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WORKING PAPER SERIES Nº 2019-46

DEPARTMENT OF ECONOMICS, FEA-USP
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Keywords: Wage inequality; evolutionary micro-dynamics; distribution of income; capacity utilization; output growth.

JEL Codes: J31; E25; E32; O41; C62.

Wage inequality as a source of endogenous macroeconomic fluctuations

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* This work was supported by CNPq (the Brazilian National Council of Scientific and Technological Development) [grant numbers 312432/2018-6 (JJS), 311811/2018-3 (GTL)]. This study was also financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

1. Introduction

Considerable empirical evidence has accumulated in support of the idea that the persistence of inter- and intra-industry wage differentials not explained by observable (and some non-observable) factors is inherent to the operation of the labor market. In fact, it has been widely empirically documented that such unexplained inter- and intra-industry wage differentials exist, are of non-negligible size and are persistent over longer periods of time. Meanwhile, there is extensive empirical evidence (using both econometric and experimental methods) that labor productivity is endogenous to the wage remuneration offered by the employer. Although the channels through which such endogeneity operates are not easily empirically identifiable, as they are likely to involve both observable and non-observable factors, several empirical studies have found that a higher wage remuneration is an effective productivity-enhancing mechanism.

Following early labor market studies by Dickens and Katz (1987), Krueger and Summers (1988), Katz and Summers (1989) and Groshen (1991), which consistently found that wage differentials between US industries remained even after accounting for observed as well as some unobserved individual heterogeneity, there has accumulated empirical evidence on such unexplained inter- and intra-industry wage differentials. In effect, the persistence of non-negligible wage differentials has been consistently found even after controlling for observable individual characteristics such as schooling or human capital, gender, years of experience, and others (see, e.g., Gittleman and Wolff, 1993; Katz and Autor, 1999; Abowd et al., 1999; Carruth et al., 2004; Caju et al., 2010; Genre et al., 2011; Abowd et al., 2018; Card et al., 2018). Meanwhile, there has also accumulated empirical evidence from laboratory experiments (see, e.g., Fehr et. al., 1998; Fehr and Falk, 1999; Charness, 2004, Charness and Kuhn, 2007; Fehr and Gächter, 2008) and field experiments (see, e.g., Gneezy and List, 2006; Bellemare and Shearer, 2009; Greiner et al., 2011) that higher wages incentivize better performance of workers on the job. Moreover, there is survey evidence (Campbell and Kamlani, 1997) and econometric evidence (see, e.g., Cappelli and Chauvin, 1991; Goldsmith et al., 2000) that receiving a higher wage remuneration positively affects workers' effort.

While most of the alternative theoretical explanations that have been offered for these two pieces of empirical evidence treat them as unrelated, a key feature of this paper is that the endogeneity of labor productivity to the wage remuneration received by the worker and the existence of a wage differential across observationally equivalent workers are integrated in a unified analytical framework. In order to accomplish such integration, the model developed herein features the possibility of heterogeneous behavior by firms as regards wage remuneration strategy to incentivize labor productivity. This behavioral heterogeneity is not parametric, however, as firms periodically revise (and possibly switch) their choice of wage remuneration strategy (which is an either-or choice) according to profitability criteria based on the seminal contributions by Herbert Simon on satisficing instead of optimizing choice. Meanwhile, the resulting evolutionary dynamics of the distribution of wage remuneration strategies in the population of firms will crucially impact on the behavior of the functional distribution of income and thereby on the dynamics of the aggregate demand-determined levels of capacity utilization and output growth.

The main results arising from our joint modelling of the micro- and macro-dynamics of the economy is that the long run features three evolutionary equilibria characterized by a stationary frequency distribution of wage remuneration strategies in the population of firms. Two of these stationary configurations are monomorphic evolutionary equilibria featuring all firms following either the higher or the lower wage remuneration strategy in each. However, both monomorphic evolutionary equilibria are locally unstable. The long-run dynamics also features a unique polymorphic evolutionary equilibria characterized by both wage remuneration strategies being followed in the population of firms. This stationary configuration, which is therefore characterized by the persistence of wage inequality across observationally similar workers, is locally stable when such wage differential is sufficiently low. However, when such wage differential slightly exceeds an empirically plausible critical (or bifurcation) value, the polymorphic evolutionary equilibrium loses its local stability. In this case, the long-run dynamics of the economy is characterized by the heterogeneity in wage remuneration strategies across firms, and hence the functional distribution of income and the average rates of capacity utilization and output growth, all undergoing stable endogenously-driven cyclical fluctuations. In effect, for not much higher (and still empirically plausible) values of the wage differential between the higher and the lower wage, the long-run behavior of those variables is characterized by chaotic dynamics.

The remainder of the paper is organized as follows. Section 2 includes a conceptual prelude and several considerations on behavioral and micro-foundational elements underlying the model set forth in this paper. Section 3 presents the structure of the model, explores its behavior in the ultra-short and short runs and derives the state transition of the distribution of wage remuneration strategies across firms as driven by evolutionarily satisficing dynamics. Section 4 then explores the emergence of multiple evolutionary equilibria and endogenous cyclical fluctuations in the average wage, relative wage differential, average labor productivity, functional distribution of income, capital capacity utilization and output growth. The final section briefly summarizes the main analytical results derived along the way and makes final comments.

2. Conceptual prelude and behavioral and micro-foundational elements

The model set forth in this paper draws considerably conceptually on the micro-foundation framework of the labor exchange elaborated in Bowles and Gintis (1990, 1993). In fact, our modelling of the labor exchange is also inspired by another interesting pioneering contribution by Herbert Simon, now on the *incompleteness* (to use a more contemporary expression) of the employment contract. In an early contribution dating back to the early 1950s, Simon reasonably argued that "in an employment contract certain aspects of the worker's behavior are stipulated in the contract terms, certain other aspects are placed within the authority of the employer, and still other aspects are left to the worker's choice" (1951, p. 305). Simon's argument proceeds by specifying satisfaction functions for the employer and the worker that each tries to maximize when the respective satisfaction levels are either certain or uncertain. Suggestively, Simon closes the paper by noting that the "most serious limitations of the model lie in the assumptions of rational utility-maximizing behavior incorporated in it." (p. 305). In the model herein, the frequency distribution of productivity-enhancing strategies across firms is endogenously time-varying as driven by satisficing evolutionary dynamics in the spirit of later contributions by Herbert Simon, with neither firms nor workers behaving as rational utility-maximizers.

Bowles and Gintis (1990) interpret the labor exchange as an archetypal example of a market exchange in a capitalist economy involving endogenous instead of exogenous claim enforcement. The labor exchange is therefore a contested exchange in that it includes the intensity of work as a contested feature of the labor process measurable only imperfectly or at considerable cost by the employer. The authors then suggestively emphasize one extremely important endogenous enforcement mechanism available to the employer: contingent renewal. In fact, when applied to a labor exchange, the employer can elicit intensity of work from the worker by promising to renew the contract in future periods if satisfied and to terminate the contract if not.

The implementation of an endogenous enforcement mechanism is typically costly to the employer, and the employer's claim against the intensity of the work performed by the worker is therefore a prime example of asset involving endogenous enforcement costs. Bowles and Gintis (1990) refer to the employment rent (or the cost of job loss to the worker) associated with the labor contract as the excess of the wage over the fallback position of the worker. Of course, the employment rent has to strictly positive for a strategy of contingent renewal of the labor contract by the employer to be effective as effort elicitation device.

In the model set forth in this paper, we build on this perceptive understanding of the labor market to further specify it as a contested environment subject to evolutionary dynamics. The model features the remuneration by a firm with a higher instead of lower wage (with the possibility of renewal of the labor contract in the next production period) to enhance labor productivity more effectively as an endogenous enforcement strategy. In effect, these two productivity-enhancing strategies (remuneration with a higher or lower wage) can be interpreted as alternative endogenous enforcement strategies in that they both provide an enforcement rent, with the higher wage strategy of course providing a higher such rent. Although, for analytical focus and simplicity, the higher and lower wages are exogenously given constants, both strategies are effective in that by following any of them the firm will obtain the labor productivity as specified in our model. Moreover, labor exchange is a contested exchange that takes place in an environment featuring firms periodically revising (and possibly switching) their choice of wage remuneration strategy as endogenous claim enforcement strategy. This gives rise to evolutionary dynamics of strategy revision crucially impacting on the dynamics of the functional distribution of income and hence the rates of capacity utilization and output growth.

In our model, the offering of either the higher or the lower wage is a take-it-or-leave-it offer under conditions of permanent excess supply of labor. For an individual worker, therefore, the alternative to accepting a given wage offer is to rejoin the large reserve army of labor and inevitably face the negligible probability of receiving another job offer. Bowles and Gintis (1993) reasonably argue that a salient feature of a contested exchange labor market is that the cost of job loss to the worker (which equals the value of the job to the worker less the expected value of the next best alternative) configures an enforcement rent. The reason is that the fear of losing the job ensures a higher level of intensity of work than the worker would deliver in the absence of such rent. In our model, since firms may choose to remunerate workers with either a lower or a higher wage, the cost of job loss and the associated employment rent across employed workers may be heterogeneous.

Bowles and Gintis (1990) alternatively refer to the employment rent as the excess of the wage over the income of an identical worker without the job. In the model herein, the possibility exists that an identical worker without a job in a firm following a certain wage remuneration strategy has a job in a firm following the other wage remuneration strategy. Yet the largeness of the reserve army of unemployed workers implies that the expected value of the next best alternative is always strictly much lower than the received wage even for workers remunerated with the lower wage. It follows that the payment of the higher wage by the firm (with the possibility of renewal of the labor contract in the next production period) can be interpreted as a productivity-enhancing strategy of attaching a higher employment rent to the job. This enforcement rent is required for the threat to terminate the relationship to succeed in enforcing the terms of exchange. As anticipated below and detailed in the following sections, in the model of this paper the enforcement rent or cost of job loss in firms remunerating workers with the higher wage is endogenously time-varying.

Bowles and Gintis (1993) term a contingent renewal strategy the offering of an enforcement rent to an exchange partner, one using the threat of termination to ensure compliance. Borrowing such terminology, the two alternative wage remuneration strategies considered in our model constitute alternative contingent renewal strategies. However, we refer to such wage compensation strategies as *evolutionarily contingent renewal strategies*. They are evolutionarily contingent in the sense that in both of them the termination of the labor contract in the end of a given production period may result not from the firm being unable to elicit as much intensity of work from the employed worker as it expected to, but from the firm having decided to switch wage remuneration strategy in light of the realized profitability. This decision, in turn, is based on a strategy revision protocol in the spirit of the contributions of Herbert Simon centered on the notion of satisficing choice. Both wage compensation strategies offer the worker the prospect of renewal of the labor contract for the next production period at a wage which nonetheless can be the same, higher (in the case of the lower wage strategy) or lower (in the case of the higher wage strategy).

In effect, a firm following the higher wage strategy may not renew the labor contract of a worker for the next production period (or may offer to renew it at the lower wage) even if in the current production period such worker has delivered a high intensity of work. The reason is that even in this case the following of the higher wage strategy to incentivize labor productivity may not yield a satisficing level of profitability for the firm under consideration. Meanwhile, as the labor productivity differential favorable to higher wage firms depends on the frequency distribution of wage remuneration strategies across firms, the intensity of work elicited by (and the resulting profitability of) a higher wage firm will vary even if such firm keeps following the higher wage strategy but the proportion of firms which follow the same strategy varies.

The renewal of the labor contract for the current production period, even if the firm was able to elicit as much intensity of work as it expected in the previous production period, is ultimately conditional on the demand for labor implied by its output production. In our model, firms face an effective demand constraint when selling their output production, and effective demand is insufficient for firms to fully utilize their capital capacity. Meanwhile, although labor productivity is constant in lower wage firms and variable (viz. endogenous to the distribution of wage strategies across firms) in higher wage firms, the reserve army of the unemployed is always large enough to ensure that labor is not a binding constraint on output production. Capital capacity is heterogeneously distributed across firms, and aggregate effective demand, which crucially depends on the functional distribution of income between wages and profits, is randomly and non-uniformly distributed across firms. As a result, if the current level of effective demand faced by the firm is lower than in the previous production period, the firm will not renew the labor contract of all its hired workers for the current production period even if they all provided the expected intensity of work in the previous production period and the firm (as well as the other firms) keeps following the same wage remuneration strategy. As detailed in the next section, we abstract from the possibility of labor hoarding by a firm, even if at the lower wage.

As capital capacity is heterogeneous across firms and aggregate effective demand (and hence aggregate output production) is randomly and non-uniformly distributed across firms, capital capacity utilization is heterogeneous across firms. As a result, the level of profits is heterogeneous in the population of firms even though the profit share (or unit profit) is homogeneous across firms following the same wage remuneration strategy, while it is heterogeneous across firms following different wage remuneration strategies. Workers do not save even if they are remunerated with the higher wage, whereas all firms save the same proportion of their heterogeneous profits irrespective of the wage remuneration strategy they have chosen to follow. All firms accumulate capital according to an accelerator-effect logic, but the rate of capital accumulation is heterogenous across firms due to capacity utilization being heterogeneous in the population on firms. Consequently, since the profit share is homogeneous across firms following the same productivity-enhancing strategy, but it is instead heterogeneous across firms following different productivity-enhancing strategies, whereas capacity utilization is heterogeneous in the population of firms, it follows that the profit rate (which is a key payoff measure in the strategy revision protocol used by firms) is heterogeneous in the population of firms.

Although the renewal of the labor contract by the firm is conditional on the demand for labor implied by its output production, which jointly depends on the effective demand it faces and the labor productivity associated with its choice of wage remuneration strategy, the firm can always use the (credible) threat of summarily terminating the labor contract at any moment of the production period if the worker fails to provide as much labor productivity as specified in the model.

In sum, suppose, for instance, that higher wage firm A decides to switch to the lower wage strategy in the next revision moment (which is at the end of the current production and selling period). In this situation, the intensity of work it elicits from workers will fall, so that its demand for labor will at least remain the same unless its effective demand-led output production falls more than proportionately. We assume that when such favorable contingency materializes and firm A places at least the same demand for labor as in the previous production period, workers hired by this firm in that production period will accept the offer to renew the labor contract at the lower wage. The reason is that they sensibly consider it a better alternative than to join the large reserve army of the unemployed and wait for a job offer by another higher wage firm. We also assume that any additional workers needed by such switching firm A will be randomly chosen from the reserve army of the unemployed to receive a take-it-or-leave-it job offer and will accept such an offer as it embodies a strictly positive (even if not the highest possible) employment rent. Yet, if the demand for labor placed by wage firm A falls due to effective demand constraints, not all of its previously hired workers who delivered the level of work intensity specified in the model will have their labor contract renewed. In this situation, we assume that workers whose labor contract are not renewed are randomly chosen and join the reserve army of the unemployed.

As a further example, suppose that lower wage firm B decides to switch to the higher wage strategy in the next revision moment. As a result, the intensity of work it elicits from workers will rise, so that its demand for labor will fall unless its effective demand-led output production rises at least as proportionately. When such favorable contingency materializes and firm B places at least the same demand for labor as in the previous production period, workers hired by this firm in that production period will of course accept the offer to renew the labor contract at the higher wage. Any additional workers needed by such switching firm B will be randomly chosen from the reserve army of the unemployed to receive a take-it-or-leave-it job offer, and will likewise definitely accept it. However, if the demand for labor placed by wage firm B falls due to effective demand constraints, not all of its previously hired workers who delivered the level of work intensity specified in the model will have their labor contract renewed. In this situation, we assume that workers whose labor contract are not renewed are randomly chosen and join the reserve army of the unemployed.

3. Structure of the model

The economy is closed to international trade and capital flows and the government does not perform any economic activity. There is production of a single good which serves both investment and consumption purposes. Such output production is carried out by imperfectly-competitive firms that combine capital and labor through a fixed-coefficient technology. Each individual firm produces (and hires labor) according to effective demand. However, the level of effective demand faced by each individual firm is assumed to be insufficient for it to produce at full capacity at the prevailing price level. The supply of labor is provided by homogeneous workers whose effort (and hence productivity) on the job is endogenous to the received wage remuneration. However, firms do not rely on labor hoarding (even if at a reduced wage) as a productivity-enhancing mechanism. Firms are homogeneous except possibly with respect to the strategy for enhancing labor productivity they have chosen to follow, which determines the wage remuneration they provide.

In a given ultra-short run each individual firm has chosen between remunerating workers with a lower wage rate $w_l \in \mathbb{R}_{++}$ or remunerating them with a higher wage rate $w_h > w_l$. A firm which has chosen to remunerate workers with a higher wage is dubbed h-firm, while a firm which has instead chosen to pay a lower wage rate is termed l-firm. An individual h-firm is motivated to pay a higher wage because doing so allows it to enhance labor productivity more effectively than otherwise. To put it specifically, in a given ultra-short run there is a proportion $\lambda \in [0,1] \subset \mathbb{R}$ of h-firms, while the remaining proportion, $1-\lambda$, is composed by l-firms. These proportions are endogenously time-varying, given that firms periodically revise (and possibly switch) wage remuneration strategy following a satisficing instead of optimizing revision protocol in the tradition of Herbert Simon. Workers hired by firms following the same wage remuneration strategy deliver the same level of productivity. Yet, while labor productivity is homogeneous across firms paying the same wage, labor productivity is heterogeneous across the two groups of firms. Moreover, the resulting labor productivity differential is endogenous, varying over time with the frequency distribution of wage remuneration strategies across firms.

Having already chosen to follow a given wage remuneration strategy, in the short run an individual firm makes a take-it-or-leave-it offer to available workers to hire the number of workers required for it to produce the effective demand-determined level of output. These workers, who are always in excess supply, take the received offer even if it is at the lower wage. To keep focus on the main issue of interest of the dynamics of the frequency distribution of wage remuneration strategies across firms and the resulting implications for wage distribution, factor income distribution, capacity utilization and output growth, we assume for simplicity that the wage rates w_h and w_l remain constant over time. The frequency distribution of wage remuneration strategies across firms, $(\lambda, 1-\lambda)$, which is predetermined both in the ultra-short run and in the short run as it results from previous individual choices, changes beyond the short run according to satisficing evolutionary dynamics. In a given ultra-short run, for predetermined values of wage differential, labor productivity differential and frequency distribution of wage remuneration strategies, individual markups vary so as to ensure that individual prices are equalized. Meanwhile, a change in the frequency distribution of wage remuneration strategies across firms, by leading to a change in the average wage, the average markup and the average labor productivity (and so in the wage share in income), causes a change in the aggregate effective demand and hence in the average values of the rates of capacity utilization and output growth in the short-run equilibrium.

In keeping with the suggestive theoretical and experimental evidence on the endogeneity of labor productivity to wage remuneration evoked in the Introduction section, and drawing on the contested exchange approach as described in the preceding section, the incentivizing of labor productivity by firms is conceptualized as a component of a contested labor exchange. In the incentivizing process considered in the model set forth here, labor productivity depends both on the received wage and the excess of the received wage over an average wage measure. In this contested labor exchange, such an average wage measure is conventionally perceived by workers as either a reliable benchmark measure of their outside opportunities or a reference point to which a given wage offer is to be compared as it embodies workers' wage expectations under uncertainty.

For concreteness, let $(w_h)^{\lambda_t}(w_l)^{1-\lambda_t}$ be a *benchmark average wage* at period $t \in \mathbb{N}$, so that the ratio of the higher wage to such benchmark measure, which we dub *relative wage* *differential*, can be expressed as $w_h \big/ \left[(w_h)^{\lambda_t} (w_l)^{1-\lambda_t} \right] = \omega^{1-\lambda_t}$ for all $\lambda_t \in [0,1] \subset \mathbb{R}$, where $\omega \equiv w_h / w_l$ is what we then term *absolute wage differential*. Therefore, it follows that $\partial(\omega^{1-\lambda_t}) / \partial w_h > 0$ for any $\lambda_t \in [0,1] \subset \mathbb{R}$. As intimated earlier, we assume that the extent to which labor productivity in h-firms is greater than labor productivity in l-firms varies positively with the relative wage differential represented by $\omega^{1-\lambda_t}$. Formally, we consider the following labor productivity differential function:

(1) $$\alpha_t \equiv \frac{a_{h,t}}{a_{l,t}} = f\left(\omega^{1-\lambda_t}\right),$$

where $a_{i,t} = X_{i,t} / L_{i,t}$, $X_{i,t}$, and $L_{i,t}$ denote labor productivity, total output production, and total employment in firms $i = h, l$ at period t, respectively. Hence, the greater the proportion of h-firms, the smaller the labor productivity differential between the two types of firms: in fact, given that the absolute wage differential $\omega = w_h / w_\ell > 1$ and $f'(\cdot) > 0$ for any $\lambda_t \in [0,1] \subset \mathbb{R}$, it then follows that $\partial \alpha_t / \partial \lambda_t = -f'\left(\omega^{1-\lambda_t}\right) \omega^{1-\lambda_t} \ln \omega < 0$ for any $\lambda_t \in [0,1] \subset \mathbb{R}$. The substance of this result is that there is strategic substitutability in the choice of the productivity-enhancing strategy featuring the remuneration of workers with the higher wage strategy. In effect, the higher the proportion of h-firms, the smaller the positive gap between the higher wage and the benchmark average wage, and therefore the smaller the labor productivity differential associated with remunerating workers with the higher wage. However, the choice of wage remuneration strategy is made by firm-owner capitalists in a decentralized and uncoordinated way. Firm-owner capitalists in the model have bounded rationality and behave according to rules of thumb, and periodically revise their choice of wage remuneration strategy based on satisficing relative profitability criteria having limited and localized knowledge concerning the system as a whole. Thus, when choosing to follow the higher wage strategy an individual firm does not take into account the negative productivity externality (represented by $\partial \alpha_t / \partial \lambda_t < 0$ in (1)) it imposes on the other firms making the same strategy choice. Paraphrasing Kalecki's (1967) perceptive observation that capitalists do many things as a class, but they certainly do not invest as a class, in our model capitalists do not act as a class either when choosing wage remuneration strategy to incentivize labor productivity more effectively.

It follows that all firms adopting the higher wage strategy $(\lambda_t = 1)$ results in the relative wage differential given by $\omega^{1-\lambda_t}$ taking its minimum value of 1. In this case, given that the labor productivity is uniform across all firms, and should be higher than the labor productivity when all firms remunerate workers with the lower wage, we assume that $f(1) > 1$. Meanwhile, if all firms switch to the l-type $(\lambda_t = 0)$, the potential relative wage differential represented by $\omega^{1-\lambda_t}$ takes its maximum value of ω, which is the absolute wage differential. In this case, an individual l-firm which decides to switch wage remuneration strategy to become an h-firm is therefore capable of obtaining the largest possible labor productivity differential, given that $f(\omega) > f\left(\omega^{1-\lambda_t}\right)$ for any $\lambda_t \in (0,1] \subset \mathbb{R}$.

Suppose that the labor productivity differential function in (1) assumes the linear form given by $f\left(\omega^{1-\lambda_t}\right) = A\omega^{1-\lambda_t}$, where $A \in (1,\infty) \subset \mathbb{R}_{++}$ is a parametric constant. For analytical convenience and without any loss of generality, we set $A = \omega^{\beta}$, where $\beta \in (0,1) \subset \mathbb{R}$ is another parametric constant. Consequently, the labor productivity differential function in (1) can be re-written as follows:

(1-a) $$\alpha_t = \omega^{\beta+1-\lambda_t}.$$

In order to better fix ideas, the properties of the labor productivity differential function in explicit form in (1-a), especially the monotonic fall in the labor productivity differential as the proportion of higher wage firms rises, are represented in Figure 1.

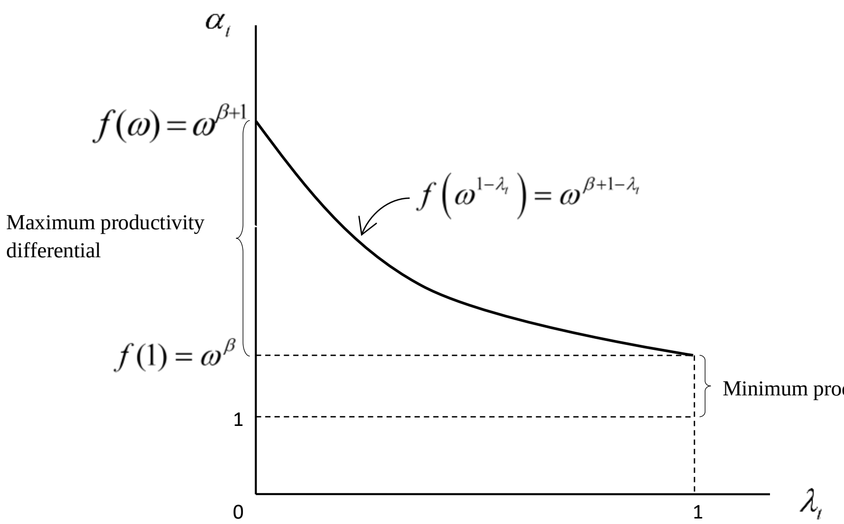

Figure 1. *Labor productivity differential function*

The substance of the specification of the labor productivity differential function in explicit form in (1-a) can be plausibly rationalized as follows. The benchmark average wage is conventionally construed by an employed worker as a reliable measure of her outside wage option. Conceivably, a worker who is employed at the higher wage conventionally interpret the benchmark average wage as a reliable estimate of at what other wage she could had been employed instead. Hence, a worker remunerated with the higher wage rate performs on the job with an additional productivity (relatively to the productivity associated with receiving the lower wage) that is the largest, the greater is the excess of the higher wage over her outside wage option as conventionally proxied by the benchmark average wage. In other words, the benchmark average wage is conventionally construed by a worker as the appropriate reference point with respect to which the higher wage should be compared when performance on the job with higher productivity is so justified. Thus, the more above-average is the higher wage, the more the latter incentivizes a worker to perform on the job with higher productivity than if instead she had been employed at the lower wage. An alternative intuitive rationale for the specification in (1-a) is that the reference point represented by the benchmark average wage is conventionally construed by a worker as a reliable estimate of her wage expectation under uncertainty. Therefore, to be (re-)employed at the higher wage is greeted by a worker as a pleasant surprise which incentivizes her to be more productive than if she had been (re-) employed at the lower wage; and the more productive, the larger is the positive gap between the higher wage and her expected wage as conventionally proxied by the benchmark average wage. In fact, Abeler et al. (2011) provide robust experimental evidence that increased wage expectations can act as a reference point positively affecting workers' performance on the job.

As far as the pricing behavior of firms is concerned, we assume that the population of firms (which is exogenously fixed) faces a binding limit-price constraint, $\bar{P}$, arising from the purpose of forestalling entry by potential competitors.[1] This common limit-price (which is an exogenously fixed constant conveniently normalized to one), is set by applying a markup over unit labor costs:

$$(2) \qquad 1=(1+z_{h,t})\frac{w_h}{a_{h,t}}=(1+z_{l,t})\frac{w_l}{a_{l,t}},$$

where $z_{l,t}\in\mathbb{R}_{++}$ and $z_{h,t}\in\mathbb{R}_{++}$ are, respectively, the markups applied by l-firms and h-firms at period t. For further simplicity, we also normalize labor productivity in firms following the lower wage strategy, $a_{l,t}$, to one for all t, which further requires that $w_l<a_{l,t}=1$ and implies that $\alpha_t=a_{h,t}$ for all t in (1). Therefore, the average labor productivity across firms is given by

[1] Early elaborations on entry and limiting pricing include Bain (1948) and Harrod (1952). The possibility of limiting-pricing behavior had already been raised in Kaldor (1935).

$\bar{a}_t = \lambda_t(\alpha_t - 1) + 1$ for all t. Besides, given the properties of the labor productivity differential function in (1), we further assume that $w_h < f(1)$, so that $w_h < \alpha_t = a_{h,t}$ for any $\lambda_t \in [0,1] \subset \mathbb{R}$. These simplifying or required assumptions about the components of the price equalization specified in (2) ensure that the income shares accruing to capital and labor remain each of them in the economically relevant open interval given by $(0,1) \subset \mathbb{R}$. As it turns out, the price equalization in (2) features the labor productivity and the markup in h-firms at period t as the sole adjusting variables. In order to better fix ideas, and recalling that a_l was innocuously normalized to one, we then re-write (2) by omitting the time t subscript where it is no longer applicable:

(2-a) $$1 = (1 + z_{h,t}) \frac{w_h}{a_{h,t}} = (1 + z_l) w_l .$$

Intuitively, an individual firm i following the higher wage strategy can be intuitively depicted as a firm willing to bet on the following prospect: such remuneration strategy will incentivize its hired workers to reciprocate with a productivity differential $\alpha_t = a_{h,t}$ which is sufficiently high to allow it to apply a markup z_h which is higher than z_l, despite charging the same price as l-firms.[2] As explored later, however, the resulting labor productivity differential, which is given by the productivity differential function in (1-a), may fall short of the extent required for the bet placed by such an individual firm i to prove successful (recall from Figure 1 that the labor productivity differential falls monotonically as the proportion of higher wage firms rises).

3.1 Ultra-short-run equilibrium

Given the predetermined distribution of wage strategies across firms, the ultra-short-run equilibrium values of the individual markups can be obtained by combining (1-a) and (2-a):

(3) $$z^*_{h,t} = \frac{a_{h,t}}{w_h} - 1 = \frac{a_{h,t}}{(w_h / w_l) w_l} - 1 = \frac{\omega^{\beta - \lambda_t}}{w_l} - 1$$

[2] Another alternative would be for an individual h-firm to use its productivity differential to charge a lower price than l-firms while applying the same markup, in an attempt to induce more effective demand for its output to be forthcoming. We abstract from this possibility by assuming that firms face a kinked demand curve, the market price (in this case, a market limit-price) for which is stable. The uniform price is sustained over time by each firm's fear that, if it undercuts, all the other firms will do the same. Thus, the absolute wage differential given by $\omega = w_h / w_l$ can be interpreted either in nominal or real terms.

and:

$$(4) \qquad z_l^* = \frac{1}{w_l} - 1.$$

Note that $z_{h,t}^* > z_l^*$ if $\omega^{\beta-\lambda_t} > 1$, a condition that is not satisfied for $\lambda_t \geq \beta$. Therefore, given that $\lambda_t \in [0,1] \subset \mathbb{R}$ and $\beta \in (0,1) \subset \mathbb{R}$, and that there is strategic substitutability in the choice of the higher wage strategy, so that the productivity differential in (1) varies negatively with the proportion of h-firms, it is possible that $z_{h,t}^* < z_l^*$.

The total real profits of the i–th firm of type τ at period t is given by:

$$(5) \qquad R_{\tau,t}^i \equiv X_{\tau,t}^i - w_\tau L_{\tau,t}^i = \begin{cases} \left(1 - \dfrac{w_h}{a_{h,t}^*}\right) X_{h,t}, \text{ if } \tau = h, \\ (1 - w_l) X_{l,t}\ , \text{ if } \tau = l. \end{cases}$$

Using (1-a) and (5), the shares of real profit in the total real output generated by the i–th firm of type τ in the ultra-short-run equilibrium t is given by:

$$(6) \qquad \pi_{\tau,t}^{i*} \equiv \frac{R_{h,t}^i}{X_{\tau,t}^i} = \begin{cases} 1 - w_l \omega^{\lambda_t - \beta} \equiv \pi_h^*(\lambda_t)\ , \text{ if } \tau = h, \\ 1 - w_l \equiv \pi_l\ , \text{ if } \tau = l. \end{cases}$$

Therefore, although the share of profits in the output generated by l-firms is constant, the ultra-short-run equilibrium value of the share of profits in the output generated by h-firms varies negatively with the proportion of these firms:

$$(7) \qquad \frac{\partial \pi_h^*(\lambda_t)}{\partial \lambda_t} = -w_l \omega^{\lambda_t - \beta} \ln \omega < 0,$$

for any $\lambda_t \in [0,1] \subset \mathbb{R}$. The substance of this result is also associated with the strategic substitutability in the choice of the higher wage strategy implied by the specification of the labor productivity differential function in (1). In fact, the higher the proportion of h-firms, the smaller the positive gap between the higher wage and the benchmark average wage, and therefore the smaller the labor productivity differential associated with remunerating workers with the higher wage.

The conditional expected value of the ultra-short run equilibrium profit share π_t^* given the type τ at period t is simply given by:

(8) $$\mathrm{E}\left(\pi_t^*\middle|\tau\right)=\begin{cases}\pi_h^*(\lambda_t)\,, \text{ if } \tau=h,\\ \pi_l\,, \text{ if } \tau=l.\end{cases}$$

Based on the law of iterated expectations (see, e.g., the simplified version in Wooldridge, 2010, Property CE.2, p. 31) and the conditional expectation in (8), we can establish the ultra-short-run average profit share $\bar{\pi}_t^*$ at period t as the expected profit share in the ultra-short run for a given frequency distribution of wage remuneration strategies across firms $(\lambda_t, 1-\lambda_t)$:

(9) $$\bar{\pi}^*(\lambda_t)\equiv\mathrm{E}(\pi_t^*)=\mathrm{E}\left[\mathrm{E}\left(\pi_t^*\middle|\tau\right)\right]=\lambda_t\mathrm{E}\left(\pi_t^*\middle|\tau=s\right)+(1-\lambda_t)\mathrm{E}\left(\pi_t^*\middle|\tau=n\right)=\lambda_t\pi_h^*(\lambda_t)+(1-\lambda_t)\pi_l\,.$$

3.2 Short-run equilibrium

In addition to what has already been assumed for other variables, the short run is defined as the time frame along which the capital stock, K, the labor supply in natural units, N, the distribution of wage strategies across firms, λ, and hence the labor productivity differential, α (which is equal to the labor productivity in h-firms, a_h) and the functional distribution of income as represented by the average profit share in (9), are all taken as predetermined. The ultra-short-run equilibrium values of the individual markups, z_h and z_l, are given by (3) and (4), which we have assumed to be achieved fast enough for them (and therefore the functional distribution of income) to be taken as predetermined in the short-run income-generating process driven by aggregate effective demand. The assumed existence of excess aggregate (and firm-level) capital capacity implies that aggregate (and firm-level) output adjusts in the short run to remove any excess aggregate (and firm-level) demand or supply in the economy (and for any individual firm). In the short-run equilibrium, therefore, aggregate savings, S, are equal to aggregate investment, I.

Let $u_t^i\equiv X_{\tau,t}^i/K_{\tau,t}^i$ be the rate of capacity utilization of the i–th firm of type $\tau=h,l$ at period t, where $K_{\tau,t}^i$ is the respective capital stock. We assume that these firm-specific rates of capacity utilization, all involving some unutilized capital capacity, are realizations of a continuous random variable with support given by $[u_t-\varepsilon, u_t+\varepsilon]\subset\mathbb{R}$, where ε is a strictly positive real constant which stands for the dispersion in the firm-specific rates of capacity utilization around the average rate of capacity utilization u_t. Therefore, given the random distribution of the aggregate capital stock across firms, aggregate effective demand (and hence aggregate output production) is randomly distributed across firms in a way that the firm-specific rates of capacity utilization are realizations of a continuous random variable with support given by $[u_t - \varepsilon, u_t + \varepsilon] \subset \mathbb{R}$ (recall from (2)-(3) that the price of the single good is homogeneous across firms). Both the admissible range of values that the exogenous constant ε can take and the endogenous value of u_t, which is always below the full capacity level due to the assumed insufficiency of effective demand, will be formally specified later.

As regards saving and consumption behavior, we assume that workers spend all of their wage income on consumption, while firm-owner capitalists save a constant proportion $\gamma \in (0,1) \subset \mathbb{R}$ of the corresponding real profits. Therefore, using (6), the amount of savings of the i–th firm of type τ normalized by its stock of capital can be expressed as follows:

(10) $$s^i_{\tau,t} \equiv \frac{S^i_{\tau,t}}{K^i_{\tau,t}} = \frac{\gamma R^i_{\tau,t}}{K^i_{\tau,t}} = \begin{cases} \gamma \pi^*_h(\lambda) u^i_t, & \text{if } \tau = h, \\ \gamma \pi_l u^i_t, & \text{if } \tau = n. \end{cases}$$

Recalling that $\mathrm{E}\left(u^i_t \middle| \tau\right) = u_t$ for $\tau = h, l$, it follows from (10) that the conditional expected value of the firm-specific saving to capital ratio s_t given the type τ is simply:

(11) $$\mathrm{E}\left(s_t \middle| \tau\right) = \begin{cases} \gamma \pi^*_h(\lambda_t) u_t, & \text{if } \tau = h, \\ \gamma \pi_l u_t, & \text{if } \tau = n. \end{cases}$$

Using again the law of iterated expectations and considering (11), we specify the short-run average savings normalized by the stock of capital $\bar{s}_t$ at each period t as the expected saving normalized by the stock of capital in the short run for a given frequency distribution of wage remuneration strategies across firms $(\lambda_t, 1-\lambda_t)$:

(12) $$\bar{s}(\lambda_t) \equiv \mathrm{E}(s_t) = \mathrm{E}\left[\mathrm{E}\left(s_t \middle| \tau\right)\right] = \lambda_t \mathrm{E}\left(s_t \middle| \tau = h\right) + (1-\lambda_t)\mathrm{E}\left(s_t \middle| \tau = l\right) = \gamma\left[\lambda_t \pi^*_h(\lambda_t) + (1-\lambda_t)\pi_l\right] u_t,$$

which can be re-written based on (9) as follows:

(13) $$\bar{s}(\lambda_t) = \gamma \bar{\pi}^*(\lambda_t) u_t.$$

Let us now turn to the derivation of the aggregate investment function. For simplicity and to keep focus on the main issue of wage inequality, we have assumed earlier that workers consume all of their income regardless of the type of firm for which they work, while firm-owner capitalists have a homogeneous saving behavior no matter what wage remuneration strategy they adopt. In the same spirit of simplicity and focus, we assume that firms behave alike as far as desired investment is concerned:

(14) $$\frac{I^i_{\tau,t}}{K^i_{\tau,t}} = \theta + \delta u^{e,i}_{\tau,t},$$

where $I^{i}_{\tau,t}$ and $u^{e,i}_{\tau,t}$ denote, respectively, the desired investment and the expected capacity utilization by the i–th firm of type τ at period t, while $\theta \in \mathbb{R}_{++}$ and $\delta \in \mathbb{R}_{++}$ are parametric constants common to all firms. We draw on Rowthorn (1982) and Dutt (1984), who in turn follow Steindl (1952), in making the (average) desired rate of capital accumulation to depend positively on the (average) rate of capacity utilization due to accelerator-type effects. Thus, the specification in (14) implies that the functional distribution of income between wages and profits impacts on effective demand only through consumption demand, as represented in (10)-(13) from the perspective of the (homogeneous) saving behavior of firm-owner capitalists. As regards expectation formation, we could suppose that firms (even when they have chosen to follow different wage remuneration strategies) have homogeneous expectations with respect to capacity utilization in the relevant future. However, as another plausible layer of heterogeneity, we assume that an individual firm, by inescapably facing an uncertain future, conventionally proxy its expected capacity utilization by its own current capacity utilization. In effect, an individual firm choosing the growth rate of its capital stock at period t is uncertain about both the (likely changing) effective demand it will face and the (revisable) wage remuneration strategy it will happen to follow over the relevant future. Reasonably, therefore, an individual firm conventionally proxy its expected capacity utilization by its own current capacity utilization, so that $u^{e,i}_{\tau,t} = u^{i}_{\tau,t}$ for any firm i of type $\tau = h,l$ at period t. Given that the firm-specific rates of capacity utilization are randomly distributed around the short-run equilibrium value u_t, it follows that the conditional expected value of the desired investment normalized by the stock of capital i_t given the type τ is simply:

(15) $$\mathrm{E}\left(i_t \middle| \tau\right) = \theta + \delta u_t .$$

Using again the law of iterated expectations and considering (11), we establish the short-run average desired investment normalized by the stock of capital $\overline{i}_t$ at each period t as the expected desired investment normalized by the stock of capital in the short run for a given frequency distribution of wage remuneration strategies across firms $(\lambda_t, 1-\lambda_t)$:

(16) $$\overline{i}_t \equiv \mathrm{E}(i_t) = \mathrm{E}\left[\mathrm{E}\left(i_t \middle| \tau\right)\right] = \lambda_t \mathrm{E}\left(i_t \middle| \tau = h\right) + (1-\lambda_t)\mathrm{E}\left(i_t \middle| \tau = l\right) = \theta + \delta u_t .$$

Finally, by substituting (13) and (16) in the product market short-run equilibrium condition given by $\overline{s}_t = \overline{i}_t$, we can then solve for the short-run equilibrium average capacity utilization to obtain:

(17) $$u_t = \frac{\theta}{\gamma\bar{\pi}^*(\lambda_t) - \delta} \equiv u^*(\lambda_t),$$

where $\bar{\pi}^*(\lambda_t)$ is given by (9). We assume that $\gamma\bar{\pi}^*(1) - \delta > 0$. As shown in the Appendix 1, $\bar{\pi}^*(1)$ is the smallest possible value of the short-run average profit share, so that $\gamma\bar{\pi}^*(\lambda_t) - \delta > 0$ for all $\lambda_t \in [0,1] \subset \mathbb{R}$, which is the standard Keynesian stability condition in effective demand-driven models like the one set forth in this paper. This means that $u^*(\lambda_t)$ is strictly positive and stable if, given the random distribution of the aggregate capital stock and the aggregate output production across firms, the average savings (as a proportion of the capital stock) is more responsive than the average desired capital accumulation to changes in the average capacity utilization, which in turn requires that the denominator of the expression in (17) is positive.

Moreover, we can substitute (17) in (16) to obtain the short-run equilibrium average output growth:

(18) $$g^*(\lambda_t) = \theta + \delta u^*(\lambda_t) = \theta\left(1 + \frac{\delta}{\gamma\bar{\pi}^*(\lambda_t) - \delta}\right).$$

As expected, the short-run equilibrium average values of capacity utilization and output growth vary positively with the (homogeneous) autonomous component θ of the (heterogeneous) firm-specific desired capital accumulation in (14), and negatively with the saving propensity of firm-owner capitalists γ in (13) and the average profit share $\bar{\pi}_t^*(\lambda_t)$ in (9). Meanwhile, as demonstrated in Appendix 1, and represented in Figure 2, the average profit share $\bar{\pi}_t^*(\lambda_t)$ varies non-monotonically with the proportion of h-firms λ_t, which in turn varies towards the long run as driven by an evolutionarily satisficing imitation dynamics to be described in the next sub-section. More precisely, as the proportion of h-firms varies from zero to one, the average profit share $\bar{\pi}_t^*(\lambda_t)$ first monotonically rises to reach a maximum at $\bar{\lambda} \in (0,1) \subset \mathbb{R}$ and then starts monotonically falling. Therefore, as the proportion of h-firms rises from zero to one, the short-run equilibrium average values of capacity utilization and output growth first falls monotonically to reach a minimum at $\bar{\lambda} \in (0,1) \subset \mathbb{R}$ and then starts rising monotonically to reach a maximum at $\lambda = 1$. As a result, the firm-specific rates of capacity utilization are randomly distributed over the range given by $[u^*(\lambda_t) - \varepsilon, u^*(\lambda_t) + \varepsilon] \subset \mathbb{R}$, where ε is a given strictly positive real constant such that $\varepsilon < \min\{u^*(\bar{\lambda}), 1 - u^*(1)\} \subset \mathbb{R}$.

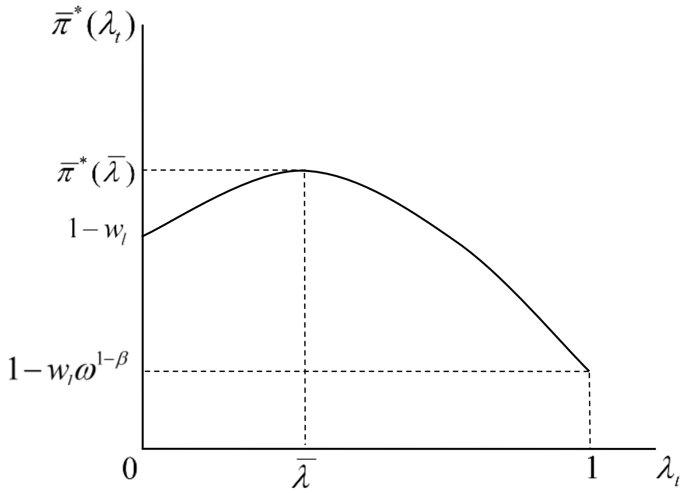

Figure 2. *Average profit share as a function of the distribution of productivity-enhancing strategies*

The profit rate of the i–th firm of type τ at period t, which will play a key role as the payoff of such i–th firm in the satisficing evolutionary dynamic derived in the next subsection, is also determined in the short-run equilibrium. In effect, using (9) and (17), the profit rate of the i–th firm of type τ in the short-run equilibrium is given by:

(19) $$r_{\tau,t}^{i*} \equiv \frac{R_{\tau,t}^{i*}}{K_{\tau,t}^{i}} = \begin{cases} \pi_h^*(\lambda_t)u_t^i \ , \text{ if } \tau = h, \\ \pi_l u_t^i \ , \text{ if } \tau = l. \end{cases}$$

3.3 State transition driven by evolutionarily satisficing imitation dynamics

The transition of the economy from the short run towards the long run is driven by exogenous changes in the available labor force, N, and endogenous changes in the frequency distribution of wage remuneration strategies across firms, λ_t, and the aggregate stock of capital, K. While the latter varies positively over time as determined by the demand-driven growth rate in (18), we assume that the growth rate of the available labor force is such that the reserve army of unemployed is always replenished to an extent sufficing to avoid that labor is a constraint to capital accumulation and output growth. As intimated earlier, the offering of either the higher or the lower wage is a take-it-or-leave-it offer under conditions of permanent excess supply of labor. We innocuously simplify matters by assuming that the available labor force grows endogenously at the same rate as capital accumulation, so that the capital to labor supply ratio remains constant.

We now describe the satisficing imitation dynamics which yield the law of motion of the proportion of *h*-firms, λ_t, following the contributions of Herbert Simon.[3] As elaborated by Simon (1955, 1956), satisficing is a theory of choice centered on the process through which available alternatives are examined and evaluated. By conceiving of choice as intending to meet an acceptability threshold rather than to select the best of all possible alternatives, satisficing theory openly contrasts with optimization theory. As imaginatively suggested by Simon, this contrast is analogous to 'looking for the sharpest needle in the haystack' (i.e., optimizing) versus 'looking for a needle sharp enough to sew with' (i.e., satisficing) (Simon 1987, p. 244). Experimental evidence on satisficing choice behavior as defined by Simon (1955) is offered in Caplin et al. (2011), who consistently find that subjects cease searching when a satisficing level of reservation utility is achieved. Meanwhile, Hey et al. (2017) find experimental evidence for Manski's (2017) theoretical contribution on when and how an agent adopts satisficing behavior as defined by Simon (1955). In fact, Manski (2017) theorizes satisficing as a class of decision strategies available to an agent who is seeking to optimize in a setting where deliberation is costly.

A firm i that has chosen to be of type h in the current period takes its current profit rate given by (19) and compares it with the profit rate it considers *satisficing*, which is denoted by ρ^i. This satisficing profit rate measure can be formally decomposed as $\rho^i = \pi^i u^i$, where $\pi^i \in [\mu - \xi, \mu + \xi] \subset (0,1) \subset \mathbb{R}$ is the *satisficing* unit profit (to which a *satisficing* markup corresponds) of the *i*–th firm, whereas μ and ξ are real constants such that $0 < \mu < 1$ and $0 < \xi < \min\{\mu, 1-\mu\}$. The unit profit (or share of profit in unit output) that is deemed *satisficing* by an individual firm depends, inter alia, on idiosyncratic features of such firm, and we reasonably assume that such *satisficing* unit profit is randomly and independently determined across firms and over time.

Taking t as the *current* period, if $r_{h,t}^{i*} = \pi_h^*(\lambda_t) u_t^i \geq \pi^i u_t^i = \rho^i$, or equivalently, if $\pi_h^*(\lambda_t) \geq \pi^i$, this h-firm i does not even consider switching wage remuneration strategy in $t+1$. Otherwise, if $\pi_h^*(\lambda_t) < \pi^i$, the h-firm i under consideration then becomes a strategy reviser. As a result, the probability of randomly choosing a firm i in the subpopulation of h-firms which considers the unit profit as not *satisficing* is given by:

(22) $$\Pr(\pi^i > \pi_h^*(\lambda_t)) = 1 - G(\pi_h^*(\lambda_t)) .$$

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[3] The formal derivation of this evolutionarily satisficing imitation dynamic is based on Vega-Redondo (1996, p. 91).

Following Vega-Redondo (1996, p. 91), we suppose that when such satisficing behavior transforms a h -firm into a potential strategy reviser it will switch to the alternative productivity-enhancing strategy of remunerating workers with the lower wage with probability given by the proportion of firms which have previously adopted such alternative strategy. This is an imitation effect, which can be associated with the idea of rule-of-thumb behavior in the present setting of choice of worker remuneration strategy triggered by satisficing. Under this premise and further assuming that the random variables related to the satisficing and imitation effects are independent from each other, the measure of higher wage firms which become lower wage firms at period t is then given by:

(23) $$\lambda_t[1-G(\pi_h^*(\lambda_t)](1-\lambda_t) .$$

Analogously, based on (19), the efflux from the population of *l*-firms which becomes *h*-firms is given by:

(24) $$(1-\lambda_t)\Pr(\pi^i > \pi_l)\lambda_t = (1-\lambda_t)[1-G(\pi_l)]\lambda_t .$$

Therefore, subtracting (23) from (24) yields the following *evolutionarily satisficing imitation dynamic*:

(25) $$\lambda_{t+1}-\lambda_t = \lambda_t(1-\lambda_t)[G(\pi_h^*(\lambda_t)-G(\pi_l)],$$

which can be re-written as follows:

(25-a) $$\lambda_{t+1} = \lambda_t + \lambda_t(1-\lambda_t)[G(\pi_h^*(\lambda_t)-G(\pi_l)] \equiv H(\lambda_t; w_l, \beta, \omega) .$$

Therefore, the map $H(\lambda_t; w_l, \beta, \omega)$ in (25-a), which is parameterized by the vector (w_l, β, ω), determines the state transition of the frequency distribution of productivity-enhancing wage strategies across firms, which crucially impacts on the macroeconomic state of the economy.

As it turns out, given that $H(\cdot)$ is a strictly increasing function with respect to λ_t, and $G(\cdot)$ is also a strictly increasing function in λ_t, a positive average unit profit differential favorable to the higher wage strategy in the current period leads to an increase in the proportion of firms following such strategy in the next period, whilst the opposite occurs when the differential in question becomes strictly negative. Hence, the satisficing evolutionary dynamics in (25) reflects the operation of a selection mechanism according to which the proportion of firms following a given productivity-enhancing strategy varies positively with the relative fitness or average unit profit differential favorable to such strategy.

4. Multiple evolutionary equilibria and endogenous cyclical fluctuations

Let us show that the satisficing evolutionary dynamic in (25) has three long-run, evolutionary equilibria. These are two monomorphic equilibria featuring survival of a single wage remuneration strategy in each, and one polymorphic equilibrium featuring the two wage remuneration strategies as survivors. It is only in the polymorphic equilibrium, therefore, that wage inequality arises as a persistent feature of the economy. Although the absolute wage differential given by $\omega = w_h / w_l$ is the same in any possible polymorphic equilibrium, the benchmark average wage given by $(w_h)^{\lambda_t}(w_l)^{1-\lambda_t}$ and the relative wage differential represented by $w_h / [(w_h)^{\lambda_t}(w_l)^{1-\lambda_t}] = \omega^{1-\lambda_t}$ both differ across polymorphic equilibria. While the former varies positively with the proportion of higher wage firms, the latter consequently varies negatively with it. Moreover, as pictured in Figure 2, the functional distribution of income, and therefore the average rates of capacity utilization and output growth, all differ across the three long-run equilibria.

It is straightforward to verify that $\lambda_{t+1} = \lambda_t = 0$ for any $t \in \mathbb{N}$ satisfies the evolutionarily satisficing imitation dynamic in (25), which means that all firms remunerating workers with the lower wage ($\lambda_t = 0$) is a fixed point of the map H in (25-a). It is straightforward to note that $\lambda_{t+1} = \lambda_t = 1$ for any $t \in \mathbb{N}$ also satisfies (25), so that all firms remunerating workers with the higher wage ($\lambda_t = 1$) is likewise a fixed point of the map H in (25-a).

Meanwhile, the existence of a polymorphic equilibrium will materialize if there is $\lambda^* \in (0,1) \subset \mathbb{R}$ such that $\lambda_{t+1} = \lambda_t = \lambda^*$ for any $t \in \mathbb{N}$. In effect, given that $H(\lambda_t)$ is continuous and strictly increasing, the satisfaction of such evolutionary equilibrium condition amounts to the equalization of the profit shares across wage remuneration strategies. Considering (6), it follows that the evolutionary equilibrium condition $\pi_h^*(\lambda^*) = \pi_l$ holds if, and only if, $\omega^{\lambda^* - \beta} = 1$. The latter evolutionary equilibrium condition holds only when $\lambda^* = \beta \in (0,1) \subset \mathbb{R}$, which is then also a fixed point of the map H in (25-a).

The local stability properties of the two monomorphic long-run equilibria can be appropriately studied by employing the first-order Taylor approximation of the map $H(\lambda_t)$ in (25-a). Taking into account (6), such approximation around a given state λ_t can be written as follows:

(26)

$$\frac{\partial H(\lambda_t;w_l,\beta,\omega)}{\partial\lambda_t}=1+(1-2\lambda_t)[G(\pi_h^*(\lambda_t))-G(\pi_l)]-\lambda_t(1-\lambda_t)G'(\pi_h^*(\lambda_t))w_l\omega^{\lambda_t-\beta}\ln\omega.$$

In the monomorphic long-run equilibrium with $\lambda_t=0$ the linear approximation in (26) reduces to:

(27) $$\frac{\partial H(0;w_l,\beta,\omega)}{\partial\lambda_t}=1+[G(\pi_h^*(0))-G(\pi_l)].$$

Given that, by assumption, $w_l>0$, $\omega>1$, and $\beta\in(0,1)\subset\mathbb{R}$, based on (6) we infer that $\pi_h^*(0)-\pi_l=w_l(1-\omega^{-\beta})>0$. Since G is a strictly increasing function, it then follows that $G(\pi_h^*(0))-G(\pi_l)>0$, so that the derivative in (27) is greater than one. Therefore, the monomorphic evolutionary equilibrium featuring the lower wage remuneration strategy as the only survivor ($\lambda_t=0$) is locally unstable.

Meanwhile, considering (26), in the monomorphic long-run equilibrium with $\lambda_t=1$ the linear approximation in (26) reduces to:

(28) $$\frac{\partial H(1;w_l,\beta,\omega)}{\partial\lambda_t}=1-[G(\pi_h^*(\lambda_t))-G(\pi_l)].$$

Recalling that, by assumption, $w_l>0$, $\omega>1$, and $\beta\in(0,1)\subset\mathbb{R}$, based on (6) we deduce that $\pi_h^*(1)-\pi_l=w_l(1-\omega^{1-\beta})<0$. Since G is a strictly increasing function, we conclude that $G(\pi_h^*(1))-G(\pi_l)<0$, so that the derivative in (28) is greater than one. Consequently, the monomorphic evolutionary equilibrium featuring the higher wage remuneration strategy as the only survivor ($\lambda_t=1$) is likewise locally unstable.

We now explore the long-run dynamics of the economy around the polymorphic evolutionary equilibrium characterized by $\lambda^*=\beta$ and hence persistent wage inequality across observationally similar workers, a dynamics which is interestingly much richer. First, note that the linearization around it by using (26) is given by:

(29) $$\frac{\partial H(\beta;w_l,\beta,\omega)}{\partial\lambda_t}=1-\beta(1-\beta)G'(1-w_l)w_l\ln\omega.$$

Since $w_l>0$, $\omega>1$, and $\beta\in(0,1)\subset\mathbb{R}$, it follows that $\dfrac{\partial H(\beta;w_l,\beta,\omega)}{\partial\lambda_t}<1$. Given the latter result, the polymorphic evolutionary equilibrium given by $\lambda^*=\beta$ is locally asymptotically stable if $\frac{\partial H(\beta; w_l, \beta, \omega)}{\partial \lambda_t} > -1$, which is satisfied when $\beta(1-\beta)G'(\pi_h^*(\beta))w_l \ln \omega < 2$. Thus, for given levels of β and w_l, the polymorphic evolutionary equilibrium characterized by $\lambda^* = \beta$ is a local attractor if the absolute wage differential measure represented by $\omega = w_h / w_l$ is sufficiently small. However, when such wage differential measure slightly exceeds certain critical (or bifurcation) value ω_c, the polymorphic evolutionary equilibrium with $\lambda^* = \beta$ loses its local stability. In this case, the long-run dynamics of the economy involves the heterogeneity in wage remuneration strategies across firms, and hence the functional distribution of income and the average rates of capital capacity utilization and output growth, all experiencing stable endogenously-driven cyclical fluctuations. This result is summarized in the proposition that follows.

Proposition 1: *For given parameters $w_l \in (0,1) \subset \mathbb{R}$ and $\beta \in (0,1) \subset \mathbb{R}$, the polymorphic evolutionary equilibrium characterized by $\lambda^* = \beta$ of the map in (25-a) exhibits the following dynamic properties:*

- *i.* *If $G'(1-w_l) > 0$ and $1 < \omega < \omega_c \equiv e^{\frac{2}{\beta(1-\beta)G'(1-w_l)w_l}}$, then the fixed point given by $\lambda^* = \beta$ of the map in (25-a) is an attractor.*
- *ii.* *If G follows an uniform distribution with support $[\mu-\xi, \mu+\xi] \subset (0,1) \subset \mathbb{R}$,* with *$\mu \in (0,1) \subset \mathbb{R}$* and *$\xi \in (0, \min\{\mu, 1-\mu\}) \subset (0,1) \subset \mathbb{R}$, then when the absolute wage differential ω passes through the critical value ω_c, the eigenvalue in (29) of the map in (25-a) goes through minus one and the fixed point $\lambda^* = \beta$ of the map in (25-a) loses its stability and a stable period-2 cycle is born, that is, a supercritical flip (or period-doubling) bifurcation occurs at the polymorphic evolutionary equilibrium given by $\lambda^* = \beta$.*

Proof: See Appendix 2.

As derived in subsection 3.2, the equilibrium average values of capacity utilization and output growth vary negatively with the average profit share. Meanwhile, it is demonstrated in Appendix 1, and represented in Figure 2, that the average profit share varies non-monotonically with the proportion of h-firms λ. Thus, the average capacity utilization and output growth are both at their highest possible long-run equilibrium levels in the monomorphic evolutionary equilibrium with all firms paying the higher wage, while they are lower in the monomorphic evolutionary equilibrium with all firms paying the lower wage. As obtained earlier, the two monomorphic equilibria of the map H in (25-a) are locally unstable, though. Consequently, as $\bar{\lambda} \in (0, \beta) \subset \mathbb{R}$ is the proportion of higher wage firms for which the average profit share is the highest possible one (per Appendix 1), the possibility then exists that the average capacity utilization and output growth are at even lower long-run equilibrium levels in the polymorphic evolutionary equilibrium. This possibility will materialize if λ^* and $\bar{\lambda}$ are sufficiently close to each other.

In order to appropriately complement the analytical results regarding local dynamics formally derived earlier, we now perform numerical simulation analyses of global dynamics of the model.[4] We have experimented extensively with the parameters related to the satisficing evolutionary dynamics before setting reasonable and plausible values for them. The exogenous variables and parameters of our dynamic model are set according to the following empirically plausible values: lower wage, $w_l = 0.4$ (recall that labor productivity in lower wage firms was normalized to one, so that 0.4 is also the wage share in the lower wage firms); parametric constant in the labor productivity differential function in (1-a), $\beta = 0.5$; saving propensity of firm-owner capitalists, $\gamma = 0.4$ (recall our assumption that workers do not save); autonomous component in the desired investment function in (14), $\theta = 0.05$; parametric constant measuring the accelerator effect in (14), $\delta = 0.18$; and parametric constants defining the support of the uniform distribution $G(\cdot)$ of satisficing unit profits (or profit shares) across firms, $\mu = 0.5$ and $\xi = 0.002$. Thus, the resulting critical (or bifurcation) value of the absolute wage differential parameter (per Proposition 1) is given by $\omega_c = 1.083$, and we have run simulations with values both below and above such critical value to explore the rich dynamic behavior of the model.

We eschew detecting only local dynamics by specifying an initial condition for the proportion of higher wage firms at $\lambda_0 = 0.1$, which is sufficiently distant from the unique polymorphic evolutionary equilibrium represented by $\lambda^* = \beta = 0.5$. Given the time sequence of the model (which is based on three "runs", namely, ultra-short, short and long), the initial values for the main endogenous variables of interest were all set at zero. These endogenous variables are the average labor productivity $\bar{a}_t = \lambda_t(\alpha_t - 1) + 1$, the profit share in higher wage firms π_h in (6), the average profit share $\bar{\pi}$ in (9), the average capacity utilization u in (17) and the average output growth rate g in (18).

[4] All numerical simulation results presented in this section (along with their graphical representation) were obtained with the open-source software iDMC, version 2.0.10, which is available at https://code.google.com/archive/p/idmc/downloads. Several routines of the iDMC software package are nicely illustrated in Lines (2007). Meanwhile, the source code used to generate the numerical results presented in what follows is obtainable upon request to the authors.

Figure 3(a) pictures a bifurcation diagram for the map H in (25-a) as a function of the parameter denoting the absolute wage differential, which is $\omega = w_h / w_l$, while Figure 3 (b) pictures the Lyapunov exponent of such map. The Lyapunov exponent of a one-dimensional discrete system such as the one in (25-a) measures the average exponential rate of divergence of neighboring initial states (Medio and Lines, 2003, subsection 7.1). More precisely, for an initial state converging to either a *stable* steady state or a stable *k*-cycle, the associated Lyapunov exponent is *negative*. Meanwhile, a chaotic trajectory features a *positive* Lyapunov exponent, which therefore measures how fast, on average, neighboring initial states diverge from each other. In a nonlinear system such as the one in (25-a) the possibility exists that two trajectories far from each other at some point in time come to close proximity at some later point in time, and subsequently resume diverging to later move close to each other again, and so on.

In effect, the two panels in Figure 3 clearly illustrate how more and more complicated the dynamic behavior of the map H in (25-a) becomes as the parameter measuring the absolute wage differential plausibly increases monotonically from $\omega = 1.08$ to $\omega = 1.16$. For each given value of the absolute wage differential, the algorithm has reliably considered 5000 transient iterations (i.e., the number of initial iterations left out in the plotting in order to disregard the transitional dynamics) and plotted only the subsequent 5000 iterations. In the top panel, for instance, for $\omega = 1.082$ all plotted points are at the polymorphic evolutionary equilibrium represented by $\lambda^* = \beta = 0.5$ (as it is a stable 1-cycle), while for $\omega = 1.12$ all plotted points are at the stable period 2-cycle, and for $\omega = 1.135$ all plotted points are at the stable period 4-cycle. In fact, the bifurcation diagram in Figure 3 (top) shows that for $\omega \approx 1.133$ a second flip (or period-doubling) bifurcation has happened, with the period 2-cycle becoming unstable and a new stable period 4-cycle being born.

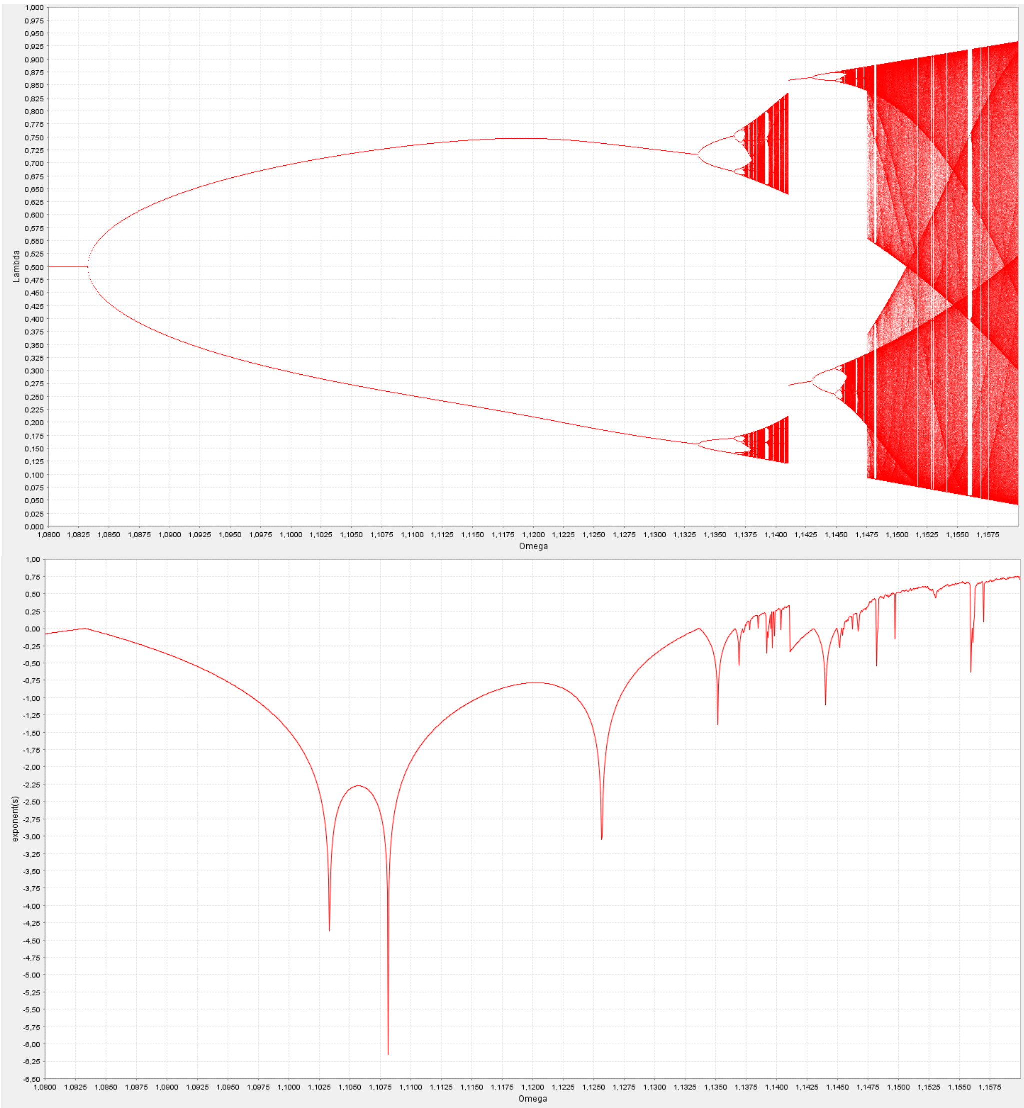
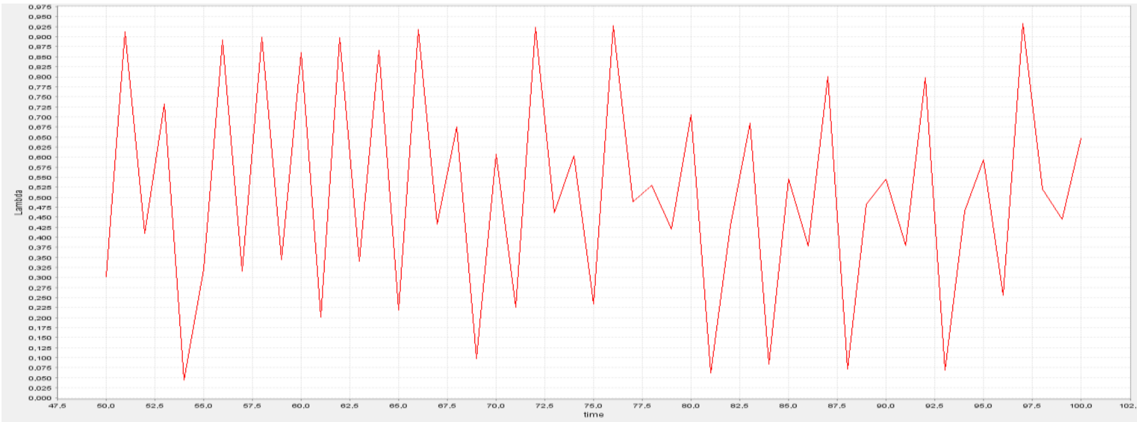
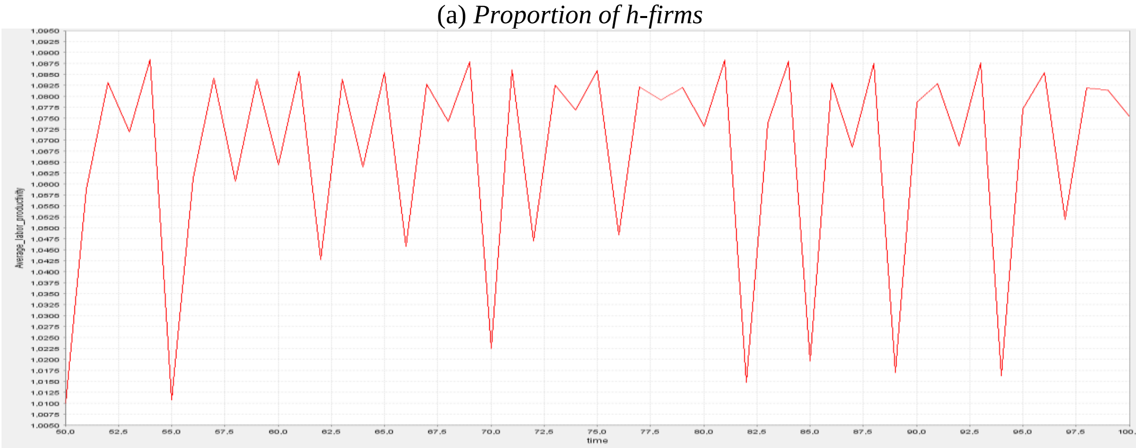


Figure 3. *Bifurcation diagram and Lyapunov exponent for the map H*

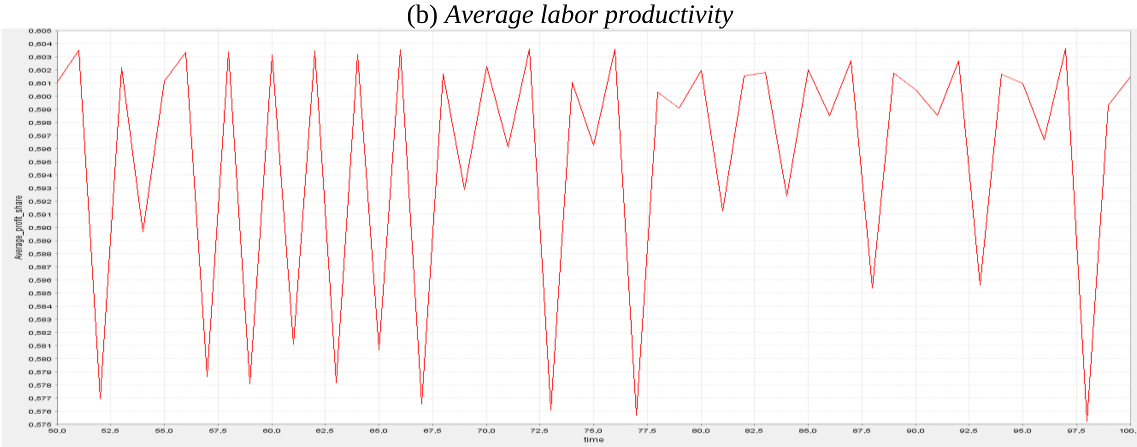
Meanwhile, the bottom panel in Figure 3 shows that for most parameter values greater than $\omega \approx 1.138$ the Lyapunov exponent is positive, revealing chaotic dynamics. The diagram of the Lyapunov exponent in Figure 3 (bottom) has infinitely many spikes, which readily indicates the occurrence of stable cycles. Note that at bifurcation of cycles the Lyapunov exponent touches 0, for instance, when a visible sequence of flip bifurcations happen from a stable period 1- to a stable period 2-cycle (for $\omega = \omega_c = 1.083$), a stable period 2- to a stable period 4-cycle (for $\omega \approx 1.133$) and a stable period 4- to a stable period 8-cycle ($\omega \approx 1.136$). After a time interval mostly dominated by chaotic dynamics, another visible sequence of flip bifurcations occur from a stable period 2- to a stable period 4-cycle (for $\omega \approx 1.142$) and a stable period 4- to a stable period 8-cycle ($\omega \approx 1.145$). For $\omega > 1.146$, which corresponds almost exclusively to blurred regions in the bifurcation diagram, some stable cycles emerge again, yet most often the



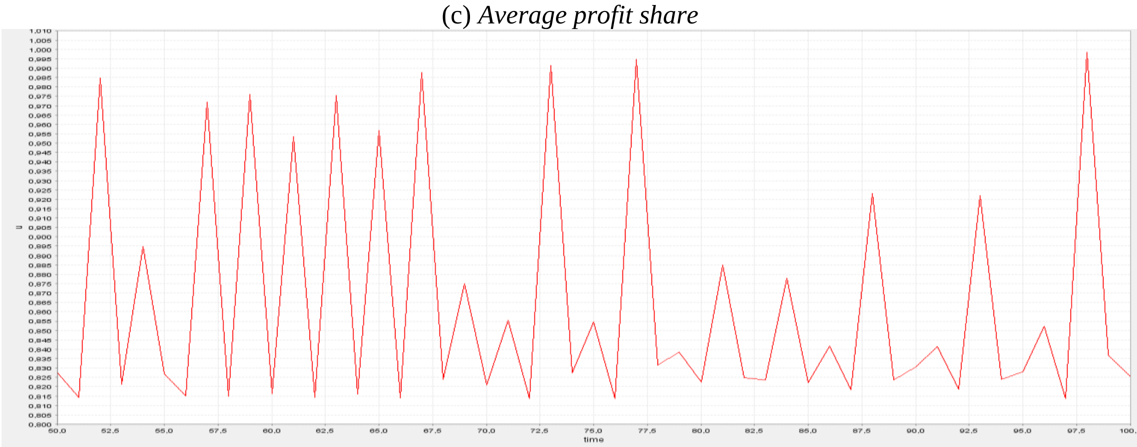
(a) *Proportion of h-firms*



(b) *Average labor productivity*



(c) *Average profit share*



(d) *Capacity utilization*

Figure 4. *Dynamic behavior of the map H for relatively high absolute wage differential* $(\omega = 1.16)$

dynamic behavior of the map H in (25-a) fails to converge to a periodic cycle and is chaotic instead.

In order to further illustrate how considerably more complicated the dynamic behavior of the map H in (25-a) becomes as the parameter measuring the absolute wage differential increases just from $\omega = 1.145$ to $\omega = 1.16$, Figure 4 describes time series for most endogenous variables when such parameter is given by $\omega = 1.16$. As depicted in panels (a)-(d), the dynamic behavior of the proportion of higher wage firms, average productivity, average profit share and capacity utilization, respectively, is characterized by recurrent and irregular fluctuations driven by the deterministic law of motion in (25-a), which is a signature of deterministic chaos.

5. Conclusions

Against the backdrop of the empirical evidence on the endogeneity of labor productivity to the wage remuneration and the persistence of wage inequality, this paper has set forth an evolutionary micro-dynamic model having these two documented features of the labor market as interconnected. Firms periodically revise their choice of remuneration with a higher or lower wage, with the resulting labor productivity differential across workers being endogenous to the distribution of wage remuneration strategies across firms.

The evolutionary micro-dynamic of the frequency distribution of wage remuneration strategies across firms crucially impacts on the behavior of the functional distribution of income and thereby on the macro-dynamic of the rates of capacity utilization and output growth. One key result derived in the paper is that the long run features persistent wage inequality. Moreover, plausibly low levels of wage inequality suffice to cause the distribution of wage remuneration strategies across firms, and therefore the distribution of income, capacity utilization and output growth, all to experience self-sustaining cyclical fluctuations. In fact, for not much higher (and still empirically plausible) levels of wage inequality, the long-run behavior of such variables is characterized by chaotic dynamics. These results were both derived analytically and illustrated with numerical simulations.

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Appendix 1: Behavior of the equilibrium average profit share

We can use the expression in (9) to evaluate how the equilibrium profit share vary with the proportion of higher wage firms:

(A-1.1) $$\frac{\partial \bar{\pi}^*(\lambda_t)}{\partial \lambda_t} = [\pi_h^*(\lambda_t) - \pi_l] + \lambda_t \frac{\partial \pi_h^*(\lambda_t)}{\partial \lambda_t}.$$

Since $\pi_h^*(\beta) - \pi_l = 0$ and, per (7), we have $\frac{\partial \pi_h^*(\lambda_t)}{\partial \lambda_t} < 0$, it follows that the derivative in (A-1.1) is strictly negative for all $\lambda_t \in (\beta, 1] \subset \mathbb{R}$. However, for any $\lambda_t \in (0, \beta) \subset \mathbb{R}$ we have $\pi_h^*(\beta) - \pi_l > 0$, given that $\frac{\partial \pi_h^*(\lambda_t)}{\partial \lambda_t} < 0$. Therefore, the sign of the derivative in (A-1.1) in the open interval represented by $\lambda_t \in (0, \beta) \subset \mathbb{R}$ requires further investigation.

Using (6) and (7), we can re-write the derivative in (A-1.1) as follows:

(A-1.1.a) $$\frac{\partial \bar{\pi}^*(\lambda_t)}{\partial \lambda_t} = w_l \left[1 - \left(1 + \lambda_t \ln \omega\right) \omega^{\lambda_t - \beta}\right].$$

It follow from the re-written expression in (A-1.1.a) that $\frac{\partial \bar{\pi}^*(0)}{\partial \lambda_t} = w_l \left(1 - \omega^{-\beta}\right) > 0$ and $\frac{\partial \bar{\pi}^*(\beta)}{\partial \lambda_t} = -w_l \beta \ln \omega < 0$. Therefore, we apply the intermediate value theorem to conclude that there is some $\bar{\lambda} \in (0, \beta) \subset \mathbb{R}$ such that $\frac{\partial \bar{\pi}^*(\bar{\lambda})}{\partial \lambda_t} = 0$. Moreover, the derivative in (A-1.1.a) is a strictly decreasing function with respect to λ_t in the unit interval, given that:

(A-1.2) $$\frac{\partial^2 \bar{\pi}^*(\lambda_t)}{\partial \lambda_t^2} = -w_l \left(1 + \lambda_t \omega^{\lambda_t - \beta} \ln \omega\right) \ln \omega < 0,$$

for all $\lambda_t \in [0,1] \subset \mathbb{R}$. Consequently, since the function in (A-1.1.a) is continuous in the real closed interval $[0,1]$, there is only one $\bar{\lambda} \in (0, \beta) \subset \mathbb{R}$ such that $\frac{\partial \bar{\pi}^*(\bar{\lambda})}{\partial \lambda_t} = 0$.

Since (A-1.1.a) is a strictly decreasing function with respect to λ_t in the unit interval and there is a unique $\bar{\lambda} \in (0, \beta) \subset \mathbb{R}$ such that $\frac{\partial \bar{\pi}^*(\bar{\lambda})}{\partial \lambda_t} = 0$, it then follows that $\frac{\partial \bar{\pi}^*(\lambda_t)}{\partial \lambda_t} > 0$ for all $\lambda_t \in [0, \beta) \subset \mathbb{R}$ and $\frac{\partial \bar{\pi}^*(\lambda_t)}{\partial \lambda_t} < 0$ for all $\lambda_t \in (\beta, 1] \subset \mathbb{R}$, so that $\bar{\lambda}$ is the global maximum in the closed interval $[0,1] \subset \mathbb{R}$.

Appendix 2: Existence of a supercritical flip bifurcation

(i) Since $w_l > 0$, $\beta \in (0,1) \subset \mathbb{R}$, $\omega > 1$ and $G'(1-w_l) > 0$, it follows from the linearization in (29) that $\frac{\partial H(\beta; w_l, \beta, \omega)}{\partial \lambda_t} < 1$. In turn, $\frac{\partial H(\beta; w_l, \beta, \omega)}{\partial \lambda_t} = 1 - \beta(1-\beta)G'(1-w_l)w_l \ln \omega > -1$ is satisfied if $\beta(1-\beta)G'(1-w_l)w_l \ln \omega < 2$, which holds for $\ln \omega < \frac{2}{\beta(1-\beta)G'(1-w_l)w_l} \equiv \ln \omega_c$. The later inequality is equivalent to $\omega < \omega_c$. Therefore, the eigenvalue in (29) is inside the unit circle for any absolute wage differential $\omega \in (1, \omega_c) \subset \mathbb{R}$.

(ii) Drawing on Medio and Lines (2003, subsection 5.4.2, pp. 156-157), a supercritical flip bifurcation at the fixed point $\lambda^* = \beta$ of the map $H(\lambda_t; w_l, \beta, \omega)$ in (25-a) occurs if, and only if, the following conditions are satisfied:

(A-2.1) $$\frac{\partial H(\beta; w_l, \beta, \omega_c)}{\partial \lambda_t} = -1;$$

(A-2.2) $$\frac{\partial^2 H^2(\beta; w_l, \beta, \omega_c)}{\partial \lambda_t^2} = 0 \text{ and } \frac{\partial^3 H^2(\beta; w_l, \beta, \omega_c)}{\partial \lambda_t^3} \neq 0,$$

where $H^2(\lambda_t; w_l, \beta, \omega) \equiv H\left(H(\lambda_t; w_l, \beta, \omega); w_l, \beta, \omega\right)$ is the second iterate of the map H;

(A-2.3) $$\frac{\partial H^2(\beta; w_l, \beta, \omega_c)}{\partial \omega} = 0 \text{ and } \frac{\partial^2 H^2(\beta; w_l, \beta, \omega_c)}{\partial \omega \partial \lambda_t} \neq 0;$$

and

(A-2.4) $$-\frac{\partial^3 H^2(\beta; w_l, \beta, \omega_c)}{\partial \lambda_t^3} \Bigg/ \frac{\partial^2 H^2(\beta; w_l, \beta, \omega_c)}{\partial \omega \partial \lambda_t} > 0.$$

We will show that all these conditions are satisfied.

Recalling that $\omega_c \equiv e^{\frac{2}{\beta(1-\beta)G'(1-w_l)w_l}}$, so that $\ln \omega_c = \frac{2}{\beta(1-\beta)G'(1-w_l)w_l}$, it then follows that $\frac{\partial H(\beta; w_l, \beta, \omega_c)}{\partial \lambda_t} = 1 - \beta(1-\beta)G'(1-w_l)w_l \left(\frac{2}{\beta(1-\beta)G'(1-w_l)w_l} \right) = -1$, which is the condition in (A-2.1).

The second iterate of the map in (25-a) is given by:[5]

[5] As the parameters w_l and β are taken as given, in order to simplify the notation, thereafter we will leave out the dependence of the second iterate with respect to these parameters, that is, from now on we will write simply $H^2(\lambda_t; \omega)$ instead of $H^2(\lambda_t; w_l, \beta, \omega)$.

(A-2.5)

$$H^2(\lambda_t;\beta) \equiv H(H(\lambda_t;\beta))$$
$$= H(\lambda_t;\beta) + H(\lambda_t;\beta)[1-H(\lambda_t;\beta)][G(\pi_h^*(H(\lambda_t;\beta)) - G(\pi_l)]$$
$$= \left\{\lambda_t + \lambda_t(1-\lambda_t)[G(\pi_h^*(\lambda_t)) - G(\pi_l)]\right\}\left\{1 + \left[(1-\lambda_t)^2[G(\pi_h^*(\lambda_t)) - G(\pi_l)]\right][G(\pi_h^*(H(\lambda_t;\beta)) - G(\pi_l)]\right\}.$$

Making use of (A.2.5) we can calculate the second order derivative evaluated at $\lambda_t = \beta$, which is given by:

(A-2.6)
$$\frac{\partial^2 H^2(\beta;\omega)}{\partial\lambda_t^2} = \left\{\left[\beta(1-\beta)G'(1-w_l)w_l\ln\omega\right]^2 + 2 - 3\beta(1-\beta)G'(1-w_l)w_l\ln\omega\right\}$$
$$\left\{\left[4\beta - 2 - \beta(1-\beta)\ln\omega\right]G'(1-w_l) + \beta(1-\beta)G''(1-w_l)w_l\ln\omega\right\}w_l\ln\omega.$$

By evaluating the derivative in (A-2.6) at the level of wage inequality $\omega = \omega_c$, we obtain $\frac{\partial^2 H^2(\beta;\omega_c)}{\partial\lambda_t^2} = \left\{2^2 + 2 - 3\times 2\right\}\left\{\left[4\beta - 2 - \frac{2}{G'(1-w_l)w_l}\right]G'(1-w_l) + G''(1-w_l)\frac{2}{G'(1-w_l)}\right\}\frac{2}{\beta(1-\beta)G'(1-w_l)} = 0$, as required by the first part of the condition in (A-2.2).

Based on the second order derivative in (A.2.5), we then obtain the third order derivative at $(\lambda_t;\omega) = (\beta;\omega_c)$ as:

(A-2.7)

$$\frac{\partial^3 H^2(\beta;\omega_c)}{\partial\lambda_t^3} = \frac{-8}{\left[\beta(1-\beta)w_l\right]^2\left[G'(1-w_l)\right]^4}\Big[9(1-2\beta)w_l\left(G'(1-w_l)\right)^3$$
$$+3\left(2-7\beta(1-\beta)\right)w_l^2\left(G'(1-w_l)\right)^4 + 6w_l^2\left(G''(1-w_l)\right)^2$$
$$+\left(G'(1-w_l)\right)^2\left(4 - 9(1-2\beta)w_l^2 G''(1-w_l)\right) - 2w_l G'(1-w_l)\left(3G''(1-w_l) + w_l G'''(1-w_l)\right)\Big].$$

Let us assume that $\pi^i \in [\mu-\xi,\mu+\xi] \subset (0,1) \subset \mathbb{R}$ is a random variable uniformly distributed across firms. More precisely, $G(\cdot)$ follows an uniform distribution with support $[\mu-\xi,\mu+\xi]$, with $0<\mu<1$ and $0<\xi<\min\{\mu,1-\mu\}$. Consequently, it follows that $G'(1-w_l) = \frac{1}{2\xi}$ and $G''(1-w_l) = G'''(1-w_l) = 0$, so that the third order derivative in (A-2.7) reduces to:

(A-2.7.a)
$$\left.\frac{\partial^3 H^2(\beta;\omega_c)}{\partial\lambda_t^3}\right|_{G'(1-w_l)=\frac{1}{2\delta}} = \frac{-8}{\left[\beta(1-\beta)w_l\right]^2}\left[9(1-2\beta)w_l 2\delta + 3\left(2-7\beta(1-\beta)\right)w_l^2 + 16\delta^2\right].$$

Note that the simplified expression within brackets in (A-2.7.a) can be conveniently further re-written as $21w_l^2\beta^2 - (36\delta + 21w_l)w_l\beta + 16\delta^2 + 18\delta w_l + 6w_l^2$. Therefore, this latter expression is null if, and only if, $\beta = \frac{(36\delta + 21w_l) \pm \sqrt{-(48\delta + 63w_l^2)}}{46w_l}$, which are complex roots. Based on this latter result and the fact the coefficient $21w_l^2$ in the previously simplified expression is strictly positive for all $w_l > 0$, we infer that $21w_l^2\beta^2 - (36\delta + 21w_l)w_l\beta + 16\delta^2 + 18\delta w_l + 6w_l^2 > 0$ for all β. Consequently, considering the third order derivative in (A-2.7.a), we conclude that $\left.\frac{\partial^3 H^2(\beta;\omega_c)}{\partial\lambda_t^3}\right|_{G'(1-w_l)=\frac{1}{2\delta}} < 0$, which satisfies the second part of the condition in (A-2.2).

Let us focus on the condition in (A-2.3). From (A-2.5) we obtain the following derivative:

(A-2.8)

$$\frac{\partial H^2(\lambda_t;\omega)}{\partial\omega} = \lambda_t(1-\lambda_t)G'(\pi_h^*(\lambda_t))\frac{\partial\pi_h^*(\lambda_t)}{\partial\omega}\left\{1+\left[(1-\lambda_t)^2[G(\pi_h^*(\lambda_t)-G(\pi_l)]\right][G(\pi_h^*(H(\lambda_t;\beta))-G(\pi_l)]\right\}$$
$$+\left\{\lambda_t + \lambda_t(1-\lambda_t)[G(\pi_h^*(\lambda_t)-G(\pi_l)]\right\}\left\{\left[(1-\lambda_t)^2 G'(\pi_h^*(\lambda_t)\frac{\partial\pi_h^*(\lambda_t)}{\partial\omega}\right][G(\pi_h^*(H(\lambda_t;\beta))-G(\pi_l)]\right.$$
$$\left.+\left[(1-\lambda_t)^2[G(\pi_h^*(\lambda_t)-G(\pi_l)]\right]G'(\pi_h^*(\lambda_t)\frac{\partial\pi_h^*(\lambda_t)}{\partial\omega}\right\}.$$

Based on (6) we have that $\frac{\partial\pi_h^*(\lambda_t)}{\partial\omega} = -w_l(\lambda_t - \beta)\omega^{\lambda_t-\beta-1}$, which is equal to zero at $\lambda_t = \beta$. Considering this latter result and recalling that $G(\pi_h^*(H(\beta;\beta)) - G(\pi_l) = 0$, it follows from (A-2.8) that $\frac{\partial H^2(\beta;\omega)}{\partial\omega} = 0$ for any ω. Therefore, the first part of the condition in (A-2.3) holds.

Meanwhile, making use of (A-2.5) we obtain the following mixed second order partial derivative at the point $(\lambda_t;\omega) = (\beta;\omega_c)$:

(A-2.9) $$\frac{\partial^2 H^2(\beta;\omega_c)}{\partial\omega\partial\lambda_t} = 2\beta(1-\beta)G'(1-w_l)w_l\omega_c.$$

As $0<\beta<1$, $\delta>0$, $G'(1-w_l)=\frac{1}{2\delta}>0$, $w_l>0$ and $\omega_c = e^{\frac{2}{\beta(1-\beta)G'(1-w_l)w_l}}>0$, it follows that the derivative in (A-2.9) is strictly positive. Hence, the second part of the condition in (A-2.3) is satisfied.

Finally, given that $\frac{\partial^2 H^2(\beta;\omega_c)}{\partial\omega\partial\lambda_t} > 0$ and, based on (A-2.7.a), we have $\left.\frac{\partial^3 H^2(\beta;\omega_c)}{\partial\lambda_t^3}\right|_{G'(1-w_l)=\frac{1}{2\delta}} < 0$, we infer that the condition in (A-2.4) holds if $G(\cdot)$ follows an uniform distribution with support $[\mu-\xi,\mu+\xi]\subset(0,1)\subset\mathbb{R}$, with $\mu\in\subset(0,1)\subset\mathbb{R}$ and $\xi\in(0,\min\{\mu,1-\mu\})\subset(0,1)\subset\mathbb{R}$.