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To Comply or not to Comply: Persistent Heterogeneity in Tax Compliance and Macroeconomic Dynamics

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We set forth an overlapping generations model in which the microdynamics of tax compliance is coupled to the macrodynamics of the economy. We specify the proportion of individuals who do not comply with their tax obligations as endogenously time-varying using the discrete choice approach, which allows considering both deterministic components and idiosyncratically subjective motivations and proclivities (such as tax morale) as drivers of tax compliance. The model replicates (and hence provides an analytical framework for a potential interpretation of) some pieces of evidence on tax evasion. First, heterogeneity in tax compliance exhibits persistence and fluctuations over the long run. Second, the proportion of non-compliant taxpayers varies positively with the tax rate and negatively with the probability of detection of tax evaders. Third, the impact of a change in the proportion of non-compliant taxpayers on the per capita output over the long run is ambiguous.

Keywords: Tax compliance; discrete choice modeling; tax morale; heterogeneous behavior; macrodynamics.

JEL Codes: H62; H40; C02; C62; E13.

To comply or not to comply: persistent heterogeneity in tax compliance and macroeconomic dynamics

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1. Introduction

The provision of essential or strategic productive public goods and services to private producers by the governments potentially plays an important role in raising living standards and social welfare by contributing to increase economic growth or per capita income. As such a provision has to be financed, when it occurs without user charges to private producers it turns out that collected tax revenues from different sources typically become an intertemporally binding constraint. Meanwhile, tax compliance is in and of itself a complex decision that has been found to be motivated by a variety of objective, subjective and social factors (see, e.g., Torgler, 2007; Alm et al., 2010).

Clearly, the threat of detection and punishment is a factor having an objective aspect to it, and existing evidence from different sources lends credence to the proposition that increased enforcement levels usually leads to increased compliance levels. Yet empirically observed compliance levels measured in a variety of ways and employing different methodologies are very often higher than warranted by the level of enforcement per se. One possible reason for such a divergence is that although the threat of detection and punishment involves objective elements such as audits and fines, it also inevitably has a subjective aspect to it. In effect, a potential tax evader may overestimate (or underestimate for that matter) the extent to which the institutional structure of enforcement to which she is subject represents a threat of detection and punishment to her specifically. Ultimately, it is the threat of detection and punishment in the eyes of a beholder who is contemplating the possibility of evading her legally due tax obligations that will influence her decision.

However, idiosyncratically subjective motivations and proclivities can also perform a significant separate role in driving tax compliance independently from an influence that subjective factors may have on the conceiving of the expected threat of detection and punishment by a potential tax evader. In fact, there is considerable evidence that subjective motivations or proclivities forming a tax morale offer a plausible explanation for what has been dubbed a puzzle, which is not why there is so much tax evasion or few tax compliance, but instead why there is less tax evasion or more tax compliance than should be plausibly expected (but of course the pertinent issue arises here of expected by whom and from the perspective of what analytical framework).

Meanwhile, additionally to the relevant issue of what are potential driving factors behind tax compliance (or the lack thereof), the literature has been also addressing theoretically and empirically the similarly relevant issue of the macroeconomic implications of tax evasion. Given the importance (and sometimes cruciality) of collected tax revenues as an effective (though not necessarily efficient and distributively fair, an issue from which we abstract) mechanism to properly finance the provision of productive public services which are essential to private production and hence income generation, and for a variety of reasons are preferably provided without user charges, tax evasion impacts on the macrodynamics of the economy. Yet the predictions that have been arising from the theoretical literature and the evidence that has been offered by the empirical literature are mixed (see, e.g., Slemrod and Weber, 2012; Pickhardt and Prinz, 2014; Alm, 2019). To some extent due to the complex nature of the two-way dynamic interactions between tax evasion and the level of macroeconomic activity, it is not surprising that conflicting theoretical results have been derived and diverging empirical evidence has been found, for example, for the question of whether a change in tax evasion impacts positively or negatively on per capita output.

In Varvarigos (2017), for example, more productive agents can conceal their true type and evade taxes due on their additional labor income. The model predicts a negative association between tax evasion and the per capita capital stock and income. Bethencourt and Kunze (2020) allow for labor and capital income tax evasion and arrive at a positive relation between tax compliance and per capita income. A complementarity between the two types of evasion explains the decrease in aggregate evasion as countries grow and accumulate capital. In Célimène et al. (2016), a positive effect of tax evasion on per capita output arises if the productivity of public spending is relatively low and evaders invest their illegal proceedings in the domestic instead of foreign equity markets. This positive effect occurs despite the negative externality created by tax evasion on the productive public expenditure. Tsakumis et al. (2007), with data for 50 countries, employ an estimate of each country's shadow economy computed by Schneider (2004) as a proxy for tax evasion. Countries with larger (smaller) shadow economies represent countries with higher (lower) tax evasion. They find a negative relationn between tax evasion and the log of output per capita. Such negative relation is robust to different measures of tax evasion reported by the World Economic Forum (WEF) and the Institute for Management Development (IMD) (see Richardson (2008)). A negative relation between the shadow economy (and tax evasion) and output per capita is reported by Schneider et al. (2010) as well.

This paper is intended to contribute to the literature on tax evasion by setting forth an overlapping generations model in which the microdynamics of tax compliance behavior across taxpayers is coupled to the macrodynamics of some variables of interest, especially the per capita output or income. Thus, our paper speaks to the branch of the literature focusing on subjective motivations in addition to objective factors as drivers of the extent to which tax compliance levels are observed, as well as to the branch exploring macroeconomic implications of tax evasion. We model the proportion of individuals who do not comply with their legally due tax obligations as endogenously time-varying using a discrete choice framework. The deterministic component of the payoff of each tax compliance strategy available to an individual (to comply or not to comply) is a well-defined measure of its performance in the recent past, while the random component broadly includes idiosyncratically subjective motivations and proclivities such as tax morale. The considered motivations and proclivities are supposed to be randomly and independently distributed across taxpayers and over time.

In accordance with the empirical evidence, heterogeneity in tax compliance behavior across taxpayers can exhibit persistence and fluctuations over the long run. Also in keeping with the empirical evidence, tax evasion (which in our model is equivalent to the proportion of non-compliant workers) varies positively with the tax rate and negatively with the probability of detection of a tax-evading worker. To some extent owing to our specification of the penalty rate levied on detected tax evaders as varying positively with the proportion of tax-evading workers, the impact of a change in such a proportion on the per capita output in the long-run equilibrium is ambiguous. Yet such an ambiguity can be intuitively traced back to the operation of the several interactions and mechanisms at play. As a result, we are able to precisely describe the economic substance of what it takes in terms of the relative strength of the several effects at play for a fall in the proportion of tax-evading workers to yield, for example, a higher per capita output in the long-run equilibrium.

We plausibly and conveniently treat tax morale as being both compliance-leaning and non compliance-leaning. A key implication is that an individual taxpayer may have mixed moral feelings with respect to complying or not complying with her legally due tax obligations, so it is on balance that she will ultimately have either a compliance-leaning tax morale or a non compliance-leaning one. Reasonably, although an individual taxpayer is not in position to question the legality of the tax obligations levied on her by the government, she may nonetheless conceive of her intrinsically subjective motivations and proclivities towards non-compliance as morally warranted. Moreover, our discrete choice approach to tax compliance interestingly allows differentiating a deepening (intensive margin) from a widening (extensive margin) of either type of overall tax morale (compliance-leaning or non compliance-leaning) across taxpayers.

The remainder of this paper is organized as follows. Section 2 describes the structure of the model and solves for its temporary equilibrium, which is the time span for which the proportion of non-compliant workers is predetermined. This section also derives the dynamics of capital formation and the dynamics of the frequency distribution of tax non-compliance strategies across workers. The dynamic behavior of the economy is explored in Section 3, in which we prove that a long-run equilibrium exists, is unique, features heterogeneity in tax compliance strategies across workers, and is a local attractor under some conditions. This section also explores how a change in the proportion of non-compliant workers impact on the per capita capital stock and the per capita output in the long-run equilibrium. Section 4 concludes.

2. Structure of the model

2.1. Temporary equilibrium

The economy is populated by individuals who live for two periods. Young individuals are endowed with one unit of labor when they are born, which is inelastically (and indifferently) supplied either to private firms or to the government. Young individuals therefore earn labor income and choose to use all of it to inelastically demand the privately produced single and homogeneous good, employing it alternatively either for immediate consumption or for saving and hence investment purposes (to wit, physical capital formation). The government levies a flat sales tax on the purchases of the single good for either purpose by young individuals. The actual public revenues (which additionally to the actual tax collection also include the amount of fines applied on tax evaders who get audited and hence detected) are entirely employed (lexicographically) by the government to remunerate the young individuals hired as tax auditors and to supply, free of charge, a flow of homogeneous public services which are used without congestion as (essential) productive inputs by private firms. Meanwhile, old individuals deliberately remain out of the labor market and are exempted by the government from paying any taxes. As a result of their saving behavior while young, old individuals own the capital stock, which is entirely and inelastically rented to firms under competitive conditions. The resulting rental income received by old individuals is all spent on the immediate consumption of the good.

In addition to making the key consumption-saving decision, young individuals also decide whether or not to comply with their legally due tax obligations, given (among other reasons) that it is common knowledge that not all taxpayers are audited.[1] However, all non-compliant taxpayers who are audited get completely detected and as a result are unable to avoid (through corruption for example) either being fined or, once fined, paying in full the ensuing fine for sales tax evasion. The resulting behavioral structure features intricate interactions involving the flow of productive public services, the wage rate, the rental rate of capital, the proportion of tax evaders in the respective

[1] Empirical analyses and/or recent examples of sales tax evasion are offered in Murray (1995), Alm et al. (2004), Keen and Lockwood (2010), Semerád and Bartůňková (2016), and Alm (2019).

subpopulation of taxpayers, the average saving rate across young individuals (detected tax evaders pay the ensuing fines out of their savings), the level of capital formation, and the level of per capita income, as will be formally explored in the following sections. The three spheres of decision-making by a young individual (saving, consumption, and tax compliance behavior) are highly intertwined and feature feedback effects in that the tax compliance behavior followed by such an individual impacts both on her expected net income and on how much she will save and consume out of her actual net income.

We start by deriving the optimal saving rate of each homogeneous type of young individual with respect to tax compliance behavior. We assume that individuals are born with no initial wealth other than their labor endowment and leave no bequest for future generations. Let c_{1t} and c_{2t+1} denote the consumption levels of an individual born in period $t \in \mathbb{N}$ when young and old, respectively. A tax-compliant young individual, who is identified by the subscript c, is levied a flat sales tax on all her purchases of the good, irrespective of whether such purchases will serve for consumption or capital formation.[2] For simplicity, we assume the price of the single good is continuously normalized to one. The unit price of the single good faced (and effectively paid) by a tax-compliant young individual, thus, is given by $1 + \tau$. The first-period budget constraint of a tax-compliant individual born in t is given by:

$$(1+\tau)c_{1t}^{c} = \big[1-(1+\tau)s_{ct}\big]w_t, \tag{1}$$

where $\tau \in (0, \frac{1}{2+\theta}) \subset \mathbb{R}$ is the flat tax rate, which is assumed to be exogenously given and constant over time, with $\theta \in \mathbb{R}_{++}$ being the one-period discount rate, $s_{ct} \in [0,1] \subset \mathbb{R}$ is the portion of the wage income of the compliant young individual which is saved in period t and $w_t \in \mathbb{R}_{++}$ is the respective wage income in period t (recall that individuals are born endowed with one unit of labor).[3] As young individuals are equally skilled and the current tax compliance behavior of a young individual does not affect her behavior either in the labor market more generally or on the job more specifically, and hence does not impact on her employment prospects, the wage rate w_t is homogeneous across young individuals (including those hired by the government as tax auditors). As shown later on, however, in the time span along which the proportion of tax evaders in the

[2] For clarity of notation, sometimes will be more convenient to employ a superscript to denote the homogeneous type of a young individual with respect to tax compliance behavior.

[3] We assume that the tax rate is strictly lower than the upper bound of the optimal saving rate of a tax-compliant young individual, given by $\frac{1}{2+\theta}$, which is achieved with $\tau = 0$, as will be derived shortly. This assumption ensures that compliant young individuals do not default on their tax obligations.

respective subpopulation of taxpayers is predetermined (which we dub temporary equilibrium), the real wage is parameterized by such a proportion.

Tax non-compliant young individuals, who are identified by the subscript n, choose to evade taxes, despite the risk of being audited and consequently detected and fined. The considered tax compliance choice is specified as an all-or-nothing choice: due taxes are either fully paid or fully evaded.[4] We assume that tax non-compliant individuals succeed in evading sales taxes since they have the complicity of the firms from which their purchases are made. Even not sharing with tax non-compliant young individuals the amount of wage income associated with the sales taxes evaded, those firms have the incentive of so collaborating with tax non-compliance as all the amount of wage income resulting from tax evasion is assumed to be spent on further purchases from them. The procedure used by the government to audit the purchases of the good made by young individuals and penalize those who evaded taxes is described later on in this section, and for now we only anticipate that a tax non-compliant individual is detected with probability $\varepsilon \in [0,1] \subset \mathbb{R}$, which is assumed to be exogenously determined, identically distributed across young individuals and constant over time. When a tax evader is detected, she has to refund the government an amount represented by $\gamma_t\big[c_{1t}^n + s_{nt}w_t\big]$, where $\gamma_t \in (\tau, \frac{1}{2+\theta}) \subset \mathbb{R}$ is the penalty rate in period t, which is applied to the value of all purchases made in her first period of life. The penalty rate is endogenously time-varying due to the government following a reactive behavior in a way that will be specified later on.[5] Yet, however the penalty rate is determined, a tax non-compliant individual takes it as given. Thus, budget constraint of a tax non-compliant individual born in t in her first period of life is represented by $c_{1t}^n = (1 - s_{nt})w_t$ with probability $1-\varepsilon$, and $(1+\gamma_t)c_{1t}^n = \big[1-(1+\gamma_t)s_{nt}\big]w_t$ with probability ε. The first-period budget constraint of a tax non-compliant individual born in t can be expressed as:

$$(1+\varepsilon\gamma_t)c_{1t}^n = \big[1-(1+\varepsilon\gamma_t)s_{nt}\big]w_t. \tag{2}$$

Although the expected unit price of the single good faced by a tax non-compliant individual in

[4] See Alm et al. (2009), Bazart and Bonein (2014), and Bazart et al. (2016) for evidence from laboratory experiments on tax compliance that the frequency of measures of tax evasion at the individual level such as the ratio of tax paid to tax owed not infrequently peaks at zero and one.

[5] While $\gamma_t > \tau$ ensures that a detected tax evader forfeits a larger portion of her wage income than she would have forfeited if she had adopted the compliance strategy, $\gamma_t < \frac{1}{2+\theta}$ guarantees that the detected tax evader has enough wage income to fulfill her penalty, given the assumption that non-compliant taxpayers who are fined are unable to avoid paying in full the ensuing fine for tax evasion. This requires that the penalty rate, γ_t, is strictly lower than the upper bound of the optimal saving rate of a tax non-compliant young individual, given by $\frac{1}{2+\theta}$, which is achieved with $\varepsilon = 0$, as will be derived shortly.

period t (hence the one she takes into account when making her consumption-saving decision) is given by $1 + \varepsilon\gamma_t$, she effectively pays, for one unit of the single good, 1 with probability $1 - \varepsilon$ or $1 + \gamma_t$ with probability ε (the case in which she is audited and inevitably fined).

In period $t + 1$, the individual born in t voluntarily leaves the labor market and retires, and her second-period budget constraint is given by:

$$c^i_{2t+1} = (1 + r_{t+1})s_{it}w_t, \tag{3}$$

where $i = c, n$ denotes the individual's type according to her tax compliance decision when young, and r_{t+1} is the rental rate of capital, which is taken as given by all individuals in both periods.[6]

Substitution of (1) into (3) yields the lifetime budget constraint faced by a compliant individual:

$$c^c_{1t} + \frac{c^c_{2t+1}}{1 + r_{t+1}} = \frac{w_t}{1 + \tau}. \tag{4}$$

Meanwhile, the lifetime budget constraint of a tax non-compliant individual can be similarly obtained by substituting (2) into (3), which yields:

$$c^n_{1t} + \frac{c^n_{2t+1}}{1 + r_{t+1}} = \frac{w_t}{1 + \varepsilon\gamma_t}. \tag{5}$$

We assume that all individuals have an instantaneous utility function taking the logarithmic form. Let $\theta \in \mathbb{R}_{++}$ be the one-period discount rate, which is assumed to be homogeneous across individuals and across generations. A young individual of type $i = c, n$, born in period t, chooses c^i_{1t} and c^i_{2t+1} that maximize her lifetime utility given by:

$$u(c^i_{1t}, c^i_{2t+1}) = \ln c^i_{1t} + \frac{1}{1 + \theta} \ln c^i_{2t+1}, \tag{6}$$

subject to the intertemporal budget constraint in (4) (if $i = c$) or in (5) (if $i = n$).

The optimal consumption plan of an individual born in t, denoted by (c^*_{1t}, c^*_{2t+1}), satisfies the

[6] This is so because, in t, the rental rate of capital in $t+1$ is yet to be determined, while in $t+1$, as in t, the market for renting capital is competitive and therefore suppliers are price takers.

following first-order condition:[7]

$$\frac{c^*_{2t+1}}{c^*_{1t}} = \frac{1 + r_{t+1}}{1 + \theta}. \tag{7}$$

Substituting (1) and (3) into (7) yields the optimal saving rate of a tax-compliant individual:

$$s^*_{ct} = \frac{1}{(2 + \theta)(1 + \tau)}, \tag{8}$$

which intuitively varies negatively with both the discount rate, θ, and the tax rate, τ.

Analogously, substituting (2) and (3) into (7) results in the optimal saving rate of a tax non-compliant individual:

$$s^*_{nt} = \frac{1}{(2 + \theta)(1 + \varepsilon\gamma_t)}, \tag{9}$$

which also intuitively varies negatively with the discount rate, the probability of detection, ε, and the penalty rate, γ_t.

As well known, the logarithmic specification for the instantaneous utility function in (6) (which in the present case applies to both tax compliant and tax non-compliant individuals) yields an optimal saving rate that does not depend on the interest rate.

On the production side of the economy, private firms produce the single good using three homogeneous inputs: labor and capital, which are supplied, respectively, by young and old individuals, and public services, the amount of which is taken as given by firms. Drawing on Barro (1990), the government provides free of charge public services that are employed as inputs by firms, the financing of which is fully done with what remains from the actual public revenues (which also include the fines levied on detected tax evaders, in addition to the actual tax collection) after tax auditors are remunerated. The product, labor and capital markets are all competitive, and for simplicity the market clearing price of the single good is continuously normalized to one. Firms carry out production subject to the following Cobb-Douglas production function:

$$Y_t = (K_t)^{\alpha} {G_t}^{\beta} \left(L^f_t\right)^{1-(\alpha+\beta)}, \tag{10}$$

where $Y_t \in \mathbb{R}_+$ is the aggregate output production, $G_t \in \mathbb{R}_+$ is the aggregate flow of productive public services, and $K_t \in \mathbb{R}_+$ and $L^f_t \in \mathbb{R}_+$ are, respectively, the aggregate quantities of capital and

[7] Since the utility function in (6) is strictly concave, the second-order conditions for utility maximization are satisfied, regardless of the tax compliance strategy adopted by the individual.

labor employed by firms in period t (the superscript f is used in L_t^f since not all young individuals work for the firms). In addition, $\alpha \in (0,1) \subset \mathbb{R}$ and $\beta \in (0,1) \subset \mathbb{R}$ denote parametric constants satisfying $\alpha+\beta < 1$. Note that not only capital and labor are essential inputs to production, but also productive public services. As in Barro (1990), a plausible rationale for including G as a separate argument of the production function in (10) is that for several reasons private inputs are not close substitutes for public inputs. We also draw on Barro (1990) in specifying the government as carrying out no production and owning no capital. As the single good can be used for multiple purposes, the reasonable simplifying assumption is that the government purchases a flow of output from firms and then make it available as productive public services to firms themselves.

Dividing both sides of the production function in (10) by the number of young individuals employed by the firms, we can express this function in intensive form as:

$$y_t = \left(k_t^f\right)^{\alpha} {g_t}^{\beta}, \tag{11}$$

where $y_t \equiv \frac{Y_t}{L_t^f}$, $k_t^f \equiv \frac{K_t}{L_t^f}$ and $g_t \equiv \frac{G_t}{L_t^f}$ are output, capital and productive public services per worker, respectively.

Considering that labor and capital are individually paid according to their marginal products, we can make use of (11) and the standard conditions for profit maximization to get:

$$w_t = [1-(\alpha+\beta)]\left(k_t^f\right)^{\alpha} {g_t}^{\beta} \tag{12}$$

and

$$r_t = \alpha\left(k_t^f\right)^{\alpha-1} {g_t}^{\beta}. \tag{13}$$

Note that the marginal product of labor in (12) and the marginal product of capital in (13) are both calculated by varying L_t^f and K_t, respectively, while holding g_t constant. This means that firms assume that changes in the amounts of labor and capital that they employ, as well as changes in their output production, do not lead to changes in the flow of productive public services provided to them by the government.

The supply of productive public services to firms is determined, in each period, according to the government budget constraint. Firms fully utilize all such supply of productive public services, as they do not face any demand constraint to sell all of their profit-maximizing production at the market price of the single good. The government has two sources of revenue: taxes collected on the purchases of the good by the tax-compliant young individuals and fines charged on the tax non-compliant young individuals who are caught evading the payment of those taxes. Meanwhile, the government incurs the cost of hiring tax auditors, who are young individuals the labor endowment of whom is employed to audit a random sample of the rest of the young individuals. From now on we refer to the former group of young individuals as *tax auditors* and to the latter simply as *workers*. Consider that in a given period t there are L_t young individuals in the economy. A relatively small fraction $\delta \in (0,1) \subset \mathbb{R}$ of these young individuals, which is supposed to be exogenously given and constant across periods, is composed of tax auditors (who always behave like tax-compliant individuals, as explained shortly), while the remaining fraction $1-\delta$ is composed of workers who behave as either compliant or non-compliant toward their tax obligations. A measure L_{nt} of workers, which is liable to vary from one period to the next one (in a fashion fully described later on), chooses to take the risk of following the non-compliance strategy, which means fully evading due taxes. We refer to this type of individual as *non-compliant worker*, identifying her by the subscript n, and define $x_t \equiv \frac{L_{nt}}{(1-\delta)L_t} \in [0,1] \subset \mathbb{R}$ as the proportion of tax evaders in the respective subpopulation. The remaining proportion $1 - x_t \equiv \frac{L_{ct}}{(1-\delta)L_t}$ follows the compliance strategy, where L_{ct} denotes the measure of workers who duly pay their taxes, and we dub this type of individual *compliant worker*, identifying her by the subscript c.

Recall that the tax imposed by the government is levied on the purchases of the good by young individuals in general. As assumed earlier, non-compliant workers succeed in evading taxes because they benefit from the complicity of the firms from which the respective purchases are made. Although non-compliant workers do not share with the respective firms the wage income associated with the evaded taxes, those firms have the incentive to collaborate with tax non-compliance since all the wage income diverted from the payment of taxes is spent on further purchases from them. We also assume that there is no secondary market (be it taxable or non-taxable) for the good, so that each produced good is transacted and taxed only once. In the event that the government could perfectly monitor every transaction carried out in the economy in a given period, it could identify those involving workers and thus subject to taxation. By the same token, if taxes were levied on the wage income with which workers are compensated, firms could be required to deduct from the gross wage the respective tax collection and then transfer it to the government. However, this would require the monitoring and auditing of the firms' compliance behavior, from which we choose to abstract.[8]

[8] We are aware that there is evidence that tax compliance is typically greater on income subject to employer withholding than on income not subject to such a feature, as recalled in Alm (2019) and confirmed yet again in a

Alternatively, if workers were formally required to fill out a tax return form when paying directly to the government their wage income tax, this would not allow perfectly identifying tax evaders either, the reason being that tax evaders would simply eschew filling out the required form, which would then demand the monitoring and auditing of the workers. In our analytical framework, all forms of tax evasion resulting in workers pocketing all of the taxes they were supposed to pay would likely yield similar qualitative results. The reason is that our assumption of monotonicity of preferences implies that all wage income is spent on purchases of the good, be it for consumption or investment purposes, as specified in (1) and (2).

It fits our analytical purposes better to consider that workers are levied a sales instead of income tax. We further assume that the economy is large in that the huge volume of purchases of the good makes it unfeasible for the government to perfectly monitor them all in real time, which therefore calls for the need of an *ex post* sampling audit conducted by tax auditors. Meanwhile, recall that the proportion of tax auditors, given by δ, is small, and we assume that (identifiable as they are) they are taxed directly on the wage income received from the government (which is equivalent to their being taxed on their purchases of the good, considering the maximizing behavior of young individuals). Therefore, tax auditors are compulsorily tax-compliant young individuals who are also assumed not to commit any wrongdoing when fulfilling their auditing duties. Although a certain measure of young individuals might prefer to be employed by a firm due to it opening the possibility of attempting to evade paying taxes, those not hired by a firm will nonetheless promptly accept a job offer received from the government. As an incentivizing device, and given our earlier assumption that all young individuals are equally skilled, the government compensates tax auditors with the same homogeneous wage received by workers hired by firms.

In a given period t, tax auditors manage to successfully audit *ex post* the purchases made by a limited, randomly selected number of workers. In addition to successfully verifying whether a given worker has complied with her legal tax obligations, the random audit carried out by the government is effective in uncovering all her unpaid sales taxes, if any. Since the probability of being audited is identically distributed across young individuals and a proportion x_t of workers behave as tax evaders, a worker is audited with probability ε and hence the probability of a non-compliant worker getting caught and inevitably fined by a tax auditor is εx_t. The estimated number of non-compliant workers

recent theory-driven experiment reported in Vossler et al. (2021). Yet it fits our purpose and modeling strategy better and more consistently to give more protagonism to the compliance behavior of individual taxpayers and to specify the behavior of the firms in a rather simplified fashion. But as explained shortly, tax auditors hired by the government compulsorily behave as tax-compliant individuals because they are taxed directly on their wage income.

who are audited in t can therefore be written as $\varepsilon x_t(1-\delta)L_t = \varepsilon L_{nt}$. Fines paid by such workers constitute the government's fine revenues, which, considering the first-period budget constraint of non-compliant workers in (2), their optimal saving rate in (9), and the maximizing behavior of young individuals, can be expressed as $\gamma_t(c_{1t}^n + s_{nt}^* w_t)\varepsilon L_{nt} = \frac{\varepsilon\gamma_t}{1+\varepsilon\gamma_t} w_t L_{nt}$. Along with fine revenues, the government receives tax revenues collected from compliant workers, which can be expressed, considering their first-period budget constraint in (1), their optimal saving rate in (8), and again the maximizing behavior of young individuals, as $\tau(c_{1t}^c + s_{ct}^* w_t)L_{ct} = \frac{\tau}{1+\tau} w_t L_{ct}$. We assume that the only cost incurred by the government in the auditing activity involves the wage compensation of tax auditors, which is the same one received by young individuals working for firms. Thus, recalling that tax auditors behave as tax-compliant young individuals, the net wage bill incurred by the government can be expressed as $[w_t - \tau(c_{1t}^c + s_{ct}^* w_t)]\delta L_t = \frac{1}{1+\tau} w_t \delta L_t$. We also assume that in each period all the actual public revenues remaining after tax auditors are remunerated is used by the government to supply productive public services. Thus, the supply of productive public services can be alternatively expressed as the sum of the tax revenues collected from compliant workers and the fine revenues collected from non-compliant workers who are audited, and subtracting the net cost of the tax auditing activity, which is the net wage bill of tax auditors incurred by the government represented above:

$$G_t = \left[\frac{\tau}{1+\tau} L_{ct} + \frac{\varepsilon\gamma_t}{1+\varepsilon\gamma_t} L_{nt} - \frac{1}{1+\tau}\delta L_t\right] w_t. \tag{14}$$

For given values of the tax rate, the probability of detecting a tax evader, and the wage bill of tax auditors, the government sets the penalty rate in period t so as to provide a strictly positive supply of productive public services and have a proper reactive response to tax evasion, as detailed shortly. This is done based on a measure of loss of tax revenues as follows. As the current wage and the price of the good (which we continuously normalize to one) are both homogeneous and publicly known, the government is able to compute the maximum amount of taxes to be collected in t should all workers follow the compliance strategy, which is given by $\frac{\tau}{1+\tau}(1-\delta)L_t w_t$. However, the actual tax collection may fall short of that maximum, and when it does, it falls short by the amount of taxes evaded by non-compliant workers, given by $\frac{\tau}{1+\tau} L_{nt} w_t$. As a result, the ratio of the latter amount to the former can be computed by the government and reactively used as a measure of loss of tax revenues in a given period t. Note that such a ratio is equal to the proportion of non-compliant workers in the subpopulation of workers, since $\frac{\frac{\tau}{1+\tau} L_{nt} w_t}{\frac{\tau}{1+\tau}(1-\delta)L_t w_t} = x_t$. Before auditing, therefore, although the government is not able to identify who, if any, the non-compliant workers are (and during the auditing process it identifies a given non-compliant worker with probability $\varepsilon \in [0,1] \subset \mathbb{R}$), it can indirectly learn the proportion of non-compliant workers in the respective subpopulation of taxpayers by computing what hereafter we dub coefficient of loss of tax revenues.

Having computed such a coefficient, the government establishes the penalty rate on tax evasion with the complementary purposes of penalizing and discouraging tax non-compliance as well as of endeavouring to compensate to some extent for the loss of public revenues represented by the evaded taxes. Accordingly, we specify the penalty rate as a strictly increasing function of the coefficient of loss of tax revenues as follows:

$$\gamma_t = \gamma(x_t), \tag{15}$$

where $\gamma'(x_t) > 0$. We further assume first that $\gamma(0) > \tau$, so that the lower bound of the penalty rate, which is achieved when $x_t = 0$, already ensures that a detected tax evader would forfeit a larger portion of her wage income than she would have forfeited if she had adopted the tax compliance strategy; and second that $\gamma(1) < \frac{1}{2+\theta}$, so that the upper bound of the penalty rate, which is achieved when ($x_t = 1$), is still strictly lower than the upper bound of the optimal saving rate of a non-compliant worker, given by $\frac{1}{2+\theta}$, which is achieved with $\varepsilon = 0$ in (9) (see footnote (5)). Given that non-compliant taxpayers who are detected are unable to default on the payment of the ensuing fine, that second further assumption ensures that even a non-compliant worker fined with the maximum penalty rate has enough savings both to pay the respective fine and to have in her second period of life a strictly positive (even if arbitrarily small) amount of capital. Note that the fulfillment of the latter condition is required to guarantee that in her second period of life the considered non-compliant worker receives a strictly positive rental income and hence has a strictly positive consumption.

Figure 1 summarizes the logical sequence of key events for young individuals and the government in period t. L_t individuals are born, a relatively small fraction of whom, δ, is hired by the government as tax auditors. They fulfill their professional duties and comply with their personal tax obligations. The remaining young individuals, $(1-\delta)L_t$, are hired by firms and choose between complying or not complying with their tax obligations. The optimal saving rate set by a young individual is conditional on her tax compliance behavior. The government levies a flat sales tax on all the purchases made by young individuals, and the actual tax collection allows the government to compute a coefficient of loss of tax revenues and thereby come to find out the proportion of tax evaders in the subpopulation of young individuals hired by firms. The penalty rate to be inevitably levied on the tax evaders identified as such in the random auditing process is set as a strictly increasing function of that coefficient of loss of tax revenues. The resulting fine is paid in full by detected tax evaders out of their savings, so that the actual saving rate of a tax evader (and therefore how much capital and rental income she will have in her second period of life) depends on whether or not she is audited. Yet the penalty rate has both a lower bound (which ensures that a detected tax evader forfeits a larger portion of her wage income than she would have forfeited if she had adopted the tax compliance strategy) and an upper bound (which ensures that a tax evader fined with the maximum penalty rate has enough savings both to pay the respective fine and to have in her second period of life a strictly positive amount of capital as source of income). The government for its part spends all the actual public revenues (collected taxes and fines) remaining after tax auditors are remunerated in supplying a flow of productive public services used as inputs by firms. It follows that a change in the penalty rate impacts on saving (and hence capital) formation through different channels. For example, given the wage rate, a higher penalty rate, by raising the fines revenue and thus the supply of productive public services, boosts aggregate output and hence aggregate saving formation, which raises capital formation. Yet the same increase in the penalty rate, by lowering the saving formation by tax evaders who are audited, causes a reduction in capital formation as well. In any case, notice that the production function in (10) implies that the marginal product of capital is diminishing.

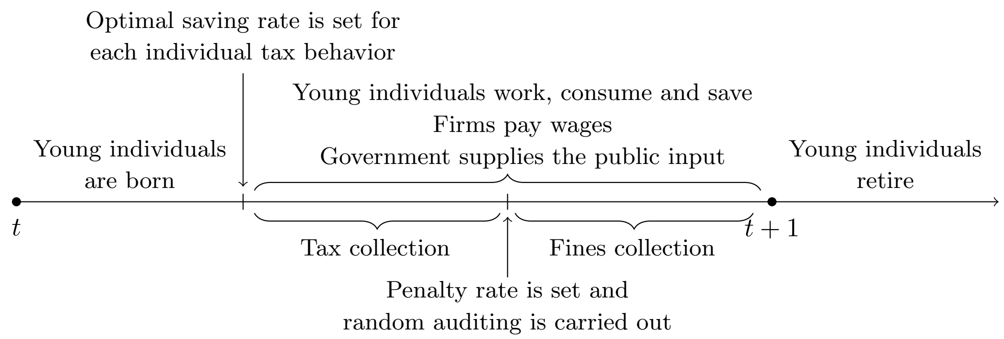

Figure 1: Logical sequence of key events for young individuals and the government in period t.

In any given period t, a temporary equilibrium is reached when all markets clear. In the product market, all output production is either consumed or saved by young individuals, or consumed by old individuals, with the price of the good freely adjusting to remove any excess supply or demand. In the capital market, the rental rate of capital freely adjusts to remove any excess supply or demand, so that the predetermined capital stock, now owned by old individuals, is all rented to firms, yielding $K_t^f = K_t$. In the competitive segment of the labor market, given the supply of labor to firms, the wage rate freely adjusts to remove any excess demand for labor by firms, thus yielding

$L_t^f = (1-\delta)L_t$ and hence $k_t^f = k_t$. Recall that the labor market is segmented, with the government placing a predetermined demand for labor given by δL_t which operates as a binding constraint on the supply of labor to firms, yet being willing to pay the same wage rate which clears the competitive segment of the labor market.

Let (w_t^*, r_t^*) be the vector of market-clearing factor prices that, together with the price of the good, which we normalize to one, establishes the temporary equilibrium in a given period t. We will now show that such an equilibrium is well-defined and parameterized by the proportion of non-compliant taxpayers in the subpopulation of workers and the capital stock per worker, which are predetermined variables in a given period t.[9]

Using the labor market equilibrium condition represented by $L_t^f = (1-\delta)L_t$ and the expressions in (1) and (2) evaluated at the optimal saving rates in (8) and (9), as well as considering (15), and recalling that $g_t = \frac{G_t}{L_t^f}$, we can use (14) to express the supply of productive public services per worker in the temporary equilibrium in period t as follows:

$$g_t^* = \phi(x_t)w_t^*, \tag{16}$$

where

$$\phi(x_t) \equiv \frac{\tau}{1+\tau}(1-x_t) + \frac{\varepsilon\gamma(x_t)}{1+\varepsilon\gamma(x_t)}x_t - \frac{\delta}{(1+\tau)(1-\delta)}, \tag{17}$$

which is the supply of productive public services per worker in wage units.

In order to ensure that, for a given $w_t^* > 0$, the supply of productive public services per worker in (16) (and hence the output per worker in (11)) only takes strictly positive values for any proportion of non-compliant workers, x_t, we assume that, for given values of the tax rate, τ, and the probability of detecting a tax evader, ε, the minimum penalty rate set by the government, $\gamma(0)$, is such that the maximum fine revenues per worker, given by $\frac{\varepsilon\gamma(0)}{1+\varepsilon\gamma(0)}w_t^*$ when $x_t = 1$, is sufficient to offset the net cost of the auditing activity per worker, $\frac{\delta}{(1+\tau)(1-\delta)}w_t^*$. This condition ensures that $g_t^* > 0$ for any $x_t \in (0,1] \subset \mathbb{R}$, if the tax policy is such that the expected punishment faced by a tax evader is relatively low (i.e., $\varepsilon\gamma(x_t) < \tau$). We also assume that the minimum tax revenues per worker, given by $\frac{\tau}{1+\tau}w_t^*$ when $x_t = 0$, is strictly greater than the net cost of the auditing activity per worker, so

[9] It can be readily seen that the temporary equilibrium in each market is stable. In the factor markets, for example, with diminishing marginal products of capital and labor, as implied by (10), the demand for capital and the demand for labor are downward-sloping in the respective markets (in the case of the labor market, we are of course referring to its competitive segment). As in each of such market the supply of the respective factor is predetermined and inelastic (and hence vertical), any excess demand (supply) will push the respective price up (down).

that, if the tax policy is such that the expected punishment faced by a tax evader is relatively high (i.e., $\varepsilon\gamma(x_t) > \tau$), then $g_t^* > 0$ for any $x_t \in [0,1) \subset \mathbb{R}$. These conditions ensuring that the supply of productive public services per worker only takes strictly positive values are formally derived in Appendix A.

Since the conditions in (A.1) and (A.2) ensure that, for a given $w_t^* > 0$, the supply of productive public services per worker is strictly positive for any proportion of non-compliant workers, we can substitute (16) into (12) to express the temporary equilibrium wage rate in period t as a function of the proportion of non-compliant workers and the capital stock per worker:

$$w_t^* = [1-(\alpha+\beta)]^{\frac{1}{1-\beta}} k_t^{\frac{\alpha}{1-\beta}} \left(\phi(x_t)\right)^{\frac{\beta}{1-\beta}} \equiv w(k_t, x_t), \tag{18}$$

which is also strictly positive for any $x_t \in [0,1] \subset \mathbb{R}$ when the conditions in (A.1) and (A.2) hold.

Substituting the latter expression into (16) yields the flow of productive public services per worker in the temporary equilibrium in period t:

$$g_t^* = \phi(x_t) w(x_t, k_t) \equiv g(k_t, x_t). \tag{19}$$

Finally, using now the temporary equilibrium conditions along with the expression in (13), the interest rate (or rental rate of capital) in the temporary equilibrium in period t is given by:

$$r_t^* = \alpha k_t^{\alpha-1} \left[\phi(x_t) w(k_t, x_t)\right]^{\beta} \equiv r(x_t, k_t). \tag{20}$$

2.2. State transition: capital formation and decision-making on tax compliance strategy

In line with our assumption that old individuals leave no bequest of any kind, we further assume that the capital stock only but fully depreciates after one period of use. Consequently, the capital stock in period $t+1$, denoted by K_{t+1}, is fully determined by the actual savings of young individuals in period t. Considering the homogeneity of the wage rate across young individuals as well as using the optimal saving rate in (8), the amount of saving realized by tax compliant workers and tax auditors together in period t is given by $s_{ct}^* w_t^* (L_{ct} + \delta L_t) = \frac{1}{(2+\theta)(1+\tau)} w_t^* (L_{ct} + \delta L_t)$. Meanwhile, considering again that the wage rate is homogeneous across young individuals but using this time the optimal saving rate in (9), the amount of saving realized by non-compliant workers who were caught evading taxes is represented by $[s_{nt}^* w_t^* - \gamma(x_t)(c_{1t}^* + s_{nt}^* w_t^*)]\varepsilon L_{nt} = \frac{1}{1+\varepsilon\gamma(x_t)} \left[\frac{1}{2+\theta} - \gamma(x_t)\right] w_t^* \varepsilon L_{nt}$, while that realized by non-compliant workers who went undetected is given by $s_{nt}^* w_t^* (1-\varepsilon) L_{nt} =$

$\frac{1}{(2+\theta)[1+\varepsilon\gamma(x_t)]} w_t^*(1-\varepsilon)L_{nt}$. The amount of saving realized by all non-compliant workers together can therefore be expressed as $\frac{1}{1+\varepsilon\gamma(x_t)}\left[\frac{1}{2+\theta}-\varepsilon\gamma(x_t)\right] w_t^* L_{nt}$. In order to sharpen the focus of this paper on the coupled dynamics of the proportion of tax non-compliant workers, on the one hand, and the capital formation and per capita income, on the other, we simplify matters by assuming that the number of individuals born in each period t, L_t, and who live for two periods, t and $t+1$, remains constant over time. By further assuming that young individuals do not incur any cost in keeping capital idle over time (a carrying cost for example), we get the following capital formation dynamics:

$$K_{t+1} = \frac{1}{(2+\theta)(1+\tau)} w_t^*(L_c+\delta L) + \frac{1}{1+\varepsilon\gamma(x_t)}\left[\frac{1}{2+\theta}-\varepsilon\gamma(x_t)\right] w_t^* L_n, \tag{21}$$

which can be written in intensive form as:

$$k_{t+1} = \varphi(x_t) w(k_t, x_t), \tag{22}$$

where $w(k_t, x_t)$ is determined by the function in (18) and

$$\varphi(x_t) \equiv \frac{1}{(2+\theta)(1+\tau)}\left(1-x_t+\frac{\delta}{1-\delta}\right) + \frac{1}{1+\varepsilon\gamma(x_t)}\left[\frac{1}{2+\theta}-\varepsilon\gamma(x_t)\right] x_t, \tag{23}$$

which is the capital formation per worker in wage units. Note that $\varphi(x_t) > 0$ for all $x_t \in [0,1] \subset \mathbb{R}$, given the assumption that $\gamma(x_t) < \frac{1}{2+\theta}$.

Having derived the equation of formation of capital per worker in (22), let us now focus on the dynamics of the frequency distribution of tax compliance strategies across workers, which will be specified as an adaptively rational equilibrium dynamics (ARED) drawing upon Brock and Hommes (1997). Workers are described as using a stochastic discrete choice model along the lines of Manski and McFadden (1981) to choose a tax compliance strategy from the finite set of available strategies, a choice which is interpreted as a purposive economic act based upon an adaptively rational decision. The deterministic component of the payoff of each tax compliance strategy is a well-defined measure of its performance in the recent past, and the resulting dynamics across strategy choice is interestingly coupled to the equilibrium dynamics of the other endogenous variables. In light of the purpose of our model, an analytically rewarding feature of its incorporation of such a discrete choice framework is that the dynamics of the frequency distribution of tax compliance strategies across workers becomes coupled to the dynamics of our macroeconomic variables of interest.

In a given period t, all workers choose in a decentralized and uncoordinated way between complying or not complying with her tax obligations. In formal terms, a worker born in period t chooses and actually implements one of the tax compliance strategies contained in the choice set represented by $\{c, n\}$. Drawing upon the literature on discrete choice modeling (see, e.g., Manski, 1993; McFadden, 2001; Train, 2009), workers are supposed to have well-defined preferences over the set of tax compliance strategies, with such preferences being represented by a payoff function which is additive in two components:

$$\pi(\omega_{jt}) = \pi^d(\omega_{jt}) + \mu(\omega_{jt}), \tag{24}$$

where $\omega_{jt} \in \{c, n\}$ is the type of the j-th worker born in t, which is defined by the tax compliance strategy chosen and implemented by her in period t; $\pi^d(\omega_{jt})$ denotes a deterministic component associated with objective and hence observable motivations of worker j based on a lagged performance measure; and $\mu(\omega_{jt})$ stands for a random component associated with subjective and hence unobservable motivations and proclivities of worker j.

Recall that workers are wage-takers and that they all receive the same wage compensation, while all young individuals make consumption, saving, and tax compliance decisions taking as given the same interest rate. Consequently, all workers born in period t face the same factor price vector (w_t^*, r_t^*), the elements of which are represented by the expressions in (18) and (20), respectively. As all individuals born in period t have the same preferences with respect to the first and the second-period consumption, the indirect utilities of tax compliant and tax non-compliant workers differ in value from each other only to the extent that the tax rate, τ, differs in value from the expected cost of the tax non-compliance strategy, which is formed in period t adaptively using information from period $t-1$, and is given by $\varepsilon\gamma(x_{t-1})$.[10] Thus, it is reasonable that a key element to feature as the deterministic component of each type of payoff function in (24) is the expected cost of the respective

[10]Using (6), (9), and (15), the indirect utilities of compliant and non-compliant workers can then be formally expressed, respectively, as follows:

$$u(c_{1t}^{c*}, c_{2t+1}^{c*}) = \ln\left(\frac{(1+\theta)w_t^*}{(2+\theta)(1+\tau)}\right) + \frac{1}{1+\theta}\ln\left(\frac{1+r_t^*}{(2+\theta)(1+\tau)}\right)$$

and

$$u(c_{1t}^{n*}, c_{2t+1}^{n*}) = \ln\left(\frac{(1+\theta)w_t^*}{(2+\theta)(1+\varepsilon\gamma(x_{t-1}))}\right) + \frac{1}{1+\theta}\ln\left(\frac{1+r_t^*}{(2+\theta)(1+\varepsilon\gamma(x_{t-1}))}\right).$$

It is straightforward to check that such indirect utilities differ in value from each other if, and only if, $\tau \neq \varepsilon\gamma(x_{t-1})$. As a result, a payoff-maximizing tax compliance strategy is also a cost-minimizing one and vice versa.

tax compliance strategy:

$$\pi^d(\omega_t^j) = \begin{cases} -\upsilon(\tau) & \text{, if } \omega_t^j = c, \\ -\upsilon\big(\varepsilon\gamma(x_{t-1})\big) & \text{, if } \omega_t^j = n, \end{cases} \tag{25}$$

where $\upsilon'(\cdot) > 0$ over the domain of its arguments.

Meanwhile, the random component of the payoff function in (24) can be interpreted as reflecting idiosyncratically subjective motivations and proclivities, such as tax morale. In fact, the presence and importance of tax morale as another determining factor of tax compliance behavior has been largely documented in a variety of studies (see, e.g., Cummings et al., 2009; Lubian and Zarri, 2011; Luttmer and Singhal, 2014; Pickhardt and Prinz, 2014; Alm, 2019). More broadly, there is considerable evidence that the decision by an individual regarding to comply or not to comply with her legally due tax obligations goes well beyond a simple amoral benefit-cost calculation predicated exclusively on narrowly defined pecuniary considerations. Tax morale is in some sense a portmanteau term covering a wide array of motivations and proclivities affecting tax compliance which are unrelated to a strict cost-benefit reasoning and are rather typically subjective and/or socially determined. In our specification of the random component of the payoff function in (24), tax morale broadly denotes an intrinsic motivation either to comply or not to comply with legally due tax obligations. Reasonably, we consider that such motivations and proclivities regarding tax compliance are idiosyncratically subjective, and hence heterogeneous across workers, as well as randomly and independently determined across individuals and over time. More precisely, we specify the random component $\mu(\omega_{jt})$ in the payoff function in (24) as a realization of a continuous random variable with support on the whole real line. However, the subjective views held by workers are homogeneous in one important respect which is that it is adaptively rational that the deterministic component of the payoff to each tax compliance strategy be represented by the expected cost associated with each strategy, as specified in (25).

As our specification in (24) supposes that both strategies available to workers feature a random component, the association of the latter with tax morale and the heterogeneity of that component across workers imply that, in principle, a given worker may ambivalently hold both compliance-leaning motivations and proclivities and non compliance-leaning ones. Therefore, we plausibly and more inclusively consider that an individual worker may have mixed moral feelings with respect to complying or not complying with her legally due tax obligations, so that it is on balance that she will ultimately have either a compliance-leaning tax morale or a non compliance-leaning one. In the context of our model, although a young individual is not in position to question the legality of the tax obligations imposed on her by the government, she may nevertheless conceive of her intrinsically subjective motivations and proclivities towards non-compliance as morally warranted. In effect, our approach to tax compliance based on a discrete choice framework allows treating the taxpayers in the model as heterogeneous along two other dimensions in regard to tax morale, which are whether on balance an individual taxpayer has a compliance-leaning tax morale or a non compliance-leaning one, and how much compliance-leaning or non compliance-leaning the tax morale of an individual taxpayer is. Of course, there may exist workers for whom the compliance-leaning motivations and proclivities and the non compliance-leaning ones offset each other, so that they have what we dub a neutral tax morale. For such workers, as a result, what is decisive in the end in the payoff function in (24) is the deterministic component specified in (25).

Recalling that the only young individuals who can choose between complying or not complying with their tax obligations are those working for firms (dubbed workers), our discrete choice approach interestingly allows differentiating a deepening (intensive margin) from a widening (extensive margin) of either type of overall tax morale (compliance-leaning or non compliance-leaning) across workers. Suppose that the overall tax morale across workers is compliance-leaning. Given the proportion of each type of individual tax morale on balance (compliance-leaning or non compliance-leaning) across workers, an increase in the average extent of compliance-leaningness on balance across workers raises the overall compliance-leaning tax morale along the intensive margin. Now suppose that the overall tax morale across workers is non compliance-leaning. Given the average extent of non compliance-leaningness on balance across workers, an increase in the frequency of workers for whom on balance the tax morale is non compliance-leaning raises the overall non compliance-leaning tax morale along the extensive margin.

In a given period t, a worker j chooses and implements a certain tax compliance strategy if such a strategy is expected to yield a payoff which is greater than or equal to the payoff associated with the alternative strategy, $\pi(\omega'_{jt})$. In formal terms, the chosen strategy $\omega_{jt} \in \{c, n\}$ has to satisfy the following payoff-maximizing condition:

$$\pi(\omega_{jt}) \geq \pi(\omega'_{jt}), \forall \omega'_{jt} \in \{c, n\}. \tag{26}$$

Using (24), the payoff-maximizing condition in (26) can be written as follows:

$$\pi^d(\omega_{jt}) - \pi^d(\omega'_{jt}) \geq \mu(\omega'_{jt}) - \mu(\omega_{jt}), \forall \omega'_{jt} \in \{c, n\}. \tag{27}$$

It follows from the inequality in (27) that for any worker j, a strategy ω_{jt} may not be the chosen one in a given period t even if the value of the deterministic component of the payoff associated with it is strictly greater than the value of the deterministic component of the payoff associated with the alternative strategy, that is, even if $\pi^d(\omega_{jt}) - \pi^d(\omega'_{jt}) > 0$. The intuitive reason is that the value of the random component of the payoff associated with strategy ω'_{jt} may be strictly greater than the value of the random component of the payoff associated with strategy ω_{jt}, and to an extent that more than offsets the advantage to strategy ω_{jt} given by $\pi^d(\omega_{jt}) > \pi^d(\omega'_{jt})$. As an illustrative example of the role that tax morale can potentially play in reducing tax non-compliance, let us pick the tax non-compliance strategy as the benchmark for the following comparison. Using (24) and (25), even if it is the case that $\tau > \varepsilon\gamma(x_{t-1})$, so that $\pi^d(\omega_{jt} = n) - \pi^d(\omega'_{jt} = c) = \upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x_{t-1})\big) > 0$, a worker j will nonetheless choose to comply with her tax obligations in period t if her idiosyncratically subjective motivations and proclivities are such that $\upsilon(\tau) - \upsilon\big(\varepsilon - \gamma(x_{t-1})\big) < \mu(\omega'_{jt} = c) - \mu(\omega_{jt} = n)$. Yet notice that the existence of a worker j for whom such a strict inequality favoring the tax compliance strategy payoff-wise is satisfied is an event which occurs with a given probability. This is because, on the one hand, those idiosyncratically subjective motivations and proclivities are heterogeneous across workers as well as randomly and independently distributed across them and over time, and on the other a tax non-compliant worker is audited with probability $\varepsilon \in [0, 1] \subset \mathbb{R}$, which is exogenously and identically distributed across workers and over time. However, our broader conception of tax morale also allows it to impact negatively on tax compliance. In effect, even if it is the case that $\varepsilon\gamma(x_{t-1}) > \tau$, so that $\pi^d(\omega'_{jt} = c) - \pi^d(\omega_{jt} = n) = \upsilon\big(\varepsilon\gamma(x_{t-1})\big) - \upsilon(\tau) > 0$, a worker j will choose not to comply with her legally due tax obligations in period t if her idiosyncratically subjective motivations and proclivities are such that $\upsilon\big(\varepsilon\gamma(x_{t-1})\big) - \upsilon(\tau) < \mu(\omega_{jt} = n) - \mu(\omega'_{jt} = c)$.

Consequently, a key implication of our discrete choice approach to tax compliance is that two workers belonging to the same generation t, and hence facing the same objective economic conditions, including the same tax system and the same amount of tax burden, may well choose to behave differently with respect to complying or not complying with their tax obligations. A worker whose tax morale on balance leans towards compliance may behave compliantly even if the value of the deterministic component of the payoff associated with noncomplying is strictly greater than the value of the deterministic component of the payoff associated with complying. Conversely, a worker with tax morale on balance favoring non-compliance (due for example to her seeing the tax system as too unfair) may behave in a non-compliant fashion even if the value of the deterministic component of the payoff to noncomplying is strictly smaller than the value of the same component of the payoff to complying.

Drawing upon the ARED approach proposed in Brock and Hommes (1997), we assume that the decision making about tax compliance strategy involves both idiosyncratically subjective motivations and proclivities as well as deterministic factors. The consideration of the latter on the part of the decision maker requires the use of recent objective information to form the (expected) cost associated with each strategy, as represented in (25). In a given period t, although the presence of the random component in (24) precludes determining the behavioral choice of each worker, we can nonetheless infer the probability with which worker j chooses not to comply with her legally due tax obligations, which from (25)-(27) is given by:

$$Prob\left(\pi(\omega_{jt} = n) \geq \pi(\omega'_{jt} = c)\right) = P\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x_{t-1})\big)\right), \tag{28}$$

where $P : \mathbb{R} \to (0,1) \subset \mathbb{R}$ denotes the cumulative distribution function of the random variable $\mu(\omega'_{jt} = c) - \mu\big(\omega_{jt} = n\big)$ embedded in (28), which is considered to be continuously differentiable with $P'(\cdot) > 0$ over its support. As a result, the probability with which worker j chooses to comply with her tax obligations in period t is simply given by $1 - P\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x_{t-1})\big)\right)$.

As all workers born in a given period t face the same expected cost associated with each tax compliance strategy, the determinants of which are τ and $\varepsilon\gamma(x_{t-1})$, as represented in (25), they share the same deterministic component in their respective payoff functions. However, workers born in a given period t are heterogeneous with respect to the expected payoff associated with each strategy alternative owing to they holding heterogeneous idiosyncratically subjective motivations and proclivities concerning tax compliance. In order to synchronize the dynamics of the proportion of non-compliant workers with the dynamics of the capital per worker in (22), we will make use of the choice probability in (28) to determine the proportion of workers born in a given period $t+1$ who choose to behave as tax evaders (or as type n workers), thus specifying the following ARED:

$$x_{t+1} = P\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x_t)\big)\right). \tag{29}$$

Therefore, for any cumulative distribution function of the random variable embedded in (28), as well as for any specific format of the function $\upsilon(\cdot)$ in (25) yielding $\upsilon'(\cdot) > 0$, we then have that $\frac{\partial x_{t+1}}{\partial \tau} = P'(\cdot)\upsilon'(\tau) > 0$ and $\frac{\partial x_{t+1}}{\partial \varepsilon} = -P'(\cdot)\upsilon'\big(\varepsilon\gamma(x_t)\big)\gamma(x_t) < 0$. Given that the tax rate τ represents the cost associated with the compliance strategy, the higher the tax rate, the higher is the proportion of non-compliant workers in period $t+1$, holding all else constant. Meanwhile, as the respective expected punishment $\varepsilon\gamma(x_t)$ represents the expected cost associated with the non-compliance strategy, the higher the expected probability of detection, given by ε, the lower is the proportion of non-compliant workers in period $t+1$, holding all else constant. Additionally, it follows from (29) that $\frac{\partial x_{t+1}}{\partial x_t} = -P'(\cdot)\upsilon'\big(\varepsilon\gamma(x_t)\big)\varepsilon\gamma'(x_t) < 0$, meaning that, holding all else constant, a change in the proportion of non-compliant workers in period t, by changing the expected punishment for period $t+1$, leads to a change in the opposite direction in the proportion of non-compliant workers in period $t+1$. The dynamic of the proportion of non-compliant workers is therefore in and of itself stable.

Note that the effects of parametric shifts explored in the preceding paragraph are mediated only by changes in the deterministic components of the payoffs associated with the strategies available to workers, which is of course due to the exogenous nature of the idiosyncratically subjective motivations and proclivities featuring the random components of those payoffs. Yet it follows from the ARED in (29) that the proportion of non-compliant workers in a given period $t+1$ also depends on those random components as reflected by the cumulative distribution function of the random variable $\mu(\omega'_{jt} = c) - \mu\big(\omega_{jt} = n\big)$ embedded in (28).

The state transition of the economy is determined by the two-dimensional map consisting of the capital formation dynamics in (22) and the ARED in (29), the state space of which is represented by $\Theta \equiv \big\{(k_t, x_t) \in \mathbb{R}^2 : k_t > 0, 0 \leq x_t \leq 1\big\}$.

3. Dynamic behavior of the economy

The model set forth in the preceding section features the microdynamics of the proportion of non-compliant workers crucially affecting the macrodynamics of the capital per worker, the output per worker, and ultimately the per capita output. For a given proportion of non-compliant workers, x, the penalty rate, γ, levied on tax evaders impacts on the supply of productive public services by affecting the amount of fines collected by the government. Yet insofar as tax evaders who are detected pay the consequent fines out of their savings, the penalty rate also impacts on aggregate saving and capital formation. As specified in Section 2.2, the proportion of non-compliant workers varies over time as each young individual chooses to adopt a certain tax compliance strategy taking into account both the subjective and the (expected) objective cost of each choosable alternative. For a given tax rate, τ, and a given probability of a worker being audited, ε, such latter costs depend on the penalty rate, $\gamma(x)$, which is increasing in the proportion of non-compliant workers. Therefore, there is a two-way relationship between the penalty rate and the proportion of workers who become tax evaders. A long-run equilibrium configuration is characterized by stationary values for both the proportion of non-compliant workers in the ARED specification in (29) and the capital per worker in (22) (and thus the output per worker and the per capita output).

In the following proposition, we establish the existence and uniqueness of the long-run equilibrium of the dynamic system given by (22) and (29).

Proposition 1 (Existence and uniqueness of a long-run equilibrium). *For a given vector of parameters $(\alpha, \beta, \varepsilon, \delta, \theta, \tau)$, the two-dimensional map consisting of (22) and (29) has a unique fixed point which is given by (k^*, x^*), with $k^* = \left[\big(1-(\alpha+\beta)\big)\big(\varphi(x^*)\big)^{1-\beta}\big(\phi(x^*)\big)^{\beta}\right]^{\frac{1}{1-(\alpha+\beta)}} \in \mathbb{R}_{++}$ and $x^* \in (0,1) \subset \mathbb{R}$ defined implicitly by $x^* = P\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x^*)\big)\right)$.*

Proof: See Appendix B.

The long-run equilibrium configuration, therefore, is characterized by heterogeneity in tax compliance behavior, with compliant workers coexisting with non-compliant ones. The proportion of tax non-compliant workers in the long-run equilibrium is determined both by deterministic factors (the tax rate, the probability of detection, and the penalty rate) and by intrinsically subjective motivations and proclivities randomly distributed across workers (such as tax morale). Meanwhile, per (17) and (23), the capital per worker in the long-run equilibrium is determined by the same random and deterministic factors (in the case of the latter both separately and through the proportion of non-compliant workers) in addition to the proportion of young individuals who are unable to choose between complying and not complying by virtue of being hired by the government, δ, the one-period discount rate, θ, and the exponents (factor elasticities) in the production function in (10), α and β.

However, a natural question that arises concerns whether the dynamics described in (22) and (29) can take the economy to the unique long-run equilibrium configuration established in Proposition 1 starting from a sufficiently small neighborhood of x^* (which can of course include either $x^* = 0$ or $x^* = 1$). The answer is yes, as formally established in the following proposition.

Proposition 2 (Stability properties of the long-run equilibrium). *For a given vector of parameters $(\alpha, \beta, \varepsilon, \delta, \theta, \tau)$, the unique long-run equilibrium (k^*, x^*) of the two-dimensional map consisting of (22) and (29) exhibits the following dynamic properties:*

i. If $\varepsilon P'\left(\upsilon(\tau)-\upsilon\big(\varepsilon\gamma(x^*)\big)\right)\upsilon'\big(\varepsilon\gamma(x^*)\big)\gamma'(x^*) < 1$*, the long-run equilibrium* (k^*, x^*) *is a local attractor.*

ii. When $\varepsilon P'\left(\upsilon(\tau)-\upsilon\big(\varepsilon\gamma(x^*)\big)\right)\upsilon'\big(\varepsilon\gamma(x^*)\big)\gamma'(x^*) = 1$*, a flip (or period-doubling) bifurcation occurs at the long-run equilibrium* (k^*, x^*)*.*

Proof: See Appendix C.

As a result, in accordance with the empirical evidence, heterogeneity in tax compliance behavior across taxpayers can exhibit considerable persistence over time, in that it can emerge as a stable long-run equilibrium of the coupled dynamics of the frequency distribution of tax compliance strategies across workers and the level of macroeconomic activity. In effect, it is as well possible that a flip (or period-doubling) bifurcation occurs at the long-run equilibrium (k^*, x^*). This is a bifurcation in which the considered system switches to a new dynamic behavior featuring twice the period of the original system when a small smooth change is experienced by one or more of its parameter values. When a parameter value crosses through a critical threshold and such a switch takes place, there come to exist two points such that applying the dynamics to each of the points yields the other point. Therefore, the dynamic behavior of the economy can be marked by fluctuating heterogeneity in tax compliance behavior on the part of workers, with the capital per worker, the output per worker, and the per capita output undergoing endogenous fluctuations as well.

Supposing that the long-run equilibrium (k^*, x^*) is a local attractor, we can interestingly explore the impact of a change in the proportion of tax-evading workers on the long-run equilibrium per capita output, which is a linear transformation of the output per worker in (11) evaluated at the long-run equilibrium (k^*, x^*).[11] Using (11), (19) and (22), we get the following expression for the per capita output in the long-run equilibrium:

$$\tilde{y} = \frac{(1-\delta)}{2}\left[\frac{\phi(x^*)}{\varphi(x^*)}\right]^{\beta}(k^*)^{\alpha+\beta}, \tag{30}$$

where $\phi(x^*)$ is the supply of productive public services per worker in wage units in (17), and $\varphi(x^*)$ is the aggregate saving formation (and therefore the aggregate capital formation) per worker in wage

[11] The level of per capita output in the long-run equilibrium, which we denote by $\tilde{y}$, can be formally expressed as $\tilde{y} = \frac{y^*(1-\delta)L}{2L} = \frac{(1-\delta)}{2}y^*$, where y^* is the output per worker in the long-run equilibrium. This linear transformation follows from our assumptions that L individuals are born in each period and live for two periods, with a constant fraction δ of those L individuals being hired by the government as auditors. The latter individuals indirectly contribute to aggregate production by allowing the government to collect fines from tax evaders, the revenue of which is entirely spent on the provision of the flow of productive public services used as input in output production.

units in (23), evaluated at the long-run equilibrium (k^*, x^*).[12]

Similarly, now from (21)-(22), the level of per capita capital stock in the long-run equilibrium, which we denote by $\tilde{k}$, can be expressed as:

$$\tilde{k} = \frac{(1-\delta)}{2} k^*. \tag{31}$$

Let η_{zx^*} denote the elasticity of variable z with respect to the proportion of non-compliant workers in the long-run equilibrium, x^*. It follows from (30) that:

$$\begin{aligned}\eta_{\tilde{y}x^*} &= (\alpha+\beta)\eta_{\tilde{k}x^*} + \beta(\eta_{\phi x^*} - \eta_{\varphi x^*}) \\ &= \frac{1}{1-(\alpha+\beta)}\left[\beta\eta_{\phi x^*} + (\alpha-\beta)\eta_{\varphi x^*}\right],\end{aligned} \tag{32}$$

where

$$\eta_{\tilde{k}x^*} = \frac{1}{1-(\alpha+\beta)}\left[\beta\eta_{\phi x^*} + (1-\beta)\eta_{\varphi x^*}\right] \tag{33}$$

is obtained from the expression for the capital per worker in the long-run equilibrium, k^*, as stated in Proposition 1.

For a given vector of parameters $(\alpha, \beta, \varepsilon, \delta, \theta, \tau)$, the impact of a change in the proportion of tax-evading workers on the per capita output in the long-run equilibrium depends on whether $\varepsilon\gamma(x^*) \geq \tau$ or $\varepsilon\gamma(x^*) < \tau$. As shown in Appendix D, $\varepsilon\gamma(x^*) \geq \tau$, which confers a payoff advantage to the tax compliance strategy from the deterministic side, implies that $\eta_{\varphi x^*} < 0 < \eta_{\phi x^*}$. Thus, when the expected punishment is greater than or equal to the tax rate, an increase in the proportion of non-compliant workers leads to an increase in the aggregate amount of fines collected by the government which more than compensates the initial loss of tax revenues, thus expanding the supply of productive public services ($\eta_{\phi x^*} > 0$) at the expense of reducing aggregate saving (and hence capital) formation ($\eta_{\varphi x^*} < 0$) (recall that a tax evader who is detected pays the resulting fines out of her savings). For a given response of the penalty rate to such an increase in the proportion of tax-evading workers, as specified in (15), an increase in such a proportion when $\varepsilon\gamma(x^*) \geq \tau$ leads to a decrease (increase) in the per capita output in the long-run equilibrium if the elasticity of the aggregate output with

[12] Note that there is no growth of the per capita output in the long-run equilibrium. The reason is that the economy lacks a source of such growth in the long-run equilibrium, as it does not experience technological progress, for example. Moreover, the only factor of production that can be accumulated from one period (at least) to the next one (capital) exhibits a diminishing marginal product, as implied by the production function in (10) displaying constant returns to scale.

respect to the supply of productive public services, given by β in (10), is strictly lower (greater) than a threshold represented by $\underline{\beta}$. In the case of the per capita capital stock, an increase in the proportion of tax-evading workers leads to a decrease (increase) in the per capita capital stock when $\beta < \overline{\beta}$ ($\beta > \overline{\beta}$), with $\underline{\beta} < \overline{\beta}$.

The rationale behind the results above is intuitive. When $\beta < \underline{\beta}$, the positive impact on output due to the expansion in the flow of supply of productive public services is not sufficient to offset the accompanying negative impact on output due to the reduction in aggregate saving and hence capital formation. When $\underline{\beta} < \beta < \overline{\beta}$, the positive impact on output due to the expansion in the flow of supply of productive public services more than offsets the accompanying negative impact on output due to the reduction in aggregate saving and hence capital formation. Meanwhile, when $\beta > \underline{\beta}$, the positive impact on output due to the expansion in the flow of supply of productive public services is instead accompanied by a further positive impact on output due to the increase in aggregate saving and hence capital formation.

Conversely, when the expected punishment is strictly lower than the tax rate, $\varepsilon\gamma(x^*) < \tau$, we show in Appendix D that a rise in the proportion of non-compliant workers leads to a fall in the per capita capital stock and the per capita output in the long-run equilibrium unless the accompanying response of the penalty rate to that same rise, denoted by $\eta_{\gamma x^*}$ is strictly greater than a threshold represented by $\overline{\eta}_\gamma$. When such a response is strictly lower than a lower threshold, $\eta_{\gamma x^*} < \underline{\eta}_\gamma$, with $\underline{\eta}_\gamma < \overline{\eta}_\gamma$, the rise in fines revenue brought about by an increase in the proportion of tax-evading workers is insufficient to offset the accompanying fall in taxes revenue, with the resulting fall in the supply of productive public services ($\eta_{\phi x^*} < 0$) exerting a downward pressure on output. Given that the resulting net effect on saving (and hence capital) formation is negative, the long-run equilibrium values of the per capita capital stock and the per capita output end up both falling in response to an increase in the proportion of non-compliant workers, as reflected by $\eta_{\tilde{k}x^*} < 0$ in (33) and $\eta_{\tilde{y}x^*} < 0$ in (32). Notice that the fall in the per capita capital stock in the long-run equilibrium occurs despite an upward pressure on aggregate savings coming from a composition effect. Recalling that a tax evader who is detected pays the ensuing fine out of her saving, this composition effect results from an increase in the proportion of tax-evading workers combined with a not too strong positive response of the penalty rate to that same increase yielding $\eta_{\varphi x^*} > 0$ (but which is not enough to offset, in (33), the fall in the supply of productive public services represented by $\eta_{\phi x^*} < 0$). Meanwhile, when $\underline{\eta}_\gamma < \eta_{\gamma x^*} < \overline{\eta}_\gamma$, it follows that $\eta_{\phi x^*} < 0$ and $\eta_{\varphi x^*} < 0$, so that the long-run equilibrium values of the per capita capital stock and the per capita output again vary negatively with the proportion of non-compliant workers. Lastly, a relatively stronger response of the penalty rate to the proportion of non-complaint workers, $\eta_{\gamma x^*} > \overline{\eta}_\gamma$, implies that $\eta_{\varphi x^*} < 0 < \eta_{\phi x^*}$. In this case, the determination of the signs of the elasticities $\eta_{\tilde{k}x^*}$ (in (33)) and $\eta_{\tilde{y}x^*}$ (in (32)) is subject to the same conditions established above for the case in which $\varepsilon\gamma(x^*) \geq \tau$. Thus, when the expected punishment is strictly lower than the tax rate, which confers a payoff advantage to the tax non-compliance strategy from the deterministic side, it takes a strong enough response of the penalty rate to an increase in the proportion of non-compliant workers to open up the possibility that the long-run equilibrium values of the per capita capital stock and the per capita output increase as well.

But recall from Proposition 1 that the value of the unique fixed point of the proportion of non-compliant workers, $x^* \in (0, 1) \subset \mathbb{R}$, is implicitly defined by $x^* = P\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x^*)\big)\right)$. Thus, the long-run equilibrium value of the proportion of non-compliant workers varies with the parameters of both components of the payoff function in (24), the deterministic and the random one. In the case of a change in the tax rate or in the probability of detection of a tax-evading worker, for example, the long-run equilibrium value of the per capita output is ultimately affected through two channels, as shown in Appendix E. The first channel is an indirect one operating through a change in the proportion of non-compliant workers. In accordance with the empirical evidence at large (including the experimental one), tax evasion (which here is equivalent to the proportion of non-compliant workers) varies positively with the tax rate and negatively with the probability of detection of a tax-evading worker (see, e.g., Alm et al., 2009; Alm, 2019). The second channel is a direct one operating through a change in both the supply of productive public services per worker in wage units in (17) and in the capital formation per worker in wage units in (23), given the proportion of non-compliant workers. Not surprisingly in light of the results of our analysis above of the effect of a given change in the proportion of non-compliant workers on the long-run equilibrium per capita output, the impact on the latter of a change in the tax rate or in the probability of detection of a tax evader is ambiguous. Yet such an ambiguity can be intuitively traced back to the operation of the several interactions and mechanisms at play. Therefore, we can precisely describe the economic substance of what it takes in terms of the relative strength of the several effects at play for a cut in the tax rate or a raise in the probability of detection of tax evasion to yield, for example, a higher per capita output in the long-run equilibrium.

4. Conclusion

Securing and maintaining an adequate tax revenue stream is crucial for ensuring a steady and satisfactory flow of provision of governmental services to society. Alongside with other public goods, this is especially true in regard to the provision of productive public services which serve as input to private production and are provided without user charges. However, there is overwhelming evidence that tax evasion is persistent, is of non-negligible magnitude and as a ratio of the aggregate output its magnitude varies over time.

Drawing upon the discrete choice approach, which allows incorporating both deterministic components and subjective motivations and proclivities such as tax morale as drivers of the compliance behavior on the part of taxpayers, the model set forth in this paper succeeds in replicating several pieces of evidence on tax evasion. Thus, the analytical framework developed in this paper provides a logically consistent description of mechanisms potentially underlying especially the persistence of tax non-compliance over time and the ambiguous impact of changes in the extent or degree of tax non-compliance on the level of per capita income over the long run.

Appendix A. Conditions for a strictly positive supply of productive public services

Let us assume that the following conditions hold:

$$\frac{\varepsilon\gamma(0)}{1+\varepsilon\gamma(0)} \geq \frac{\delta}{(1+\tau)(1-\delta)}, \tag{A.1}$$

$$\frac{\tau}{1+\tau} > \frac{\delta}{(1+\tau)(1-\delta)}. \tag{A.2}$$

Considering the properties of the penalty rate function in (15), we can infer that for all $x_t \in [0,1] \subset \mathbb{R}$ we have that:

$$\frac{\partial}{\partial x_t}\left(\frac{\varepsilon\gamma(x_t)}{1+\varepsilon\gamma(x_t)}\right) = \frac{\varepsilon\gamma'(x_t)}{[1+\varepsilon\gamma(x_t)]^2} > 0, \tag{A.3}$$

so that it is possible to establish the following inequality associated with the function $\phi(x_t)$ in (17) for any $x_t \in (0,1] \subset \mathbb{R}$:

$$\phi(x_t) > \frac{\tau}{1+\tau}(1-x_t) + \frac{\varepsilon\gamma(0)}{1+\varepsilon\gamma(0)}x_t - \frac{\delta}{(1+\tau)(1-\delta)}. \tag{A.4}$$

Note that the right-hand side (RHS) of the inequality in (A.4) varies linearly with respect to x_t.

Let us show that the conditions in (A.1) and (A.2), taken together, are sufficient to ensure that $\phi(x_t) > 0$ for all $x_t \in [0, 1] \subset \mathbb{R}$, regardless of the severity of the minimum expected punishment for tax evasion, given by $\varepsilon\gamma(0)$, as compared to the tax rate. If $\varepsilon\gamma(0) < \tau$, we have that $\frac{\varepsilon\gamma(0)}{1+\varepsilon\gamma(0)} < \frac{\tau}{1+\tau}$. In this case, the RHS in (A.4) reaches its minimum at $x_t = 1$, given by $\frac{\varepsilon\gamma(0)}{1+\varepsilon\gamma(0)} - \frac{\delta}{(1+\tau)(1-\delta)}$, which, per (A.1), is positive. This implies that $\phi(x_t) > 0$ for all $x_t \in (0, 1] \subset \mathbb{R}$, given (A.4). In particular, when $\varepsilon\gamma(0) = \tau$, it follows that $\frac{\varepsilon\gamma(0)}{1+\varepsilon\gamma(0)} = \frac{\tau}{1+\tau}$, so that the RHS in (A.4) is a constant function of x_t and its value is $\frac{\tau}{1+\tau} - \frac{\delta}{(1+\tau)(1-\delta)}$, which is strictly positive considering (A.2). Therefore, based on the condition in (A.4), we can infer that $\phi(x_t) > 0$ for all $x_t \in (0, 1] \subset \mathbb{R}$. Meanwhile, if $\varepsilon\gamma(0) > \tau$, we have that $\frac{\varepsilon\gamma(0)}{1+\varepsilon\gamma(0)} > \frac{\tau}{1+\tau}$. In this case, the RHS in (A.4) reaches its minimum at $x_t = 0$, given by $\frac{\tau}{1+\tau} - \frac{\delta}{(1+\tau)(1-\delta)}$, which, per (A.2), is strictly positive. This also ensures that $\phi(x_t) > 0$ for all $x_t \in (0, 1] \subset \mathbb{R}$, given (A.4).

Finally, at $x_t = 0$ the RHS in (A.4) takes the value represented by $\frac{\tau}{1+\tau} - \frac{\delta}{(1+\tau)(1-\delta)}$, which, per (A.2), is strictly positive. As a result, considering the inequality in (A.4), we have that $\phi(0) > 0$. This completes the demonstration that the conditions in (A.1) and (A.2), taken together, are sufficient to ensure that, in any period t, for a given $w_t^* > 0$, and for any fraction of noncompliant workers, the supply of productive public services per worker only takes strictly positive values.

Appendix B. Existence and uniqueness of a long-run equilibrium with heterogeneity in tax compliance strategies across workers

A long-run equilibrium solution (k^*, x^*) of the map represented by (21) and (29) has to satisfy the following system of equations:

$$\begin{cases} k^* = \varphi(x^*) w(k^*, x^*), & \text{(B.1)} \\ x^* = P\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x^*)\big)\right), & \text{(B.2)} \end{cases}$$

where $P : \mathbb{R} \to (0, 1) \subset \mathbb{R}$ is the cumulative distribution function in (28), assumed to be continuously differentiable with $P'(\cdot) > 0$ over its support.

Given (17) and (22)-(23), we can use (B.1) to express k^* as a function of x^* and get:

$$k^* = \left[\big(1 - (\alpha + \beta)\big)\big(\varphi(x^*)\big)^{1-\beta}\big(\phi(x^*)\big)^{\beta}\right]^{\frac{1}{1-(\alpha+\beta)}}. \tag{B.3}$$

Therefore, in order to demonstrate both the existence and the uniqueness of the long-run equilibrium solution (k^*, x^*) we need to show that there is a unique $x^* \in [0,1] \subset \mathbb{R}$ which solves (B.2). Specifying $\psi(x_t) = x_t - P\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x^*)\big)\right)$, the condition in (B.1) is satisfied if, and only if, $\psi(x^*) = 0$.

Considering that, by assumption, $0 < P(\cdot) < 1$ over its support, it is straightforward to see that $0 < P\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x^*)\big)\right) < 1$ for all $x \in [0,1] \subset \mathbb{R}$. Thus, we can establish that:

$$\psi(0) = -P\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(0)\big)\right) < 0 \quad \text{and} \quad \psi(1) = 1 - P\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(1)\big)\right) > 0. \tag{B.4}$$

We can demonstrate that there is a unique $x^* \in (0,1) \subset \mathbb{R}$ such that $\psi(x^*) = 0$. Considering the inequalities in (B.4) and that the function $\psi(x_t)$ in (15) is continuous over the domain represented by $[0,1] \subset \mathbb{R}$, we can apply the intermediate value theorem to conclude that there is some $x^* \in (0,1) \subset \mathbb{R}$ such that $\psi(x^*) = 0$.

Moreover, since $P'(\cdot) > 0$ over its support and the function $\psi(x_t)$ in (15) is continuously differentiable over the domain given by $[0,1] \subset \mathbb{R}$, it follows that:

$$\psi'(x) = 1 + \varepsilon P'\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x_t)\big)\right) \upsilon'\big(\varepsilon\gamma(x_t)\big)\gamma'(x_t) > 0, \tag{B.5}$$

for all $x \in [0,1] \subset \mathbb{R}$. As a result, given that the function in (B.5) is continuous in the closed interval $[0,1] \subset \mathbb{R}$, there is only one $x^* \in (0,1) \subset \mathbb{R}$ such that $\psi(x^*) = 0$.

Appendix C. Stability properties of the long-run equilibrium with heterogeneity in tax compliance strategies across the economy

The Jacobian matrix evaluated around the long-run equilibrium (k^*, x^*) is given by:

$$J(k^*, x^*) = \left[\begin{array}{c|c} \frac{\alpha}{1-\beta} & \varphi'(x^*)w(k^*, x^*) + \varphi(x^*)\frac{\partial w(k^*,x^*)}{\partial x_t} \\ \hline 0 & -\varepsilon P'\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x^*)\big)\right) \upsilon'\big(\varepsilon\gamma(x^*)\big)\gamma'(x^*) \end{array}\right] \tag{C.1}$$

Let ξ be an eigenvalue of the Jacobian matrix in (C.1). We can set the characteristic equation of the linearization around the long-run equilibrium as follows:

$$\begin{vmatrix} a - \xi & \varphi'(x^*)w(k^*, x^*) + \varphi(x^*)\frac{\partial w(k^*,x^*)}{\partial x_t} \\ 0 & c - \xi \end{vmatrix} = \xi^2 - (a + c)\xi + ac = 0, \tag{C.2}$$

where $a \equiv \frac{\alpha}{1-\beta}$ and $c \equiv -\varepsilon P'\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x^*)\big)\right)\upsilon'\big(\varepsilon\gamma(x^*)\big)\gamma'(x^*)$.

The second-order characteristic equation in (C.2) has two eigenvalues inside the unit circle if, and only if, the following conditions are satisfied (Medio and Lines, 2001, p. 52):

$$1 + \mathrm{tr}J(k^*, x^*) + \det J(k^*, x^*) = \left(1 + \frac{\alpha}{1-\beta}\right)\left[1 - \varepsilon P'\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x^*)\big)\right)\upsilon'\big(\varepsilon\gamma(x^*)\big)\gamma'(x^*)\right] > 0, \tag{C.3}$$

$$1 - \mathrm{tr}J(k^*, x^*) + \det J(k^*, x^*) = \left(\frac{1-(\alpha+\beta)}{1-\beta}\right)\left[1 + \varepsilon P'\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x^*)\big)\right)\upsilon'\big(\varepsilon\gamma(x^*)\big)\gamma'(x^*)\right] > 0, \tag{C.4}$$

$$1 - \det J(k^*, x^*) = 1 + \left(\frac{\alpha}{1-\beta}\right)\left[\varepsilon P'\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x^*)\big)\right)\upsilon'\big(\varepsilon\gamma(x^*)\big)\gamma'(x^*)\right] > 0 \tag{C.5}$$

Since $\alpha \in (0,1) \subset \mathbb{R}$ and $\beta \in (0,1) \subset \mathbb{R}$, we know that $1 + \frac{\alpha}{1-\beta} > 0$, so that the condition in (C.3) holds if:

$$\varepsilon P'\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x^*)\big)\right)\upsilon'\big(\varepsilon\gamma(x^*)\big)\gamma'(x^*) < 1 \tag{C.6}$$

Under the assumptions that $\alpha \in (0,1) \subset \mathbb{R}$ and $\beta \in (0,1) \subset \mathbb{R}$ and further assuming that $(\alpha+\beta) \in (0,1) \subset \mathbb{R}$, we can readily infer that $0 < \frac{\alpha}{1-\beta} < 1$ and $\frac{1-(\alpha+\beta)}{1-\beta} > 0$. Considering this latter inequality, we know that the condition in (C.4) is satisfied, since $1+\varepsilon P'\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x^*)\big)\right)\upsilon'\big(\varepsilon\gamma(x^*)\big)\gamma'(x^*) > 0$. Finally, as $0 < \frac{\alpha}{1-\beta} < 1$ and $1 + \varepsilon P'\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x^*)\big)\right)\upsilon'\big(\varepsilon\gamma(x^*)\big)\gamma'(x^*) > 0$, the condition in (C.5) is also satisfied. This completes the demonstration that the long-run equilibrium (k^*, x^*) is a local attractor if the condition in (C.6) holds.

As just depicted, the stability conditions in (C.4) and (C.5) are both satisfied irrespective of whether or not the condition in (C.6) is satisfied. Therefore, only the stability condition in (C.3) can be violated. In fact, when $\varepsilon P'\left(\upsilon(\tau) - \upsilon\big(\varepsilon\gamma(x^*)\big)\right)\upsilon'\big(\varepsilon\gamma(x^*)\big)\gamma'(x^*) = 1$, we have a violation of the condition in (C.3), which results in $1+\mathrm{tr}J(k^*, x^*)+\det J(k^*, x^*) = 0$, where $\mathrm{tr}J(k^*, x^*) \in (-1, 0) \subset \mathbb{R}$ and $\det J(k^*, x^*) \in (-1, 0) \subset \mathbb{R}$. In this case, one of the eigenvalues of the Jacobian matrix in (C.1) is equal to -1, which then yields a flip (or period-doubling) bifurcation, as demonstrated in (Medio and Lines, 2001, p. 160).

Appendix D. Effect of a change in the proportion of non-compliant workers on the long-run equilibrium value of the per capita output

Using (17) and (23), we can compute the following elasticities associated with the long-run equilibrium:

$$\eta_{\phi x^*} = \frac{x^*}{\phi(x^*)}\left[-\frac{\tau}{1+\tau} + \frac{\varepsilon\gamma(x^*)}{1+\varepsilon\gamma(x^*)}\left(\frac{\eta_{\gamma,x^*}}{1+\varepsilon\gamma(x^*)}+1\right)\right], \tag{D.1}$$

$$\eta_{\varphi x^*} = \frac{x^*}{\varphi(x^*)}\left\{-\frac{1}{2+\theta}\left(\frac{1}{1+\tau} - \frac{1}{1+\varepsilon\gamma(x^*)}\right) - \frac{\varepsilon\gamma(x^*)}{1+\varepsilon\gamma(x^*)}\left[\frac{\eta_{\gamma,x^*}}{1+\varepsilon\gamma(x^*)}\left(1+\frac{1}{2+\theta}\right)+1\right]\right\}, \tag{D.2}$$

where $\eta_{\gamma x^*} > 0$ is the elasticity of the penalty rate with respect to the proportion of non-compliant workers in the long-run equilibrium, obtained from the penalty function in (15) and hence strictly positive.

We first claim that when $\varepsilon\gamma(x^*) \geq \tau$, an increase in the proportion of non-compliant workers results in the per capita capital stock and hence the per capita output in the long-run equilibrium decreasing (increasing) if the elasticity of the aggregate output with respect to the flow of supply of productive public services, given by β in (10), is lower (higher) than a certain threshold (which is different for each such macroeconomic variable, though).

It is straightforward from the elasticity expressions in (D.1) and (D.2) that $\varepsilon\gamma(x^*) \geq \tau$ implies that $\eta_{\varphi x^*} < 0 < \eta_{\phi x^*}$. Thus, in order to determine the signs of the elasticities in (32) and (33), let $\underline{\beta} = \frac{-\alpha\eta_{\varphi x^*}}{\eta_{\phi x^*}-\eta_{\varphi x^*}}$ and $\overline{\beta} = \frac{-\eta_{\varphi x^*}}{\eta_{\phi x^*}-\eta_{\varphi x^*}}$, and note that $0 < \underline{\beta} < \overline{\beta} < 1$, given that $\eta_{\varphi x^*} < 0 < \eta_{\phi x^*}$ and $\alpha \in (0,1) \subset \mathbb{R}$. Note as well that $\beta = \underline{\beta}$ is a necessary (and sufficient) condition for $\eta_{\tilde{y}x^*} = 0$ in (32), and $\beta = \overline{\beta}$ is a necessary (and sufficient) condition for $\eta_{\tilde{k}x^*} = 0$ in (33).

Consider that $\beta \in \left(0, \underline{\beta}\right] \subset \mathbb{R}$. If $\beta \neq \underline{\beta}$, it follows from (33) and (32) that $\eta_{\tilde{k}x^*} < 0$ and $\eta_{\tilde{y}x^*} < 0$, respectively. And if $\beta = \underline{\beta}$, we have that $\eta_{\tilde{k}x^*} < 0$ and $\eta_{\tilde{y}x^*} = 0$. Thus, with $\varepsilon\gamma(x^*) \geq \tau$, both the per capita capital stock and the per capita output vary negatively with the proportion of non-compliant workers in the long-run equilibrium when the elasticity of the aggregate output with respect to the supply of productive public services is strictly lower than the threshold given by $\underline{\beta}$. Now consider instead that $\beta \in \left(\underline{\beta}, \overline{\beta}\right] \subset \mathbb{R}$. If $\beta \neq \overline{\beta}$, it follows that $\eta_{\tilde{k}x^*} < 0 < \eta_{\tilde{y}x^*}$. Meanwhile, if $\beta = \overline{\beta}$, we have that $\eta_{\tilde{k}x^*} = 0$ and $\eta_{\tilde{y}x^*} > 0$. Finally, considering that $\beta \in \left(\overline{\beta}, 1\right) \subset \mathbb{R}$, it follows that $\eta_{\tilde{k}x^*} > 0$ and $\eta_{\tilde{y}x^*} > 0$. Thus, with $\varepsilon\gamma(x^*) \geq \tau$, the per capita capital stock varies negatively (positively) with the proportion of non-compliant workers in the long-run equilibrium when the elasticity of the aggregate output with respect to the supply of productive public services is strictly lower (greater) than the threshold represented by $\overline{\beta}$. Meanwhile, the per capita output varies negatively (positively) with the proportion of non-compliant workers in the long-run equilibrium when the elasticity of the aggregate output with respect to the supply of productive public services is strictly lower (greater) than the threshold represented by $\underline{\beta}$.

Our second claim is that when the expected punishment is strictly lower than the tax rate, $\varepsilon\gamma(x^*) < \tau$, the per capita capital stock and the per capita output fall in response to an increase in the proportion of non-compliant workers in the long-run equilibrium if the accompanying response of the penalty rate to that same increase, as specified in (15), is not sufficiently strong. To see that, let:

$$\overline{\eta}_\gamma = \frac{1+\varepsilon\gamma(x^*)}{1+\tau}\left(\frac{\tau}{\varepsilon\gamma(x^*)} - 1\right) \tag{D.3}$$

and

$$\underline{\eta}_\gamma = \frac{\overline{\eta}_\gamma}{3+\theta} - \frac{2+\theta}{3+\theta}\big(1+\varepsilon\gamma(x^*)\big), \tag{D.4}$$

and note that $0 < \underline{\eta}_\gamma < \overline{\eta}_\gamma$. Note as well, from (D.1), that $\eta_{\gamma x^*} = \overline{\eta}_\gamma$ is a necessary (and sufficient) condition for $\eta_{\phi x^*} = 0$ and, now from (D.2), that $\eta_{\gamma x^*} = \underline{\eta}_\gamma$ is a necessary (and sufficient) condition for $\eta_{\varphi x^*} = 0$. Moreover, the expression for $\eta_{\phi x^*}$ in (D.1) is monotonically increasing in $\eta_{\gamma x^*}$, while the expression for $\eta_{\varphi x^*}$ in (D.2) is monotonically decreasing in $\eta_{\gamma x^*}$. Thus, when the response of the penalty rate to a change in the proportion of tax-evading workers is relatively weaker, that is, $\eta_{\gamma x^*} < \underline{\eta}_\gamma$, it follows from (D.1) and (D.2) that $\eta_{\phi x^*} < 0 < \eta_{\varphi x^*}$. We will show that in this case, $\eta_{\tilde{k}x^*} < 0$. For the sake of contradiction, assume otherwise. From (33), it follows that $\beta \geq \frac{-\eta_{\varphi x^*}}{\eta_{\phi x^*} - \eta_{\varphi x^*}}$, but, given that $\beta \in (0,1) \subset \mathbb{R}$, this implies that $\frac{-\eta_{\varphi x^*}}{\eta_{\phi x^*} - \eta_{\varphi x^*}} \leq 1$, which leads to a contradiction, since $\eta_{\phi x^*} < 0 < \eta_{\varphi x^*}$. It is straightforward to infer from (32) that $\eta_{\phi x^*} < 0 < \eta_{\varphi x^*}$ along with $\eta_{\tilde{k}x^*} < 0$ implies that $\eta_{\tilde{y}x^*} < 0$.

Now consider the case of a relatively moderate response of the penalty rate to a change in the proportion of non-compliant workers, that is, $\underline{\eta}_\gamma < \eta_{\gamma x^*} < \overline{\eta}_\gamma$. From (D.3) and (D.4), it follows that $\eta_{\phi x^*} < 0$ and $\eta_{\varphi x^*} < 0$, which in turn, from (33), imply that $\eta_{\tilde{k}x^*} < 0$. Considering (32), we will show that $\eta_{\tilde{y}x^*} < 0$ as well. Suppose that $\eta_{\phi x^*} \leq \eta_{\varphi x^*} < 0$. It trivially follows from (32) that $\eta_{\tilde{y}x^*} < 0$. Now suppose, for the sake of contradiction, that $\eta_{\varphi x^*} < \eta_{\phi x^*} < 0$. Given (33), it must be the case that $\beta < \underline{\beta}$, but this contradicts our assumption, since $-\eta_{\varphi x^*} > 0$ and $\beta \in (0,1) \subset \mathbb{R}$. Therefore, if $\underline{\eta}_\gamma < \eta_{\gamma x^*} < \overline{\eta}_\gamma$, it follows that $\eta_{\phi x^*} \leq \eta_{\varphi x^*} < 0$, which results in both the per capita capital stock and the per capita output varying negatively with the proportion of non-compliant workers in the long-run equilibrium, so that we have $\eta_{\tilde{k}x^*} < 0$ in (33) and $\eta_{\tilde{y}x^*} < 0$ in (32).

Finally, consider the case of a relatively stronger response of the penalty rate to a change in the proportion of tax-evading workers, that is, $\eta_{\gamma x^*} > \underline{\eta}_\gamma$. From (D.3) and (D.4), it follows that $\eta_{\varphi x^*} < 0 < \eta_{\phi x^*}$. In this case, the signs of the elasticities $\eta_{\tilde{k}x^*}$ (in (33)) and $\eta_{\tilde{y}x^*}$ (in (32)) are determined according to the same conditions stated above in our first claim, in which we consider that $\varepsilon\gamma(x^*) \geq \tau$.

Appendix E. Effect of a change in the tax rate or in the probability of detection of tax evasion on the long-run equilibrium value of the per capita output

Let us consider the effect of a change in the tax rate, τ, on the long-run equilibrium value of the per capita output. Let $\eta_{z\tau}$ denote the elasticity of variable z with respect to τ. We can use (17) and (23) to compute the following elasticities associated with the long-run equilibrium:

$$\eta_{\phi\tau} = \frac{\tau}{\phi(x^*)(1+\tau)^2}\left(1 - x^* + \frac{\delta}{1-\delta}\right) > 0, \tag{E.1}$$

$$\eta_{\varphi\tau} = -\frac{\tau}{\varphi(x^*)(2+\theta)(1+\tau)^2}\left(1 - x^* + \frac{\delta}{1-\delta}\right) < 0. \tag{E.2}$$

It follows from (30) that:

$$\eta_{\tilde{y}\tau} = \frac{1}{1-(\alpha+\beta)}\left[\beta\left(\eta_{\phi x^*}\frac{\partial x^*}{\partial\tau} + \eta_{\phi\tau}\right) + (\alpha-\beta)\left(\eta_{\varphi x^*}\frac{\partial x^*}{\partial\tau} + \eta_{\varphi\tau}\right)\right]. \tag{E.3}$$

Recalling the elasticity of per capita output with respect to the proportion of non-compliant workers in (32), the expression for the elasticity measure in (E.3) can be rearranged to yield:

$$\eta_{\tilde{y}\tau} = \frac{\partial x^*}{\partial\tau}\eta_{\tilde{y}x^*} + \frac{1}{1-(\alpha+\beta)}[\beta\eta_{\phi\tau} + (\alpha-\beta)\eta_{\varphi\tau}]. \tag{E.4}$$

The expression in (E.4) shows that a change in the tax rate impacts on per capita output in the long-run equilibrium through an indirect channel (by changing the proportion of non-compliant workers in the long-run equilibrium), represented by the first term on the RHS of (E.4), and a direct channel, represented by the second term on the RHS of (E.4). Using (E.1) and (E.2), we find that the second term is positive (negative) if β is strictly greater (lower) than $\frac{-\eta_{\varphi\tau}}{\eta_{\phi\tau}-\eta_{\varphi\tau}}$. The sign of the first term is the same as the sign of $\eta_{\tilde{y}x^*}$, as we have from (29) that $\frac{\partial x^*}{\partial\tau} > 0$ for any $x^* \in (0,1) \subset \mathbb{R}$. Considering the conditions for $\eta_{\tilde{y}x^*} > 0$ derived in Appendix D, an increase in the tax rate has a positive indirect effect on per capita output in the long-run equilibrium if $\beta > \overline{\beta}$ and either $\varepsilon\gamma(x^*) \geq \tau$, or $\varepsilon\gamma(x^*) < \tau$ and $\eta_{\gamma x^*} > \overline{\eta}_\gamma$. Otherwise, it follows that $\eta_{\tilde{y}x^*} \leq 0$ and the indirect effect of an increase in the tax rate on per capita output is either negative or null.

Let us now consider the effect of an increase in the probability of detection of a tax-evading worker, ε, on the long-run equilibrium value of the per capita output. Let $\eta_{z\varepsilon}$ denote the elasticity of variable z with respect to ε. We can again use (17) and (23) to compute the following elasticities associated with the long-run equilibrium:

$$\eta_{\phi\varepsilon} = \frac{\varepsilon\gamma(x^*)x^*}{\phi(x^*)\left[1+\varepsilon\gamma(x^*)\right]^2} > 0, \tag{E.5}$$

$$\eta_{\varphi\varepsilon} = -\frac{\varepsilon x^*}{\varphi(x^*)\left[1+\varepsilon\gamma(x^*)\right]^2}\left[\frac{1}{2+\theta} - \gamma(x^*)\right] < 0. \tag{E.6}$$

It follows from (30) that:

$$\eta_{\tilde{y}\varepsilon} = \frac{1}{1-(\alpha+\beta)}\left[\beta\left(\eta_{\phi x^*}\frac{\partial x^*}{\partial\varepsilon} + \eta_{\phi\varepsilon}\right) + (\alpha-\beta)\left(\eta_{\varphi x^*}\frac{\partial x^*}{\partial\varepsilon} + \eta_{\varphi\varepsilon}\right)\right]. \tag{E.7}$$

Recalling again the elasticity of per capita output with respect to the proportion of non-compliant workers in (32), we can rearrange the expression for the elasticity measure in (E.7) to get:

$$\eta_{\tilde{y}\varepsilon} = \frac{\partial x^*}{\partial\varepsilon}\eta_{\tilde{y}x^*} + \frac{1}{1-(\alpha+\beta)}[\beta\eta_{\phi\varepsilon} + (\alpha-\beta)\eta_{\varphi\varepsilon}]. \tag{E.8}$$

The expression in (E.8) shows that a change in the probability of detection of a tax-evading worker impacts on per capita output in the long-run equilibrium through an indirect channel (by affecting the proportion of tax-evading workers in the long-run equilibrium), captured by the first term on the RHS of (E.4), and a direct channel, captured by the second term on the RHS of (E.4). The effect through the direct channel is positive (negative) if β is strictly greater (lower) than $\frac{-\eta_{\varphi\varepsilon}}{\eta_{\phi\varepsilon}-\eta_{\varphi\varepsilon}}$. As for the sign of the first term, which captures the effect operating through the indirect channel, we have from (29) that $\frac{\partial x^*}{\partial\varepsilon} < 0$ for any $x^* \in (0,1) \subset \mathbb{R}$. Considering the conditions for $\eta_{\tilde{y}x^*} > 0$ derived in Appendix D, an increase in the probability of detection has a negative indirect effect on per capita output in the long-run equilibrium if $\beta > \overline{\beta}$ and either $\varepsilon\gamma(x^*) \geq \tau$, or $\varepsilon\gamma(x^*) < \tau$ and $\eta_{\gamma x^*} > \overline{\eta}_{\gamma}$. Otherwise, it follows that $\eta_{\tilde{y}x^*} \leq 0$ and the indirect effect of an increase in the probability of detection on per capita output is either positive or null.

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