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Education quality and non-convergence

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This paper assesses the role of education quality in the convergence process of GDP per capita through teachers quality impact in human capital formation. The simple two-period OLG model suggests initial level of teacher's human capital is important to explain non-convergence, even when education quality return is decreasing. This non-convergence arises because an initially low level of teachers' human capital translates into a low level of human capital transferred to students, which means a low level of teachers' human capital in the next period, and so on. It is also shown an education quantity-quality trade-off, despite all dynamics coming from quality evolution. This trade-off helps to explain why developing countries did not reached high GDP levels, despite recent evolution of average years of schooling in these countries. The paper, therefore, provides an alternative explanation for why countries income does not converge, even when differences in other inputs, such as capital stock, are not accounted for.

Keywords: human capital; education quality; economic growth.

JEL Codes: O40; J24; I25.

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1 Introduction

Differences in human capital stock have been indicated for decades as one of the major factors that explain why some countries are richer than others. More recently, however, the quality embodied in this human capital stock began to be analyzed more carefully. Hanushek (2013), Hanushek and Woessmann (2012), Tamura (2001), and Schoellman (2012), just to name a few works, assess the role of human capital quality[1] on explaining output per worker (or wages, in the latter) variability: quality seems to be at least as important as quantity (empirically and theoretically), and in some specifications much more important than only quantity.

Figure 1 shows that the vast majority of countries have approached the United States in education attainment in the past decades. Thus, one should expect log GDP convergence if human capital quantity is viewed as much more important than others inputs. However, figure 2 shows a stable GDP distribution, with no convergence at all. Along with this fact, Hanushek (2013) presents evidence that when controlling for cognitive skills acquired during education process (a measure of education quality), the initial value of years of schooling does not have any effect on average annual growth rate of GDP per capita in the period 1960-2010.

Growth models that account for education quality, such as Tamura (2001), usually take it as exogenous or as a policy variable that responds to an arbitrary central planner objective function. This paper tries to contribute by making education quality endogenous, which arises through a simple two-period OLG model with occupational choice. Similar to Gilpin and Kaganovich (2012), education quality is measured by average human capital of teachers, with the proportion of teachers on population being assumed to be constant. Convergence process is analyzed with and without the presence of human capital externalities, which is pointed by the literature as an

[1] Human capital defined as education (quality and quantity) only, leaving health and other components of a human capital's broader definition aside.

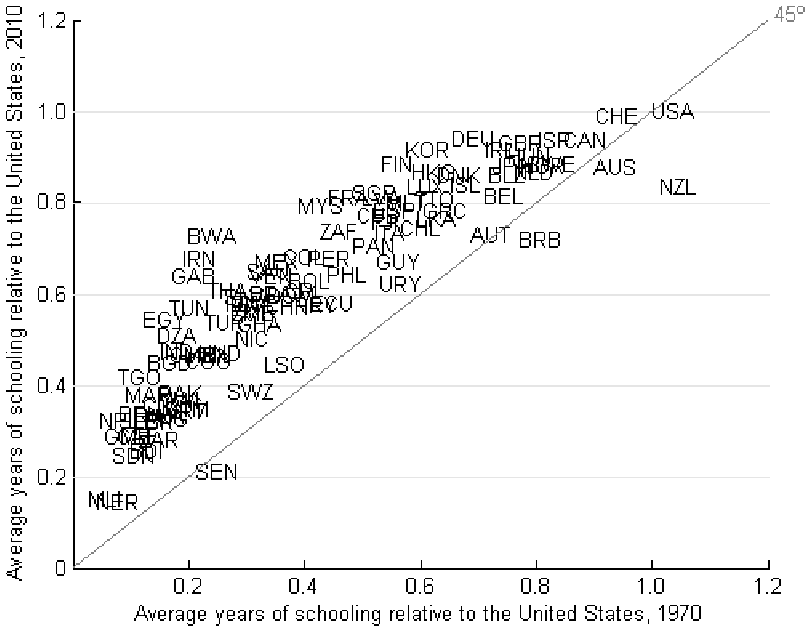

Figure 1: Average years of schooling in 1970 *versus* average years of schooling in 2010.

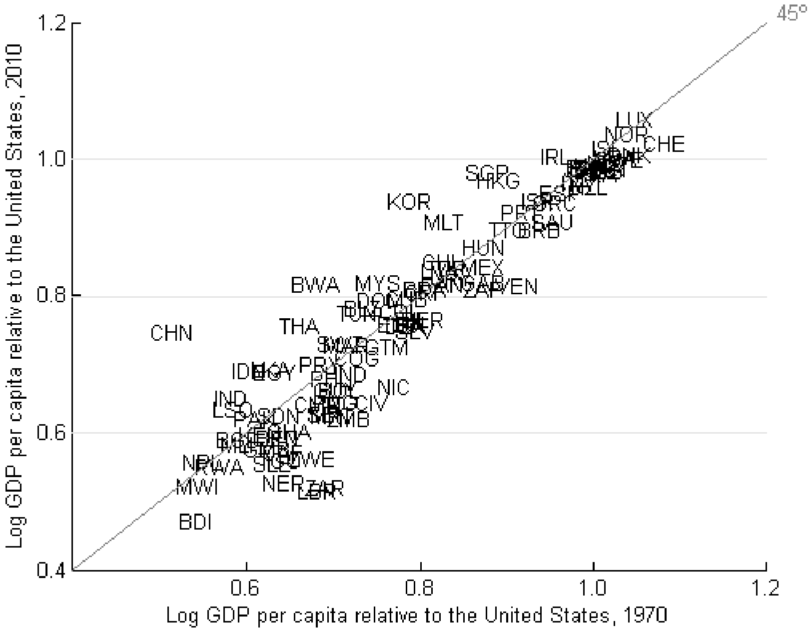

Figure 2: Log GDP per capita in 1970 *versus* log GDP per capita in 2010.

important channel of higher returns to education[2].

The modeling strategy proposed here differs from previous works when, under some conditions, even when countries differ only in their education quality level we could still observe non-convergence or, at least, convergence to different steady-states values. It is shown that in the presence of a certain kind of human capital externalities, even with decreasing returns to education quality, there could be a multiple steady-state environment, where the initial distribution of human capital in a country could define to which steady-state this country would converge in the long-run. The explanation presented in this paper is that an initially low level of human capital would lead to an initially low level of teachers' human capital, which translates into a low level of human capital transfered to students. Then, some of these students would chose the teacher career in the next period, which would mean a still low level of teachers' human capital, and so on. Thus, under a set of conditions that are presented in the next sections, the shape of human capital's initial distribution defines the long-run equilibrium through the channel of education quality (figure 3).

This teacher quality mechanism appears to be reasonable and in accordance with the economics of education literature. Cheety et al. (2014) show that the quality of teaching is of great importance in the students' future through an estimation of long-term impacts of high value-added teachers. Moreover, Rivkin et al. (2005) and Rockoff (2004) show that, when dealing with the determinants of students' academic achievement, teacher fixed effects (a measure of teacher quality) are very important predictors of student outcome[3].

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[2]Despite Acemoglu and Angrist (2000) founding evidence that human capital externalities of high school are statistically insignificant and also not large in magnitude, Iranzo and Peri (2009) found that social returns of an increase in the proportion of population that have a college degree is significant and of magnitude similar to private returns.

[3]According to Rivkin et al. (2005), *"the results demonstrate quite clearly that the observable school and teacher characteristics explain little of the between-classroom variation in achievement growth despite the fact that a substantial share of the overall achievement gain variation occurs between teachers. Importantly, even though the sample includes just*

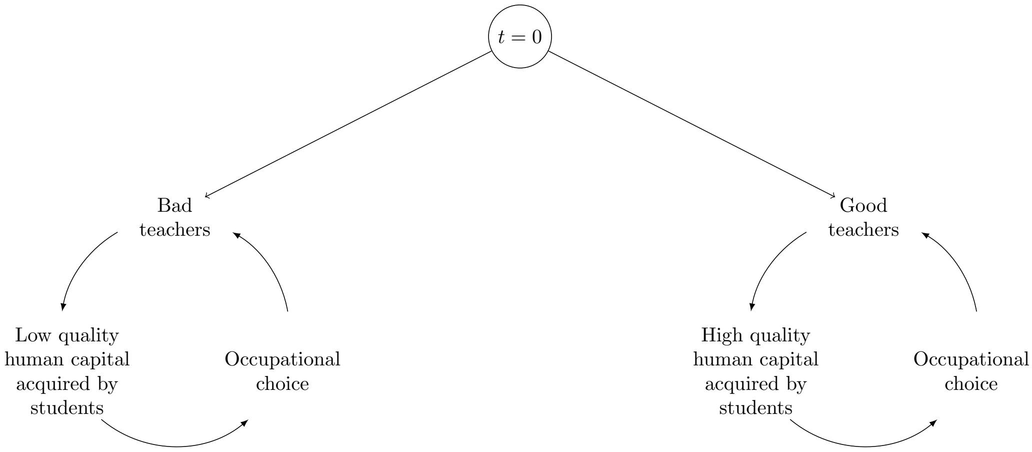

Figure 3: Cycles of human capital accumulation conditional on initial human capital's distribution.

Going further and trying to dialog with the "real world" data, we perform a simulation exercise to assess the model and its predictions. With calibration grounded on micro literature evidence and taking USA as benchmark, it is also shown that the required policy through which the poorer country would converge to the steady-state (or growth path) of the richer country, could be very costly for current generations as the consumption level in the period of the policy change would certainly fall.

This paper tries to explain the well debated problem of non convergence, but using education quality as the main channel of human capital dynamics and through which multiple steady-states arises. The paper is closer to Azariadis and Drazen (1990) when accounting for human capital externalities, but also dialogs with the literature that deals with non-convergence problem through some market imperfection in OLG models, e.g. Galor and Zeira (1993) and Galor and Moav (2004). A similarity between all of these models and the one presented in this paper is the need of a non-convexity in some accumulation function to generate multiple steady-states.

schools with a single teacher per grade, the inclusion of school rather than teacher fixed effects reduces the explanatory power by over forty percent" (p. 421).

The rest of this paper is organized as follows. Section 2 presents the basic model. Section 3 describes the initial conditions, its underlying hypothesis and what is the predicted long-run equilibrium. Section 4 relies on the same initial conditions to analyze the long-run equilibrium when human capital externalities are accounted for. Section 5 draws some comparative dynamics to help us understand the underlying mechanisms through which aggregate variables adjust to policy shocks. Section 6 uses calibrated parameters to simulate the model and perform some quantitative exercises, such as policy analysis. Section 7 presents conclusions.

2 The Basic Model

2.1 Individuals

Consider a simple two-period OLG economy with no population growth and a continuum of individuals of measure 1. Individuals accumulate human capital in the first period and choose a career (teacher or market employee), work and consume their entire income in the second period. Accumulation of human capital is compulsory and taken in public schools. Preferences are identical across individuals and assumed to exhibit a constant relative risk aversion (CRRA):

$$U_t^i = \frac{(c_t^i)^{1-\sigma} - 1}{1-\sigma} \quad \text{subject to} \quad \begin{cases} c_t^i \leq (1-\tau_M).w_t^M.h_t^i + \phi_t, & \text{if } i \in M \\ c_t^i \leq (1-\tau_T).w_t^T + \phi_t, & \text{if } i \notin M \end{cases} \tag{1}$$

where $\sigma > 0$, τ_M and τ_T are income tax rates, $i \in M$ means that individual i works at the private market and $i \notin M$ means that individual i works as a teacher. ϕ_t is a lump-sum transfer granted by government according to its budget constraint. Note that all teachers earn the same income independently of their human capital level h_t^i, which is in accordance with the teacher's hiring process presented later in the text.

Individuals, however, may differ in their innate ability a_t^i, which is distributed uniformly over the interval $[0, \bar{B}]$[4]. This innate ability is a component of human capital accumulation function along with the proportion of first period spent in school ($s_{t-1} \in [0,1]$), which is the same for all individuals and chosen by government, and education quality, which is given by average human capital of individuals that have chosen to be teachers in previous period (h^T_{t-1}). Thus,

$$h^i_t = C_t.a^i_t.(s_{t-1})^\eta.(h^T_{t-1})^v \tag{2}$$

where $\eta, v > 0$, and C_t is a time-dependent productivity term, which is assumed to be constant when externalities are not acting and is assumed to be a function of aggregate human capital when externalities are present. We assume that s_t is a function of the proportion of teachers in population and other variables that are not under the control of individuals[5]. Given this structure of preferences and human capital accumulation, individuals will choose only what career to take as a response to their level of human capital and so to their level of innate ability. The analysis of the individual who is indifferent between choosing teacher career or market career give us the innate ability threshold which define who will be part of teachers group and market group.

$$U^{iM}_t = U^{iT}_t \iff (1-\tau_M).w^M_t.h^i_t = (1-\tau_T).w^T_t$$

$$a^\star_t = \frac{(1-\tau_T).w^T_t}{(1-\tau_M).w^M_t}.\frac{1}{C_t.(s_{t-1})^\eta.(h^T_{t-1})^v} \tag{3}$$

Therefore, everyone who has a level of innate ability such that $a^i_t > a^\star_t$ will choose to be part of private market. All individual i who has $a^i_t < a^\star_t$

[4]This assumption of uniform distribution follows Galor and Moav (2000) and Gilpin and Kaganovich (2012).

[5]Despite being a restrict formulation, a simple cross-country estimation (using 2005 World Bank data) with 55 countries shows that the adjusted R^2 of $ln(s_t)$ on the ln of the proportion of teachers in population and on the ln of the well-known pupil-teacher ratio is around 67%.

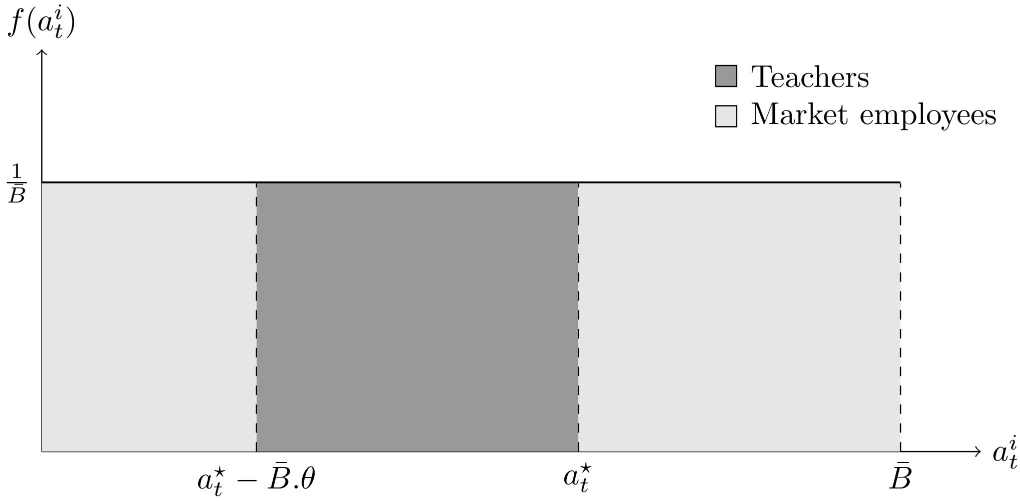

Figure 4: Probability density function of innate ability. The dark grey area is the proportion θ of teachers in the population. Every individual whose innate ability is outside this dark grey area will be a market employee.

would chose to be a teacher, but we impose as a restriction that the fraction of teachers in the population is constant and equal to θ for every t. This restriction can be thought as a physical restriction: for a given educational infrastructure (e.g., a given number of schools), the government must hire θ teachers to make its educational system work properly. It can not hire less than θ and would not be able to accommodate more than θ. In addition, these θ teachers are hired through a public tender that occurs in every period and offers only θ jobs, which, through competition among all applicants, are filled by the θ most qualified individuals. Figure 4 shows what this constant θ implies: all individuals with a sufficiently low level of innate ability will be market employees despite their initial desire of being teachers, since all θ teachers would have already been hired. It is also implied by a constant θ that $s_t = s, \forall t$. Therefore, the proportion of time spent on school will change only with a change in θ.

2.2 Production of final output

Production is described by:

$$Y_t = A_t.H_t^{\beta}$$

where $H_t = \int_{i \in M} h_t^i.di$, A_t is the exogenous level of aggregate productivity at time t and $\beta \in (0, 1]$. Thus, the marginal productivity of human capital (and also market wage as we assume perfect competition and profit maximization) is given by:

$$w_t^M = A_t.\beta.H_t^{\beta-1} \tag{4}$$

2.3 Government

Teachers are hired by a government that collects a τ_M fraction of market employees income and a τ_T fraction of teachers income. As a policy choice, government spends a p_t fraction of all its tax revenue paying teachers and $1 - p_t$ with a lump-sum transfer that complements consumers' income in equation (1). Despite the fact that p_t can vary with time (e.g., following a government policy of gradual increase in teachers' salaries), we assume a constant $p_t = p$ throughout the paper. Thus, the government budget constraint is given by:

$$(\tau_M.w_t^M.H_t + w_t^T.\tau_T.\theta).p = \theta.w_t^T$$

Teacher's salary (w_t^T) can be set as a function of other variables and parameters so that

$$w_t^T = \frac{p.\tau_M.w_t^M.H_t}{\theta.(1 - p.\tau_T)} \tag{5}$$

Notice that despite w_t^T being the same for all teachers, it evolves according to the whole economy: given equation (4), the salary of teachers will be higher the higher economy's Y_t. Therefore, given model parameters and government budget constraint, w_t^T will evolve endogenously.

2.4 Aggregate human capital

Aggregate human capital is obtained by simply integrating the individual human capital function in the ability interval that defines who works at the market and who works as teacher.

$$H_t = \int_0^{a_t^\star - \bar{B}.\theta} h_t^i(a_t^i).dF(a_t^i) + \int_{a_t^\star}^{\bar{B}} h_t^i(a_t^i).dF(a_t^i)$$

$$h_t^T = \theta^{-1}. \int_{a_t^\star - \bar{B}.\theta}^{a_t^\star} h_t^i(a_t^i).dF(a_t^i)$$

Hence, it follows from (3)-(5) (see Appendix A) that

$$H_t = \frac{C_t.\bar{B}}{2}.M.s^\eta.(h_{t-1}^T)^v \tag{6}$$

$$h_t^T = \frac{C_t.\bar{B}}{2.\theta}.\left[1 - M\right].s^\eta.(h_{t-1}^T)^v \tag{7}$$

where $M \equiv \left[\frac{(1+\theta^2).(1-p.\tau_T).(1-\tau_M)}{(1-p.\tau_T-\tau_M.(1-p))}\right]$.

Since aggregate human capital of whole population (teachers and market employees) is obtained by taking the integral of individual human capital function over the whole ability interval, we can represent it by the sum $H_t + \theta.h_t^T$, which is equal to $\frac{C_t.\bar{B}}{2}.s^\eta.(h_{t-1}^T)^v$. Then, equation (6) shows that H_t is a constant fraction M of whole population's aggregate human capital. However, what is the intuition behind M?

For simplicity, let's assume for a moment that $p = 1$. In this case, M will be reduced to $(1+\theta^2).(1-\tau_M)$, which represents the incentives that individuals face in the occupational choice process. Note that for a greater θ, the same government budget will pay less for each teacher and the teacher career will be less attractive to all individuals in t, including those with higher values for innate ability. Therefore, H_t will be higher through a greater M, and the average teacher will be less qualified for the same reason. Nevertheless, a greater τ_M will act on the contrary direction, making the market career less attractive in t: H_t will be smaller due to a smaller M.

Going further, a technical assumption about innate ability threshold is required in order to guarantee that we will have θ teachers in all periods, to guarantee that the dark grey area in figure 4 will be equal to θ.

Assumption 1. *(Assumption to guarantee θ teachers for every t). In order to have a proportion of θ teachers in the population for every t, $a_t^\star$ must lie inside innate ability interval such that $\theta.\bar{B} < a_t^\star < \bar{B}$*[6]*.*

Finally, under Assumption (1), the complete solution of the model depends only on initial values for h_0^T and a function which defines C_t.

3 Initial conditions and convergence

Let's assume that individuals' human capital on $t = 0$ (i.e., h_0^i) is distributed uniformly over the interval $[x-\lambda; x+\lambda]$ such that $E[h_0^i] = x$ and $Var[h_0^i] = \frac{\lambda^2}{3}$. Using the human capital threshold of career choice, given by relative (teacher-market) income, and equation (5), h_0^T can be restated as a function only of structural parameters, fraction p, x and λ (see Appendix A for all steps).

$$h_0^T = \frac{(1-\tau_T).p}{(1-p.\tau_T-\tau_M.(1-p))}.\frac{\tau_M.x}{\theta} - (1-\tau_M).\theta.\lambda.\frac{(1-p.\tau_T)}{(1-p.\tau_T-\tau_M.(1-p))} \tag{8}$$

The model structure presented so far, along with initial conditions given by equation (8) above and the assumption of $C_t = C$ for now, leads to the following result:

Proposition 1. *(Convergence process under $C_t = C$, and different h_0^T). Consider two economies that are identical in all aspects except for their initial*

[6]Plugging equations (5) and (6) into equation (3), we can restate $\theta.\bar{B} < a_t^\star < \bar{B}$ as

$$2.\theta^2 < \frac{(1-\tau_T).p}{(1-p.\tau_T-\tau_T.\tau_M.(1-p))}.\tau_M.(1+\theta^2) < 2.\theta$$

level of teacher's human capital, which can be result of different x and/or λ. Given these different initial conditions:

1. $v \in (0,1)$*: both economies would converge to the same unique steady-state equilibrium.*
2. $v \geq 1$*: divergence would take place if*

$$\frac{C.\bar{B}}{2.\theta}.[1-M].s^{\eta} > 1$$

Proof. Since the whole model's dynamic is driven by h_t^T evolution, its behavior is a necessary and sufficient condition to define behavior of all real variables. Growth of h_t^T is given by:

$$g(h_{t-1}^T) \equiv \frac{h_t^T}{h_{t-1}^T} = \frac{C.\bar{B}}{2.\theta}.[1-M].s^{\eta}.(h_{t-1}^T)^{v-1}$$

1. $v \in (0,1)$.

   By definition, h_{ss}^T is a steady state if and only if $g(h_{ss}^T) = 1$, i.e. $h_t^T = h_{t-1}^T = h_{ss}^T$. Since $\lim_{h^T \to 0} g(h^T) = \infty$, $\lim_{h^T \to \infty} g(h^T) = 0$ and $g(.)$ is a continuous function, by the Intermediate Value Theorem there is a h_{ss}^T such that $0 < g(h_{ss}^T) = 1 < \infty$. Moreover, note that $g(.)$ is everywhere decreasing, i.e. $\partial g(h^T)/\partial h^T < 0 \; \forall h^T \in (0,\infty)$. Therefore, there can only exist a unique h_{ss}^T such that $g(h_{ss}^T) = 1$. Every country with the same structural and policy parameters will converge to this h_{ss}^T, regardless the initial conditions.

2. $v \geq 1$.

   In this case, $g(h^T) > 1 \; \forall h^T \geq 1$. For countries with the same structural and policy parameters, but with different initial conditions, divergence would take place, since an initial difference would persist (or even be amplified) through a $g(h^T)$ that is

   - equal across countries if $v = 1$.
   - always higher for an initially richer country if $v > 1$.

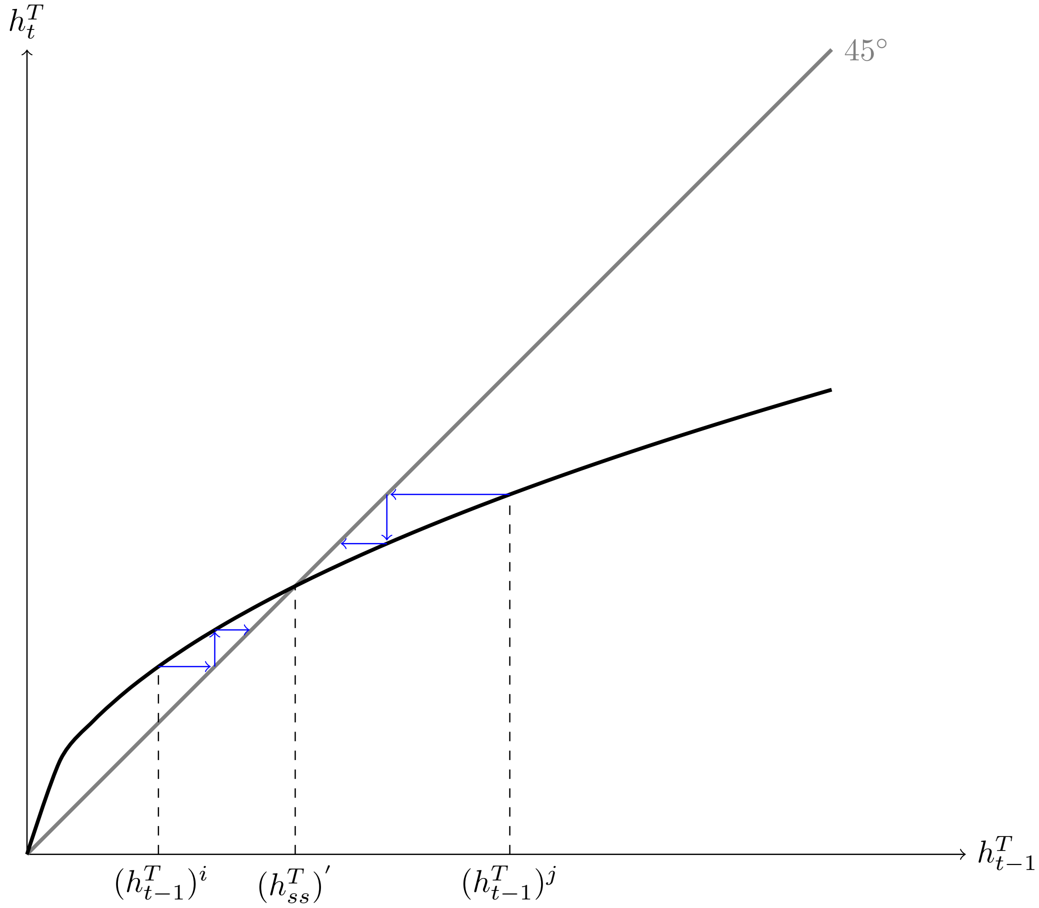

Figure 5: Average teachers' human capital evolution when $v \in (0, 1)$ according to equation (7). Both country i and country j converge to the same and unique steady-state equilibrium.

□

Proposition 1, however, does not bring any result that has not already been obtained in the literature: figure 5 shows the well known case of convergence when we have decreasing returns in the human capital accumulation function. The next section, which introduces the human capital externality idea, is more insightful.

4 Human capital externalities and convergence

It is usual to assume that aggregate human capital generates some kind of externality to the individual earnings, although the magnitude and at which point externality begins to act remains unclear. Acemoglu and Angrist (2000) present evidence that this externality isn't strong in magnitude when dealing with changes in secondary schooling. Iranzo and Peri (2009) reinforce this result about changes in secondary schooling, but show that there is strong and statistically significant external returns to education when there are positive changes in higher education. This comes from the fact that highly educated workers may facilitate the adoption of better and more productive technologies. However, for a sufficiently high level of aggregate human capital, children's accumulation of human capital could also benefit from more skilled teachers, parents, neighbors, etc. that would be able to help them in the learning process, which is subject to the adoption of newer technologies too. Therefore, the presence of this externality could boost not only the Total Factor Productivity of an economy (Iranzo and Peri (2009)) but also the accumulation of human capital itself[7].

Following this evidence, we now assume externality plays a role in the model and begins to act when human capital is above a certain threshold[8]. This externality is introduced via C_t, which is defined as a function of aggregate human capital of whole population[9] such that

$$C_t = D.\left(\frac{C_t.\bar{B}.s^{\eta}.(h_{t-1}^T)^v}{2}\right)^{\gamma}$$

and, thus,

[7]See Acemoglu (1996) for a model where the rate of return on human capital is increasing in the average human capital of the workforce but not through effects in the TFP.

[8]This threshold externality structure is directly related to Azariadis and Drazen (1990)

[9]Just as showed in section 2.4, this is given by $H_t + \theta.h_t^T = \frac{C_t.\bar{B}}{2}.s^{\eta}.(h_{t-1}^T)^v$

$$C_t = D^{\frac{1}{1-\gamma}}.\left(\frac{\bar{B}}{2}\right)^{\frac{\gamma}{1-\gamma}}.s^{\frac{\eta.\gamma}{1-\gamma}}.(h_{t-1}^T)^{\frac{v.\gamma}{1-\gamma}} \quad \forall t \text{ when } E[h_t^i] \geq \bar{h}$$

where $\gamma \in (0,1)$ and D is an arbitrary positive constant. Thus, C_t will have a discontinuity point at the human capital threshold defined as $\bar{h}$.

$$C_t = \begin{cases} C & \text{if } E[h_t^i] < \bar{h} \\ D^{\frac{1}{1-\gamma}}.\left(\frac{\bar{B}}{2}\right)^{\frac{\gamma}{1-\gamma}}.s^{\frac{\eta.\gamma}{1-\gamma}}.(h_{t-1}^T)^{\frac{v.\gamma}{1-\gamma}} & \text{if } E[h_t^i] \geq \bar{h} \end{cases} \tag{9}$$

When $\bar{h}$ is sufficiently high this strutcure is in accordance with empirical evidence of statistically insignificant external returns to secondary school but statistically significant external returns to schooling at higher levels, since for an initially high level of human capital and a given distribution variance a positive change in the average human capital of the whole population means, with high probability, an increase in the proportion of college graduates.

Therefore, h_t^T and H_t are given by

$$h_t^T = \begin{cases} \frac{C}{\theta}.\frac{\bar{B}}{2}.[1-M].s^{\eta}.(h_{t-1}^T)^{v} & \text{if } E[h_t^i] < \bar{h} \\ \frac{D^{\frac{1}{1-\gamma}}}{\theta}.\left(\frac{\bar{B}}{2}\right)^{\frac{1}{1-\gamma}}.[1-M].s^{\frac{\eta}{1-\gamma}}.(h_{t-1}^T)^{\frac{v}{1-\gamma}} & \text{if } E[h_t^i] \geq \bar{h} \end{cases} \tag{10}$$

$$H_t = \begin{cases} C.\frac{\bar{B}}{2}.M.s^{\eta}.(h_{t-1}^T)^{v} & \text{if } E[h_t^i] < \bar{h} \\ D^{\frac{1}{1-\gamma}}.\left(\frac{\bar{B}}{2}\right)^{\frac{1}{1-\gamma}}.M.s^{\frac{\eta}{1-\gamma}}.(h_{t-1}^T)^{\frac{v}{1-\gamma}} & \text{if } E[h_t^i] \geq \bar{h} \end{cases} \tag{11}$$

Note that this discontinuity in equations (10) and (11), along with different initial conditions, could lead to an environment where countries would converge to different growth paths or non-zero steady-state values: this phenomenon has been recently defined as the *middle-income trap*. Despite the development of literature on middle-income trap being recent, Eichengreen et al. (2013) and Aiyar et al. (2013) try to assess what variables constitute active restraints to further growth in middle-income countries that have fallen in this trap. Eichengreen et al. (2013) also present evidence that human capital level is a determinant in growth slowdowns episodes, which is in accordance with the externality structure proposed here.

Following this definition and the presence of externality, the proposition below sums up the main result of the paper.

Proposition 2. *(Convergence process under the presence of aggregate human capital externality, and different h_0^T). Consider two economies that are identical in all aspects except for their initial level of teachers' human capital, which can be result of different x and/or λ for example. These different initial conditions could lead to different steady-state equilibria, which would translate into a middle-income trap, if:*

1. *$v \in (0, 1)$:*

   (a) *Human capital externality isn't strong enough to promote non-decreasing returns, i.e., $v + \gamma < 1$, but is sufficiently high to guarantee that*

$$\frac{D^{\frac{1}{1-\gamma}}}{\theta} . \left(\frac{\bar{B}}{2}\right)^{\frac{1}{1-\gamma}} . [1-M] . s^{\frac{\eta}{1-\gamma}} > (\bar{h^T})^{\frac{1-\gamma-v}{1-\gamma}}$$

   (b) *Human capital externality is strong enough to guarantee non-decreasing returns to education quality, i.e., $v+\gamma \geq 1$, and constants are such that*

$$\frac{D^{\frac{1}{1-\gamma}}}{\theta} . \left(\frac{\bar{B}}{2}\right)^{\frac{1}{1-\gamma}} . [1-M] . s^{\frac{\eta}{1-\gamma}} > 1$$

2. *$v \geq 1$: divergence would take place just as in the model without externality, but differences in the long run GDP could be amplified.*

Proof. This follows almost immediately from proof of Proposition 1 with only some particularities.

1. $v \in (0, 1)$.

(a) $v + \gamma < 1$

Following the same steps of the first part of Proposition 1, there are two values for h_{ss}^T where $g(.) = 1$: a low h_{ss}^T in the interval given by $[0, \bar{h^T})$ and a high h_{ss}^T in the interval given by $[\bar{h^T}, \infty)$. Thus, h_0^T define, in the absence of any parameters change, to which of these two steady-states the economy would converge.

(b) $v + \gamma \geq 1$

In this case, there is a steady-state h_{ss}^T value in the interval given by $[0, \bar{h^T})$, but $g(.) > 1\ \forall h^T$ values in the interval $[\bar{h^T}, \infty)$. Therefore, countries with a $h_0^T < \bar{h^T}$ would converge to h_{ss}^T, but countries with a greater h_0^T would grow indefinitely.

2. $v \geq 1$.

Just as in Proposition 1, when v is sufficiently high, divergence would take place if we face countries with different initial conditions. However, the presence of externality amplifies the distance between the poorer and the richer country in comparison to the case without externality, since the initially richer country would grow faster for at least one period $(\frac{v}{1-\gamma} > v)$.

□

Proposition 2 shows that when there is an aggregate human capital externality in the individual human capital accumulation function, if this externality is strong enough (but not necessarily strong enough to generate non-decreasing returns) countries that have an initially low average human capital (or even an extremely unequal initial distribution of human capital) would converge to a steady-state in the long-run that is lower than the steady-state to which a country that have better initial conditions would converge. This non-convergence is result of the human capital transfer effect through which *bad* teachers in the present make *bad* teachers and also less skilled workers in the future (see figure 3). The fact that non-convergence arises with differences only in education quality, all else assumed being equal, even human capital quantity given by s, is the main contribution of this paper

To illustrate this, figures 6 and 7 show two cases where the presence of this externality makes convergence (to an unique steady-state) of the same countries i and j in figure 5 unfeasible.

5 Comparative dynamics results

Our main interest in this section is to understand what are the trade-offs that some policies in poor countries are subject to. Moreover, we would like to understand the mechanisms through which these trade-offs arise. Therefore, we would like to answer questions such as: *i*) what are the mechanisms through which government decisions affect individuals' occupational choice? *ii*) does a policy to enhance human capital's quantity, such as the ones that we observed in the last decades in emerging economies, affect the human capital's quality and/or long-run output equilibrium? *iii*) does a change in the income tax rate and/or in the proportion of the government's budget spent with teachers' salaries affect the human capital's quality? *iv*) what are the costs to current generations of a policy that aims to reach a higher steady-state equilibrium in the long-run?[10]

Note that all model's dynamics come from the evolution of teachers' average quality, which is deeply affected by the process of occupational choice and, therefore, by the parameters which define who, in the innate ability's distribution, will be a teacher. In order to assess this question, use equation (3) and substitute equations (5), (6) and assume from now on, for the sake of simplicity, that $\tau_M = \tau_T$ to reach:

$$a^{\star} = \frac{p.\tau}{\theta.(1 - p.\tau)} . \frac{\bar{B}}{2} . M \tag{12}$$

where M is now equal to $[(1 + \theta^2).(1 - p.\tau)]$. Note that this M definition implies that: $\frac{\partial M}{\partial \theta} > 0$, $\frac{\partial M}{\partial \tau} < 0$, and $\frac{\partial M}{\partial p} < 0$. Following these derivatives

[10]All results, and especially derivatives signs, presented in this section are easily obtained noting that $M > 0$, $\theta \in (0, 1)$, τ_M and τ_T are both in the interval $(0, 1)$, and proportion $p \in (0, 1]$. Derivation steps, however, are available upon request.

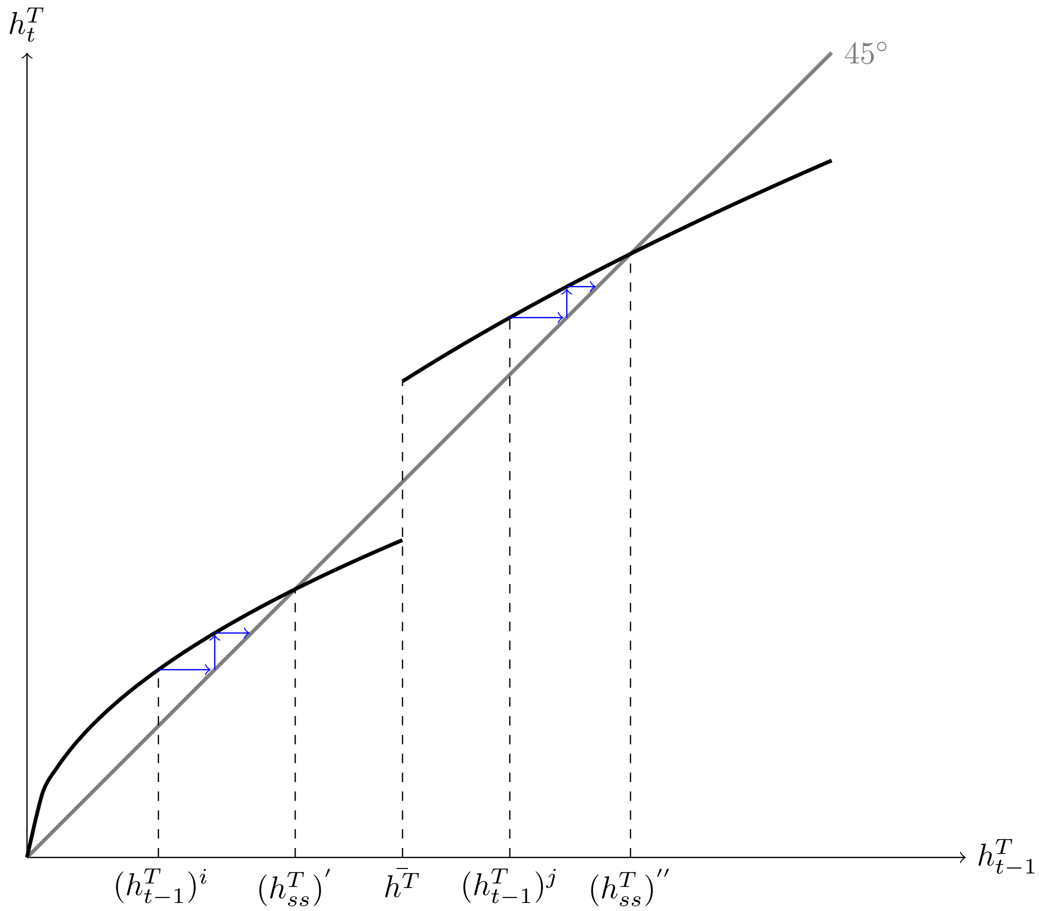

Figure 6: Average teachers human capital evolution under the presence of aggregate human capital externality with $v + \gamma < 1$ according to equation (10). Now country i converge to the same steady-state equilibrium as before but country j converge to a higher steady-state equilibrium. There isn't convergence to the same steady-state in this case.

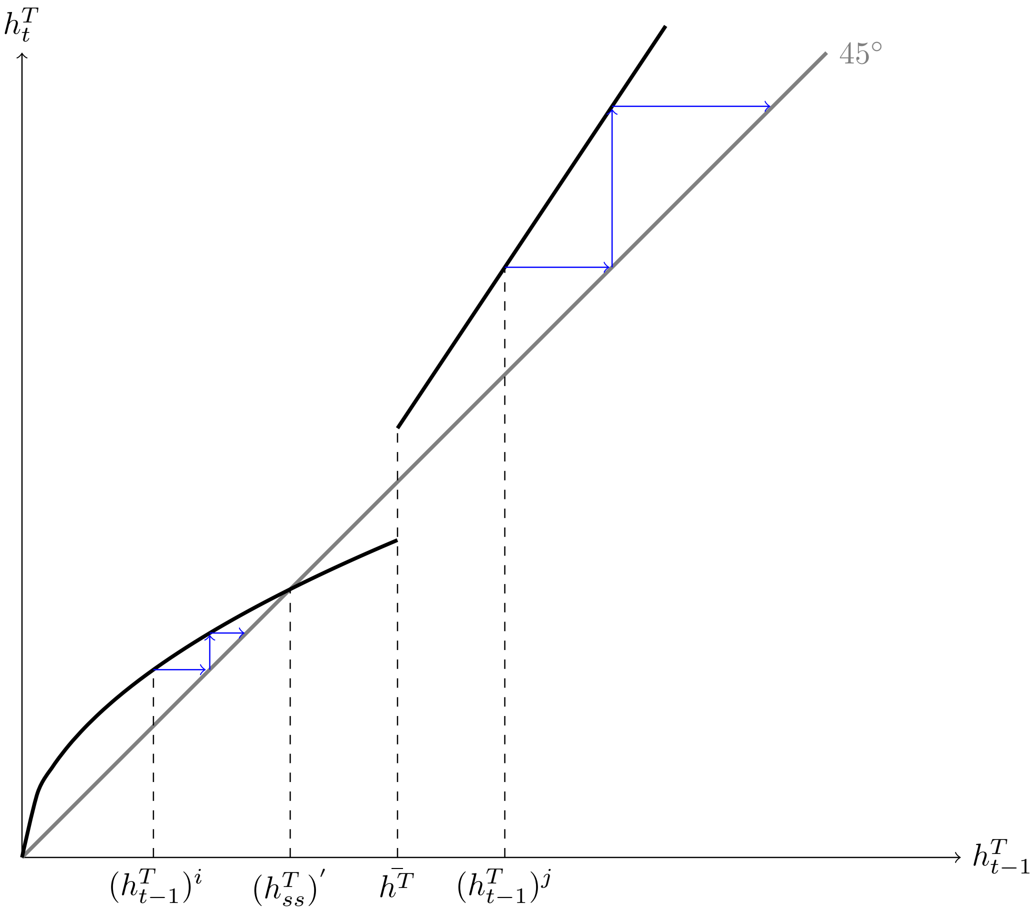

Figure 7: Average teachers human capital evolution under the presence of aggregate human capital externality with $v + \gamma = 1$ according to equation (10). Now country i converge to the same steady-state equilibrium as before but country j grows at a permanently constant rate given by parameters, just as in AK model.

signs, we are able to answer how policy shocks affect individuals' occupational choice through changes on the innate ability threshold $a^\star$:

$$\frac{\partial a^\star}{\partial \theta} < 0, \quad \frac{\partial a^\star}{\partial \tau_M} > 0, \quad \frac{\partial a^\star}{\partial p} > 0$$

Note that, just as previously discussed for the case when $p = 1$, a greater θ (result of a public policy aiming to increase the average years of schooling of the population as a whole, for example) will make the teacher career less attractive to all individuals: $\frac{\partial a^\star}{\partial \theta} < 0$. The same government budget will have to be divided by a larger number of teachers, i.e., government will pay less for each teacher and the individual that was indifferent between career options will now strictly prefer the market career.

Changes in τ and p also impact the occupational choice's process through the innate ability's threshold. Note that a greater τ will make the market career less attractive since government will require an increasing income tax in the human capital level from those who work as a market employee. A greater τ means that potential good teachers will face a worse outside option in the market. This fact is in accordance with Nagler et al. (2015) evidence: they show that more able individuals prefer teacher career during recessions due to a weaker private market and, therefore, lower expected earnings in non-teacher occupations. A greater p will also affects positively $a^\star$, just as a greater τ, but by making earnings in the teacher career higher and not earnings in the private market lower. However, both changes in τ and p affect occupational choice by modifying relative earnings (teacher/market) in the same direction.

But what about the effect of these policies in aggregate variables, especially in teachers' quality and in the GDP *per capita* evolution?

5.1 A change in θ

Taking the partial derivative of equations (7) and (6) with respect to θ can help us understand how the proportion of teachers in the population affects teachers' quality h_t^T and, therefore, how it affects H_t, which is the main component of GDP *per capita*. Thus,

1. $\frac{\partial h_t^T}{\partial \theta} < 0, \quad \frac{\partial h_{t+1}^T}{\partial \theta} < 0, \quad \frac{\partial (h_{ss}^T)'}{\partial \theta} < 0$

2. $\frac{\partial H_t}{\partial \theta} > 0, \quad \frac{\partial H_{t+1}}{\partial \theta} \lesseqgtr 0, \quad \frac{\partial (H_{ss})'}{\partial \theta} \lesseqgtr 0$

Note that derivatives of h_t^T show a trade-off between quantity and quality of human capital: an investment in increasing average years of schooling ($\frac{\partial s}{\partial \theta} > 0$) leads to a reduction in average teachers' quality in the short and long-run (i.e., an increase in θ leads to a reduction in h_t^T and h_{ss}^T). Intuitively, a government decision of hiring a greater number of teachers ($\theta \uparrow$), in a public tender hiring process, will mean more positions being filled by the same pool of candidates, and, therefore, a lower competition among candidates, which will translate into a lower average quality of hired teachers[11]. In the long-run, this investment in quantity of education (higher θ) will lead to a lower steady-state value for the average teachers' quality, since government will hire, period after period, less qualified teachers on average.

The impact of a higher θ over H_t, however, is not uniform along time: despite a positive impact in the period of the policy change, the impact in the near generations and also in the long-run equilibrium is parameter-dependent. To see this dependency even in the long-run, lets take the partial derivative of H steady-state value with respect to θ. For the sake of simplicity, lets also assume $v = 0.5$, which leaves us with the equation below.

$$H_{ss} = \underbrace{\left[\frac{C.\bar{B}}{2}\right]^2}_{=K_1} . \frac{M.(1-M)}{\theta} . s^{2\eta}$$

which has, as partial derivative

[11]Despite being generated by other mechanisms, this quantity-quality trade-off is also discussed in Gilpin and Kaganovich (2012).

$$\frac{\partial H_{ss}}{\partial \theta} = K_1.s^{2\eta}\left[\underbrace{\frac{2}{s}.\frac{\partial s}{\partial \theta}.\frac{M.(1-M)}{\theta}}_{>0} + \underbrace{\frac{\frac{\partial M}{\partial \theta}.(1-2.M).\theta - M + M^2}{\theta^2}}_{<0 \ if \ p\tau<0.5+\theta^2(1-p.\tau)}\right]$$

Since reasonable values for parameters guarantee that $p\tau < 0.5 + \theta^2(1 - p.\tau)$, $\frac{\partial H_{ss}}{\partial \theta}$ will be positive only if the first term inside brackets is sufficiently high. This will be the case for a sufficiently low s, i.e., a country with a low initial level of average years of schooling could experience steady-state gains from hiring more teachers as a policy to enhance average years of schooling. However, this positive effect fade out with a higher s and, therefore, $\frac{\partial H_{ss}}{\partial \theta}$ become negative for a sufficiently high s and so on. These dynamics are in accordance with the empirical evidence of the last decades in emerging countries, which are summarized in the figures 1 and 2.

Duflo (2001) assess an unusual policy of school construction and teacher hiring in Indonesia in which investment in education quantity lead to an increase in individuals human capital, despite its possible negative effects on education quality. However, her results appears to favor the derivatives signs above, since the (direct and indirect, through years of schooling channel) effect of the policy on wages is only positive and statistically significant in regions that had a below median preprogram education (table 6 in the paper), which is in accordance with our above conclusion that $\frac{\partial H_{ss}}{\partial \theta} > 0$ only for a sufficiently low s.

5.2 A change in τ[12]

In order to understand how income tax rate affects teachers' quality and H_t, consider a change in τ, with all other parameters held constant. Thus, partial derivative of equations (7) and (6) yields

1. $\frac{\partial h_t^T}{\partial \tau} > 0, \quad \frac{\partial h_{t+1}^T}{\partial \tau} > 0, \quad \frac{\partial (h_{ss}^T)'}{\partial \tau} > 0$

[12]Changes in p have all the same effects of τ and arise through the same channels.

2. $\frac{\partial H_t}{\partial \tau} < 0, \quad \frac{\partial H_{t+1}}{\partial \tau} \lesseqgtr 0, \quad \frac{\partial (H_{ss})'}{\partial \tau} > 0$

Unlike changes in θ, an increase in τ will lead to a greater teachers' average quality in t and also in the long-run. A higher τ affects the attractiveness of market career by increasing relative (teacher/market) earnings, and, then, pushing away the more able applicants from the market ($\frac{\partial a^\star}{\partial \tau} > 0$). For a given θ, this new τ will dislocate the *teachers' area* in figure 4 to the right and, therefore, will lead to an also higher h_t^T. The steady-state value of teachers' average quality will be higher and can also be sufficient to make a poor country escape from the middle-income trap imposed by its initial conditions.

However, a higher τ has negative impacts over current generations by lowering individual income and, therefore, individual consumption level. Despite the positive long-run impact in H_t through a higher h_t^T, the cost of an increase in income tax rate for current generations can be high enough to make it unfeasible politically if government policy is subject to the median voter choice, for example. A future extension for the model could be a government that chooses the quantity of education it would provide according to voters preferences and, therefore, the probability to be reelected. This choice, however, would be subject to government budget constraint and the quantity-quality trade-off developed in this paper.

6 Simulation

We will now go through a calibration effort in order to simulate the model and perform some quantitative exercises. Parameters of policy are taken from USA data, which constitute our benchmark economy, and education quality return, and externality parameter are calibrated according to micro literature evidence. To focus only in education differences, A_t is normalized so that $A_t = A = 1$, but we maintain it in the model so that questions about exogenous technological shocks could be addressed in future research. We describe all calibration steps below, whose values are summarized in table 1.

The elasticity of output with respect to human capital (β) is set to match the same elasticity in Erosa et al. (2010) model, where it is equal to two thirds. Preference parameter σ is set to 2, just as in Erosa et al. (2010) too, given the similar model period[13]. The proportion of teachers θ is taken from World Bank Databank and is set to 1.51%, which is equal to the average of US teachers population ratio in the period 1993-2010[14]. The income tax rate, which we assume being the same to market employees and teachers, is set to 9.92% according to 1970-2010 average of the individuals plus corporations income taxes as percentage of GDP[15]. The fraction p is set to 33.83%, which is equal to the ratio between 2000-2010 average of all staff compensation in primary, secondary and tertiary public institutions as percentage of GDP[16] and government revenue as percentage of GDP, which is the same as our 9.92% income tax rate.

The human capital threshold structure proposed here does not have, to our knowledge, an empirical counterpart in the literature, which makes the calibration of $\bar{h}$ a real challenging task given all empirical issues that we have to deal with when measuring human capital externalities[17]. Therefore, we perform the quantitative exercises as a function of the distance between $(h^T_{ss})'$ and $\bar{h^T}$ by assuming $(h^T_{ss})' = z.\bar{h^T}$ where $z \in (0, 1)$.

In order to reach the whole calibrated model we now need to define a value for the education quality and quantity returns (υ and η), the externality magnitude γ, and how θ translate into s (all calculations steps are at

[13]Erosa et al. (2010) model is a 3-period OLG, so they set the model period to 20 years to reach an individual life expectancy of 60 years. Since individuals live only two periods in our model the model period is set to 30 years to reach the same life expectancy of 60 years.

[14]Teachers in all educational levels, i.e., primary, secondary, and tertiary.

[15]Available at White House website, table 2.3: `https://www.whitehouse.gov/omb/budget/Historicals/`.

[16]Available in World Bank Databank - Education Statistics. p would be equal to 26.43% if we considered only primary and secondary public institutions.

[17]As an addition to the calibration, we could try to reach $\bar{h}$ using the threshold estimation method proposed by Hansen (2000). We leave this for future research.

Appendix A). Given empirical issues on estimating returns to teacher quality while controlling for teacher quantity, there isn't a consensus in the literature about what would be a reasonable v, although, just as noted in Gilpin and Kaganovich (2012), there are evidences that $v > 0$. However, γ can be inferred by the estimations of Iranzo and Peri (2009): since external effects of an increase in the proportion of college graduates are around $3\% - 9\%$ and, according to micro literature, return to one more year of schooling[18] is around $6\% - 12\%$, γ is set to 0.38. Following this $6\% - 12\%$ interval for education quantity return, we set $\eta = 0.10$, which is also the world average return in Psacharopoulos (1994).

Given this absence of a consensual v, we simulate our model with three possible values for this parameter in order to dialog with micro evidence, and reach (when the externality is acting) decreasing and constant returns for education quality. Following Schoellman (2012) evidence of human capital quality as much important as human capital quantity, first we assumed $v = \eta = 0.10$, then we arbitrarily assumed a v of 0.5 ($v + \gamma < 1$), and finally a v of 0.62 in order to reach $v + \gamma = 1$.

Finally, to avoid relying in a non-consensus functional form for s and how it relates to θ we perform a simple cross-country estimation with 2005 World Bank Data. For a 55 countries sample, we estimate the following regression using OLS:

$$log(AYS30_i) = \omega_0 + \omega_1.log(\theta_i) + \omega_2.log(PUPIL_i) + \epsilon_i$$

where $AYS30_i$ is country i's average years of schooling divided by 30, which is our s in the model, and $PUPIL_i$ is the well-know pupil-teacher ratio in all education levels for the same country. This simple estimation yields a positive and statistically significant ω_1, and, even with standard errors robust to heterokedasticity, can't reject the hypothesis that $\omega_1 + \omega_2 = 1$. Despite these only two explanatory variables, the R^2 is more than 0.66[19].These evidences

[18]A good survey of the estimates is in Psacharopoulos (1994).

[19]The results are similar if we considered only 2010 data, with only 43 countries in the

Table 1: Calibrated parameters

Parameter		Value	Source
	η	0.10	Micro literature
s function parameters	ω	0.90	OLS estimation
	K	7.7906	OLS estimation
	r	14.456	World Bank
		0.10	Micro literature
Education quality return	v	0.50	-
		0.62	-
Human capital externality magnitude	γ	0.38	Micro literature
Upper bound of innate ability	$\bar{B}$	1	-
CRRA	σ	2	Erosa et al. (2010)
Exogenous level of aggregate productivity	A	1	-
Human capital share in the production function	β	0.67	Erosa et al. (2010)
Proportion of teachers in the population	θ	0.0151	World Bank
Income tax rate	τ_M	0.0992	White House
Proportion of govt. budget spent with teachers	p_t	0.3383	World Bank
Constant	C	2.7	-
Constant	D	2.7	-

made us comfortable in assuming a s given by a Cobb-Douglas function in which $s = K.\theta^{\omega}.r^{1-\omega}$, where r is the pupil-teacher ratio and K is just a constant. For the quantitative exercises, we set $r = 14.456$, which is the 2005 value for pupil-teacher ratio in all education levels in the US.

We are able now to do some simulations and evaluate the policy effectiveness in this calibrated economy. First, two economies with distinct initial conditions are compared in the absence of any policy: we compare an economy initially below the human capital threshold and an economy initially above this threshold. Table 2 illustrates what would be the steady-state rich/poor ratio for teachers' average human capital, aggregate private market human capital, and GDP per capita, for all v values. The case when $v + \gamma = 0.88$ is the one presented in figure 6 where $\frac{(h^T_{ss})''}{(h^T_{ss})'} = 32.21$.

Then, we calculate what would be the minimum positive change in policy

sample. We also estimate a fixed effects model when including both 2005 and 2010 data: despite some changes, ω_1 still positive and statistically significant at the 5% level when not controlling for heterokedasticity.

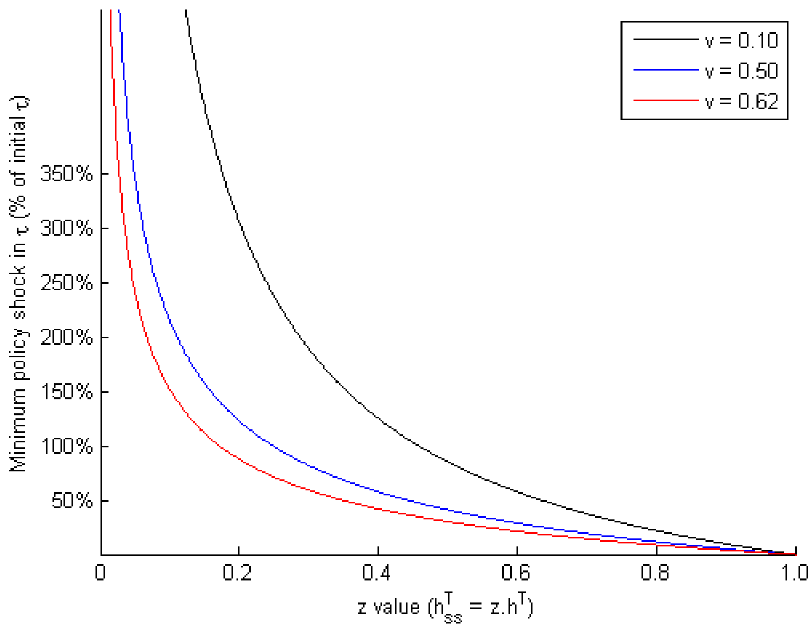

Figure 8: Minimum required policy changes as a function of z.

parameters, i.e. $(\Delta\tau, \Delta p)$, which would make the poor economy escape the middle-income trap[20]. Table 3 summarizes the results of this exercise with $z = 0.25$, $z = 0.75$ and $z = 0.90$. This exercise shows that a considerable increase in the tax income rate and/or in the proportion of the government budget spent with teachers' salaries is required to let the poor country overcome the middle-income trap[21]. Moreover, these minimum $\Delta\tau$ and Δp are higher for a lower v and increases in an exponential way the poorer the country: figure 8 illustrates it.

These minimum policy changes, however, are just the required ones to guarantee that, without externality, $h^T_{ss} > \bar{h^T}$. Figure 9 illustrates this exercise: the minimum policy changes are such that the low steady-state equilib-

[20]Note that we do not evaluate variations of θ as a government policy when this government is aiming to escape the middle-income trap since θ should be drastically reduced to achieve this goal, and this is not what we observe in the data.

[21]In percent of initial value, the minimum required policy is the same for τ and p

Table 2: Steady-state rich/poor ratio for real variables

	h^T	H	Y
$v + \gamma = 0.48$	1.21	1.21	1.14
$v + \gamma = 0.88$	32.21	32.21	10.35
$v + \gamma = 1.00$	∞	∞	∞

Table 3: Minimum required policy shock in τ (% of initial τ)

	$z = 0.25$	$z = 0.75$	$z = 0.90$
$v = 0.10$	236.85%	29.32%	3.34%
$v = 0.50$	99.34%	15.37%	1.82%
$v = 0.62$	71.39%	11.50%	1.37%

rium moves from A to B and B is greater than human capital threshold $\bar{h^T}$. Thus, the poor country may have to sustain these higher tax rates and higher spending with teachers' salaries for a prolonged period. This may imply an unsustainable burden for current generations since a higher tax rate means a lower level of consumption in t, just as showed in the previous section. Moreover, the cost of the required policy grows in magnitude and also in the number of periods that have to be sustained when v and z fall. In summary, the required policy to overcome the middle-income trap has a considerable cost for current generations in all cases.

Finally, we perform another simulation exercise to try to understand the recent movement in the average years of schooling in emerging markets accompanied by a stable GDP per capita distribution. We evaluate the dynamical response of h_t^T and H_t in two cases: 1) an increase in θ that would led a country with an initial average years of schooling of 3 to an average of 7 years, and 2) a further increase of 50% in θ. Figures 10 and 11 summarizes the results of this exercise, which are in line with our derivatives signs in the previous section and also, and more important, with figures 1 and 2, which represent one of the main motivations of this paper. Investments in education quantity without any improvement in education quality seems to be short lived if one is aiming GDP per capita improvement: the net effect

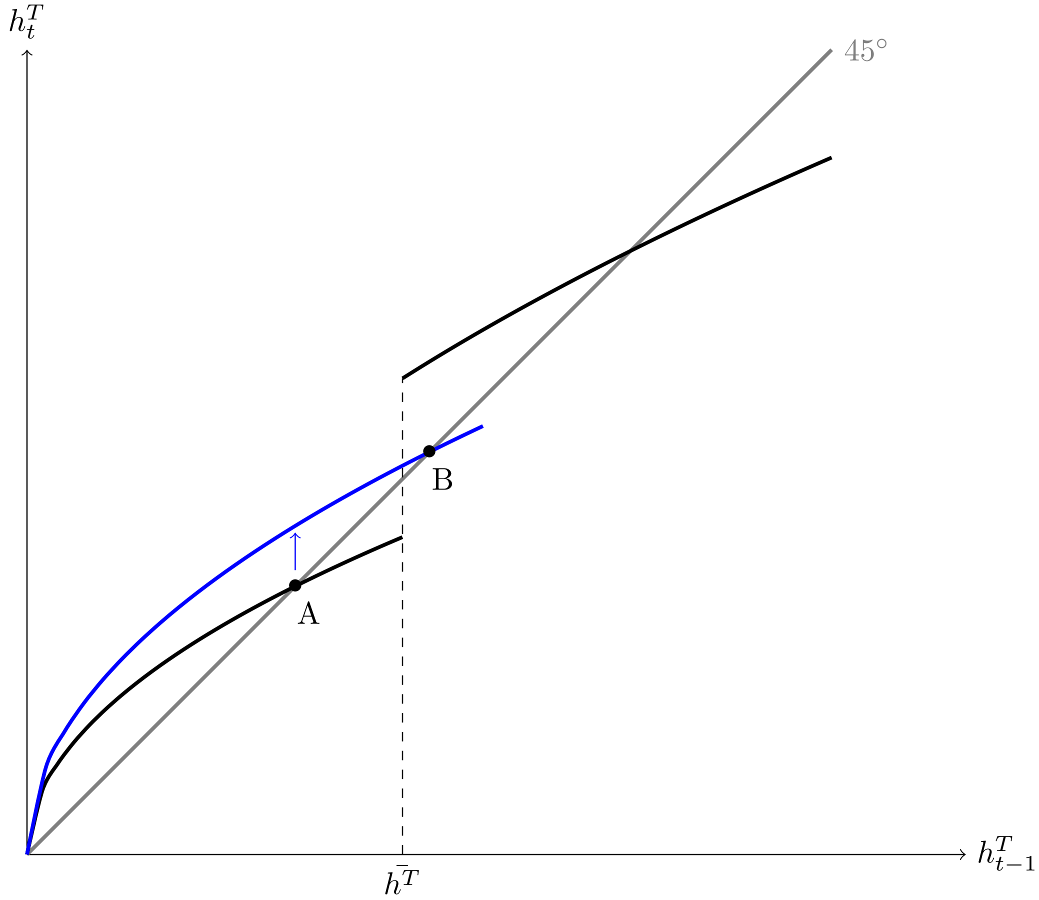

Figure 9: Simulation exercise of required minimum policy changes.

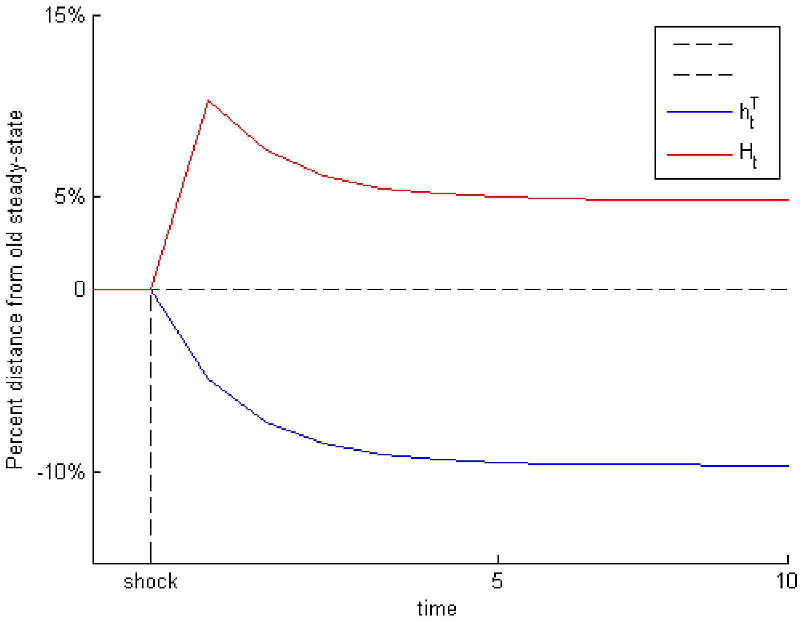

Figure 10: Impact of an increase in θ for a country with an initial average years of schooling of 3.

of an increase in θ on H_{ss} (and, therefore, on steady-state GDP *per capita*) is positive only for a sufficiently low initial s.

7 Concluding Remarks

This paper assesses the role of education quality in the non-convergence phenomenon of GDP *per capita*. By making education quality equal to the average of teachers' human capital, just as in Gilpin and Kaganovich (2012), the paper analyzes how the quality (and not only quantity) of human capital contributes to GDP variability in response to model parameters.

It is shown that differences in education quality by itself can generate multiple steady-states or different endogenous growth paths. This paper reaches this result by making education quality endogenous and not only a calibrated parameter or a policy variable as in Tamura (2001), for example. This paper also shows that, even when education quality return is decreasing,

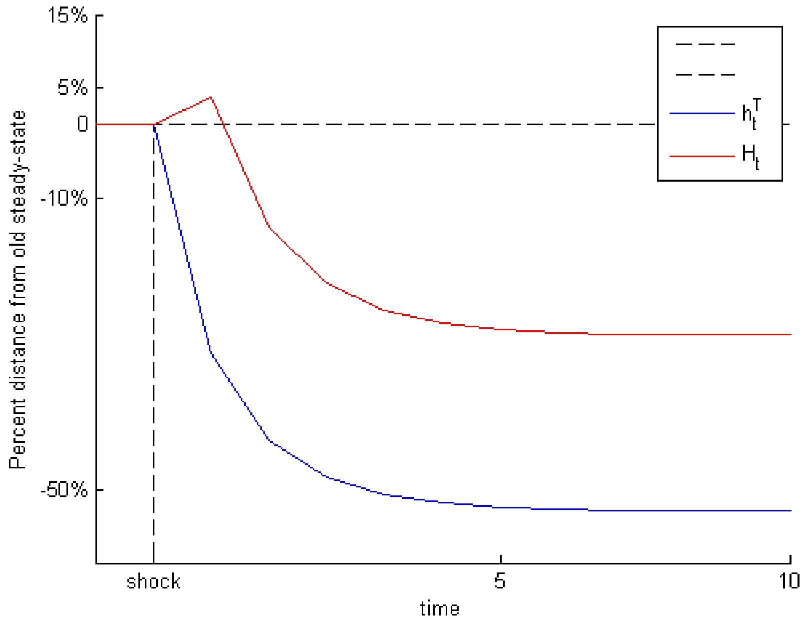

Figure 11: Impact of an increase in θ for a country with an initial average years of schooling of 7.

multiple steady-states environment arises if human capital externalities are acting. Therefore, initial distribution of human capital is of great importance in forecasting what will be a country's long-run equilibrium.

Moreover, it is shown a quality-quantity trade-off that could help explain why emerging economies, which have converged to rich economies in terms of average years of schooling, have not reached a high level of GDP also, just as showed in figures 1 and 2.

Some simulation exercises are done in order to gain some insights about what would be the required public policy if a poor country is aiming convergence. Using USA parameters to build a benchmark economy, it is shown that the required minimum changes in income tax rate and in the proportion of government budget spent with teachers' salaries are big in magnitude in all specifications. The cost to current generations of this required policy are exponentially higher for poorer countries, which is a disturbing implication of the model and a worrisome problem for policymakers of the international community.

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Appendix A

- **How to reach equations (6) and (7)**

  Equations (6) and (7) are the result of integrating h_t^i, equations (3), (4), and (5), which represents, respectively, individual ability threshold, perfectly competitive markets and government's budget constraint.

  Thus,

$$
\begin{aligned}
H_t &= \int_0^{a_t^\star - \bar{B}.\theta} h_t^i(a_t^i).dF(a_t^i) + \int_{a_t^\star}^{\bar{B}} h_t^i(a_t^i).dF(a_t^i) \\
&= \int_0^{a_t^\star - \bar{B}.\theta} .C_t.a_t^i.(s)^\eta.(h_{t-1}^T)^v.f(a_t^i).da_t^i + \int_{a_t^\star}^{\bar{B}} .C_t.a_t^i.(s)^\eta.(h_{t-1}^T)^v.f(a_t^i).da_t^i \\
&= C_t.(h_{t-1}^T)^v.(s)^\eta.\left(\int_0^{a_t^\star - \bar{B}.\theta} a_t^i.\frac{1}{\bar{B}}.da_t^i + \int_{a_t^\star}^{\bar{B}} a_t^i.\frac{1}{\bar{B}}.da_t^i\right) \\
&= \frac{C_t.(s)^\eta.(h_{t-1}^T)^v}{\bar{B}}.\left(\left[\frac{(a_t^i)^2}{2}\right]_0^{a_t^\star - \bar{B}.\theta} + \left[\frac{(a_t^i)^2}{2}\right]_{a_t^\star}^{\bar{B}}\right) \\
&= \frac{C_t.(s)^\eta.(h_{t-1}^T)^v}{2.\bar{B}}.\left((a_t^\star)^2 - 2.a_t^\star.\bar{B}.\theta + \bar{B}^2.\theta^2 + \bar{B}^2 - (a_t^\star)^2\right) \\
&= \frac{C_t.\bar{B}.(1+\theta^2).(s)^\eta}{2}.(h_{t-1}^T)^v - C_t.(s)^\eta.(h_{t-1}^T)^v.\theta.a_t^\star \\
&= \frac{C_t.\bar{B}.(1+\theta^2).(s)^\eta}{2}.(h_{t-1}^T)^v - C_t.(s)^\eta.(h_{t-1}^T)^v.\theta.\frac{(1-\tau_T).w_t^T}{(1-\tau_M).w_t^M}.\frac{1}{C_t.(s)^\eta.(h_{t-1}^T)^v} \\
&= \frac{C_t.\bar{B}.(1+\theta^2).(s)^\eta}{2}.(h_{t-1}^T)^v - \theta.\frac{(1-\tau_T).\frac{p.\tau_M.w_t^M.H_t}{\theta.(1-p.\tau_T)}}{(1-\tau_M).w_t^M} \\
&= \frac{C_t.\bar{B}.(1+\theta^2)}{2}.(h_{t-1}^T)^v - \theta.\frac{(1-\tau_T).p.\tau_M.w_t^M.H_t}{(1-\tau_M).w_t^M.\theta.(1-p.\tau_T)} \\
&= \frac{C_t.\bar{B}.(1+\theta^2)}{2}.(h_{t-1}^T)^v - \frac{p.(1-\tau_T)}{(1-p.\tau_T)}.\frac{\tau_M}{(1-\tau_M)}.H_t
\end{aligned}
$$

  and finally

$$
H_t = \frac{C_t.\bar{B}}{2}.\left[\frac{(1+\theta^2).(1-p.\tau_T).(1-\tau_M)}{(1-p.\tau_T-\tau_M.(1-p))}\right].(s)^\eta.(h_{t-1}^T)^v
$$

Equation (7) just come from equation (6) and the fact that $H_t+\theta.h_t^T = \int_0^{\bar{B}} h_t^i(a_t^i).dF(a_t^i) = \frac{C_t.(s)^\eta.(h_{t-1}^T)^v.\bar{B}}{2}$.

- **Rewritting h_0^T as a function of initial conditions**

  Since we assumed that h_0^i is distributed uniformly over the interval $[x-\lambda; x+\lambda]$, h_0^T and H_0 could be calculated directly, noting only that individual ability threshold that defines career choice can be restated as individual human capital threshold.

$$
\begin{aligned}
h_0^T &= \theta^{-1}.\int_{h_0^\star-2.\lambda.\theta}^{h_0^\star} h_0^i.dF(h_0^i)\\
&= \theta^{-1}.\int_{h_0^\star-2.\lambda.\theta}^{h_0^\star} h_0^i.f(h_0^i).dh_0^i\\
&= \theta^{-1}.\int_{h_0^\star-2.\lambda.\theta}^{h_0^\star} h_0^i.\frac{1}{2.\lambda}.dh_0^i\\
&= \frac{1}{2.\lambda.\theta}.\left[\frac{(h_0^i)^2}{2}\right]_{h_0^\star-2.\lambda.\theta}^{h_0^\star}\\
&= \frac{1}{4.\lambda.\theta}.\left[(h_0^i)^2-(h_0^i)^2+4.\lambda.\theta.h_0^\star-4.\lambda^2.\theta^2\right]\\
h_0^T &= h_0^\star-\theta.\lambda
\end{aligned}
$$

It is straightforward to show that

$$H_0 = -\theta.h_0^\star+\theta^2.\lambda+x$$

Now, recall equations (3) and (5), the assumption of θ teachers $\forall t$, and note that at $t=0$ we have:

$$
\begin{aligned}
h_0^\star &= \frac{(1-\tau_T).w_0^T}{(1-\tau_M).w_0^M} \\
&= \frac{(1-\tau_T)}{(1-\tau_M).w_0^M}.\frac{p.\tau_M.w_0^M.H_0}{\theta.(1-p.\tau_T)} \\
&= \frac{(1-\tau_T)}{(1-\tau_M)}.\frac{p.\tau_M.H_0}{\theta.(1-p.\tau_T)} \\
h_0^\star &= \frac{(1-\tau_T)}{(1-\tau_M)}.\frac{p.\tau_M}{\theta.(1-p.\tau_T)}.[-\theta.h_0^\star+\theta^2.\lambda+x] \\
h_0^\star &= \frac{(1-\tau_T).p.\tau_M}{\theta.(1-p.\tau_T-\tau_M.(1-p))}.[\theta^2.\lambda+x]
\end{aligned}
$$

Finally, replacing this $h_0^\star$ function on h_0^T

$$
\begin{aligned}
h_0^T &= \frac{(1-\tau_T).p.\tau_M}{\theta.(1-p.\tau_T-\tau_M.(1-p))}.[\theta^2.\lambda+x]-\theta.\lambda \\
h_0^T &= \frac{(1-\tau_T).p}{(1-p.\tau_T-\tau_M.(1-p))}.\frac{\tau_M.x}{\theta}-(1-\tau_M).\theta.\lambda.\frac{(1-p.\tau_T)}{(1-p.\tau_T-\tau_M.(1-p))}
\end{aligned}
$$

- **Reaching calibrated** γ

  We can reach the calibrated value of γ by targeting the return to education when the externality of Iranzo and Peri (2009) is acting, and if the individual years of schooling were added in the human capital production function. Given the intervals of externality magnitude and returns to schooling (cited previously in the text), we can thus set a lower bound and upper bound to γ.

  1. *Lower bound:* $\frac{0.06}{1-\underline{\gamma}} = 0.06 + 0.03 \implies \underline{\gamma} \approx 0.33$
  2. *Upper bound:* $\frac{0.12}{1-\overline{\gamma}} = 0.12 + 0.09 \implies \overline{\gamma} \approx 0.43$

  Thus, our calibrated γ is set to equal the middle point between these bounds, i.e., 0.38.