflation predictor switching when inflation forecast errors have to cross a threshold in order to induce a switch. Our alternative modeling is intended to incorporate such threshold effect in a more natural and fitting way than has been done in the existing literature (Branch, 2004, 2007, and Lines and Westerhoff, 2010), in which such threshold is conceived of as a component of the cost associated with the use of a predictor (and a cost which is usually taken as exogenously fixed and deterministic).

In fact, as better elaborated in the subsequent section, our formal methodological approach, which follows the spirit of the contributions of Herbert Simon, conceives of choice as intending to meet an acceptability threshold rather than to select the best of all alternatives, hence the notion of *satisficing behavior*. Therefore, while in Branch (2004, 2007) and Lines and Westerhoff (2010) the predictor selection process follows a discrete choice model (the multinomial logit model) set forth in Brock and Hommes (1997), in the model of this paper the predictor choice follows satisficing evolutionary dynamics governed by performance differentials.

In keeping with the empirical evidence on inflation expectations and following Lines and Westerhoff (2010), the aggregate expected inflation is modeled in this paper as a weighted average of the inflation forecast of agents who follow an *ex ante* costly full rationality or perfect foresight strategy and the inflation forecast of agents who use an *ex ante* costless bounded rationality or adaptive foresight strategy. However, differently from Lines and Westerhoff's discrete choice framework, our proposed satisficing evolutionary dynamics generates *pervasive adaptive foresight* (all agents eventually use the bounded rationality inflation forecasting strategy) as a long-run equilibrium in the absence of a perturbation (noise) analogous to mutation in natural environments. Meanwhile, in the presence of this mutation the satisficing evolutionary dynamics yields heterogeneity in inflation expectations as a long-run equilibrium outcome (a result which is in keeping with the empirical evidence reported above on the persistence of heterogeneous inflation expectations). In either case (with and without mutation) the long-run equilibrium is nonetheless characterized by the natural values of the output growth, unemployment and inflation being achieved. In fact, a long-run equilibrium solution consistent with the achievement of the natural values of the endogenous macroeconomic variables does not emerge because all agents adopt the perfect foresight inflation forecasting strategy. Instead, all agents (or the majority of them) eventually adopt the bounded rationality inflation forecasting strategy without making any inflation forecasting errors because the natural values of the output growth, unemployment and inflation are being continuously reached.

As the general case, our model features "mutation" as an exogenous noise in the satisficing evolutionary mechanism, leading some decision makers to select an inflation forecasting strategy at random. We should emphasize that "mutation" is a metaphor for all sorts of reasons, outside the more explicit part of the model, for why agents might switch inflation predictor, and we take mutations as being independent across agents. Two rationales for the presence of mutation are that an agent exits the economy with some (fixed) probability and is replaced with a new agent knowing nothing about the decision-making environment, or that each agent simply experiments occasionally with exogenously fixed probability. A question that arises is whether the presence of such noise component happens to prevent convergence towards a long-run equilibrium consistent with the achievement of the natural values of the endogenous macroeconomic variables, and the answer is no. Yet, the long-run equilibrium frequency distribution of inflation forecasting strategies *does* depend on whether the satisficing evolutionary dynamics are perturbed: in the absence (presence) of mutant agents, the long-run equilibrium configuration, which is an attractor, is given by the natural values of the endogenous macroeconomic variables and the extinction of the full rationality inflation forecasting strategy (survival of both inflation forecasting strategies). Hence, pervasive perfect foresight is neither a necessary condition for the achievement of the natural values of the output growth, unemployment and inflation, nor an inevitable consequence of the achievement of these natural values. In fact, our analytical results show that heterogeneous inflation expectations across the population of private decision makers need not prevent the successful handling of macroeconomic stabilization policy. This is actually true even when inflation expectations are not only heterogeneous but also, as a direct consequence of the propensity of decision makers to switch between inflation forecasting strategies based on satisficing criteria, endogenously time-varying.

The remainder of the paper is organized as follows. The next section presents the motivating empirical evidence and briefly discusses (and broadly compares the present contribution to) the related literature, while Section 3 lays out the structure of the basic macroeconomic model on which our investigation is based. Section 4 examines the consequences for macroeconomic stability and the achievement of the natural values of the output growth, unemployment and inflation in the long-run equilibrium outcome of heterogeneous inflation expectations that decision makers switch between according to satisficing evolutionary dynamics. Section 5 then concludes.

2. Motivating evidence and related literature

The already extensive empirical literature on inflation expectations formation offers support for the motivation of this paper as stated in the preceding section. In fact, this literature can be divided into two groups depending on the type of evidence used in the investigation: studies based on survey data; and laboratory experiments with human subjects. In either case, a common finding is the strong empirical support for time-varying heterogeneity in the formation of inflation expectations with predominance of some form of bounded rationality (see, e.g., Duffy, 2008, and Hommes, 2011, 2013, for comprehensive overviews of experiments dealing with the formation of expectations about macroeconomic variables). The main reasons for this heterogeneity that have been offered in the literature are that agents rely on different models, have access to different information sets, or have different cognitive capacities for processing information.

In the experimental literature on inflation expectations, for instance, Adam (2007) finds that subjects' inflation expectations are not captured by the rational expectations predictor, while predictors based on lagged inflation capture inflation expectations quite well. Assenza et al. (2011) find that subjects use simple inflation forecasting heuristics that anchor their predictions on past observations, with individual learning taking the form of switching from one heuristic to another. Pfajfar and Zakelj (2011) provide substantial evidence in support of heterogeneity in the inflation forecasting process both across subjects and time. They find that subjects form expectations in accordance with different theoretical models, and although the most popular rule is trend extrapolation, a significant share of the population uses adaptive expectations, adaptive learning (as in Evans and Honkapohja, 2001) or sticky information type models (as in Mankiw and Reis, 2002). The authors also find that rather than sticking to one single model, subjects frequently switch between alternative models. Pfajfar and Zakelj (2014) also find evidence that switching between different rules to form inflation expectations describes the behavior of subjects better than a single rule. Although they cannot reject rational expectations for about 30-45% of subjects, they also find support for simpler heuristics such as trend extrapolation, adaptive expectations and sticky information for about 35-50% of subjects. Experimental evidence for inflation expectations heterogeneity is also found in Roos and Luhan (2013), who verify that adaptive and static (or naïve) inflation expectations are frequently observed.

Heterogeneous and time-varying inflation expectations have been also found in empirical analyses based on survey data. In Carroll (2003), for instance, the typical US household is found to update expectations infrequently, and does so to the most recently reported past statistics rather than to the rational forward-looking forecast. Mankiw, Reis and Wolfers (2004) employ survey data for the US to find extensive heterogeneity in inflation expectations, and to obtain only modest confirmation of perfect rationality. Capistrán and Timmermann (2009) find heterogeneity in inflation expectations across US professional forecasters that varies over time depending on the level and the variance of current inflation. Blanchflower and MacCoille (2009), employing survey data for the UK, find strong support for heterogeneous and backward-looking inflation expectations. Weber (2010) uses survey data for five core European countries to detect that there is very little evidence that the inflation expectations of both households and professional are rational. Pfajfar and Santoro (2010) study the heterogeneity in inflation forecasts by exploring time series of percentiles from the empirical distribution of US survey data. They identify three regions of the distribution that correspond to different mechanisms of expectation formation: a static or highly autoregressive region on the left hand side of the median, a nearly rational or unbiased region around the median and a proportion of forecasts on the right hand side of the median formed according with adaptive behavior and sticky information.

The recent literature also features macroeconomic models with heterogeneous inflation expectations from which predictions are sometimes tested empirically. Branch (2004) develops a model where agents form their forecasts of inflation by selecting a predictor from a set of costly alternatives whereby they may rationally choose a method other than the most accurate. Agents are seen as rationally heterogeneous in the sense that each predictor choice is optimal for them; strictly speaking, agents' expectations are seen as boundedly rational and consistent with optimizing behavior. When the model is used to test whether US survey data exhibit these rationally heterogeneous expectations, it is found that there is dynamic switching that depends on the relative mean squared errors of the predictors. Agents are identified with three forecast mechanisms: a vector autoregressive forecast (which is seen as a boundedly rational predictor that is in the spirit of rational expectations); adaptive expectations; and static expectations. The author finds that, on average, agents adopt the vector autoregressive method more often than the other methods. However, when a costly rational predictor is included instead of the vector autoregressive forecast, the vast majority of agents behave adaptively. Branch (2007) uses US survey data on inflation expectations to compare two models of sticky information against the rationally heterogeneous expectations model set forth in Branch (2004). The author finds robust evidence that the distribution of agents across predictors is time varying.

Meanwhile, in Brazier et al. (2008) agents employ two heuristics to forecast inflation: one is based on one-period lagged inflation, the other on the inflation target set by the monetary authority (which is the steady-state inflation). Agents observe the performance of these heuristics with noise, but the better the true past performance of a heuristic, the greater chance there is that an agent employs it to make the next period's forecast. The authors find that, on average, the majority of agents use the inflation-target heuristic, although there are times when everyone does, and times when no one does. While Brazier et al. (2008) embed those heuristics to forecast inflation in a monetary overlapping-generations model and heuristic switching is described by a discrete choice model à la Brock and Hommes (1997), in this paper a different pair of forecasting strategies is embedded in a macroeconomic model with output growth, unemployment and inflation as endogenous variables, and (more importantly) agents switch between forecasting strategies according to satisficing evolutionary dynamics with and without noise. De Grauwe (2010) develops a macroeconomic model with agents featuring cognitive limitations and use simple but biased heuristics to forecast future inflation. The author follows Brazier et al. (2008) in assuming the same two inflation forecasting rules. The market forecast is a weighted average of these two forecasts, with these weights being subject to predictor selection dynamics based on discrete choice theory. While De Grauwe's (2010) is a three-equation model generating endogenous and self-fulfilling waves of optimism and pessimism, the model of this paper explores whether there is convergence towards an equilibrium configuration featuring the output growth, unemployment and inflation at their natural levels when private agents switch between inflation forecasting strategies based on satisficing evolutionary dynamics. Branch and McGough (2009) set forth a model with agents (exogenously) split between rational and adaptive forecasters and monetary policy follows a Taylor-type rule. As it turns out, the dynamic properties of the model depend crucially on the frequency distribution of agents across forecasting models. Even though in the model of this paper inflation forecasting heterogeneity also involves rational and adaptive expectations and is endogenously varying rather than exogenously fixed, agents switch between forecasting strategies according to satisficing evolutionary dynamics with and without noise. Branch and McGough (2010) embed dynamic predictor selection in a model featuring heterogeneity in inflation expectations. They extend Branch and McGough (2009) by following Brock and Hommes (1997), setting the degree of heterogeneity to vary over time depending on past forecast errors (net of a fixed cost), thereby coupling predictor choice with the dynamics of output and inflation. Agents choose between using a costly rational predictor and using a costless adaptive predictor. It is found that for sufficiently low costs, the model's steady state is stable. For higher costs, however, the steady state may destabilize and the system may bifurcate. In the model in this paper, the choice of foresight strategy is also coupled with the dynamics of inflation and output (and unemployment as well), and involves costly rational and costless adaptive forecasts, but (as already established above) agents switch between available forecasting strategies in accordance with satisficing evolutionary dynamics (with and without noise) rather than the discrete choice model developed in Brock and Hommes (1997).

As intimated earlier, our contribution is mostly related to the model set forth in Lines and Westerhoff (2010), in which the interplay between heterogeneity in inflation expectations and the coupled dynamics of inflation, output growth and unemployment is explored. Agents switch between trend-following (which is costless) and perfect foresight (which is costly) strategies to form inflation expectations, and the proportions of agents using one or the other predictor is updated over time in accordance with the discrete choice model set forth in Brock and Hommes (1997). The authors show analytically that the unique equilibrium of inflation, output growth and unemployment features the natural values of these macroeconomic variables, but this equilibrium outcome loses stability through a bifurcation at a critical value which depends on the cost associated with perfect foresight and the parameter measuring how sensitive agents are to selecting the most attractive predictor. The authors also conduct numerical simulations of the model to investigate its global dynamic behavior and discuss conditions under which it leads to endogenous, complex fluctuations in macroeconomic variables and the proportions of agents using one or the other predictor.

Our contribution differs from the interesting one offered in Lines and Westerhoff (2010) in several respects. First, although we employ the same macroeconomic model (which features Okun's law, an expectations-augmented Phillips curve and an aggregate demand relation), and likewise assume that agents switch between costless boundedly rational foresight and costly perfect foresight to form inflation expectations, in our model instead of trend-followers there are adaptive forecasters who base their inflation expectations on all past values of inflation, which implies the well-known adaptive expectations' error-adjustment mechanism which involves just one past period (from which we obtain a lower-order law of motion of the inflation rate). Second, the two models differ considerably in the way they incorporate the empirical evidence found in Branch (2004) that inflation forecast errors must cross a threshold in order to induce a switch of inflation predictors. Lines and Westerhoff (2010) follow the suggestion made in Branch (2004, 2007) to conceive of the cost associated with the use of a predictor in the Brock and Hommes (1997) framework as also reflecting (in addition to information collection and processing costs) this inertial behavior (or predisposition effect) based on acceptable ranges of forecast errors. In this paper, although we maintain the assumption that there is a cost associated with perfect foresight, this inertial behavior is modeled in a more natural and fitting way by incorporating Herbert Simon's appealing notion of satisficing, which conceives of choice as intending to meet an acceptability threshold rather than to select the best of all alternatives. Furthermore, we treat the threshold (or predisposition) effect found empirically in Branch (2004) as randomly agent-specific. In Brock and Hommes (1997), agents employ a discrete choice model to select a predictor where the deterministic part of the utility of the predictor is the performance measure. While the standard discrete choice model features deterministic and random individual-specific characteristics, in our model the payoff of an individual agent playing a given foresight strategy features both a deterministic component and a random agent-specific threshold (or predisposition effect). In our model such random component is therefore not random shocks faced by the agent, but random agent-specific thresholds, although in both cases the random component influences the payoff of each of the possible choices.

Third, in Lines and Westerhoff (2010) the frequency distribution of inflation predictors in the population of agents evolves over time in accordance with the discrete choice, predictor switching mechanism set forth in Brock and Hommes (1997), while in this paper this frequency distribution follows satisficing evolutionary dynamics with and without mutation. As a result, in Lines and Westerhoff (2010), as in Brock and Hommes (1997), for a strictly positive and finite intensity of choice (or intensity with which firms react to increases in relative net benefit) and information cost, there is a strictly positive share of firms using the strictly dominated perfect foresight strategy even in the longer-run equilibrium. In our model without mutation, however, strictly dominated foresight strategies vanish asymptotically. Indeed, in our model, the long-run evolutionarily satisficing equilibrium is characterized either by all firms playing the adaptive foresight strategy (in the absence of mutation) or by most firms playing such strategy (in the presence of mutation). In either case, however, as in Lines and Westerhoff (2010), the long-run equilibrium is characterized by the natural values of the output growth, unemployment and inflation being achieved.

Fourth, the model in Lines and Westerhoff (2010) can exhibit local instability of the longer-run equilibrium and complicated global equilibrium dynamics, while in our model (with and without mutation) the long-run evolutionary equilibrium is unique and locally stable. In Lines and Westerhoff (2010), as in Brock and Hommes (1997), a rational choice between a cheap destabilizing (adaptive foresight) predictor and a costly (perfect foresight) stabilizing predictor, leads to the existence of a very complicated dynamics when the intensity of choice to switch predictors is high. In other words, the model in Lines and Westerhoff (2010) shows that with information costs it may be rational for firms to select methods other than perfect foresight, with the conflict between cheap free riding and costly sophisticated prediction being a potential source of instability and complicated global dynamics. In our model, meanwhile, the conflict between free riding and costly sophisticated prediction is instead a source of stability even if there is mutation, so that the assertion by Brock and Hommes (1997) that instability is inherent in such situations when more sophisticated prediction methods are more expensive is not confirmed in our model. In fact, our model embeds satisficing learning as an evolutionary dynamics that succeeds in taking the strategy-revision process to a stable long-run equilibrium configuration where, despite either the majority or even all of the agents play the costless adaptive foresight strategy, the rates of output growth, unemployment and inflation all achieve their natural values.

3. The model

3.1. The macroeconomic setting

As in Lines and Westerhoff (2010), the basic macroeconomic model on which the analysis in this paper is conducted can be stated as follows:

(1) $$u_t - u_{t-1} = -\beta(g_t - g_n),$$

(2) $$\pi_t = \pi_t^e - \alpha(u_t - u_n),$$

(3) $$g_t = m - \pi_t,$$

where u_t denotes the unemployment rate in period *t*, u_n represents the natural (or normal) unemployment rate, g_t is the output growth rate in period *t*, g_n is the natural (or normal) output growth rate, m stands for the nominal money growth rate, π_t is the actual rate of inflation in period *t*, π_t^e is the aggregate expected rate of inflation for period *t*, which is formed by agents in period $t-1$, and lower case Greek letters denote strictly positive parameters. Equation (1) is simply Okun's law, according to which changes in the rate of unemployment are negatively related to the rate of output growth, equation (2) is an expectations-augmented Phillips curve, and equation (3) is a simple linear aggregate demand relation.

Inserting (2) into (3) and the resulting expressions into (1), we get:

(4) $$u_t - u_n = \frac{1}{1+\alpha\beta}(u_{t-1} - u_n) + \frac{\beta}{1+\alpha\beta}(\pi_t^e - \pi_n),$$

where $\pi_n = m - g_n$ is the natural (or normal) inflation rate. Substituting (4) in (2), we obtain:

(5) $$\pi_t - \pi_n = -\frac{\alpha}{1+\alpha\beta}(u_{t-1} - u_n) + \frac{1}{1+\alpha\beta}(\pi_t^e - \pi_n).$$

Therefore, for a given vector of structural and policy parameters given by $(\alpha, \beta, u_n, \pi_n = m - g_n)$, the state transition of the unemployment rate and the actual inflation depends not only on the current unemployment rate, but also on the aggregate expected inflation.

3.2. Inflation expectations formation

At a given period *t*, each agent either follows the costly perfect foresight strategy, so that her expected rate of inflation is given by:

(6) $$\pi_t^R = \pi_t,$$

or follows the costless adaptive foresight strategy and form inflation expectations given by:

(7) $$\pi_t^A = \pi_{t-1}^A + \gamma(\pi_{t-1} - \pi_{t-1}^A),$$

where $\gamma \in (0,1) \subset \mathbb{R}$ is a parameter.[1]

At a given period *t* there is a fraction $x_t \in [0,1] \subset \mathbb{R}$ of the population of agents, which may vary from one period to the next one, forming inflation expectations adaptively, that is, they use the bounded rationality predictor in (7). These agents are *adaptive forecasters*. The remaining fraction, $1 - x_t$, is made up of fully informed agents that pay an exogenously fixed and strictly positive cost to play the perfect foresight strategy in (6). These agents are *perfect forecasters*. Given this predictor distribution

[1] The error-adjustment mechanism in (7) is obtained under the assumption that the current expected rate of inflation is a weighted average of all past inflation rates given by $\pi_t^A = \sum_{\tau=1}^{\infty} f(\tau)\pi_{t-\tau}$, where $f(\tau) = (1-\gamma)\gamma^{\tau-1}$ is a weighting function.

across the population of agents, the average expected rate of inflation rate can then be expressed as a weighted linear combination of these two predictors as follows:

(8) $$\pi_t^e = x_{t-1}\pi_t^A + (1 - x_{t-1})\pi_t^R .$$

Therefore, the cost associated with perfect foresight is actually the cost associated with a potential heterogeneity in the choice of inflation forecasting strategy. In fact, as shown in (8), playing the perfect foresight strategy requires knowing the frequency distribution of inflation forecasting strategies across agents.

Using (6), (7) and (8), the gap between the average expected rate of inflation and the natural (or normal) rate of inflation can be expressed as follows:

(9) $$\pi_t^e - \pi_n = x_{t-1}[\gamma(\pi_{t-1} - \pi_n) + (1-\gamma)(\pi_{t-1}^A - \pi_n)] + (1 - x_{t-1})(\pi_t - \pi_n) .$$

3.3 Average inflation expectations dynamics as satisficing evolutionary dynamics

Although the frequency distribution of predictors is given at a given period *t*, it evolves over time in accordance with satisficing evolutionary dynamics. Yet, as in Lines and Westerhoff (2010), we assume that agents prefer inflation predictors with a high forecasting accuracy and, therefore, rely on squared prediction error as a (publicly observable) fitness measure. Using the same nomenclature as in Lines and Westerhoff (2010), the attractiveness of the adaptive forecasting strategy can be defined as follows:

(10) $$a_t^A = -(\pi_t^A - \pi_t)^2 .$$

Given (6), it follows that $\pi_t^R - \pi_t = 0$. Hence, the attractiveness of the perfect foresight strategy is given by:

(11) $$a_t^R = -(\pi_t^R - \pi_t)^2 - \kappa = -\kappa ,$$

where $\kappa \in \mathbb{R}_{++}$ stands for the cost associated with playing the perfect foresight strategy.

Let us now describe the satisficing evolutionary dynamics which yield the law of motion of the proportion of adaptive forecasters, x_t, and which is in the spirit of the contributions of Herbert Simon.[2] As classically elaborated by Simon (1955, 1956), satisficing is a theory of choice centered on the process through which available

[2] The formal derivation of this satisficing evolutionary dynamics is based on Vega-Redondo (1996, p. 91).

alternatives are examined and evaluated. By conceiving of choice as intending to meet an acceptability threshold rather than to select the best of all alternatives, satisficing theory contrasts with optimization theory. As Simon suggests, this contrast is analogous to 'looking for the sharpest needle in the haystack' (i.e., optimizing) versus 'looking for a needle sharp enough to sew with' (i.e., satisficing) (Simon, 1987, p. 244). In our specification, an adaptive forecaster *i* takes the current attractiveness given by (10) and compares it with the attractiveness she considers *acceptable*, which is denoted by a^i. Taking $t-1$ as the *current* period, if $a^i \leq a^A_{t-1}$ (i.e., if $\left|a^i\right| \geq \left|a^A_{t-1}\right|$), the adaptive forecasters *i* does not consider changing her strategy for forming inflation expectations in *t*. Otherwise, the adaptive forecaster *i* becomes a strategy reviser. The attractiveness that is acceptable to an agent depends, inter alia, on idiosyncratic features. We assume that acceptable attractiveness is randomly and independently determined across agents and over time. More precisely, we assume that a^i is a random variable with cumulative distribution function $F : \mathbb{R}_{-} \rightarrow [0,1] \subset \mathbb{R}$ which is continuously differentiable. As a result, the probability of randomly choosing an agent *i* in the subpopulation of adaptive forecasters who considers the current attractiveness a^A_{t-1} as acceptable is given by:

(12) $$\Pr\left(\left|a^i\right| \geq \left|a^A_{t-1}\right|\right) = \Pr(a^i \leq a^A_{t-1}) = F(a^A_{t-1}).$$

Consequently, the probability that a randomly drawn agent *i* in the subpopulation of adaptive forecasters will come to consider that the currently observed attractiveness is unacceptable is simply:

(13) $$\Pr\left(\left|a^i\right| < \left|a^A_{t-1}\right|\right) = \Pr(a^i > a^A_{t-1}) = 1 - F(a^A_{t-1}).$$

The measure of adaptive forecasters who become perfect forecasters is then given by:

(14) $$x_{t-1}[1 - F(a^A_{t-1})].$$

Analogously, the measure of perfect forecasters who becomes adaptive forecasters is given by:

(15) $$(1 - x_{t-1})\Pr\left(\left|a^i\right| < \left|a^R_{t-1}\right|\right) = (1 - x_{t-1})\Pr(a^i > a^R_{t-1}) = (1 - x_{t-1})[1 - F(a^R_{t-1})].$$

Therefore, subtracting (14) from (15) and using (10)-(11) yield the following satisficing evolutionary dynamics:

(16) $$x_t - x_{t-1} = (1 - x_{t-1})[1 - F(-\kappa)] - x_{t-1}[1 - F(-(\pi^A_{t-1} - \pi_{t-1})^2)].$$

As it turns out, an increase in the relative attractiveness of the adaptive foresight strategy in the current period leads to an increase in the proportion of agents playing this strategy in the next period, while the opposite occurs when the relative attractiveness of the perfect foresight strategy rises. Hence, the satisficing evolutionary dynamics in (16) reflects the operation of a selection mechanism according to which the proportion of agents playing a given inflation forecasting strategy varies positively with the relative fitness or attractiveness of such strategy.

To gain in generality, we also consider the possibility that the satisficing evolutionary dynamics in (16) operate in the presence of a perturbation term, analogous to mutation in natural environments. In a biological setting, mutation is interpreted literally as consisting of random changes in genetic codes. In economic settings, as pointed out by Samuelson (1997, ch. 7), mutation refers to a situation in which a player refrains from comparing payoffs and changes strategy at random. Therefore, the present extension features mutation as exogenous perturbation in the satisficing evolutionary mechanism leading some decision makers to choose an inflation foresight strategy at random. Following Kandori, Mailath and Rob (1993), two rationales for random choice are that an agent exits the economy with some (fixed) probability and is replaced with a new agent who happens to know nothing about (or is inexperienced in) the decision-making process, or that each agent simply "experiments" occasionally with exogenously fixed probability. This noise component could also capture the effect, for instance, of exogenous institutional factors such as changes of administration in the monetary authority or other changes in the policy-making framework.

Drawing on Gale, Binmore and Samuelson (1995), mutation can be incorporated into the satisficing evolutionary mechanism in (16) as follows. Let $\varepsilon \in (0,1) \subset \mathbb{R}$ be the measure of mutant agents that choose an inflation foresight strategy in a given revision period independently of the respective payoffs. Therefore, there are εx_{t-1} adaptive forecasters and $\varepsilon(1 - x_{t-1})$ perfect forecasters behaving as mutants. We assume that mutant agents choose either one or the other of the two inflation foresight strategies with equal probability, so that there are $\varepsilon x_{t-1}/2$ adaptive forecaster mutant agents and $\varepsilon(1 - x_{t-1})/2$ perfect forecaster mutant agents changing foresight strategy. The net flow of mutant agents becoming adaptive forecaster agents in a given revision period, which can be either positive or negative, is then the following:

$$\varepsilon\left(1-x_{t-1}\right)\frac{1}{2}-\varepsilon x_{t-1}\frac{1}{2}=\varepsilon\left(\frac{1}{2}-x_{t-1}\right). \tag{17}$$

Following Gale, Binmore and Samuelson (1995), this perturbation can then be added to the evolutionary mechanism (16) to yield the following *noisy satisficing evolutionary dynamics*:

$$x_t-x_{t-1}=(1-\varepsilon)\left\{(1-x_{t-1})[1-F(-\kappa)]-x_{t-1}[1-F(-(\pi_{t-1}^A-\pi_{t-1})^2)]\right\}+\varepsilon\left(\frac{1}{2}-x_{t-1}\right). \tag{18}$$

4. Behavior of the model in the long-run evolutionary equilibrium

Note that the macrodynamics of the rates of output growth, unemployment and inflation is coupled with the microdynamics of the frequency distribution of inflation foresight strategies across firms. In fact, plugging (9) into (5), we can establish that:

$$\pi_t-\pi_n=\frac{1}{\alpha\beta+x_{t-1}}\left\{-\alpha(u_{t-1}-u_n)+x_{t-1}\left[\gamma(\pi_{t-1}-\pi_n)+(1-\gamma)(\pi_{t-1}^A-\pi_n)\right]\right\}. \tag{19}$$

Meanwhile, inserting (19) into (9) and the resulting expressions into (4), we get:

$$u_t-u_n=\frac{x_{t-1}}{\alpha\beta+x_{t-1}}\left\{u_{t-1}-u_n+\beta\left[\gamma(\pi_{t-1}-\pi_n)+(1-\gamma)(\pi_{t-1}^A-\pi_n)\right]\right\}. \tag{20}$$

Therefore, the state transition of the economy is determined by the difference equation system given by (18)-(20) and the difference equation in (7), which can be re-written in terms of gaps with relation to the natural (or normal) inflation as follows:

$$\pi_t^A-\pi_n=\gamma(\pi_{t-1}-\pi_n)+(1-\gamma)(\pi_{t-1}^A-\pi_n). \tag{21}$$

As it turns out, the state transition of the economy is determined by the dynamic system given by (18)-(21), whose state space is $\Theta=\{(x_t,\pi_t,u_t,\pi_t^A)\in\mathbb{R}^4:0\le x_t\le 1,0\le u_t\le 1\}$.

In the following proposition we establish the existence and uniqueness of the evolutionary long-run equilibrium of the dynamic system given by (18)-(21).

Proposition 1: *For a given vector of parameters* $(\alpha, \beta, u_n, \pi_n = m - g_n, \varepsilon)$ *, the dynamic system given by* (18)-(21) *has a unique long-run evolutionary equilibrium which is given by* $(x^*, \pi_n, u_n, \pi_n) \in \Theta$*, with* $x^* = \dfrac{(1-\varepsilon)[1-F(-\kappa)] + \varepsilon/2}{(1-\varepsilon)[1-F(-\kappa)] + \varepsilon} \in (1/2, 1] \subset \mathbb{R}$.

<u>Proof</u>: See Appendix 1.

In the long-run evolutionary equilibrium, therefore, the adaptive forecasting strategy is either the only to survive ($x^* = 1$) in the absence of mutation ($\varepsilon = 0$) or the most played strategy ($1/2 < x^* < 1$) in the presence of mutation ($0 < \varepsilon < 1$). Note that the equilibrium values of the endogenous macroeconomic variables (π_t, u_t, and π_t^A) are identical in both cases. Although pervasive adaptive foresight (all agents eventually use the boundedly rational inflation forecasting strategy) is an evolutionarily satisficing long-run equilibrium outcome when mutation is absent, this equilibrium configuration is nonetheless characterized by the natural values of the rates of inflation, output growth and unemployment being achieved. The intuition is straightforward: when mutation is absent, once the economy converges to the long-run equilibrium, the adaptive foresight strategy fares as well as the perfect foresight strategy, but it is less costly. In fact, given the nature of the information imperfection which imposes a cost on perfect foresight, all agents playing the perfect foresight strategy ($x = 0$) is not a long-run equilibrium since it means that all agents are paying the cost to learn the measure of a heterogeneity that does not exist. Meanwhile, when there is no mutation, $x^* = 1$ is a long-run equilibrium since it means that no agent is paying the cost of an inexistent heterogeneity, and when there is mutation, $1/2 < x^* < 1$ is a long-run equilibrium because there is a heterogeneity cost to be paid.

However, a natural question that arises regards whether the dynamics given by (18)-(21) actually takes the economy to the long-run equilibrium outcome established in Proposition 1. Or, in other words, in an economy where initially there are heterogeneous inflation expectations, do adaptive forecasters come to learn to predict perfectly the actual inflation without ever having to pay the cost associated with the perfect foresight strategy? The answer is yes, as formally established in the following proposition.

Proposition 2: *For a given vector of parameters* $(\alpha, \beta, u_n, \pi_n = m - g_n, \varepsilon)$, *the unique long-run evolutionary equilibrium* given by $(x^*, \pi_n, u_n, \pi_n) \in \Theta$ *of the dynamic system* given by (18)-(21) *is locally asymptotically stable*.

Proof: See Appendix 2.

Consequently, the predictor switching game investigated in this paper is subject to an evolutionary learning dynamics which takes it to a long-run equilibrium in which, albeit either the majority or even all of the agents play the adaptive foresight strategy to form inflation expectations, the rates of inflation, output growth and unemployment all achieve their natural values. Therefore, inflation expectations that are persistently heterogeneous need not prevent the successful conduct of macroeconomic stabilization policy. Furthermore, when there is mutation, so that the two inflation forecasting strategies survive in the long-run equilibrium (yet the adaptive forecasting strategy predominates), such long-run equilibrium heterogeneity varies expectedly with some parameters:

(22) $$\frac{\partial x^*}{\partial \kappa} = \frac{\varepsilon(1-\varepsilon)F'(-\kappa)}{2\{(1-\varepsilon)[1-F(-\kappa)]+\varepsilon\}^2} > 0,$$

(23) $$\frac{\partial x^*}{\partial \varepsilon} = \frac{-[1-F(-\kappa)]}{\{(1-\varepsilon)[1-F(-\kappa)]+\varepsilon\}^2} < 0.$$

Intuitively, in the evolutionary long-run equilibrium with coexistence of the two inflation forecasting strategies, the higher the cost associated with perfect foresight, the greater the predominance of agents playing the adaptive foresight strategy (recall from Proposition 1 that, in the presence of mutation, the adaptive forecasting strategy is the most played strategy in the long-run equilibrium, $1/2 < x^* < 1$). On the other hand, the higher the measure of mutant agents, the smaller the predominance of agents following the adaptive foresight strategy. The intuition is that, the greater the noise in the selection process of inflation forecasting strategy, the greater has to be the proportion of perfect forecasters in the long-run equilibrium to ensure that the aggregate expected inflation is consistent with the natural rate of inflation.[3]

[3] Moreover, since the stable eigenvalue corresponding to the evolutionary dynamics is given by $\lambda_1 = (1-\varepsilon)F(-\kappa)$ (see Appendix 2), the lower the measure of mutant agents or the higher the

5. Conclusions

Drawing on an extensive empirical literature that documents persistent and time-varying heterogeneity in the formation of inflation expectations, this paper embeds two inflation forecasting strategies – one based on costly perfect foresight, the second based on costless adaptive foresight – in a simple macroeconomic model. Moreover, the paper draws on the robust evidence from survey data that forecast errors have to pass some threshold before agents abandon their previously selected inflation forecasting strategy to fittingly postulate that decision makers switch between these forecasting strategies based on satisficing evolutionary dynamics. The resulting model allows us to study whether or not a long-run equilibrium consistent with the output growth, unemployment and inflation at their natural levels can be achieved, and whether or not this involves the extinction of either of the inflation forecasting strategies with which the population of decision makers may begin.

As regards the existence of a long-run evolutionary equilibrium, our results show that the adaptive forecasting strategy is either the only one to survive (which is the case in the absence of mutation) or the most played forecasting strategy when both strategies survive (which is the case in the presence of mutation). However, in either case the evolutionarily satisficing long-run equilibrium is characterized by the natural values of the rates of inflation, output growth and unemployment being achieved.

As regards the stability of the long-run evolutionary equilibrium, we find that the predictor switching game explored in this paper is subject to satisficing evolutionary dynamics which takes the economy to a long-run equilibrium in which, albeit either the majority or even all of the agents play the adaptive foresight strategy to form inflation expectations, inflation, output growth and unemployment achieve their natural values. Moreover, in the evolutionary equilibrium with coexistence of both inflation forecasting strategies, the higher the cost associated with playing the perfect foresight strategy, the higher the proportion of decision makers playing the adaptive foresight strategy. Meanwhile, in the same evolutionary equilibrium, the higher the measure of mutant decision makers, the lower the proportion of decision makers employing the adaptive foresight strategy.

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cost associated with perfect foresight, the lower the speed of convergence of the distribution of inflation forecasting strategies to its long-run equilibrium value. The intuition is that perfect forecasters play a crucial role in guiding the satisficing evolutionary dynamics.

All in all, the analytical results summarized above suggest that we should not be as concerned about heterogeneity in inflation expectations as one might have thought. After all, persistently heterogeneous inflation expectations (which is fully in keeping with the empirical evidence) as stable long-run equilibrium configuration is not inimical to macroeconomic stabilization. Instead, the satisficing evolutionary dynamics which drive the frequency distribution of inflation forecasting strategies across agents succeed in diminishing the cost of macroeconomic stabilization in the long-run equilibrium. In fact, in the long-run equilibrium the output (and thus employment) cost associated with stable inflation at its natural level is equal to zero, while the heterogeneity cost is either equal to zero (when no agent is playing the perfect foresight strategy) or strictly positive but nonetheless diminished by the predominance of agents playing the costless adaptive foresight strategy (when both forecasting strategies survive).

Even though this paper explores implications of an evolutionarily time-varying heterogeneity in the formation of inflation expectations using a specific macroeconomic model, the likely prospect that other kinds of heterogeneity of behavior are also subject to satisficing evolutionary dynamics with relevant implications deserves future research.

Appendix 1: Proof of Proposition 1

An equilibrium of the dynamic system given by (18)-(21) should satisfy the condition given by $\pi_t = \pi_{t-1} \equiv \pi^*$ and $\pi_t^A = \pi_{t-1}^A = \pi^{A*}$ for any $t \in \{1,2,3,...\} \subset \mathbb{N}$. Therefore, we can use (21) to establish that:

(A-1) $$\pi^{A*} - \pi_n = \gamma(\pi^* - \pi_n) + (1-\gamma)(\pi^{A*} - \pi_n).$$

It then follows from (A-1) that:

(A-2) $$\pi^{A*} = \pi^*.$$

An equilibrium of the dynamic system given by (18)-(21) should also satisfy the condition given by $x_t = x_{t-1} \equiv x^*$ and $u_t = u_{t-1} \equiv u^*$ for any $t \in \{1,2,3,...\} \subset \mathbb{N}$. Therefore, we can use (A-2) and (20) to obtain:

(A-3) $$u^* - u_n = \frac{x^*}{\alpha}(\pi^* - \pi_n).$$

Now, we can substitute (A-2) and (A-3) in (19) to obtain:

(A-4) $$\pi^* = \pi_n.$$

Therefore, based on (A-2), (A-3) and (A-4), it follows that:

(A-5) $$\pi^{A*} = \pi_n \text{ and } u^* = u_n.$$

Considering (A-5), it follows that $F(-(\pi_{t-1}^A - \pi_{t-1})^2) = F(0) = 1$ in the long-run equilibrium. Therefore, from the noisy satisficing evolutionary dynamics in (18) the following condition has to be satisfied:

(A-6) $$(1-\varepsilon)(1-x^*)[1-F(-\kappa)] + \varepsilon\left(\frac{1}{2} - x^*\right) = 0.$$

The solution this condition is given by:

(A-7) $$x^* = \frac{(1-\varepsilon)[1-F(-\kappa)] + \varepsilon/2}{(1-\varepsilon)[1-F(-\kappa)] + \varepsilon}.$$

Let us prove that $x^* \in (1/2,1] \subset \mathbb{R}$ for all $\varepsilon \in [0,1) \subset \mathbb{R}$. Since $\varepsilon \in [0,1) \subset \mathbb{R}$ and $F(-\kappa)] \subset (0,1) \subset \mathbb{R}$, it follows that $(1-\varepsilon)[1-F(-\kappa)] > 0$. Hence, we can use the latter inequality to establish that:

(A-8) $2(1-\varepsilon)[1-F(-\kappa)]+\varepsilon > (1-\varepsilon)[1-F(-\kappa)]+\varepsilon \Rightarrow x^{*} = \dfrac{(1-\varepsilon)[1-F(-\kappa)]+\varepsilon/2}{(1-\varepsilon)[1-F(-\kappa)]+\varepsilon} > \dfrac{1}{2}$,

for all $\varepsilon \in [0,1) \subset \mathbb{R}$, and:

(A-9) $(1-\varepsilon)[1-F(-\kappa)]+\varepsilon/2 < (1-\varepsilon)[1-F(-\kappa)]+\varepsilon \Rightarrow x^{*} = \dfrac{(1-\varepsilon)[1-F(-\kappa)]+\varepsilon/2}{(1-\varepsilon)[1-F(-\kappa)]+\varepsilon} < 1$,

for all $\varepsilon \in (0,1) \subset \mathbb{R}$, and $x^{*} = 1$ for $\varepsilon = 0$. This completes the proof that there is one, and only one, equilibrium $(x^{*}, \pi_n, u_n, \pi_n) \in \Theta$ of the system (18)-(21). □

Appendix 2: Proof of Proposition 2

Consider the Jacobian matrix of the linearization around the long-run evolutionary equilibrium of the system given by (18)-(21):

(A-10) $$J(x^{*}, \pi_n, u_n, \pi_n) = \begin{bmatrix} (1-\varepsilon)F(-\kappa) & 0 & 0 & 0 \\ 0 & \dfrac{\gamma x^{*}}{\alpha\beta + x^{*}} & \dfrac{-\alpha}{\alpha\beta + x^{*}} & \dfrac{(1-\gamma)x^{*}}{\alpha\beta + x^{*}} \\ 0 & \dfrac{\beta\gamma x^{*}}{\alpha\beta + x^{*}} & \dfrac{x^{*}}{\alpha\beta + x^{*}} & \dfrac{\beta(1-\gamma)x^{*}}{\alpha\beta + x^{*}} \\ 0 & \gamma & 0 & 1-\gamma \end{bmatrix}.$$

Let λ be an eigenvalue of the Jacobian matrix (A-10) and I the 4×4 identity matrix. We can then set the following characteristic equation of the linearization around the equilibrium:

(A-11) $$|J - \lambda I| = \begin{vmatrix} (1-\varepsilon)F(-\kappa)-\lambda & 0 & 0 & 0 \\ 0 & \dfrac{\gamma x^{*}}{\alpha\beta + x^{*}} - \lambda & \dfrac{-\alpha}{\alpha\beta + x^{*}} & \dfrac{(1-\gamma)x^{*}}{\alpha\beta + x^{*}} \\ 0 & \dfrac{\beta\gamma x^{*}}{\alpha\beta + x^{*}} & \dfrac{x^{*}}{\alpha\beta + x^{*}} - \lambda & \dfrac{\beta(1-\gamma)x^{*}}{\alpha\beta + x^{*}} \\ 0 & \gamma & 0 & 1-\gamma-\lambda \end{vmatrix} = 0$$

.

We can use the Laplace expansion to express (A-11) as follows:

(A-12) $$|J-\lambda I| = [(1-\varepsilon)F(-\kappa)-\lambda]\begin{vmatrix} \frac{\gamma x^*}{\alpha\beta+x^*}-\lambda & \frac{-\alpha}{\alpha\beta+x^*} & \frac{(1-\gamma)x^*}{\alpha\beta+x^*} \\ \frac{\beta\gamma x^*}{\alpha\beta+x^*} & \frac{x^*}{\alpha\beta+x^*}-\lambda & \frac{\beta(1-\gamma)x^*}{\alpha\beta+x^*} \\ \gamma & 0 & 1-\gamma-\lambda \end{vmatrix} = 0.$$

Consequently, one of the eigenvalues can already be explicitly determined, which is given by $\lambda_1 = (1-\varepsilon)F(-\kappa)$. Since $\varepsilon \in [0,1) \subset \mathbb{R}$ and $F(-\kappa)] \subset (0,1) \subset \mathbb{R}$, it follows that $0 < \lambda_1 < 1$..

Considering (A-12), the remaining eigenvalues have to satisfy:

(A-13) $$\begin{vmatrix} \frac{\gamma x^*}{\alpha\beta+x^*}-\lambda & \frac{-\alpha}{\alpha\beta+x^*} & \frac{(1-\gamma)x^*}{\alpha\beta+x^*} \\ \frac{\beta\gamma x^*}{\alpha\beta+x^*} & \frac{x^*}{\alpha\beta+x^*}-\lambda & \frac{\beta(1-\gamma)x^*}{\alpha\beta+x^*} \\ \gamma & 0 & 1-\gamma-\lambda \end{vmatrix} = 0,$$

which can be re-written as:

(A-13-a) $$\lambda(\lambda^2 + a\lambda + b) = 0,$$

where:

(A-14) $$a = -trJ = -\left[\frac{(1+\gamma)x^*}{\alpha\beta+x^*} + 1 - \gamma\right],$$

and:

(A-15) $$b = \begin{vmatrix} \frac{x^*}{\alpha\beta+x^*} & \frac{\beta(1-\gamma)x^*}{\alpha\beta+x^*} \\ 0 & 1-\gamma \end{vmatrix} + \begin{vmatrix} \frac{\gamma x^*}{\alpha\beta+x^*} & \frac{(1-\gamma)x^*}{\alpha\beta+x^*} \\ \gamma & 1-\gamma \end{vmatrix} + \begin{vmatrix} \frac{\gamma x^*}{\alpha\beta+x^*} & \frac{-\alpha}{\alpha\beta+x^*} \\ \frac{\beta\gamma x^*}{\alpha\beta+x^*} & \frac{x^*}{\alpha\beta+x^*} \end{vmatrix} = \frac{x^*}{\alpha\beta+x^*}.$$

Given (A-13-a), we know that one eigenvalue is always zero. We can then make use of the Samuelson stability conditions for a second order characteristic equation to determine if the remaining two eigenvalues are inside the unit circle. In Farebrother

(1973, p. 396, inequalities 2.4 and 2.5) we obtain the following set of simplified Samuelson conditions for the quadratic polynomial within parentheses in (A.13-a):

(A-16) $b < 1$ and $1 + b > |a|$.

Let us prove that $b < 1$. Since $1/2 < x^* \leq 1$ (see proof of the Proposition 1), $\alpha > 0$, and $\beta > 0$, it follows that $x^* < \alpha\beta + x^*$ such that:

(A-17) $$b = \frac{x^*}{\alpha\beta + x^*} < 1.$$

Let us now prove that $1 + b > |a|$. Given that $\gamma > 0$, it follows from (A-17) that:

(A-18) $$\gamma > \frac{\gamma x^*}{\alpha\beta + x^*}.$$

Adding and subtracting b from the right side of (A-17), we obtain:

(A-19) $$\gamma > \frac{(1+\gamma)x^* - x^*}{\alpha\beta + x^*}.$$

Finally, we can add one to both sides of (A-18) and after some algebraic manipulation we arrive at:

(A.20) $$1 + \frac{x^*}{\alpha\beta + x^*} > \frac{(1+\gamma)x^*}{\alpha\beta + x^*} + 1 - \gamma,$$

which, considering (A-14) and (A-15), implies that $1 + b > |a|$.

Therefore, we have demonstrated that all the eingenvalues of (A-11) are strictly less than one in absolute value, so the equilibrium configuration given by $(x^*, \pi_n, u_n, \pi_n) \in \Theta$ of the system given by (18)-(21) is locally asymptotically stable. □

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