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Keywords: Heterogeneous expectations; evolutionary dynamics; satisficing behavior; reference dependence; output growth rate.

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PERSISTENT HETEROGENEITY IN WORKING HOUSEHOLDS' EXPECTATIONS REGARDING ECONOMIC CONDITIONS AND MACROECONOMIC DYNAMICS*

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Abstract

The paper develops a novel evolutionary microdynamic of expectation switching on the part of working households and embeds it in a macrodynamic of capacity utilization and output growth driven by aggregate demand. The evolutionary microdynamic is specified by drawing on two important contributions to behavioral economics: the approach to satisficing choice developed by Herbert Simon and the approach to choice centered on the notion of reference dependence advanced by Daniel Kahneman and Amos Tversky. An essential role is performed in the evolutionary protocol of expectation switching by the focal notion of satisficing reference point. Notably, the microdynamics of the frequency distribution of expectations about future economic conditions across working households and the macrodynamics of capacity utilization and output growth arise as co-evolutionary phenomena. The evolutionary protocol grounded in satisficing reference dependence offers a plausible explanation for two closely related empirical regularities: the persistence of heterogeneity in working households' expectations about future economic conditions, formally derived herein as a unique and asymptotically stable evolutionary equilibrium, and the procyclical role played by these expectations as determinants of the actual level of economic activity.

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1 Introduction

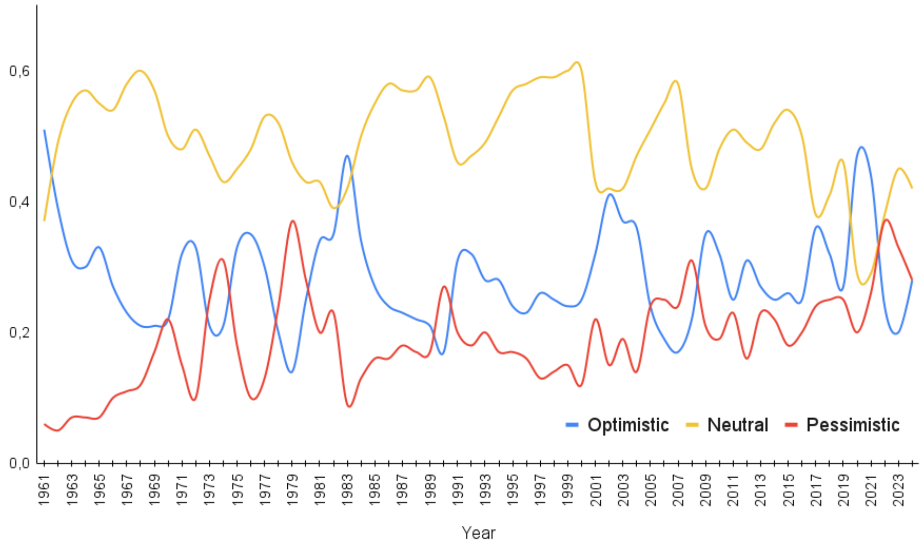
A robust finding emerging from a variety of survey data for the U.S. and European countries is the persistent and time-varying heterogeneity in expectations about future economic conditions across households. These household expectations are typically elicited and reported in a qualitative fashion that can be interpreted as ranging from pessimism to optimism (or close words), also including neutral households who instead expect economic conditions to remain about the same in the considered future (see the motivating Figure 1). Meanwhile, there is also extensive evidence from household micro data for the U.S. and European countries of considerable variation in the marginal propensity to consume across income levels and other observable and unobservable predictors.

This paper conceives as analytically plausible to interpret those two pieces of empirical evidence as causally interrelated in a novel evolutionary microdynamic of expectation switching by working households, which is then embedded in a demand-led macrodynamic of capacity utilization and output growth. Interestingly for our purposes, the main takeaway from the detailed study by Colarieti et al. (2024) is that survey responses can be reliable predictors of households' behaviors, while D'Acunto and Weber (2024) review an extensive literature showing that subjective expectations elicited via surveys contribute to explain heterogeneous consumption and saving choices. In fact, recent research has highlighted the advantages of using survey measures of expectations in macroeconomics (Manski, 2018) and also the important insights gained from using micro data to rebuild macroeconomic models to feature heterogenous behavior and more appropriate microfoundations (Vines and Wills, 2018).

Our novel evolutionary microdynamic driving the frequency distribution of expectations about future economic conditions across workers is specified by drawing on two important approaches to behavioral economics. The first is the approach to choice behavior centered on the notion of *satisficing* choice developed by Herbert Simon in several contributions (see, e.g., Simon, 1955, 1956, 1987), whereas the second is based on the notion of *reference dependence*, one of the fundamental principles of prospect theory advanced by Daniel Kahneman and Amos Tversky (Kahneman and Tversky, 1979). Combining elements of these two analytical approaches to behavioral economics, a key role is performed in our evolutionary protocol of expectation switching by the focal notion of *satisficing reference point*. Under plausible circumstances, our evolutionary protocol replicates the empirical evidence on the persistence and time-varying nature of heterogeneity in households' expectations about future economic conditions. In effect, there is considerable evidence on persistent heterogeneity in consumers' expectations about future economic conditions in both the U.S. and the European Union (Bissonnette and Van Soest, 2015; Curtin, 2019; Syed, 2021; Colarieti et al., 2024), and also on the time-varying nature of such heterogeneity (Curtin, 2019; Claus and Nguyen, 2020).

Meanwhile, there is also evidence based on U.S. and European micro and macro data showing that households' expectations about future economic conditions, by influencing their consumption behavior, are an important determinant of the actual level of economic activity, with an increase in pessimism (optimism) typically leading to a fall (increase) in the level and/or rate of growth of output. Against the backdrop of all such empirical evidence on household expectations and consumption behavior, the ambition of this paper is to suggest a further plausible candidate explanation for the interrelationship amongst such motivating pieces of evidence by

Figure 1: U.S. Michigan Survey of Consumers: expectations about future economic conditions (1961-2024)



Source: own elaboration (see footnote 2).

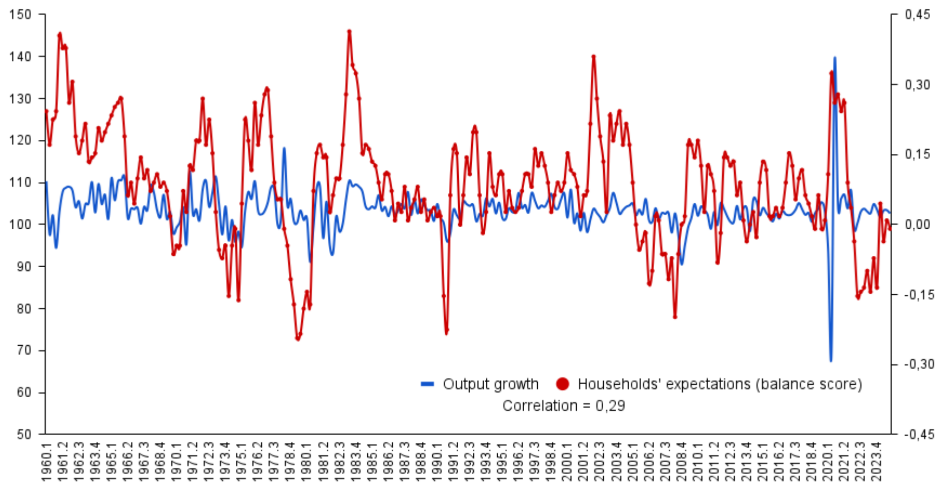
inevitably abstracting from several other potential sources of alternative explanations.

The evolutionary microdynamic centered on satisficing reference dependence which will be developed and explored herein implies that the microdynamics of the frequency distribution of expectations about future economic conditions across working households and the macrodynamics of capacity utilization and output growth arise as co-evolutionary phenomena. As an outcome of this co-evolutionary process, the key analytical result derived in this paper—the existence of a unique, asymptotically stable evolutionary equilibrium in which optimistic, pessimistic, and neutral expectations coexist—offers a plausible and behaviorally sound rationalization for the temporal pattern suggested by Figure 1, namely that the frequency distribution of these expectations fluctuates around a steady-state level.

According to Keynes (1936, 1937), people's propensity to consume is influenced by several factors such as the distribution of income and their expectations about an uncertain future. In effect, individuals may refrain from spending out of their incomes with a view to build up a reserve against unforeseen future contingencies. Several authors provide empirical evidence that there is considerable variation in the marginal propensity to consume across individual income levels, with the propensity to consume being higher for lower-income households (see, e.g., Jappelli and Pistaferri, 2010, 2014; Carroll et al., 2014; Carroll et al., 2017). Therefore, it seems reasonable and plausible to suppose that working households who are optimistic about the economic conditions in the near future have a higher (lower) propensity to consume (save) than households who are pessimistic about those conditions.

Using micro data for the U.S. and the Euro Area, Jang and Sacht (2021) find that consumers employ simple forecasting heuristics when forming expectations about future economic conditions. They also detect that there is a strong correlation between consumer confidence and household expenditure, with waves of optimism and pessimism playing an important role in the expectation formation process of consumers and their resulting spending decisions. Consumer spending can then be seen as a primary driving force behind fluctuations in the level of economic activity and the formation of expectations about future economic conditions is an essential and complex task inevitably undertaken by all consumers (Curtin, 2019). Using household survey data for the U.S., Barnes and Olivei (2017) find that consumers' expectations about future economic conditions have predictive power for future aggregate real consumption growth. Roth and Wohlfart (2020) employ a representative online panel from the U.S. and experimental methods to explore how individuals' macroeconomic expectations causally affect their personal economic prospects and their behavior. They find that respondents update their macroeconomic outlook in response to the professional forecasts to which they are exposed, extrapolate to expectations about their personal economic circumstances, and adjust their consumption plans in a heterogeneous manner. In effect, consumers' expectations can be seen as an important leading economic indicator, as suggested by the motivating Figure 2.

Figure 2: Expectations about future economic conditions and output growth in the U.S. (1960.1-2024.4)



Source: own elaboration with data from the U.S. Michigan Survey of Consumers and Bureau of Economic Analysis (see footnote 2).

Benhabib and Spiegel (2019) use survey expectations data for the U.S. concerning national output growth and future economic activity at the state level. The authors detect a statistically significant relationship between current consumers' expectations about future economic conditions and consumption expenditures and the level of economic activity in the next year, with more optimistic expectations leading to more consumption and a higher level of economic activity. Öztürk and Stokman (2019) use survey data for Europe and the U.S. to explore whether consumer expectations have an independent effect on household spending, with results suggesting that such expectations have a considerable impact on spending growth. Qualitatively similar results for the U.S. and/or the euro area are found in Souleles (2004), Aarle and Kappler (2012), and Dees and Brinca (2013).

We should emphasize that our paper is not intended to provide a plausible explanation specifically for the U.S. experience illustrated in Figure 1 and Figure 2 or a similar experience in some other country. Those figures are instead intended to motivate the importance of exploring the coupled interplay between time-varying heterogeneity in household expectations about future economic conditions and macroeconomic dynamics using an evolutionary framework. Yet as it is inevitably and justifiably the case in a formal modelling contribution, we abstract from several other determinants of macrodynamics of capacity utilization and output growth to focus and gain insight on the effects of time-varying heterogeneity in household expectations about future economic conditions operating through consumption demand, for which there is extensive empirical evidence. We also reasonably abstract from several other causal mechanisms underlying the endogenous switching of expectations about future economic conditions on the part of working households to focus and gain insight on the operation of our focal mechanism of satisficing reference dependence. Thus, our analytical strategy of focusing on and confining attention to only a few causal transmission channels and mechanisms underlying the co-evolution of expectation switching by workers and the macrodynamics of economic activity to derive definite and rationalizable results is analogous to some extent to the need to control for other covariates when empirically testing for causality.

The remainder of this paper is organized as follows. Section 2 describes the macroeconomic setting and solves for the short-run equilibrium, assuming a configuration in which the frequency distribution of workers' expectations about future economic conditions is predetermined. Section 3 outlines the protocol of expectation formation within an evolutionary dynamic framework based on the notion of satisficing reference dependence. Section 4 examines the persistence of heterogeneity in workers' expectations regarding future economic conditions and the resulting macrodynamics over the evolutionary long run. Section 5 concludes.

2 Macroeconomic setting and short-run equilibrium

The model economy is closed and features no government activities, producing only one good/service for both investment and consumption purposes. Output production is carried out by imperfectly-competitive firms that combine capital and labor through a fixed-coefficient technology:

$$X = min\left\{K\nu, \frac{L}{a}\right\}, \tag{1}$$

where X is the output level, K is the stock of capital, L is the employment level, $\nu \in \mathbb{R}$ is the full-capacity output to capital ratio, which is an exogenously fixed technical parameter, and $a \in \mathbb{R}$ is the labor to output ratio (or the inverse of labor productivity). Although workers may hold heterogeneous expectations about future economic conditions, the productivity of labor and the working day are homogeneous across workers and exogenously fixed constants. As the technical coefficient ν is constant, we measure the rate of capacity utilization, σ, by the proxy represented by the output to capital ratio, X/K.

The economy is composed of two social classes, firm-owner capitalists and workers, who earn profits and wages, respectively. The functional distribution of aggregate income is given by:

$$X = VL + R, \tag{2}$$

where V is the real wage and R is the level of aggregate profits.

Firms produce (and hire labor and utilize their installed capital stock) according to aggregate effective demand. As we model only the case featuring excess productive capacity (in labor and capital), the level of employment for a given value of the technical coefficient a is determined by output production:

$$L = aX. \tag{3}$$

Using (1)-(3), the share of profits in aggregate income, $\pi \in (0, 1) \subset \mathbb{R}$, is given by:

$$\pi = 1 - Va. \tag{4}$$

Firms sell their output production in oligopolistic markets. The price level of the single good, P, is set as a constant markup factor, $z \in (1, \infty) \subset \mathbb{R}$, over nominal unit labor costs measured as the nominal wage bill per unit of output, WL/X, where W is the nominal wage, an exogenously given constant. Hence the price level and the real wage, $V = W/P$, as well as the functional distribution of aggregate income, all remain constant. In the model developed in this paper, an individual firm is unable to perfectly observe what expectation about future economic conditions a given household member holds in her capacity as a worker or, for that matter, as a consumer. As a result, though working households have heterogeneous propensities to consume depending on their expectations about future economic conditions, with to more optimism corresponding higher propensity to consume, as detailed shortly, they all face the same price level when spending their wage income on consumption. Meanwhile, since firms are unable to perfectly observe whether an individual worker holds a pessimistic expectation about future economic conditions (and could possibly provide relatively more work intensity by having a higher expected cost of job loss) or an optimistic expectation (and hence could possibly deliver relatively less work intensity by having a lower expected cost of job), all workers are compensated with the same real wage. Yet the resulting heterogeneity in the expected cost of job loss across working households, despite translating into heterogeneity in their propensity to consume, does not translate into heterogeneity in their intensity of work, so that labor productivity (and hence the real unit labor cost and the corresponding markup) is uniform across firms.[1]

Workers' propensity to consume depends on their expectations about future economic conditions. There is a continuum of workers (labor is always in excess supply), but we group together three types of employed workers as regards their expectations about future economic conditions and respective consumption propensities. Yet there is heterogeneity in consumption propensities not only across types, but also within types, although within-type average consumption propensities remain constant. More precisely, $c_{w,\tau} \in (0, 1) \subset \mathbb{R}$ is the expected (average) propensity to consume of a worker of type $\tau \in T = \{o, n, p\}$, with o, n, and p standing for optimistic, neutral, and pessimistic, respectively. This taxonomy is based on (but not expected to be fully representative of) the U.S. Michigan Survey of Consumers, in which households are monthly asked (among other questions): "And how about a year from now, do you expect that in the country as a whole, business conditions will be better, or worse than they are at present, or just about the same?"[2] In the context of this paper, the current output growth rate and the worker-

[1] In Silveira and Lima (2021), the positive correlation between pessimistic unemployment expectations and actual unemployment consistently observed with household survey data can arise in a novel heterogeneous expectations-augmented short-run efficiency wage model featuring heterogeneity in labor effort on the job across workers through a composition effect. Yet that heterogeneity is parametric instead of endogenously time-varying.

[2] In addition to 'better', 'about the same' and 'worse', another possible answer is 'don't know', usually comprising a very small percentage of all the answers. On the basis of the distribution of all these answers, a measure dubbed balance score is calculated as the percentage of households who thought the economic conditions would be better minus the percentage who thought it would be worse, plus 100, thus varying between 0 and 200 (https://data.

idiosyncratic reference output growth rate perform a key role in the formation of expectations about future economic conditions in the country as whole on the part of workers, as detailed in Section 3.

In accordance with the robust empirical evidence from survey data on persistent heterogeneity in economic conditions expectations across workers reported earlier, where such expectations range cardinally from more optimistic to more pessimistic ones, we assume the following well-defined ordering for the expected consumption propensities of workers:

$$0 < c_{w,p} < c_{w,n} < c_{w,o} \leq 1. \tag{5}$$

Admittedly, no matter how pessimistic about the future economic conditions workers become, they simply cannot afford to save much of their wage income. In fact, it is quite reasonable to assume that even the most pessimistic workers will have a lower saving propensity than firm-owner capitalists, the saving behavior of whom is assumed to be homogeneous. Thus, we further assume that $c_c \in (0, c_{w,p}) \subset \mathbb{R}$ is capitalists' propensity to consume.

For concreteness, for a given propensity to consume of optimistic workers represented by $c_{w,o} \in (0, 1] \subset \mathbb{R}$, we assume the following specific form for the well-defined ordering for the consumption propensities out of wage income of neutral and pessimistic workers, respectively:

$$c_{w,n} = \alpha c_{w,o} \tag{6}$$

and

$$c_{w,p} = \alpha c_{w,n} = \alpha^2 c_{w,o}, \tag{7}$$

where $\alpha \in (0, 1) \subset \mathbb{R}$ is a parametric constant. Note that the specifications in (6) and (7) allow us to differentiate a deepening (intensive margin) from a widening (extensive margin) of the effects of workers' expectations about future economic conditions on their consumption behavior and thereby the level of economic activity in such an aggregate demand-led economy. In fact, given the frequency distribution of expectations about future economic conditions across workers, a fall (increase) in the proportionality parameter α is equivalent to an increase (fall) in workers' average pessimism (optimism) along the intensive margin.[3] Meanwhile, given the proportionality parameter α, a rise in the proportion of pessimistic workers is equivalent to an increase (fall) in workers' average pessimism (optimism) along the extensive margin.

The proportions of optimistic, neutral and pessimistic workers are denoted, respectively, by $\theta \in [0, 1] \subset \mathbb{R}$, $\eta \in [0, 1] \subset \mathbb{R}$, and $\rho \in [0, 1] \subset \mathbb{R}$, such that $\theta + \eta + \rho = 1$. The expected

sca.isr.umich.edu/data-archive/mine.php, Table 26). In Figure 1 and Figure 2, the proportion of each type of expectation and the balance score are calculated ignoring the category 'don't know' and using respectively annual and quarterly averages of the monthly data (but monthly data are made available only starting in 1978).

[3]Given the qualitative nature of the respective question posed in the U.S. Michigan and European Union surveys of future economic conditions expectations by households, as described in footnote 2, a more precise empirically based specification of such an effect along the intensive margin is not available, thus the simplified (but qualitatively representative) specification in (6) and (7). The main survey of economic conditions expectations in the European Union countries (The Joint Harmonised EU Programme of Business and Consumer Survey) monthly asks households how do they expect the general economic situation in the country to develop over the next 12 months. Answers include 'a lot better' (PP), 'a little better' (P), 'a little worse' (M), 'a lot worse' (MM), 'stay the same', and 'don't know'. The composite indicator for the EU is built from surveys conducted by national institutes in the Member States and the candidate countries (https://economy-finance.ec.europa.eu/system/files/2023-02/bcs_user_guide.pdf). The balance score measure for the EU survey is given by $(PP + 0.5P) - (0.5M + MM)$.

(average) propensity to consume out of wage income across types of working households can then be defined as $\overline{c}_w = \theta c_{w,o} + \eta c_{w,n} + \rho c_{w,p}$, which, considering that $\eta = 1 - \theta - \rho$, can be re-written as a function of the frequency distribution of working households' types as follows:

$$\overline{c}_w = c_{w,o}\Phi(\theta, \rho, \alpha), \tag{8}$$

where

$$\Phi(\theta, \rho, \alpha) \equiv \alpha + (1-\alpha)\theta - \alpha(1-\alpha)\rho. \tag{9}$$

From now on we will simplify matters by assuming that optimistic working households spend in consumption all of their wage income, that is, $c_{w,o} = 1$, so that $\overline{c}_w = \Phi(\theta, \rho, \alpha)$. Thus, it follows from (8) and (9) that for any $(\theta, \eta, \rho) \in \Delta \equiv \{(\theta, \eta, \rho) \in \mathbb{R}^3_+ : \theta + \eta + \rho = 1\}$, we have:

$$\frac{\partial \Phi}{\partial \theta} = 1 - \alpha > 0, \tag{10}$$

$$\frac{\partial \Phi}{\partial \rho} = -\alpha(1-\alpha) < 0, \tag{11}$$

and

$$\frac{\partial \Phi}{\partial \alpha} = 1 - \theta - \rho + 2\alpha\rho = \eta + 2\alpha\rho > 0. \tag{12}$$

Therefore, (10) and (11) show that a rise in workers' optimism (pessimism) with respect to future economic conditions along the extensive margin raises (lowers) workers' average propensity to consume, whereas (12) shows that a fall (rise) in workers' pessimism (optimism) with respect to future economic conditions along the intensive margin also raises (lowers) workers' average propensity to consume. In addition, given that (10) and (11) imply that (9) is a strictly increasing (decreasing) linear function of the proportion of optimistic (pessimistic) workers, we have that $\Phi(1, 0, \alpha) = 1$ is the maximum value and $\Phi(0, 1, \alpha) = \alpha^2 < 1$ is the minimum value of the function in (9), for a given α. Thus, we have that $\overline{c}_w = \Phi(\theta, \rho, \alpha) \in [\alpha^2, 1] \subset \mathbb{R}_{++}$ for any $(\theta, \eta, \rho) \in \Delta$.

As a result, recalling that $c_{w,o} = 1$, aggregate saving, which is composed of savings by both workers, S_w, and capitalists, S_c, is given by:

$$S = S_w + S_c = [1 - \Phi(\theta, \rho, \alpha)]VL + (1 - c_c)(X - VL), \tag{13}$$

where we have used (2) and (8), and where c_c denotes the propensity to consume out of profit income, assumed to be homogeneous across capitalists. Normalizing (13) by the capital stock, we have:

$$\begin{aligned} g^S = \frac{S}{K} &= [1 - \Phi(\theta, \rho, \alpha)]\frac{VL}{X}\frac{X}{K} + (1 - c_c)\left[\frac{(X - VL)}{X}\frac{X}{K}\right] \\ &= \{[1 - \Phi(\theta, \rho, \alpha)](1 - \pi) + (1 - c_c)\pi\}\sigma, \end{aligned} \tag{14}$$

where we have used (3) and recalled that $\sigma = X/K$ denotes the rate of capacity utilization.

Firms make capital accumulation plans described by the following desired investment function (expressed as a proportion of the capital stock):

$$g^I \equiv \frac{I}{K} = \gamma + \delta\sigma, \tag{15}$$

where $\gamma \in \mathbb{R}_{++}$ and $\delta \in \mathbb{R}_{++}$ are parametric constants.

We draw on Rowthorn (1982) and Dutt (1984) in making the desired rate of capital accumulation by firms to depend positively on the rate of capacity utilization due to accelerator-type effects. It follows that the specification in (15) implies that the functional distribution of income between wages and profits (which is anyway constant over time) impacts directly on aggregate effective demand only through the consumption demand by workers and capitalists. As explored later, an exogenous change in the functional distribution of income will nonetheless also impact indirectly on aggregate effective demand through the capacity utilization effect on firms' desired investment function in (15).

Reasonably, not all variables of the model vary at the same time, and some variables are taken to be predetermined at any given moment in time. Recalling that the real wage and the labor input (which are both homogeneous across workers) have already been assumed to remain constant throughout, we further assume that the capital stock, K, the labor supply, N, and the frequency distribution of expectations about future economic conditions in the population of workers, (θ, η, ρ), are all predetermined at any given moment in time. The assumed existence of excess productive capacity in capital and labor implies that aggregate output adjusts at any given moment in time to remove any existing excess aggregate demand for or supply of the single good/service produced in the economy. Thus, in the short-run equilibrium configuration, aggregate saving is always equal to aggregate investment. Using (14)-(15) to properly solve for the short-run equilibrium value of capacity utilization by means of the product market equilibrium condition given by $g^S = g^I$, we obtain:

$$\sigma^* = \frac{\gamma}{[1-\Phi(\theta,\rho,\alpha)](1-\pi) + (1-c_c)\pi - \delta} \equiv \sigma^*(\theta,\rho;\vec{\mu}), \tag{16}$$

where $\vec{\mu} = (\alpha, \gamma, \pi, c_c, \delta)$ is a vector of parametric constants.

Assuming for innocuous simplicity that the capital stock does not depreciate, the short-run equilibrium value of the output growth rate can be obtained by substituting (16) into (15):

$$g^* = \gamma\left[1 + \frac{\delta}{[1-\Phi(\theta,\rho,\alpha)](1-\pi) + (1-c_c)\pi - \delta}\right] \equiv g^*(\theta,\rho;\vec{\mu}). \tag{17}$$

We suppose that the short-run values in (16) and (17) are stable at any given moment in time by further assuming that the aggregate saving (as a proportion of the capital stock) in (14) is more responsive than the desired capital accumulation in (15) to changes in the rate of capacity utilization, which in turn requires that the denominator of the expression in (16) is strictly positive. This condition is the standard Keynesian stability condition typically assumed in aggregate demand-led models like the one set forth in this paper. In the present model, note that the satisfaction of this condition also ensures that the short-run equilibrium value of capacity utilization in (16) is strictly positive. Note also that the response of the aggregate saving in (14) to a change in capacity utilization crucially depends on the frequency distribution of expectations about future economic conditions in the population of workers, given that workers' average saving propensity depends on that frequency distribution. As we have assumed earlier that optimistic workers spend in consumption of all their wage income, it follows that for the Keynesian stability condition given by $[1-\Phi(\theta,\rho,\alpha)](1-\pi)+(1-c_c)\pi>\delta$ to be satisfied it is sufficient to assume that:

$$[1-\Phi(1,0,\alpha)](1-\pi)+(1-c_c)\pi=(1-c_c)\pi>\delta, \tag{18}$$

which corresponds to a situation in which all workers hold optimistic expectations with respect to future economic conditions (with the result that workers' average saving propensity is at its lower bound of zero) and only firm-owner capitalists save. As the Keynesian stability condition is valid for the maximum value of $\Phi(\theta,\rho,\alpha)$, which is $\Phi(1,0,\alpha)=1$, the same holds true for any distribution of workers' expectations $(\theta,\eta,\rho)\in\Delta$, given that $\Phi(\theta,\rho,\alpha)\in[\alpha^2,1]\subset\mathbb{R}_{++}$ for any $(\theta,\eta,\rho)\in\Delta$.

Regarding comparative statics effects, it can be easily algebraically verified (and intuitively explained by the aggregate demand-led nature of the model) that the short-run equilibrium values of the rates of capacity utilization and output growth in (16) and (17) both vary positively with the parameters of the desired investment function in (15). These short-run equilibrium values also intuitively vary positively with the average propensity to consume of workers in (8), recalling that we have assumed that optimistic workers spend in consumption of all their wage income, so that $c_{w,o}=1$:

$$\frac{\partial\sigma^*}{\partial\Phi}=\frac{\sigma^*(1-\pi)}{[1-\Phi(\theta,\rho,\alpha)](1-\pi)+(1-c_c)\pi-\delta}>0 \tag{19}$$

and

$$\frac{\partial g^*}{\partial\Phi}=\delta\frac{\sigma^*(1-\pi)}{[1-\Phi(\theta,\rho,\alpha)](1-\pi)+(1-c_c)\pi-\delta}=\delta\frac{\partial\sigma^*}{\partial\Phi}>0. \tag{20}$$

Meanwhile, the short-run equilibrium values of the rates of capital capacity utilization and output growth in (16) and (17) vary negatively with the profit share in income:

$$\frac{\partial\sigma^*}{\partial\pi}=\frac{[c_c-\Phi(\theta,\rho,\alpha)]\sigma^*}{[1-\Phi(\theta,\rho,\alpha)](1-\pi)+(1-c_c)\pi-\delta}<0$$

and

$$\frac{\partial g^*}{\partial\pi}=\delta\frac{\partial\sigma^*}{\partial\pi}<0. \tag{21}$$

Recall our assumption that even the most pessimistic workers cannot afford to have a higher saving propensity than firm-owner capitalists, so that $c_{w,p}>c_c$. Moreover, the average propensity to consume of workers when they are all pessimistic is given by $\Phi(0,1,\alpha)=\alpha^2<1$, and when they are all optimistic is given by $\Phi(1,0,\alpha)=1$. It follows that $0<c_c<\alpha^2<\Phi(\theta,\rho,\alpha)\leq 1$, so that the numerator (and hence the resulting comparative statics effect) in (21) is strictly negative. The substance of this result is that an exogenous redistribution of income from workers to capitalists reduces aggregate consumption and thereby aggregate demand even when the average propensity to consume of workers is at its lower bound represented by $\Phi(0,1,\alpha)=\alpha^2<1$, which corresponds to an extreme situation with all workers holding pessimistic expectations about future economic conditions. In addition, such functional redistribution of income in favor of capitalists is aggregate demand-reducing also by negatively impacting on firms' desired investment spending through the capacity utilization effect on their desired capital accumulation.

Recall that our specification of the ordering of the consumption propensities out of wage income in (6) and (7) interestingly allows us to differentiate a deepening (intensive margin) from a widening (extensive margin) of the effects of workers' expectations about future economic conditions on the level of economic activity. Intuitively, a rise in the proportion of optimistic workers is equivalent to an increase in workers' average optimism with respect to future economic conditions along the extensive margin, which raises the short-run equilibrium values of the rates of capacity utilization and output growth in (16) and (17) by raising workers' average propensity to consume. Recalling (10) and (20), we have:

$$\frac{\partial g^*}{\partial \theta} = \frac{\partial g^*}{\partial \Phi}\frac{\partial \Phi}{\partial \theta} = \frac{\partial g^*}{\partial \Phi}(1-\alpha) = \delta\frac{\partial \sigma *}{\partial \Phi}(1-\alpha) > 0. \tag{22}$$

Analogously, a rise in the proportion of pessimistic workers is equivalent to an increase in workers' average pessimism about future economic conditions along the extensive margin, which lowers the short-run equilibrium values of capacity utilization and output growth by reducing workers' average propensity to consume. Recalling (11) e (20), note that:

$$\frac{\partial g^*}{\partial \rho} = \frac{\partial g^*}{\partial \Phi}\frac{\partial \Phi}{\partial \rho} = -\frac{\partial g^*}{\partial \Phi}\alpha(1-\alpha) = -\delta\frac{\partial \sigma *}{\partial \Phi}\alpha(1-\alpha) < 0. \tag{23}$$

Meanwhile, a rise in the proportionality parameter α featuring in (6) and (7) is equivalent to a decrease (increase) in workers' average pessimism (optimism) with respect to future economic conditions along the intensive margin, which raises the short-run equilibrium values of the rates of capacity utilization and output growth by increasing workers' average propensity to consume. Recalling (12) and (20), note that:

$$\frac{\partial g^*}{\partial \alpha} = \frac{\partial g^*}{\partial \Phi}\frac{\partial \Phi}{\partial \alpha} = \delta\frac{\partial \sigma *}{\partial \Phi}(\eta + 2\alpha\rho) > 0. \tag{24}$$

3 Expectation formation in an evolutionary dynamic based on satisfacing reference dependence

In the transition to the evolutionary long run our earlier assumptions with respect to stability ensure that the short-run equilibrium values of the capacity utilization rate in (16) and the output growth rate in (17) are always attained. The dynamics of the economy are driven by changes in the labor supply, N, the aggregate stock of capital, K, and the frequency distribution of expectations about future economic conditions in the population of workers, (θ, η, ρ), which is a predetermined variable at any given moment in time previously referred to as short run. While the capital stock varies positively over time as determined by the aggregate demand-driven output growth rate in (17), we assume that the growth rate of labor supply is such that the existing labor surplus is continually replenished to an extent sufficing to avoid that labor ever becomes a constraint to capital accumulation and output growth. More specifically, we simplify matters by assuming that the labor supply grows endogenously at the same rate as capital accumulation, so that the capital to labor supply ratio, $k = K/N$, remains constant over time. In Harrodian parlance, the aggregate demand-determined warranted growth rate in (17) is unique, stable (per (18)) and equal to the natural growth rate (which is equal to the growth rate of the labor supply, as labor productivity, $1/a$, is constant) through the endogenous adjustment of the latter. Therefore, the dynamics of the economy are crucially driven by changes in the frequency distribution of expectations about future economic conditions across workers, as explored in what follows.[4]

Working households are assumed to form expectations about future economic conditions constrained by their inescapable uncertain knowledge about the future. As a result, working households revise (and possibly switch) their expectations about economic conditions for the relevant future under conditions of bounded rationality. In this context, heterogeneity in working households' views about economic conditions even in the proximate future cannot be considered to reflect ignorance or irrationality on the part of them, but should instead be seen as reflecting their reasonably different perceptions and beliefs with respect to an uncertain future.

We specify the evolutionary microdynamic driving the distribution of expectations about future economic conditions across workers by drawing on two approaches to behavioral economics. The first is the approach to choice behavior based on the notion of *satisficing* choice developed by Herbert Simon. According to Simon (1955, 1956), the choice behavior is inescapably subject to the limited information collecting and processing capacities of the individuals, who are therefore unable to optimize based on perfect knowledge. Individuals must instead rely on boundedly rational heuristics, rules-of-thumb, conventions, routines and other satisficing criteria and procedures as the reliable bases for their decision making. As a result, choice is a process of meeting a satisficing threshold instead of choosing the best of all existing alternatives within an optimization program. As Simon fittingly suggests, this contrast is analogous to 'looking for the sharpest needle in the haystack' (i.e., optimizing) versus 'looking for a needle sharp enough to sew with' (i.e., satisficing) (Simon, 1987, p. 244).[5] The other approach to behavioral economics on which we draw is based on the notion of *reference dependence*, which is one of the fundamental principles of prospect theory developed by Kahneman and Tversky (1979). In prospect theory, when deciding between alternatives that involve risk and uncertainty, individuals evaluate outcomes relative to a *reference point* instead of in absolute terms. The underlying intuition is that outcomes are not experienced by individuals on an absolute scale, but instead are experienced relative to some point of reference.[6] Combining elements of these two approaches to behavioral economics, a key role will be played in our evolutionary

[4] Note that a stationary value for the frequency distribution of expectations about future economic conditions across workers will imply stationary values not only for the capacity utilization rate in (16) and the output growth rate in (17), but for the employment rate as well. In fact, note that the employment rate is given by $L/N = (L/X)(X/K)(K/N) = a\sigma k$. Therefore, the rates of capacity utilization, σ, and employment always vary in the same direction in response to changes in parameters or predetermined variables.

[5] Caplin and Dean (2011) and Hey et al. (2017) provide experimental evidence on satisficing choice behavior as defined in Simon (1955, 1956), while Artinger et al. (2022) survey advances in satisficing behavior in economics, psychology, and management following Simon's pioneering contributions.

[6] Early evidence of such behavior come from laboratory experiments run by Kahneman and Tversky (1979). These experiments have been replicated and extended in several ways, in parallel with the development of an extensive theoretical literature seeking to model behavior based on reference dependence (see O'Donoghue and Sprenger, 2018, for a review). Recent evidence of reference dependence has been found in studies involving, for example, the daily labor supply of taxi drivers (Thakral and Tô, 2021), job search (DellaVigna et al., 2017), behavioral responses to taxation (Homonoff, 2018), and the timing of retirement (Seibold, 2021).

protocol of expectation formation and switching by the notion of *satisficing reference point*, a term we borrow from Schubert et al. (2018) who used it in a quite different context.

In a given point in time to which we have already referred to as short run, a worker i takes the publicly known output growth rate in (17) and compares it to her reference output growth rate, g_i^r. In the spirit of the evolutionary contributions of Simon (1955, 1956) on bounded rationality, the focal reference point represented by g_i^r is determined by boundedly rational heuristics, rules-of-thumb, conventions, routines and other satisficing criteria and procedures. The individual satisficing reference point represented by g_i^r depends, inter alia, on idiosyncratic features, and we assume that the set of satisficing reference points are randomly and independently determined across workers and over time. More precisely, we assume that the satisficing reference output growth rate g_i^r is a random variable with cumulative distribution function $H : \mathbb{R} \to [0, 1] \subset \mathbb{R}$ which is continuously differentiable and strictly increasing. Thus, the probability of randomly finding a worker for whom her satisficing reference output growth rate is lower than or equal to the current output growth rate in (17) is given by:

$$Pr(g_i^r \leq g^*) = H(g^*). \tag{25}$$

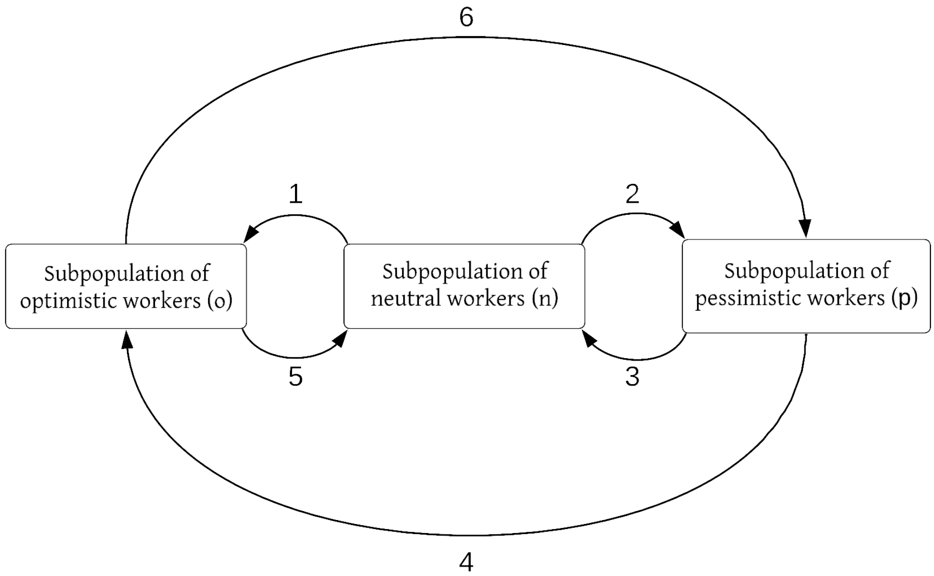
It follows that the probability that a randomly found worker i will consider that the current output growth rate in (17) is strictly lower than her satisficing reference output growth rate is simply:

$$Pr(g_i^r > g^*) = 1 - H(g^*). \tag{26}$$

As previously noted, the dynamic behavior of the economy is driven by changes in the frequency distribution of expectations about future economic conditions in the population of workers. The frequency distribution of optimistic, neutral and pessimistic workers is predetermined in a given short run, changing over time under the influence of the satisficing probabilities in (25) and (26). These probabilities will perform a key role in the transition flows between expectation types occurring at the end of a given short run, which will result in the frequency of optimistic, neutral and pessimistic workers featuring as predetermined variable in the next short run.[7] These several transition flows between expectation types are represented diagramatically in Figure 3 and algebraically in Table 1.

[7]Note that the output growth rate in (17) is strictly positive for any frequency distribution of expectation types in the population of workers, so that the employment level (which is measured by the number of employed workers) is always growing over time (recall that the productivity of labor and the working day are both constant). Therefore, there are always employed workers in a given short run who were not employed in the preceding short run. We assume that the satisficing probabilities in (25) and (26) also apply to such newly employed workers, and in this case are determined at the very beginning of a given short run using the output growth rate of the preceding short run.

Figure 3: Diagram of the flows between expectation types held by workers



Source: own elaboration.

Table 1: Formal expressions of the flows between expectation types held by workers

Identification	**Transition flows between expectation types held by workers**
1	$\eta H(g^*)$
2	$\eta[1 - H(g^*)]$
3	$\rho H(g^*)(1 - \varepsilon)$
4	$\rho H(g^*)\varepsilon$
5	$\theta[1 - H(g^*)](1 - \varepsilon)$
6	$\theta[1 - H(g^*)]\varepsilon$

Source: own elaboration.

Recall that the probability of randomly finding a worker whose satisficing reference output growth rate is less than or equal to the current output growth rate g^* is given by $H(g^*)$, so that the probability that a randomly found worker i will consider that the current output growth rate is strictly lower than her satisficing reference output growth rate is given by $1 - H(g^*)$. Thus, Flow 1 in Figure 3, the formal expression of which is given by $\eta H(g^*)$ in Table 1, indicates the mass of neutral workers inflowing into the optimistic type as a result of observing that the current output growth rate is equal to or greater than their satisficing reference output growth rate. Meanwhile, Flow 2, whose formal expression is given by $\eta[1 - H(g^*)]$, describes the mass of neutral workers inflowing into the pessimistic type due to observing that the current output growth rate is strictly lower than their satisficing reference output growth rate. It can be noted that adding together the masses of pessimistic workers represented by Flows 1 and 2 yields exactly the total outflow from the subpopulation of workers having neutral expectations about economic conditions in the near future, which is given by η.

Flows 3 and 4 indicate the mass of pessimistic workers whose satisficing reference output growth rate is less than or equal to the current output growth rate, which induces them to switch expectation about the future economic conditions. The probability with which a given such pessimistic worker will make an extreme switch of expectation type to now become optimistic is given by $\varepsilon \in [0, 1] \subset \mathbb{R}$, an exogenously fixed parameter, so that the probability of a moderate switch to now have neutral expectations is given by $1 - \varepsilon$. Therefore, Flow 3, whose formal expression is given by $\rho H(g^*)(1 - \varepsilon)$ in Table 1, indicates the mass of pessimistic workers who will switch to neutral expectations, while Flow 4, whose formal expression is given by $\rho H(g^*)\varepsilon$, describes the mass of pessimistic workers who will switch to optimistic expectations. Note that adding together the masses of pessimistic workers indicated by Flows 3 and 4 yields exactly the total outflow from the subpopulation of workers having pessimistic expectations about economic conditions in the near future, which is given by $\rho H(g^*)$.

Finally, Flows 5 and 6 indicate the mass of optimistic workers whose satisficing reference output growth rate is strictly greater than the current output growth rate, which then induces them to switch expectation about the future economic conditions. Analogously to the treatment given to the extreme (moderate) shift from pessimism to optimism (neutrality), the probability with which a given such optimistic worker will make an extreme switch of expectation type to become pessimistic is given by $\varepsilon \in [0, 1] \subset \mathbb{R}$, so that the probability of a moderate switch to have neutral expectations is given by $1 - \varepsilon$. Therefore, Flow 5, whose formal expression is given by $\theta[1 - H(g^*)](1 - \varepsilon)$ in Table 1, indicates the mass of optimistic workers who will turn to having neutral expectations, whereas Flow 6, whose formal expression is given by $\theta[1 - H(g^*)]\epsilon$, describes the mass of optimistic workers who will turn to having pessimistic expectations. Note that adding together the masses of optimistic workers indicated by Flows 5 and 6 yields exactly the total outflow from the subpopulation of workers with optimistic expectations about economic conditions in the near future, which is given by $\theta[1 - H(g^*)]$.

Notice that the case with $\varepsilon = 0$ corresponds to an expectation switching behavior according to which workers make only moderate switches of expectation about economic conditions in the near future, in that there are no expectation switches from pessimism to optimism and vice versa. In contrast, the polar opposite case featuring $\varepsilon = 1$ corresponds to an expectation switching behavior according to which workers make only extreme switches of expectation about economic conditions in the near future, switching from pessimism to optimism and vice versa. Meanwhile, the case featuring $\varepsilon \in (0, \frac{1}{2}) \subset \mathbb{R}$ refers to a situation in which pessimistic and optimistic workers make both moderate and extreme expectation switches, but the former expectation switch is made with higher probability than the latter. In contrast, the case featuring $\varepsilon \in \left(\frac{1}{2}, 1\right) \subset \mathbb{R}$ corresponds to a situation in which pessimistic and optimistic workers make both moderate and extreme expectation switches, but the extreme switches are made with higher probability than the moderate ones. Finally, the particular case with $\varepsilon = \frac{1}{2}$ corresponds to an expectation switching behavior according to which pessimistic and optimistic workers make both moderate and extreme expectation switches, with such expectation switches being equiprobable.[6]

The rate of change of the mass of optimistic workers, $\dot{\theta} = \frac{d\theta}{dt}$, is given by the net result of the inflow of workers to the optimistic type, which is the sum between Flows 1 and 4 in Table 1,

[6] A natural and interesting extension which we nonetheless leave for future research involves endogenising the switch propensity parameter ε. A necessary (and possibly sufficient) condition for implementing such an extension is a plausible and well-founded rationale, along with a tractable formal specification, for treating the parameter in question as endogenous.

and the outflow from such an expectation type, which is the sum between Flows 5 e 6:

$$\dot{\theta} = \eta H(g^*) + \rho H(g^*)\varepsilon - \theta[1 - H(g^*)]. \tag{27}$$

Analogously, the rate of change of the mass of pessimistic workers, $\dot{\rho} = \frac{d\rho}{dt}$, is given by the net result of the inflow of workers to the pessimistic type, which is the sum between Flows 2 and 6 in Table 1, and the outflow from such an expectation type, which in turn is the sum between Flows 3 and 4:

$$\dot{\rho} = \eta[1 - H(g^*)] + \theta[1 - H(g^*)]\varepsilon - \rho H(g^*). \tag{28}$$

The state transition of the frequency distribution of expectations in the population of workers, for a given output growth rate g^*, is driven by the dynamical system formed by the ordinary differential equations (27) and (28). It is important to emphasize that, for a given $\varepsilon \in (0, 1] \subset \mathbb{R}$, a given change in the current output growth rate in (17) results in a change in the same (opposite) direction of the proportion of optimistic (pessimistic) workers, as we are assuming that $H(g^*)$ is strictly increasing. However, recall the output growth rate in (17) varies positively with the proportion of optimistic workers (per (22)) and negatively with the proportion of pessimistic workers (per (23)). It follows that the microdynamics of the frequency distribution of expectations about future economic conditions in the population of workers and the macrodynamics of the output growth rate are co-evolutionarily coupled.

4 Persistence of heterogeneity in workers' expectations and macrodynamics in the evolutionary long run

The evolutionary dynamics based on satisficing reference dependence specified in (27) and (28), combined with the macroeconomic setting specified in Section 2, imply that the microdynamics of the frequency distribution of expectations about future economic conditions across workers and the macrodynamics of capacity utilization and output growth are co-evolutionary phenomena. Formally, we can use (17) and the definition $\eta = 1 - \theta - \rho$ to rewrite the dynamical system formed by (27) and (28) as follows:

$$\dot{\theta} = \phi_1(\theta, \rho) - \theta \tag{29}$$

and

$$\dot{\rho} = \phi_2(\theta, \rho) - \rho, \tag{30}$$

where $\phi_1(\theta, \rho) \equiv [1 - (1 - \varepsilon)\rho]H\big(g^*(\theta, \rho; \vec{\mu})\big)$ denotes the mass of workers transitioning from a non-optimistic into an optimistic expectation state, and $\phi_2(\theta, \rho) \equiv [1-(1-\varepsilon)\theta][1-H\big(g^*(\theta, \rho; \vec{\mu})\big)]$ denotes the mass of workers transitioning from a non-pessimistic into a pessimistic expectation state. Note that the state space of the dynamical system (29)-(30) is given by $\Theta = \{(\theta, \rho) \in \mathbb{R}^2_+ : (\theta, \eta, \rho) \in \Delta\}$, which is the projection of the unit simplex $\Delta \equiv \{(\theta, \eta, \rho) \in \mathbb{R}^3_+ : \theta + \eta + \rho = 1\}$. The properties of such coupled evolutionary dynamics are explored in this section, where it is formally demonstrated that the persistent heterogeneity in expectations about future economic conditions shown in Figure 1 emerges as an evolutionary equilibrium.

An evolutionary equilibrium solution arises when the condition represented by $\dot{\theta} = \dot{\rho} = 0$ is satisfied by the dynamical system formed by (29) and (30). In principle, such an evolutionary equilibrium may feature the survival in forming of only a single type of expectation, meaning that all the population of workers has either only optimistic or neutral or pessimistic expectations about future economic conditions, a configuration denoting a *monomorphic* evolutionary equilibrium. The dynamical system composed of (29) and (30) may also have an evolutionary equilibrium solution featuring the survival in forming of two types of expectation, a configuration characterizing a *partial polymorphic* evolutionary equilibrium. It is as well possible the existence of an evolutionary equilibrium configuration featuring the survival in forming of all three types of expectation, a solution characterizing a *full polymorphic* evolutionary equilibrium.

As demonstrated in Appendix A and formally summarized in Proposition 1 below, the dynamical system composed of (29) and (30) does not have an evolutionary equilibrium configuration featuring either one of the three possible types of monomorphic equilibrium ($\theta = 1$, $\eta = 1$, and $\rho = 1$) or one of the three possible types of partial polymorphic equilibrium ($\theta + \eta = 1$, $\eta + \rho = 1$, and $\theta + \rho = 1$). However, the dynamical system composed of (29) and (30) features a full polymorphic evolutionary equilibrium, $(\theta^*, \rho^*) \in \Theta$, with $\theta^* > 0$, $\rho^* > 0$ and $\theta^* + \rho^* < 1$.

Proposition 1 *For a given vector of parameters $\vec{\mu} = (\alpha, \gamma, \pi, c_c, \delta)$, with $\alpha \in (0, 1) \subset \mathbb{R}$, $\gamma \in \mathbb{R}_{++}$, $\pi \in (0, 1) \subset \mathbb{R}$, $c_c \in (0, \alpha^2) \subset \mathbb{R}_{++}$ and $\delta \in (0, (1 - c_c)\pi) \subset \mathbb{R}$, the evolutionary dynamics given by (29)-(30) have:*

(i) Neither a monomorphic evolutionary equilibrium nor a partial polymorphic evolutionary equilibrium for any $\varepsilon \in (0, 1] \subset \mathbb{R}$; and

(ii) A full polymorphic evolutionary equilibrium (θ^*, ρ^*) for any $\varepsilon \in (0, 1) \subset \mathbb{R}$.

Proof: See Appendix A.

As formally shown in Appendix B and synthesized in Proposition 2 below, a unique full polymorphic evolutionary equilibrium emerges if the following inequality is satisfied:

$$\frac{\partial \phi_1(\theta^*, \rho^*)}{\partial \theta} + \frac{\partial \phi_2(\theta^*, \rho^*)}{\partial \rho} + \frac{\partial \phi_1(\theta^*, \rho^*)}{\partial \rho}\frac{\partial \phi_2(\theta^*, \rho^*)}{\partial \theta} - \frac{\partial \phi_1(\theta^*, \rho^*)}{\partial \theta}\frac{\partial \phi_2(\theta^*, \rho^*)}{\partial \rho} < 1. \quad (31)$$

The term $\frac{\partial \phi_1(\theta^*, \rho^*)}{\partial \theta} = [1 - (1 - \varepsilon)\rho^*]H'\big(g^*(\cdot)\big)\frac{\partial g^*(\cdot)}{\partial \theta}$ in (31), featuring as (B.4) in Appendix B, captures the marginal effect of a given variation of the proportion of optimistic workers on the mass of workers transitioning from a non-optimistic into an optimistic expectation state. This term therefore represents an *indirect marginal effect of optimism on itself*, namely, an indirect marginal feedback effect of optimism operating through the transition protocol represented by $\phi_1(\theta^*, \rho^*)$ in (29). By contrast, the corresponding direct level effect of optimism on itself is given by the one-to-one variation in the existing proportion of optimistic workers induced by a given variation in that same proportion; namely, the compositional effect generated by the current proportion of workers already in the optimistic expectation state. The indirect marginal effect of optimism on itself is strictly positive, since $1 - (1 - \varepsilon)\rho^* > 0$, $H'\big(g^*(\cdot)\big) > 0$, and, per (22), $\frac{\partial g^*(\cdot)}{\partial \theta} > 0$. Analogously, the term $\frac{\partial \phi_2(\theta^*, \rho^*)}{\partial \rho} = -[1 - (1 - \varepsilon)\theta^*]H'\big(g^*(\cdot)\big)\frac{\partial g^*(\cdot)}{\partial \rho}$ in (31), featuring as (B.7) in Appendix B, reflects the marginal effect of a given variation of the proportion of pessimistic workers on the mass of workers transitioning from a non-pessimistic into a pessimistic expectation state. This term thus represents an *indirect marginal effect of* *pessimism on itself*, namely, an indirect marginal feedback effect of pessimism operating through the transition mechanism represented by $\phi_2(\theta^*, \rho^*)$ in (30). In contrast, the corresponding *direct level effect of pessimism on itself* is given by the one-to-one variation in the existing proportion of pessimistic workers; namely, the compositional effect generated by the current proportion of workers already in the pessimistic expectation state. The indirect marginal effect of pessimism on itself is strictly positive, since $1 - (1 - \varepsilon)\theta^* > 0$, $H'\big(g^*(\cdot)\big) > 0$, and, per (23), $\frac{\partial g^*(\cdot)}{\partial \rho} < 0$.

Meanwhile, the term $\frac{\partial \phi_1(\theta^*,\rho^*)}{\partial \rho}$ in (31) captures the marginal effect of the proportion of pessimistic workers on the mass of workers transitioning from a non-optimistic into an optimistic expectation state. As shown in (B.5) in Appendix B, this marginal effect is strictly negative. Finally, the term $\frac{\partial \phi_2(\theta^*,\rho^*)}{\partial \theta}$ in (31) reflects the marginal effect of the proportion of optimistic workers on the mass of workers transitioning from a non-pessimistic into a pessimistic expectation state. As shown in (B.6) in Appendix B, this marginal effect is strictly negative. Nonetheless, Appendix B further establishes that the difference between the last two positive terms on the left-hand side of (31), $\frac{\partial \phi_1(\theta^*,\rho^*)}{\partial \rho}\frac{\partial \phi_2(\theta^*,\rho^*)}{\partial \theta} - \frac{\partial \phi_1(\theta^*,\rho^*)}{\partial \theta}\frac{\partial \phi_2(\theta^*,\rho^*)}{\partial \rho}$, is strictly negative.

The following proposition establishes this result concerning the uniqueness the full polymorphic evolutionary equilibrium represented by $(\theta^*, \rho^*) \in \Theta$.

Proposition 2 *For a given vector of parameters $\vec{\mu} = (\alpha, \gamma, \pi, c_c, \delta)$, with $\alpha \in (0, 1) \subset \mathbb{R}$, $\gamma \in \mathbb{R}_{++}$, $\pi \in (0, 1) \subset \mathbb{R}$, $c_c \in (0, \alpha^2) \in \mathbb{R}_{++}$, and $\delta \in (0, (1 - c_c)\pi) \subset \mathbb{R}$, if the extreme-switch probability is given by $\varepsilon \in (0, 1) \subset \mathbb{R}$ and the condition (31) holds, the evolutionary dynamics in (29)-(30) features a unique equilibrium $(\theta^*, \rho^*) \in \Theta$, which is a full polymorphic evolutionary one, that is, an evolutionary equilibrium characterized by $\theta^* > 0$, $\rho^* > 0$ and $\theta^* + \rho^* < 1$.*

Proof: See Appendix B.

We now examine the stability properties of the satisficing reference dependence dynamics jointly defined by (29) and (30). The associated Jacobian matrix, denoted by $\mathbf{J}(\theta, \rho)$, is reported in (C.1) in Appendix C. As formally demonstrated therein, the uniqueness condition in (31) guarantees that the unique full polymorphic evolutionary equilibrium (θ^*, ρ^*) is locally asymptotically stable. To interpret the uniqueness condition in (31) as a stability condition, it is necessary to consider both the nature and the magnitude of the various effects at play. Let us begin by considering the indirect marginal effect of optimism on itself, given by $\frac{\partial \phi_1(\theta^*,\rho^*)}{\partial \theta}$, and the indirect marginal effect of pessimism on itself, given by $\frac{\partial \phi_2(\theta^*,\rho^*)}{\partial \rho}$. Both of these *indirect marginal effects*, due to their self-reinforcing nature, act as potentially destabilizing forces when considered in isolation, as reflected in the relative contribution of the first two terms on the left-hand side of the condition in (31). Specifically, starting from an evolutionary equilibrium, an increase in the proportion of optimists (pessimists) raises (lowers) the growth rate of output, which in turn induces a larger inflow of non-optimistic (non-pessimistic) workers transitioning into optimism (pessimism), thereby further amplifying optimism (pessimism) along the extensive margin. Nonetheless, the same increase in the proportion of optimists (pessimists) entails a strictly negative *direct level effect* on itself, thereby dampening optimism (pessimism) along the extensive margin. Moreover, relative to the other terms in the condition, the multiplicative interaction between these two indirect marginal effects—the last term on the left-hand side of the respective condition in (31), which is strictly positive—contributes positively to its satisfaction.

The strictly negative marginal effect of the proportion of pessimists on the mass of non-optimists transitioning into optimism, $\frac{\partial \phi_1(\theta^*,\rho^*)}{\partial \rho} < 0$, and the strictly negative marginal effect of the proportion of optimists on the mass of non-pessimists transitioning into pessimism, $\frac{\partial \phi_2(\theta^*,\rho^*)}{\partial \theta} < 0$, also act as potentially destabilizing forces when considered in isolation. Specifically, starting from an evolutionary equilibrium, an increase in the proportion of pessimists (optimists), accompanied, for example, by a proportional decrease in the proportion of optimists (pessimists), by implying a lower (higher) growth rate of output, leads to a decrease in the mass of non-optimists (non-pessimists) transitioning into optimism (pessimism). In fact, these potentially destabilizing forces reinforce each other. In the case of the further decrease in the proportion of optimists due to the decrease in the mass of non-optimists transitioning into optimism, the proportion of pessimists increases further, since $\frac{\partial \phi_2(\theta^*,\rho^*)}{\partial \theta} < 0$. Similarly, in the case of the further decrease in the proportion of pessimists due to the decrease in the mass of non-pessimists transitioning into pessimism, the mass of non-optimists transitioning into optimism increases, the proportion of optimists increases further, since $\frac{\partial \phi_1(\theta^*,\rho^*)}{\partial \rho} < 0$. Not surprisingly, therefore, relative to the other terms in the condition, the multiplicative interaction between these two cross marginal effects—the penultimate term on the left-hand side of the respective condition in (31), which is strictly positive—contributes negatively to its satisfaction.

The Jacobian matrix $\mathbf{J}(\theta, \rho)$ associated with the satisficing reference dependence dynamics defined by (29) and (30), as presented in (C.1) in Appendix C, incorporates all the direct, indirect, and cross effects discussed above. The diagonal elements represent the effects of each state variable (θ and ρ) on its own rate of change (both direct and indirect effects), while the off-diagonal elements capture the effects of each variable on the rate of change of the other variable (cross effects). Therefore, the uniqueness condition in (31) reflects the relative strengths of these effects, ensuring—as shown in Appendix C—that $\det \mathbf{J}(\theta^*, \rho^*) > 0$ and $\operatorname{tr} \mathbf{J}(\theta^*, \rho^*) < 0$, so that the unique full polymorphic equilibrium (θ^*, ρ^*) is locally asymptotically stable. Moreover, as also demonstrated in Appendix C, if this balance of forces holds for any frequency distribution of expectations about future economic conditions across working households, the full polymorphic evolutionary equilibrium is globally asymptotically stable.

We can summarize these possible local and global behaviors of the satisficing reference dependence dynamics described by (29) and (30), under the assumption that the full polymorphic evolutionary equilibrium $(\theta^*, \rho^*) \in \Theta$ is unique, as follows.

Proposition 3 *For a given parameter vector $\vec{\mu} = (\alpha, \gamma, \pi, c_c, \delta)$, with $\alpha \in (0,1) \subset \mathbb{R}$, $\gamma \in \mathbb{R}_{++}$, $\pi \in (0,1) \subset \mathbb{R}$, $c_c \in (0, \alpha^2) \in \mathbb{R}_{++}$, and $\delta \in (0, (1-c_c)\pi) \subset \mathbb{R}$, and for an extreme-switch probability $\varepsilon \in (0,1) \subset \mathbb{R}$, the evolutionary protocol in (29)-(30) exhibits the following dynamic properties:*

- *(i) If the uniqueness condition in (31) holds, then $\det \mathbf{J}(\theta^*, \rho^*) > 0$, implying the existence of a unique evolutionary equilibrium $(\theta^*, \rho^*) \in \Theta$, which is fully polymorphic ($\theta^* > 0$, $\rho^* > 0$, and $\theta^* + \rho^* < 1$) and locally asymptotically stable; and*
- *(ii) If $\frac{\partial \phi_1(\theta,\rho)}{\partial \theta} + \frac{\partial \phi_2(\theta,\rho)}{\partial \rho} < 1$ for all $(\theta, \rho) \in \Theta$, then there exists a unique evolutionary equilibrium (θ^*, ρ^*), which is fully polymorphic and globally asymptotically stable.*

Proof: See Appendix C.

To complement the analytical results on stability derived above, we present below the phase diagram resulting from a merely illustrative numerical simulation, computed using the *Mathematica* software. The exogenous variables and parameters are set according to the following plausible values: propensity to consume of optimistic working households, $c_{w,o} = 1$; propensity to consume of capitalists, $c_c = 0.6$; intensive margin parameter, $\alpha = 0.9$; profit share, $\pi = 0.4$; accelerator coefficient in the desired rate of capital accumulation, $\delta = 0.1$; autonomous component in the desired rate of capital accumulation, $\gamma = 0.05$; and probability of extreme switch of expectation state, $\varepsilon = 0.01$. Recall that the satisficing reference output growth rate g_i^r is a random variable with cumulative distribution function $H : \mathbb{R} \rightarrow [0, 1] \subset \mathbb{R}$ which is continuously differentiable and strictly increasing. Consequently, both transition probabilities in (25) and (26) depend on $H(g^*)$, where $g^* = g^*(\theta, \rho; \vec{\mu})$ denotes the (stable) short-run equilibrium value of the output growth rate defined in (17). For illustrative purposes only, we assume a cumulative distribution function $H(g^*)$ corresponding to an exponential distribution, $H\big(g^*(\theta, \rho; \vec{\mu})\big) = 1 - e^{-\lambda g^*(\theta,\rho;\vec{\mu})]}$, with rate parameter $\lambda = 8$. Figure 4 illustrates the unique full polymorphic evolutionary equilibrium (θ^*, ρ^*) identified in Proposition 2 as a global sink, where $(\theta^*, \rho^*) \cong (0.37, 0.30)$, implying $\eta^* \cong 0.33$, $\sigma^* \cong 0.44$ and $g^* \cong 0.09$.

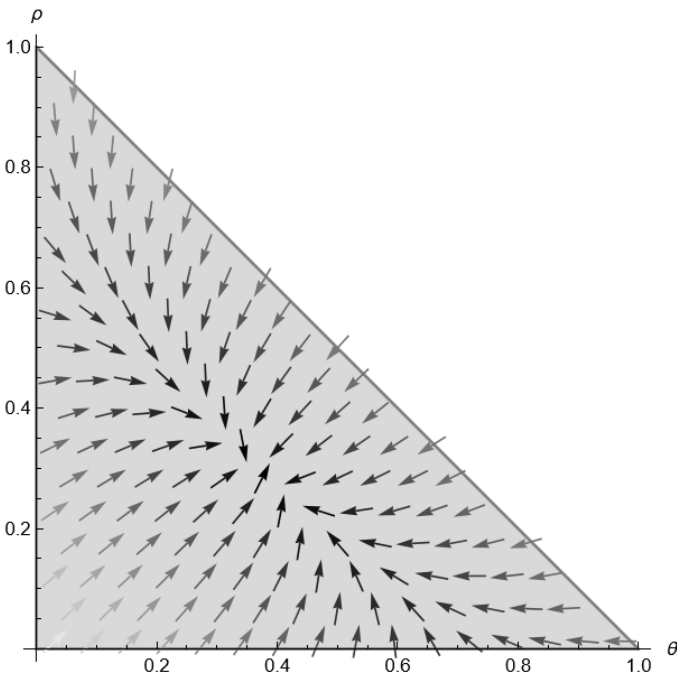


Figure 4: Unique full polymorphic evolutionary equilibrium (θ^*, ρ^*) as a global sink.

5 Conclusions

There is considerable survey evidence on persistent and time-varying heterogeneity in expectations about future economic conditions across working households. There is also extensive robust evidence using micro and macro data that such expectations, by influencing households' consumption behavior, are an important driver of the actual level of economic activity. Against this empirical backdrop, this paper develops a novel evolutionary microdynamic of expectation switching by working households, which is then embedded in a demand-led macrodynamic of capacity utilization and output growth.

Our novel evolutionary microdynamic governing the frequency distribution of expectations about future economic conditions across workers is specified by drawing on two analytical approaches to behavioral economics. One is the approach to satisficing choice developed by Herbert

Simon, while the other is the approach to choice based on the notion of reference dependence developed by Daniel Kahneman and Amos Tversky. Combining elements of these two approaches to behavioral economics, a central role is performed in our evolutionary protocol of expectation switching by the notion of satisficing reference point. Significantly, the microdynamics of the frequency distribution of expectations about future economic conditions in the population of workers and the macrodynamics of the output growth rate are co-evolutionarily coupled.

Our novel evolutionary protocol based on satisficing reference dependence offers a plausible qualitative explanation for two pieces of empirical evidence, with an emphasis on the underlying causal mechanisms: the persistence of heterogeneity in working households' expectations about future economic conditions and the procyclical role performed by such expectations as drivers of the actual level of economic activity.

Specifically, the key analytical result derived in this paper—the existence of a unique, asymptotically stable evolutionary equilibrium in which optimistic, pessimistic, and neutral expectations all coexist—provides a plausible and behaviorally sound rationalization for the temporal pattern suggested by Figure 1, namely that the frequency distribution of these expectations fluctuates around a steady-state level.

References

Aarle, B. and Kappler, M. (2012). Economic sentiment shocks and fluctuations in economic activity in the euro area and the USA. *Intereconomics: Review of European Economic Policy*, 47(1):44–51.

Artinger, F. M., Gigerenzer, G., and Jacobs, P. (2022). Satisficing: Integrating two traditions. *Journal of Economic Literature*, 60(2):598–635.

Barnes, M. L. and Olivei, G. P. (2017). Consumer attitudes and their forecasting power for consumer spending. *Journal of Money, Credit and Banking*, 49:1031–1058.

Benhabib, J. and Spiegel, M. M. (2019). Sentiments and economic activity: Evidence from US States. *The Economic Journal*, 129(618):715–733.

Bissonnette, L. and Van Soest, A. (2015). Heterogeneity in consumers' income and pension expectations. *Journal of Pension Economics and Finance*, 14:439–465.

Caplin, A. and Dean, M. (2011). Search, choice, and revealed preference. *Theoretical Economics*, 6(1):19–48.

Carroll, C. D., Slacalek, J., and Tokuoka, K. (2014). The distribution of wealth and the MPC: Implications of new european data. *The American Economic Review*, 104:107–111.

Carroll, C. D., Slacalek, J., White, M. N., and Tokuoka, K. (2017). The distribution of wealth and the marginal propensity to consume. *Quantitative Economics*, 8:977–1020.

Claus, E. and Nguyen, V. H. (2020). Monetary policy shocks from the consumer perspective. *Journal of Monetary Economics*, 114:159–173.

Colarieti, R., Mei, P., and Stantcheva, S. (2024). The how and why of household reactions to income shocks. Working Paper 32191, National Bureau of Economic Research.

Curtin, R. T. (2019). *Consumer Expectations: Micro Foundations and Macro Impact*. Cambridge University Press.

D'Acunto, F. and Weber, M. (2024). Why survey-based subjective expectations are meaningful and important. *Annual Review of Economics*, 16(1):329–357.

Dees, S. and Brinca, P. S. (2013). Consumer confidence as a predictor of consumption spending: Evidence for the United States and the Euro area. *International Economics*, 134:1–14.

DellaVigna, S., Lindner, A., Reizer, B., and Schmieder, J. F. (2017). Reference-dependent job search: Evidence from Hungary. *The Quarterly Journal of Economics*, 132(4):1969–2018.

Dutt, A. (1984). Stagnation, income distribution and monopoly power. *Cambridge Journal of Economics*, 8(1):25–40.

Hey, J. D., Permana, Y., and Rochanahastin, N. (2017). When and how to satisfice: An experimental investigation. *Theory and Decision*, 83(3):337–353.

Homonoff, T. A. (2018). Can small incentives have large effects? The impact of taxes versus bonuses on disposable bag use. *American Economic Journal: Economic Policy*, 10(4):177–210.

Jang, T.-S. and Sacht, S. (2021). Forecast heuristics, consumer expectations, and New-Keynesian macroeconomics: A Horse race. *Journal of Economic Behavior & Organization*, 182(C):493–511.

Jappelli, T. and Pistaferri, L. (2010). The consumption response to income changes. *The Annual Review of Economics*, 2:479–506.

Jappelli, T. and Pistaferri, L. (2014). Fiscal policy and MPC heterogeneity. *American Economic Journal: Macroeconomics*, 6:107–136.

Kahneman, D. and Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, 47(2):263–291.

Kehoe, T. (1998). Uniqueness and stability. In Kirman, A., editor, *Elements of General Equilibrium Analysis*. Oxford: Basil Blackwell.

Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*. Macmillan, London.

Keynes, J. M. (1937). The general theory of employment. *Quarterly Journal of Economics*, 51(2):209–223.

Manski, C. F. (2018). Survey measurement of probabilistic macroeconomic expectations: Progress and promise. *NBER Macroeconomics Annual*, 32:411–471.

Medio, A. and Lines, M. (2001). *Nonlinear Dynamics: A Primer*. Cambridge University Press, Cambridge, UK.

Olech, C. (1963). On the global stability of an autonomous system on the plane. *Contributions to differential equations*, 1(3):389–400.

Öztürk, B. and Stokman, A. (2019). Animal spirits and household spending in Europe and the US. *Economics Letters*, 185:108697.

O'Donoghue, T. and Sprenger, C. (2018). *Reference-dependent preferences. In Handbook of Behavioral Economics: Applications and Foundations*, volume 1. Elsevier.

Roth, C. and Wohlfart, J. (2020). How do expectations about the macroeconomy affect personal expectations and behavior? *The Review of Economics and Statistics*, 102(4):731–748.

Rowthorn, B. (1982). Demand, real wages and economic growth. *Studi Economic*, pages 3–54.

Schubert, T., Baier, E., and Rammer, C. (2018). Firm capabilities, technological dynamism and the internationalisation of innovation: A behavioural approach. *Journal of International Business Studies*, 49(1):70–95.

Seibold, A. (2021). Reference points for retirement behavior: Evidence from German pension discontinuities. *American Economic Review*, 111(4):1126–65.

Silveira, J. J. and Lima, G. T. (2021). Can workers' increased pessimism about the labor market conditions raise unemployment? *International Review of Economics & Finance*, 72:125–134.

Simon, H. A. (1955). A behavioral model of rational choice. *The Quarterly Journal of Economics*, 69(1):99–118.

Simon, H. A. (1956). Rational choice and the structure of the environment. *Psychological Review*, 63(2):129.

Simon, H. A. (1987). Satisficing. In Eatwell, J., Milgate, M., and Newman, P., editors, *The New Palgrave: A Dictionary of Economics*, volume 4. Stockton Press, New York.

Souleles, N. S. (2004). Expectations, heterogeneous forecast errors, and consumption: Micro evidence from the Michigan Consumer Sentiment Surveys. *Journal of Money, Credit and Banking*, (1):39–72.

Syed, S. A. S. (2021). Heterogeneous consumers in the euro-area, facing homogeneous monetary policy: Tale of two large economies. *The Journal of Economic Asymmetries*, 24.

Thakral, N. and Tô, L. T. (2021). Daily labor supply and adaptive reference points. *American Economic Review*, 111(8):2417–43.

Vines, D. and Wills, S. (2018). The rebuilding macroeconomic theory project: an analytical assessment. *Oxford Review of Economic Policy*, 34(1-2):1–42.

Appendix A Existence of evolutionary equilibrium

(i) Let us start by formally demonstrating that the dynamical system composed of (29) and (30) does not have either a monomorphic evolutionary equilibrium or a partial polymorphic evolutionary equilibrium.

- Inexistence of a monomorphic evolutionary equilibrium:
  - Assume that $\eta = 1$ and hence $(\theta, \rho) = (0, 0)$. We have by construction that $H(g^*) \in (0, 1) \subset \mathbb{R}$ for any $g^* > 0$. Besides, we know from (9) that $\Phi(0, 0, \alpha) = \alpha$ and hence from (17) that $g^*(0, 0; \vec{\mu}) = \gamma \left[1 + \frac{\delta}{(1-\alpha)(1-\pi)+(1-c_c)\pi-\delta}\right] > 0$. Thus, it follows from (29) that $\dot{\theta} = H(g^*(0, 0; \vec{\mu})) > 0$, with the result that there are neutral workers inflowing to the subpopulation of optimistic workers. In effect, there are also neutral workers inflowing to the subpopulation of pessimistic workers, as it follows from (30) that $\dot{\rho} = 1 - H(g^*(0, 0; \vec{\mu})) > 0$, considering that $H(g^*(0, 0; \vec{\mu})) < 1 \in (0, 1) \subset \mathbb{R}$. Hence the frequency distribution of expectations in the population of workers would tend to $int\Theta = \{(\theta, \rho) \in \mathbb{R}^2_+ : \theta > 0, \rho > 0, \theta + \rho < 1\}$.
  - Suppose that $\rho = 1$, so that $(\theta, \rho) = (0, 1)$. We know from (9) that $\Phi(0, 1, \alpha) = \alpha^2$ and hence from (17) that $g^*(0, 1; \vec{\mu}) = \gamma \left[1 + \frac{\delta}{(1-\alpha^2)(1-\pi)+(1-c_c)\pi-\delta}\right] > 0$, so that it follows from (29) that $\dot{\theta} = \varepsilon H(g^*(0, 1; \vec{\mu})) > 0$ for any $\varepsilon \in (0, 1] \subset \mathbb{R}$ and from (30) that $\dot{\rho} = -H(g^*(0, 1; \vec{\mu})) < 0$ for any $g^* > 0$. Hence the vector field determined by (29) and (30) would point to $int\Theta$.
  - Consider that $\theta = 1$, which means that $(\theta, \rho) = (1, 0)$. We know from (9) that $\Phi(1, 0, \alpha) = 1$ and hence from (17) that $g^*(1, 0; \vec{\mu}) = \gamma \left[1 + \frac{\delta}{(1-c_c)\pi-\delta}\right] > 0$. Thus, it follows from (29) that $\dot{\theta} = H(g^*(1, 0; \vec{\mu})) - 1 < 0$ and from (30) that $\dot{\rho} = \varepsilon \left[1 - H(g^*(1, 0; \vec{\mu}))\right] > 0$, provided that $\varepsilon > 0$, given that $g^*(1, 0; \vec{\mu}) > 0$ and $H(g^*(1, 0; \vec{\mu})) \in (0, 1) \subset \mathbb{R}$ for any $g^* > 0$. Therefore, the vector field determined by (29) and (30) would point to $int\Theta$.
- Inexistence of a partial polymorphic evolutionary equilibrium:
  - The subset of the state space given by $\Theta_1 \equiv \{(\theta, \rho) \in \mathbb{R}^2_+ : 0 < \theta < 1, \rho = 0\}$ is characterized by the coexistence only of workers with optimistic and neutral expectations about economic conditions in the near future. In this case, the dynamics of the subpopulations of optimistic and pessimistic workers are given respectively by $\dot{\theta} = H(g^*(\theta, 0; \vec{\mu})) - \theta$, the sign of which is ambiguous, and $\dot{\rho} = [1 - (1 - \varepsilon)\theta] \left[1 - H(g^*(\theta, 0; \vec{\mu}))\right] > 0$, given that $[1 - (1 - \varepsilon)\theta] > 0$ for any $\varepsilon \in (0, 1] \subset \mathbb{R}$, $\theta \in (0, 1) \subset \mathbb{R}$ and $H(g^*(\theta, 0; \vec{\mu})) < 1$, with $g^* > 0$. Considering that $\dot{\rho} > 0$, if the economy would happen to be in an initial situation located at any point in Θ_1, an inflow to the subpopulation of pessimistic workers would take place and the vector field determined by (29) and (30) would point to $int\Theta$.
  - In the subset of the state space given by $\Theta_2 \equiv \{(\theta, \rho) \in \mathbb{R}^2_+ : \theta = 0, 0 < \rho < 1\}$, meanwhile, there coexist exclusively workers holding either pessimistic or neutral expectations about future economic conditions. In this situation, the dynamics of the subpopulation of pessimistic workers is given by $\dot{\rho} = [1 - H(g^*(0, \rho; \vec{\mu}))] - \rho$, the sign of which is ambiguous, with $H(g^*(0, \rho; \vec{\mu})) \in (0, 1) \subset \mathbb{R}$. However, the dynamics of the subpopulation of optimistic workers is given by

$\dot{\theta} = [1-(1-\varepsilon)\theta]\, H(g^*(0,\rho;\vec{\mu})) > 0$, as $[1-(1-\varepsilon)\theta] > 0$ for any $\varepsilon \in (0,1] \subset \mathbb{R}$, $\theta \in (0,1) \subset \mathbb{R}$, and $H(g^*(0,\rho;\vec{\mu})) > 0$ for any $g^* > 0$. Considering that $\dot{\theta} > 0$, if the economy would happen to be located at any point in Θ_2, an inflow to the subpopulation of optimistic workers would take place and the vector field determined by (29) and (30) would point to $int\Theta$.

– It is also possible that the initial situation is located in the subset of the state space given by $\Theta_3 \equiv \{(\theta,\rho) \in \mathbb{R}^2_+ : 0 < \theta < 1, 0 < \rho < 1, \theta+\rho = 1\}$, which features the coexistence of only optimistic and pessimistic workers. It follows that the dynamics of the subpopulations of optimistic and pessimistic workers are represented by $\dot{\theta} = [1-(1-\varepsilon)(1-\theta)]\, H(g^*(\theta, 1-\theta;\vec{\mu})) - \theta$ and $\dot{\rho} = [1-(1-\varepsilon)\theta]\, [1 - H(g^*(\theta,1-\theta;\vec{\mu}))] - (1-\theta)$, respectively. When $\theta \to 0^+$, the inflow to the subpopulation of otimistic workers is given by $\dot{\theta} \to \varepsilon H(g^*(0,1;\vec{\mu})) > 0$, provided that $\varepsilon > 0$, while the inflow to the subpopulation of pessimistic workers is given by $\dot{\rho} \to -H(g^*(0,1;\vec{\mu})) < 0$. Note that the speed of the inflow to the subpopulation of pessimistic workers is higher than the speed of the inflow to the subpopulation of optimistic workers, that is, $|H(g^*(0,1;\vec{\mu}))| > |\varepsilon H(g^*(0,1;\vec{\mu}))|$ for any $\varepsilon \in (0,1] \subset \mathbb{R}$, so that the system remains in the $int\Theta$. When $\theta \to 1^-$, it follows that $\dot{\theta} \to H(g^*(1,0;\vec{\mu})) - 1 < 0$ and $\dot{\rho} \to \varepsilon\, [1 - H(g^*(1,0;\vec{\mu}))] > 0$ for any $\varepsilon > 0$, so that there is an outflow from the subpopulation of optimistic workers and an inflow to the subpopulation of pessimistic workers. Given that the speed of the outflow from the subpopulation of optimistic workers is higher than the speed of the inflow to the subpopulation of pessimistic workers, that is, $|H(g^*(1,0;\vec{\mu})) - 1| > |\varepsilon[1 - H(g^*(1,0;\vec{\mu}))]|$ for any $\varepsilon \in (0,1] \subset \mathbb{R}$, the vector field determined by (29) and (30) points to $int\Theta$.

(ii) Let us examine the conditions for the existence of a full polymorphic evolutionary equilibrium for the satisficing reference dependence dynamics described by (29)–(30). We will demonstrate that a full polymorphic evolutionary equilibrium exists; that is, a pair $(\theta^*, \rho^*) \in \Theta$ featuring $0 < \theta^* < 1$, $0 < \rho^* < 1$, and $0 < \theta^* + \rho^* < 1$, such that:

$$\phi_1(\theta^*,\rho^*) - \theta = [1-(1-\varepsilon)\rho^*] H\big(g^*(\theta^*,\rho^*;\vec{\mu})\big) - \theta^* = 0 \tag{A.1}$$

and

$$\phi_2(\theta^*,\rho^*) - \theta = [1-(1-\varepsilon)\theta^*]\, \big[1 - H\big(g^*(\theta^*,\rho^*;\vec{\mu})\big)\big] - \rho^* = 0. \tag{A.2}$$

By solving for $H\big(g^*(\cdot)\big)$ in the equilibrium conditions (A.1) and (A.2), we obtain:

$$H\big(g^*(\theta^*,\rho^*;\vec{\mu})\big) = \frac{\theta^*}{1-(1-\varepsilon)\rho^*} \tag{A.3}$$

and

$$H\big(g^*(\theta^*,\rho^*;\vec{\mu})\big) = 1 - \frac{\rho^*}{1-(1-\varepsilon)\theta^*}. \tag{A.4}$$

By setting the right-hand sides of (A.3) and (A.4) equal, we obtain:

$$\frac{\theta^*}{1-(1-\varepsilon)\rho^*} = 1 - \frac{\rho^*}{1-(1-\varepsilon)\theta^*}, \tag{A.5}$$

the solution of which is given by the following expression:

$$\rho^* = \frac{1}{2}\left[\frac{-2+\varepsilon+\theta^*-2\varepsilon\theta^*+\varepsilon^2\theta^*}{\varepsilon-1} - \sqrt{\frac{(4-3\theta^*)\theta^*+2\varepsilon^2(1-2\theta^*)\theta^*+\varepsilon^4\theta^{*2}+2\varepsilon\theta^*(2\theta^*-1)+\varepsilon^2(1-4\theta^*+2\theta^{*2})}{(\varepsilon-1)^2}}\right] \equiv h(\theta^*,\varepsilon). \tag{A.6}$$

Using (A.6), the right-hand side of the condition in (A.3) can be expressed solely as a function of θ^*, namely:

$$\frac{\theta^*}{1-(1-\varepsilon)h(\theta^*,\varepsilon)} = \frac{2\theta^*}{\varepsilon+\theta^*-2\varepsilon\theta^*+\varepsilon^2\theta^*-(\varepsilon-1)\sqrt{\frac{4\theta^*+(\varepsilon+(\varepsilon-1)^2\theta^*)(\varepsilon+(\varepsilon+3)(1+\varepsilon)\theta^*)}{(\varepsilon-1)^2}}} \equiv f(\theta^*,\varepsilon). \tag{A.7}$$

Note that the result in (A.7) can equivalently be obtained by substituting (A.6) into the right-hand side of the condition in (A.4).

We can now use (A.6) and (A.7) to reduce the equilibrium conditions in (A.3)–(A.4) to a single, consolidated condition, given by:

$$H\big(g^*(\theta^*,h(\theta^*,\varepsilon);\vec{\mu})\big) = \frac{\theta^*}{1-(1-\varepsilon)h(\theta^*,\varepsilon)} \equiv f(\theta^*,\varepsilon). \tag{A.8}$$

To demonstrate the existence of a full polymorphic evolutionary equilibrium, let us define the following function:

$$\varphi(\theta^*;\vec{\mu},\varepsilon) \equiv H\big(g^*(\theta^*,h(\theta^*,\varepsilon);\vec{\mu})\big) - f(\theta^*,\varepsilon). \tag{A.9}$$

Thus, if we show that there exists a $\theta^* \in (0,1) \subset \mathbb{R}$ such that $\varphi(\theta^*;\vec{\mu},\varepsilon) = 0$, thereby satisfying (A.8), it follows that a full polymorphic evolutionary equilibrium $(\theta^*,\rho^*) = (\theta^*,h(\theta^*,\varepsilon)) \in \Theta$ exists.

Indeed, given that $h(0,\varepsilon) = 1$ and $h(1,\varepsilon) = 0$, it follows that $f(0,\varepsilon) = 0$ and $f(1,\varepsilon) = 1$, so that from (A.9) we can infer:

$$\varphi(0;\vec{\mu},\varepsilon) \equiv H\big(g^*(0,1;\vec{\mu})\big) > 0 \tag{A.10}$$

and

$$\varphi(1;\vec{\mu},\varepsilon) \equiv H\big(g^*(1,0;\vec{\mu})\big) - 1 < 0. \tag{A.11}$$

Under the Keynesian stability condition assumed in Section 2, the output growth rate $g^*(\cdot)$ in (17) is well defined for any pair $(\theta,\rho) \in \Theta$ and for any parameter vector $\vec{\mu}$ satisfying the restrictions specified in Section 2, which implies that the function in (A.9) is continuous for all $\varepsilon \in [0,1) \subset \mathbb{R}$. Consequently, by applying inequalities (A.10) and (A.11) together with the intermediate value theorem, we deduce the existence of a $\theta^* \in (0,1) \subset \mathbb{R}$ such that $\varphi(\theta^*;\vec{\mu},\varepsilon) = 0$, thereby satisfying (A.8) and establishing the existence of at least one full polymorphic evolutionary equilibrium $(\theta^*,\rho^*) = \big(\theta^*,h(\theta^*,\varepsilon)\big) \in \Theta$.

Appendix B Uniqueness of the polymorphic evolutionary equilibrium

We now derive the condition that guarantees the uniqueness of the polymorphic evolutionary equilibrium. First, noting that $\eta = 1-(\theta+\rho)$, and based on the system defining an evolutionary equilibrium in (A.1)–(A.2) within the state space Θ, we can express the corresponding system in the simplex Δ as follows:

$$\begin{cases} \theta = [1-(1-\varepsilon)\rho]H\big(g^*(\theta,\rho;\vec{\mu})\big) \equiv \phi_1(\theta,\rho), \\ \rho = [1-(1-\varepsilon)\theta][1-H\big(g^*(\theta,\rho;\vec{\mu})\big)] \equiv \phi_2(\theta,\rho), \\ \eta = \{\rho H\big(g^*(\theta,\rho;\vec{\mu})\big) + \theta[1-H\big(g^*(\theta,\rho;\vec{\mu})\big)]\}(1-\varepsilon) \equiv \phi_3(\theta,\rho,\eta). \end{cases} \tag{B.1}$$

The system in (B.1) defines a vector function $\phi : \Delta \to \Delta$ represented by $\phi(\theta,\rho,\eta) = \big(\phi_1(\cdot),\phi_2(\cdot),\phi_3(\cdot)\big)$, and by construction, an evolutionary equilibrium $(\theta^*,\rho^*) \in \Theta$ corresponds to a fixed point of ϕ, that is, $(\theta^*,\rho^*,\eta^*) = \phi(\theta^*,\rho^*,\eta^*)$, where $\eta^* = 1-(\theta^*+\rho^*)$.

Suppose that the satisficing reference dependence evolutionary dynamics characterized by (A.1)–(A.2) feature only regular equilibria, that is, a finite number of equilibria that are locally unique. In this case, according to Kehoe (1998, p. 53), the Index Theorem provides the following sufficient condition for the uniqueness of the full polymorphic evolutionary equilibrium under consideration:

$$det[\mathbf{I} - \mathbf{D}\phi(\theta^*,\rho^*,\eta^*)] > 0, \tag{B.2}$$

where $\mathbf{I}$ denotes the identity matrix of order 3 and $\mathbf{D}\phi(\theta^*,\rho^*)$ denotes the Jacobian matrix of the function $\phi(\cdot)$.

Using the system in (B.1), the condition in (B.2) becomes:

$$det[\mathbf{I} - \mathbf{D}\phi(\theta^*,\rho^*,\eta^*)] = det\begin{bmatrix} 1-\frac{\partial\phi_1}{\partial\theta} & -\frac{\partial\phi_1}{\partial\rho} & 0 \\ -\frac{\partial\phi_2}{\partial\theta} & 1-\frac{\partial\phi_2}{\partial\rho} & 0 \\ -\frac{\partial\phi_3}{\partial\theta} & -\frac{\partial\phi_3}{\partial\rho} & 1 \end{bmatrix} = 1 - \frac{\partial\phi_1}{\partial\theta} - \frac{\partial\phi_2}{\partial\rho} + \frac{\partial\phi_1}{\partial\theta}\frac{\partial\phi_2}{\partial\rho} - \frac{\partial\phi_1}{\partial\rho}\frac{\partial\phi_2}{\partial\theta}. \tag{B.3}$$

Meanwhile, based on (B.1) and using the derivatives in (22)–(23), we obtain:

$$\frac{\partial\phi_1(\theta^*,\rho^*,\eta^*)}{\partial\theta} = [1-(1-\varepsilon)\rho^*]H'\big(g^*(\cdot)\big)\frac{\partial g^*(\cdot)}{\partial\theta} > 0, \tag{B.4}$$

$$\frac{\partial\phi_1(\theta^*,\rho^*,\eta^*)}{\partial\rho} = -(1-\varepsilon)H\big(g^*(\cdot)\big) + [1-(1-\varepsilon)\rho^*]H'\big(g^*(\cdot)\big)\frac{\partial g^*(\cdot)}{\partial\rho} < 0, \tag{B.5}$$

$$\frac{\partial\phi_2(\theta^*,\rho^*,\eta^*)}{\partial\theta} = -(1-\varepsilon)[1-H\big(g^*(\cdot)\big)] - [1-(1-\varepsilon)\theta^*]H'\big(g^*(\cdot)\big)\frac{\partial g^*(\cdot)}{\partial\theta} < 0, \tag{B.6}$$

and

$$\frac{\partial\phi_2(\theta^*,\rho^*,\eta^*)}{\partial\rho} = -[1-(1-\varepsilon)\theta^*]H'\big(g^*(\cdot)\big)\frac{\partial g^*(\cdot)}{\partial\rho} > 0. \tag{B.7}$$

By substituting (B.4)–(B.7) into (B.3) and imposing the restriction in (B.2) on the resulting expression, we obtain the condition guaranteeing the uniqueness of the full polymorphic evolutionary equilibrium in (31) in Section 4.

It is important to note that, beyond the signs of the derivatives in (B.4) and (B.5) appearing in (31), the sign of the expression $A \equiv \frac{\partial \phi_1(\theta^*,\rho^*)}{\partial \rho}\frac{\partial \phi_2(\theta^*,\rho^*)}{\partial \theta} - \frac{\partial \phi_1(\theta^*,\rho^*)}{\partial \theta}\frac{\partial \phi_2(\theta^*,\rho^*)}{\partial \rho}$ is also determined and is strictly negative. This can be verified by using the derivatives in (B.4)-(B.7) to simplify the expression A as follows:

$$A = -(1-\varepsilon)^2 H\big(g^*(\cdot)\big)[1 - H\big(g^*(\cdot)\big)] - (1-\varepsilon)[1-(1-\varepsilon)\theta^*]H'\big(g^*(\cdot)\big)H\big(g^*(\cdot)\big)\frac{\partial g^*(\cdot)}{\partial \theta} + (1-\varepsilon)[1-(1-\varepsilon)\rho^*]H'\big(g^*(\cdot)\big)\big[1 - H\big(g^*(\cdot)\big)\big]\frac{\partial g^*(\cdot)}{\partial \rho}. \quad \text{(B.8)}$$

Based on this expression we can infer that $A < 0$ for all $(\theta^*, \rho^*) \in \Theta$ since $1-(1-\varepsilon)\theta^* > 0$, $1-(1-\varepsilon)\rho^* > 0$, $H'\big(g^*(\cdot)\big) > 0$, $0 < H\big(g^*(\cdot)\big) < 1$, and, per (22)-(23), $\frac{\partial g^*(\cdot)}{\partial \theta} > 0$ and $\frac{\partial g^*(\cdot)}{\partial \rho} < 0$ for all $(\theta^*, \rho^*) \in \Theta$.

Appendix C Stability properties of the full polymorphic evolutionary equilibrium

The Jacobian matrix of the dynamical system (29)-(30) evaluated at any state $(\theta, \rho) \in \Theta$ can be expressed as follows:

$$\mathbf{J}(\theta,\rho) = \begin{bmatrix} \frac{\partial \phi_1(\theta,\rho)}{\partial \theta} - 1 & \frac{\partial \phi_1(\theta,\rho)}{\partial \rho} \\ \frac{\partial \phi_2(\theta,\rho)}{\partial \theta} & \frac{\partial \phi_2(\theta,\rho)}{\partial \rho} - 1 \end{bmatrix}, \quad \text{(C.1)}$$

the determinant and trace of which are, respectively, given by:

$$det\mathbf{J}(\theta,\rho) = 1 - \frac{\partial \phi_1(\theta,\rho)}{\partial \theta} - \frac{\partial \phi_2(\theta,\rho)}{\partial \rho} + \frac{\partial \phi_1}{\partial \theta}\frac{\partial \phi_2}{\partial \rho} - \frac{\partial \phi_1}{\partial \rho}\frac{\partial \phi_2}{\partial \theta} \quad \text{(C.2)}$$

and

$$tr\mathbf{J}(\theta,\rho) = \frac{\partial \phi_1(\theta,\rho)}{\partial \theta} + \frac{\partial \phi_2(\theta,\rho)}{\partial \rho} - 2. \quad \text{(C.3)}$$

From the uniqueness condition in (31), it follows that $\det \mathbf{J}(\theta^*, \rho^*) > 0$ and $\operatorname{tr} \mathbf{J}(\theta^*, \rho^*) < 0$. Therefore, by the Hartman–Grobman Theorem (see Medio and Lines (2001, p. 42)), the linearization around the unique full polymorphic equilibrium (θ^*, ρ^*) implies that this equilibrium is locally asymptotically stable.

Furthermore, given (C.1)–(C.3), if the expression in (C.2) is strictly positive for all states $(\theta, \rho) \in \Theta$, with the result that $\det \mathbf{J}(\theta, \rho) > 0$ for all $(\theta, \rho) \in \Theta$, it follows that $\operatorname{tr} \mathbf{J}(\theta, \rho) < 0$ and $\left(\frac{\partial \phi_1(\theta,\rho)}{\partial \theta} - 1\right)\left(\frac{\partial \phi_2(\theta,\rho)}{\partial \rho} - 1\right) > 0$ for all $(\theta, \rho) \in \Theta$. Under these conditions, Theorem 3 in Olech (1963) implies that the unique full polymorphic equilibrium $(\theta^*, \rho^*) \in \Theta$ is globally asymptotically stable.