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Can Workers' Increased Pessimism about the Labor Market Conditions Raise Unemployment?

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Can workers' increased pessimism about the labor market conditions raise unemployment?

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1. Introduction

There is experimental evidence that higher wages elicit more effort from workers (Gneezy and List, 2006; Charness and Kuhn, 2007) and survey evidence that firms see wages as affecting effort (Campbell and Kamlani, 1997). There is also empirical evidence that the wage-effort elasticity is positive (Cappelli and Chauvin, 1991) and that workers providing greater effort earn higher wages (Goldsmith et al., 2000). Meanwhile, as there is also survey evidence on persistent heterogeneity in unemployment expectations across workers, it is reasonable to presume that the expected cost of job loss and the resulting provision of effort are heterogeneous across workers.

In the model herein, the firm cannot perfectly observe whether a worker holds a pessimistic unemployment expectation (and provides relatively more effort by having a higher expected cost of job loss) or an optimistic unemployment expectation (and delivers relatively less effort by having a lower expected cost of job). There are also neutral workers who are more optimistic than pessimistic workers, but more pessimistic than optimistic ones. Facing such non-observable behavioral heterogeneity on the part of workers, each firm sets the uniform wage that minimizes the cost of labor per unit of *average* effort, which is the wage that satisfies what we dub *weighted Solow condition*.

Thus, optimistic workers are more costly per unit of effort than neutral and (to a greater extent) pessimistic workers, as all workers receive the same wage but neutral and (to a greater extent) pessimistic workers provide more effort than optimistic workers. The latter impose a negative externality on neutral and (to a greater extent) pessimistic workers in the form of a lower wage per unit of effort. In effect, pessimistic workers are also imposed a negative externality by neutral workers, as the latter are more costly per unit of effort.

It follows that the equilibrium wage and unemployment rate depend on the distribution of unemployment expectations across workers, which is predetermined. It is then worth exploring whether a higher proportion of pessimistic workers can substitute for equilibrium unemployment as worker discipline device (borrowing the title of the related paper by Shapiro and Stiglitz, 1984). Meanwhile, there is correlation evidence based on the major U.S. and European surveys of households which has been shown econometrically to mean that unemployment expectations are an important driver of actual unemployment, in that a rise (fall) in pessimism (optimism) leads to an increase in unemployment (Leduc and Sill, 2013; Girardi, 2014). In this context, the main result coming out of our heterogeneous expectations-augmented efficiency wage model is that whether a higher proportion of workers holding pessimistic unemployment expectations will lower or increase actual unemployment depends to a great extent on the prevailing distribution of unemployment expectations across workers. Leduc and Sill (2013) suggest that their estimated result that a fall in expected unemployment leads to a fall in actual unemployment rate squares well with the predictions of a standard labor matching model. The intuition is that less pessimistic unemployment expectations raise the marginal benefit of a match and lead to a fall in actual unemployment as more vacancies are posted. Alternatively, our model shows that a similar positive relationship between pessimistic unemployment expectations and actual unemployment can arise in a heterogeneous expectations-augmented efficiency wage model through a composition effect which is empirically testable with available survey data.

2. The model

Although workers' unemployment expectations and resulting effort exerted at work are not perfectly observed by the firm, workers nonetheless care about the possibility of being fired if they are caught shirking. Workers' expected cost of job loss depends on the wage received in the current job and how likely they expect to be re-employed along with the alternative wage as determinants of the wage associated with the expected labor market conditions. We draw on Romer (2019, ch. 11) to postulate the following functional form for the effort function:

$$(1) \qquad \varepsilon_\tau = \begin{cases} \left(\dfrac{w_\tau - \mu_\tau}{\mu_\tau} \right)^\gamma, & \text{for } w_\tau > \mu_\tau, \\ 0, & \text{otherwise,} \end{cases}$$

where ε_τ is the level of effort exerted by a worker of type $\tau = n, o, p$ (which stands for neutral, optimistic and pessimistic, respectively), $w_\tau \in \mathbb{R}_{++}$ is the wage received by a worker of type $\tau = n, o, p$, while $\mu_\tau \in [0,1] \subset \mathbb{R}_{++}$ is an indicator of the wage associated with the expected labor market conditions by a worker of type $\tau = n, o, p$, and the parameter $\gamma \in (0,1/2) \subset \mathbb{R}$ denotes an measure (reasonably empirically constrained in value) of the effort-enhancing effect of paying to a worker of type $\tau = n, o, p$ a wage which is higher than the wage associated with her expected labor market conditions. We assume that the latter is given by:

(2) $\mu_\tau = (1-u_\tau^e)w_{a,\tau}$,

where $u_\tau^e \in [0,1] \subset \mathbb{R}$ is the expected unemployment rate by a worker of type $\tau = n,o,p$ and $w_{a,\tau} \in \mathbb{R}_{++}$ is the alternative wage of a worker of type $\tau = n,o,p$. Differently from the standard specification in Romer (2019), the workers' expected likelihood of re-employment in (2) is not homogeneously proxied by the current rate of unemployment. In accordance with the empirical evidence from survey data on persistent heterogeneity in unemployment expectations across workers, with such expectations ranging from more optimistic to more pessimistic ones, we assume the following well-defined ordering for the unemployment expectations of workers of type $\tau = o,n,p$:

(3) $0 \le u_o^e < u_n^e = u < u_p^e \le 1$,

where u is the rate of unemployment. The ordering in (3) is mostly based on the U.S. Michigan Survey of Consumers, in which households are asked: "How about people out of work during the coming 12 months — do you think that there will be more unemployment than now, about the same, or less?"[1] Therefore, in equilibrium, it is only the unemployment expectations of neutral workers that are confirmed. In fact, a standard shirking version of the efficiency wage model with homogeneous unemployment expectations across workers can be seen as a special case of the model herein featuring all workers holding neutral unemployment expectations.

Each firm is assumed to be small with respect to the economy, and therefore takes workers' expected cost of job loss as given. Firms are unable to either detect perfectly whether a given worker holds optimistic, neutral or pessimistic unemployment expectations, or monitor perfectly workers' resulting effort on the job. Thus, firms set the homogeneous wage w (i.e., $w_\tau = w$ for any $\tau = o,n,p$) that minimizes the cost of labor per unit of *average* effort ε. This wage can be equivalently obtained by specifying the choice problem of a firm as being to compute the amount of labor L (workers are homogeneous in all respects other than the unemployment expectation they hold), and the wage w that maximize its profits given by:

[1] See https://data.sca.isr.umich.edu/. The main survey of unemployment expectations in the EU countries similarly asks households how do they expect the number of people unemployed in the country to change over the next 12 months. Answers include 'increase sharply, 'increase slightly', 'remain the same', 'fall slightly', and 'fall sharply' (https://ec.europa.eu/info/sites/info/files/bcs_user_guide_en_0.pdf).

(4) $\pi = F(\varepsilon L) - wL$,

where $F(\cdot)$ is a production function with $F'(\cdot) > 0$ and $F''(\cdot) < 0$, and such maximization problem is subject to the following constraint represented by the parametric composition of the average effort provided by workers:

(5) $\varepsilon = \varepsilon_n^{\eta} \varepsilon_o^{\theta} \varepsilon_p^{\rho}$,

where η, θ, and ρ denote the proportions of neutral, optimistic, and pessimistic workers, respectively, with $(\eta, \theta, \rho) \in \Theta$, where $\Theta = \{(\eta, \theta, \rho) \in \mathbb{R}_+^3 : \eta + \theta + \rho = 1\}$ is the set (simplex) composed of all possible distributions of unemployment expectations across workers.

Assuming that $w > \mu_\tau$, the first-order conditions for an interior solution which determines the optimal combination (w, L) in (4) are:

(6) $$\frac{\partial \pi}{\partial w} = F'(\varepsilon L) L \frac{\partial \varepsilon}{\partial w} - L = 0,$$

(7) $$\frac{\partial \pi}{\partial L} = F'(\varepsilon L) \varepsilon - w = 0.$$

Substituting (6) in (7) we obtain the so-called Solow condition, according to which the profit-maximizing pair (w, L) implies a unitary wage elasticity of effort:[2]

(8) $$\frac{\partial \varepsilon}{\partial w} \frac{w}{\varepsilon} = 1.$$

Differently from a standard shirking version of the efficiency wage model with homogeneous unemployment expectations across workers, the effort level in (8) is the average level specified in (5). Thus, considering (1), we re-write (8) as follows:

(8-a) $$\left[\rho \varepsilon_p^{-1} \varepsilon \frac{\partial \varepsilon_p}{\partial w} + \theta \varepsilon_\theta^{-1} \varepsilon \frac{\partial \varepsilon_\theta}{\partial w} + \eta \varepsilon_n^{-1} \varepsilon \frac{\partial \varepsilon_n}{\partial w} \right] \frac{w}{\varepsilon} = \eta \frac{\partial \varepsilon_n}{\partial w} \frac{w}{\varepsilon_n} + \theta \frac{\partial \varepsilon_o}{\partial w} \frac{w}{\varepsilon_o} + \rho \frac{\partial \varepsilon_p}{\partial w} \frac{w}{\varepsilon_p} = 1.$$

[2] As shown in Appendix A, the average effort in (5) is a strictly concave function of the wage. Since the production function is also assumed to be strictly concave, it follows that the second-order conditions for profit maximization are satisfied.

Therefore, given that firms set the homogeneous wage that minimizes the cost of labor per unit of *average* effort under heterogeneity in workers' unemployment expectations (and accordingly heterogeneity in workers' expected cost of job loss and their effort provision on the job), such cost-minimizing wage satisfies the condition in (8-a) that we dub *weighted Solow condition*.

The model in (1)-(8-a) can be solved for the equilibrium rate of unemployment u^* as follows. The symmetric Nash equilibrium features all firms paying the wage w that satisfies the weighted Solow condition in (8-a), so that $w_{a,\tau} = w > \mu_\tau$ for any $\tau = n, o, p$. Taking μ_τ as given for any $\tau = n, o, p$, it follows from (1) that:

(9) $$\frac{\partial \varepsilon_\tau}{\partial w}\frac{w}{\varepsilon_\tau} = \gamma\left(\frac{w-\mu_\tau}{\mu_\tau}\right)^{\gamma-1}\frac{1}{\mu_\tau}\frac{w}{\left(\frac{w-\mu_\tau}{\mu_\tau}\right)^{\gamma}} = \frac{\gamma w}{w-\mu_\tau}.$$

Using (2), the expression in (9) can be re-written as:

(9-a) $$\frac{\partial \varepsilon_\tau}{\partial w}\frac{w}{\varepsilon_\tau} = \frac{\gamma w}{w-(1-u_\tau^e)w} = \frac{\gamma}{u_\tau^e}.$$

Substituting (9-a) in (8-a), we obtain:

(8-b) $$\eta\frac{\gamma}{u_n^e} + \theta\frac{\gamma}{u_o^e} + \rho\frac{\gamma}{u_p^e} = 1.$$

We assume the following specific form for the well-defined ordering for the unemployment expectations of workers of type $\tau = o, n, p$ in (3):

(10) $$u_\tau^e = \begin{cases} u/2\ , & \text{for } \tau = o, \\ u\ , & \text{for } \tau = n, \\ (u+1)/2\ , & \text{for } \tau = p. \end{cases}$$

We can then substitute (10) in (8-b) to obtain the condition implicitly defining the equilibrium unemployment rate u^*:

(11) $$\frac{\eta}{u^*} + \frac{2\theta}{u^*} + \frac{2\rho}{u^*+1} - \frac{1}{\gamma} = 0.$$

All workers holding either optimistic or neutral unemployment expectations yield economically meaningful values for the equilibrium rate of unemployment, namely $u^*\big|_{\theta=1} = \gamma/2$ and $u^*\big|_{\eta=1} = \gamma$. Yet all workers holding pessimistic unemployment expectations generates $u^*\big|_{\rho=1} = 2\gamma - 1$, which does not take on economically meaningful values for our empirically grounded assumption that $\gamma \in (0,1/2) \subset \mathbb{R}$. In the following proposition we establish the conditions ensuring the existence and uniqueness of the equilibrium unemployment rate for any $(\eta,\theta,\rho) \in \Theta$ *without complete predominance of pessimistic workers*.

Proposition 1. *For any* $(\eta,\theta,\rho) \in \Theta$ *with* $\rho < 1$ *and* $\gamma \in (0,1/2] \subset \mathbb{R}$, *the equilibrium rate of unemployment is given by* $u^* = \dfrac{-[1-\gamma(1+\theta+\rho)] + \sqrt{[1-\gamma(1+\theta+\rho)]^2 + 4\gamma(1+\theta-\rho)}}{2} \in (0,1) \subset \mathbb{R}$. *Moreover, for every* u^* *so determined there exists a unique profit maximizing choice of wage and employment for all firms which, by normalizing the labor supply to one, is given by* $(w^*, L^*) = \left(F'(\varepsilon^* L^*)\varepsilon^*, 1-u^*\right)$, *where* $\varepsilon^* = \left[\left(\dfrac{u^*}{1-u^*}\right)^{\gamma}\right]^{\eta} \left[\left(\dfrac{u^*}{2-u^*}\right)^{\gamma}\right]^{\theta} \left[\left(\dfrac{1+u^*}{1-u^*}\right)^{\gamma}\right]^{\rho}$.

Proof: See Appendix B.

The following proposition establishes how the equilibrium unemployment rate varies with an exogenous change in the distribution of unemployment expectations across workers. This inquiry is in line with the econometric evidence offered in Leduc and Sill (2013) and Girardi (2014) that shifts in unemployment expectations are an exogenous source of moves in actual unemployment.

Proposition 2. *Let* $\alpha \in \mathbb{R}_+$ *be a parametric constant so that* $(\eta, \theta = \alpha\rho, \rho) \in \Theta$. *Therefore, if the proportions of pessimistic and optimistic workers* (ρ,θ) *move through the simplex* Θ *along the ray* α, *a rise (fall) in the proportion of pessimistic workers is accompanied by an increase (decrease) in the proportion of optimistic workers. Since* $\gamma \in (0,1/2) \subset \mathbb{R}$, *for a given pair* (α,γ), *the equilibrium unemployment rate exhibits the following properties:*

i. *If* $\alpha \in (0,\alpha_c) \subset \mathbb{R}$, *where* $\alpha_c \equiv \dfrac{1-\gamma}{1+\gamma} \in (0,1) \subset \mathbb{R}$, *then* $\dfrac{du^*}{d\rho} < 0$;

ii. *If* $\alpha = \alpha_c$, *then* $\dfrac{du^*}{d\rho} = 0$;

iii. *If* $\alpha \in (\alpha_c, +\infty) \subset \mathbb{R}$, *then* $\dfrac{du^*}{d\rho} > 0$.

Proof: See Appendix C.

The U.S. Michigan survey measure which tracks the changes in the unemployment rate quite well (Leduc and Sill, 2013) is defined as a balance score equal to the percentage of households who thought the unemployment rate would increase minus the percentage who thought it would fall, plus 100.[3] In terms of our model, such measure is given by $BS = 100(\rho - \theta + 1)$, which is well defined for any $(\eta, \theta, \rho) \in \Theta$. Since $-1 \le \rho - \theta \le 1$, it follows that $0 \le BS \le 200$.

Let us explore the implications of a change in the proportions of pessimistic and optimistic workers (ρ, θ) through the simplex Θ along the ray $\alpha \in \mathbb{R}_+$, where $\theta = \alpha\rho$. The balance score is given by:

(12) $$BS = 100[(1-\alpha)\rho + 1].$$

We can then explore the behavior of *BS* along any ray α by differentiating (12) with respect to the proportion of pessimistic workers:

(13) $$\frac{\partial BS}{\partial \rho} = 100(1-\alpha).$$

When $\alpha = 1$ the derivative in (13) is null and BS is constant along such ray with $\theta = \rho$, and (12) implies that $BS = 100$ for any $(\eta = 1 - 2\rho, \theta = \rho, \rho) \in \Theta$. For any ray $\alpha \in [0,1) \subset \mathbb{R}$, along which $\theta < \rho$, the derivative in (13) is strictly positive and, per (12), $BS > 100$ for any $(\eta = 1 - 2\rho, \theta = \alpha\rho, \rho > 0) \in \Theta$. Thus, the higher the proportion of pessimistic workers, the higher BS along any ray $\alpha \in [0,1) \subset \mathbb{R}$. And for any finite ray $\alpha > 1$, with $\theta > \rho$, the derivative in (13) is strictly negative and, per (12), $BS < 100$ for any $(\eta = 1 - 2\rho, \theta = \alpha\rho, \rho > 0) \in \Theta$. Thus, the higher the proportion of pessimistic workers, the lower BS along any finite ray $\alpha > 1$.

In our model, therefore, along any ray $\alpha \in (\alpha_c, 1) \subset \mathbb{R}$, an exogenous rise in the proportion of pessimistic workers generates the positive correlation between BS and the actual unemployment rate which has been found using data from the Michigan survey. In effect, it follows from (13) in conjunction with Proposition 2 that $\dfrac{\partial BS}{\partial \rho} > 0$ and $\dfrac{du^*}{d\rho} > 0$ along any ray $\alpha \in (\alpha_c, 1) \subset \mathbb{R}$.

[3] See https://data.sca.isr.umich.edu/data-archive/mine.php for time series monthly data for these percentages back to 1978.

3. Conclusions

Our model has shown that the distribution of heterogeneous unemployment expectations across workers impacts non-linearly on unemployment. In effect, whether a higher proportion of workers holding pessimistic unemployment expectations (and hence facing a higher expected cost of job loss) leads to a lower or higher unemployment rate depends on the prevailing distribution of unemployment expectations across workers.

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Appendix A: Strict concavity of the average effort function

Taking μ_τ as given for any $\tau = o, n, p$ and considering (9), it follows from (5) that:

$$\text{(A.1)} \qquad \frac{\partial \varepsilon}{\partial w} = \eta \frac{\varepsilon}{\varepsilon_n}\frac{\partial \varepsilon_n}{\partial w} + \theta \frac{\varepsilon}{\varepsilon_o}\frac{\partial \varepsilon_o}{\partial w} + \rho \frac{\varepsilon}{\varepsilon_p}\frac{\partial \varepsilon_p}{\partial w} = \frac{\varepsilon}{w}\left[\eta \frac{\gamma w}{w-\mu_n} + \theta \frac{\gamma w}{w-\mu_o} + \rho \frac{\gamma w}{w-\mu_p}\right].$$

Recalling from (8) that $\frac{\partial \varepsilon}{\partial w} w - \varepsilon = 0$, differentiation of (A.1) with respect to the wage yields:

(A.2) $$\frac{\partial^2 \varepsilon}{\partial w^2} = \frac{\varepsilon}{w}\left[\eta \frac{-\gamma\mu_n}{(w-\mu_n)^2} + \theta \frac{-\gamma\mu_o}{(w-\mu_o)^2} + \rho \frac{-\gamma\mu_p}{(w-\mu_p)^2}\right] < 0,$$

for all $w-\mu_\tau > 0$ with $\tau = o, n, p$.

Appendix B: Proof of Proposition 1

The equilibrium condition in (11) can be re-written as a quadratic equation given by $(u^*)^2 + [1-\gamma(1+\theta+\rho)]u^* - \gamma(1+\theta-\rho) = 0$. Thus, u^* can be defined as a root of the following quadratic function:

(B.1) $$\phi(u) = u^2 + [1-\gamma(1+\theta+\rho)]u - \gamma(1+\theta-\rho).$$

Let u' and u'' be the roots of (B.1). Since $(\eta,\theta,\rho) \in \Theta$ and $\gamma \in (0,1/2) \subset \mathbb{R}$, it follows that $u'u'' = -\gamma(1+\theta-\rho) < 0$ if $\rho \in [0,1) \subset \mathbb{R}$. Hence, these roots have different signs and the positive one is given by:

(B.2) $$u'' = \frac{-[1-\gamma(1+\theta+\rho)] + \sqrt{[1-\gamma(1+\theta+\rho)]^2 + 4\gamma(1+\theta-\rho)}}{2} > 0.$$

Besides, we obtain $u'' \le 1$ if $-[1-\gamma(1+\theta+\rho)] + \sqrt{[1-\gamma(1+\theta+\rho)]^2 + 4\gamma(1+\theta-\rho)} \le 2$. The latter inequality can be simplified to $\gamma \le \frac{1}{1+\theta}$, which is satisfied for any $(\eta,\theta,\rho) \in \Theta$ and $\gamma \in (0,1/2) \subset \mathbb{R}$. Therefore, we have $u^* = u'' \in (0,1) \subset \mathbb{R}$.

Having determined u^*, we can use (1), (2), (5) and (10) to obtain the equilibrium average effort as $\varepsilon^* = \left[\left(\frac{u^*}{1-u^*}\right)^\gamma\right]^{1-(\theta+\rho)} \left[\left(\frac{u^*}{2-u^*}\right)^\gamma\right]^\theta \left[\left(\frac{1+u^*}{1-u^*}\right)^\gamma\right]^\rho > 0$. By normalizing the labor supply to one, the equilibrium employment can be expressed as $L^* = 1-u^*$, so that the equilibrium employment in effort units is $\varepsilon^* L^* = \left(u^*\right)^{\gamma[1-(\theta+\rho)]} \left(\frac{u^*}{2-u^*}\right)^{\gamma\theta} \left(1+u^*\right)^{\gamma\rho} \left(1-u^*\right)^{1-\gamma(1-\theta)} > 0$ for any $u^* \in (0,1) \subset \mathbb{R}$.

We can then substitute ε^* and $\varepsilon^* L^*$ in (7) to obtain the equilibrium wage as $w^* = F'\left(\varepsilon^* L^*\right)\varepsilon^* > 0$

for any $u^* \in (0,1) \subset \mathbb{R}$. And given that $0 < \frac{u^*}{2} \leq u_\tau^e \leq \frac{u^*}{2} < 1$ for any $\tau = o, n, p$, it is the case that $w^* - \mu_\tau^* = w^* - (1 - u_\tau^e) w^* = u_\tau^e w^* > 0$ for any $\tau = o, n, p$, as assumed earlier to establish (6)-(7).

Appendix C: Proof of Proposition 2

By construction, we have $\theta = \alpha\rho$. The equilibrium unemployment rate specified in Proposition 1 can then be re-written as follows:

(C.1) $$u^* = \frac{-\left[1-\gamma\left(1+(\alpha+1)\rho\right)\right] + \sqrt{\left[1-\gamma\left(1+(\alpha+1)\rho\right)\right]^2 + 4\gamma\left(1+(\alpha-1)\rho\right)}}{2}.$$

The derivative of (C.1) with respect to α is given by:

(C.2) $$\frac{\partial u^*}{\partial \rho} = \frac{\gamma(\alpha+1) + \frac{1}{2}\left\{\left[1-\gamma\left(1+(\alpha+1)\rho\right)\right]^2 + 4\gamma\left(1+(\alpha-1)\rho\right)\right\}^{-1/2}\left\{-2\left[1-\gamma\left(1+(\alpha+1)\rho\right)\right]\gamma(\alpha+1) + 4\gamma(\alpha-1)\right\}}{2}.$$

For any $\rho \in [0,1) \subset \mathbb{R}$, it follows that the derivative in (C.2) is null only when it is evaluated at $\alpha = \alpha_c \equiv \frac{1-\gamma}{1+\gamma} \in (0,1) \subset \mathbb{R}$. We also have that:

(C.3) $$\frac{\partial}{\partial \alpha}\left(\frac{\partial u^*}{\partial \rho}\right)\Bigg|_{\alpha=\alpha_c} = \frac{\partial^2 u^*}{\partial \alpha \partial \rho}\Bigg|_{\alpha=\alpha_c} = \frac{\gamma}{2}\left\{1 + \frac{(1+\gamma)^2 + 4\gamma\rho}{1+2\gamma(1-\rho)+\gamma^2} + \frac{2\gamma\rho[(1+\gamma)^2 + 2\gamma\rho]}{[(1+2\gamma(1-\rho)+\gamma^2]^2}\right\} > 0.$$

Therefore, for any $\rho \in [0,1) \subset \mathbb{R}$ and $\gamma \in (0,1/2) \subset \mathbb{R}$, the derivative in (C.2) is strictly negative if $\alpha \in (0, \alpha_c) \subset \mathbb{R}$ and strictly positive if $\alpha \in (\alpha_c, +\infty) \subset \mathbb{R}$.